The Visible Hand, The Invisible Hand and Efficiency

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Abstract

When a firm forms a market closes. Resources that were previously allocated via the price system are allocated by managerial authority within the firm. We explore this choice of organizational form using a model of price formation in which agents negotiate prices on behalf of their principals when there is trade in a market. Principals motivate agents to make efforts and form prices by writing contracts contingent on the prices that the agents themselves negotiate. Admitting agency issues into price formation introduces a need for a principal to have the authority to coordinate economic activity. This can be achieved by closing a market and forming a firm, thereby contracting directly with both agents, and centrally directing trade. Closing a market, however, results in a loss of information from market prices, information that can be used to reduce the cost of contracting. This information cannot be replicated by internally generated “transfer prices.” Hence, when the market is internalized within the firm, information from market prices is lost. Choice of organizational form, a market or a firm, is then determined by the relative value of central authority over agents (the “visible” hand) versus information from market prices (the “invisible” hand).
“In many sectors of the economy the visible hand of management replaced what Adam Smith referred to as the invisible hand of market forces. The market remained the generator of demand for goods and services, but modern business enterprises took over the function of coordinating flows of goods…’’

I. Introduction

A basic tenet of modern economics is the proposition that prices allocate resources efficiently. The invisible hand is a system of prices that contain information, without each market participant having to bear the full cost of producing that information. But, when a firm forms, a market closes; resources that were previously allocated via the invisible hand of the price system are now allocated by the visible hand of managerial authority within the firm. Such “centrally planned” internal firm economies can be very large. Table 1 shows that General Motors has an internal economy roughly the size of a small, developed, capitalist country. The existence of these planned economies poses the question: What advantages do firm managers have over markets in allocating resources?

Providing a satisfactory answer to this question requires explaining the essential features of the modern corporation. Since it is managers, not owners, who allocate resources inside corporations, we must have a model in which there is a separation of ownership and control, first discussed by Berle and Means (1932) and later by Jensen and Meckling (1976). But who are the owners? Managers, in fact, contract with “the corporation.” So, another defining characteristic is the fact that the modern corporation is a separate entity from its shareholders. It has the power to contract and own property in the corporate name and, therefore, the shareholders have limited liability. Finally, as a separate legal entity, the corporation owns the assets.

To be clear, it is difficult to envision General Motors as corresponding to a large number of entrepreneurs who have banded together to form a “firm.” Of course, such “firms” exist, as partnerships. But, for a firm to have an internal economy the size of a large company like General Motors requires that there be agents of the “firm” who are directed by someone who may be the owner, or an agent of the owner, and who, in any case, has the authority or right to direct how resources (e.g., agents’ efforts) are to be allocated. Then again we are led to ask: What is “authority”? What is the source of this demand for authority? Why doesn’t the market work?

We address these questions by exploring a principal-agent model with multiple principals and agents in which agents negotiate prices on behalf of their principals when there is trade between firms. Principals motivate agents to make efforts and form prices by writing contracts contingent on the prices that the agents negotiate. These prices are potentially informative with regard to the effort choice of the agents, thus enhancing allocative efficiency. But, admitting agency issues into price formation introduces a coordination problem for the principals. Each principal can only write the contract with his own agent.
and therefore cannot control the effort choice of the other firm's agent.\textsuperscript{1} This externality can be internalized if a single principal has the authority to coordinate economic activity by closing the market and forming a firm, thereby contracting directly with both agents. Closing the intermediate market however, results in a loss of the information revealed by those market prices and leads to a moral hazard in teams problem.\textsuperscript{2} Choice of organizational form, a market or a firm, is then determined by comparing the relative value of having managerial authority over agents versus having information from market prices.

The model economy is one in which intermediate goods are produced to be inputs into the production of final goods. While the production technology is owned by the principal(s), the production of each type of good, intermediate and final, requires labor inputs from agents. Each agent makes a non-verifiable effort choice, which contributes to the quality of the output good and hence to its final price. One way of organizing production consists of two firms and a market for inputs. In this case one principal hires the agent that produces the intermediate good while the other principal hires the agent that produces the final good. In order for the final good to be produced the upstream firm has to buy the intermediate input good from the downstream firm. Having made efforts the two agents meet and form a price at which ownership of the input good is transferred. Compensation for the agents is based on the prices that they, themselves, negotiate in the input market, as well as the final output price. Alternatively, production may be organized within a single firm in which case one principal hires both agents and there is no input market. Rather, the intermediate input good is transferred from one agent to another inside the firm.

When one principal hires both agents we ask why production is organized this way when there is the alternative of producing in separate firms. With separate firms, trade of inputs involves prices that, presumably, are valuable in the allocation of resources (here, agents' efforts). If production is organized in a single firm, losing the information from the price (information that cannot be replicated internally, as we show) and creating a moral hazard in teams problem, what is the countervailing benefit? We answer this by developing a notion of managerial authority.

A theory of the firm is equivalently a theory of frictions in the price formation process. Complicated inputs are not sold in Walrasian auctions. How then are such prices formed? We address this by extending the standard principal-agent model to include an expanded role for the agent. In the standard principal-agent model an agent makes an unobservable effort choice that, together with a random realization of nature, results in a “price” for the output. The problem is that the agent can shirk because he can credibly claim that a low price is the result of a high effort by him and a bad realization of the state

\textsuperscript{1} As will be seen, below, this is one of the characteristics that defines the boundary of the firm.
variable. But this story is not complete. In fact, the output of the agent is sold to someone at a price. Who is this counterparty and how is the price formed? In our model, when there are two firms, the input good produced by the first agent is sold to the second agent. The two agents meet, form prices, and trade. The randomness of nature is replaced by the endogenous effort choice of the ‘other agent.’ Extending the principal-agent model in this way allows us to ask how information, valuable to the principals for contracting and inducing effort from agents, comes to be in the price.

The trade-off determining the organizational form depends on these agency issues. Each agent is motivated by the compensation contract that his principal designs. When there are two firms, this contract, however, is partly based on the prices that the agents negotiate. Since principals cannot coordinate their contract choices for their agents ex-ante, we say that each principal lacks the authority to contract with the other principal's agent. Such authority is the benefit of organizing production in a single firm. The cost of a single firm is the potential loss of information from input prices. This information is valuable to the principals since it allows them to write less expensive contracts with their agents. But it is not in the interests of the agents to produce this information. Thus, while these prices give principals an additional source of information (regarding the investment choice of effort made by their agent), because of the agency problem in price formation, the principals must sometimes bear a cost if they want prices to be informative. The cost is one of enforcing constraints that induce agents to form prices that maximize the profits of their principals. Prices are only valuable for principals if they can obtain some of the information for free. This is the essence of a price system.\(^3\) In the context of our paper, getting some information for free means that each principal need not enforce some constraints on his agent’s compensation contracts all of the time.

We develop the price formation process in a series of steps to clarify how principals may obtain some information that they do not pay for from the price. We start with a simple benchmark model in which agents are symmetrically informed about each other's effort choices when they meet. We show that, in this case, the input price contains no additional informational value relative to the final output price. Thus, the input price is of no incremental value in allocating effort resources and, consequently, a (single) firm is the optimal organizational form. We then introduce asymmetric information by assuming that (without loss of generality) one of the agents may be uninformed about the effort choice of the other agent. We find that input prices have an informational role (beyond that of output prices) but, depending on the bargaining game played by the agents when they form prices, principals may or may not benefit from this information because of possible strategic behavior by the agents. Finally, we focus on the most

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\(^2\) This terminology, “moral hazard in teams,” is due to Holmstrom (1982). Holmstrom assumed that the firm could not be split up into separate firms that then trade with each other.

\(^3\) The classic paper on this subject is by Hayek (1945). Also related is Grossman and Stiglitz (1976) who address the issue of how there can be information in prices if information is costly to produce.
interesting case in which input prices provide principals some information for free. In this scenario, the trade-off between the two organizational forms becomes nontrivial. There is a benefit to having the market, namely there is some “free,” but valuable, information in prices, but there is a cost to having the market as well, namely, principals are unable to coordinate their contract choices. They lack authority over each other’s agent.

We then explore which organizational form is socially efficient and which is privately efficient from the viewpoint of the agents and from the viewpoint of the principals. In our model principals may be thought of as the outside shareholders of a firm (or their representatives, e.g., the board of directors). Agents may be viewed as CEOs/managers. Perhaps surprisingly we find that agents always make the socially efficient organizational choice, but principals do not. The reason is this. When forming prices each agent can extract rents from his principal due to the externality provided by the other principal’s agent. In the single firm, the lack of a market price allows agents to extract some rents because of the moral hazard in teams problem. In either organizational form rents are highest when contracts are consistent with both agents making high effort choices (this is when the incentive compatibility constraints bind). Consequently, agents choose the organizational form where principals will give them contracts that induce high effort choices. From the point of view of the principals, however, a different organizational form may be privately optimal. If agents can extract too much from principals when they both make high efforts, principals might prefer an organizational form where low effort is made. Since in our model high effort is socially efficient, the result follows.

Finally, we ask why is it not possible to replicate the information in market prices through internally generated “transfer” prices? The answer is related to property rights and limited liability. In the market, a necessary condition for there to be a transfer of ownership of the input good from one firm to the other is that both agents agree on a price. If agents disagree on a price, there is no trade, the principals earn no money and therefore, cannot pay their agents (due to limited liability of principals). In fact, the principals have formed a “corporation,” and it is this entity that (implicitly) has contracted with the agents. Agents understand this and must therefore agree on a price in order to transfer property rights to the good. In this way, competition between principal-agent pairs in the market induces informative prices.

On the other hand, internal “trade” involves no transfer of property rights. When there is one principal/corporation who hires both agents, that principal may instruct her agents to send her signals so as to replicate market prices. But such signals are not informative (equivalently, no signals will be sent) since the principal’s threat of withholding compensation if no signals are reported is not a credible one. The principal’s limited liability constraint does not bind. Internally, a trade between divisions of a firm can take place without there being an agreement on a transfer price. Since no transfer of property rights is
involved the principal, faced with uninformative signals, will always choose to continue with production and get positive profits. Thus, while she gains the authority to contract with both agents, she loses information.

The notion that the principal has limited liability is critical to our argument about transfer prices; it means that the principal cannot be forced to pay agents if the “firm” has not sold any output. In other words, the principal is not a person but a “corporation.” That is, there exists a legal notion, “incorporation,” which allows the firm to be distinct from the individual person who is the principal. Incorporation means that the entity, the “firm,” has limited liability. Though the legal notion of a limited liability corporation is a rather recent invention (see Hunt (1936)), the ability to incorporate effectively allowed for limited liability. In our analysis the existence of firms, in the sense of legal corporations—the entities that Berle and Means studied—depends on this legal concept. The existence of partnerships or entrepreneurial firms does not require this legal notion and, indeed, these are the only ‘firms” that existed prior to the development of the idea of the “corporation.” In this sense there is a close link between law and finance in our analysis, somewhat akin to recent ideas of La Porta, Lopez-de-Silanes, Shleifer and Vishny (1997, 1998). At the end of the paper we briefly discuss further the role of the legal notion of a corporation.

This paper is related to the vast literature on the theory of the firm. See Holmstrom and Tirole (1989) and Hart (1989) for surveys. Earlier work on the theory of the firm includes Coase (1937, 1988), Alchian and Demsetz (1972), Williamson (1970, 1975, 1979, 1985, 1988), and Jensen and Meckling (1976). Coase was the first to draw attention to the idea that firms and markets allocate resources differently. He identified the role of the firm as coordinating economic activity in situations where the market fails to do so. Alchian and Demsetz, as well as Williamson, argue that firms form when the cost of transacting via the market is too high. They identify features of market transactions that are costly, including asset specificity, the duration of the relationship between the transacting parties, and the ability to identify the marginal productivity of inputs. Recent theories of the firm include Grossman and Hart (GH) (1986), and Hart and Moore (HM) (1990).

Limited liability per se is not the issue as this concept was essentially an after-thought to the legal notion of the “corporation.” In English common law an express concept of limited liability was not needed because the idea that the corporation was a separate person allowed for the contractual construction of limited liability. Henn and Alexander (1983) write that: “Limited liability of the shareholder with respect to claims of creditors of the corporation is illusory because the corporation usually … had the power to make levitations (calls and assessments) on its shareholders for money to pay its liabilities and its creditors could derivatively assert such corporate power directly against the shareholders by a process resembling subrogation. By legal ingenuity, the corporation’s power to make levies on its shareholders was expressly excluded or limited—thus achieving limited liability” (p. 19).

More recent work, extending the GH and HM ideas, include Rajan and Zingales (1998) and de Meza and Lockwood (1998).
In many ways, this literature provides an entrepreneurial theory of the firm (or a theory of the entrepreneurial firm). In criticizing some of the earlier work, Fama (1980) writes that: “…this literature fails to explain the large modern corporation in which control of the firm is in the hands of managers who are more or less separate from the firm’s security holders” (p.289). Our work is distinct from the previous work in several important ways. First, we focus on explaining the important attributes of the modern corporation. The modern corporation is a separate entity from its shareholders and has the power to contract and own property in the corporate name; there is a separation of ownership and control; there is limited liability. Thus, we do not focus on an owner/manager; it is not an entrepreneurial theory of the firm. Thus, as opposed to GH and HM, we start with principal-agent relationships as already given, and do not explain why they have arisen. Our model formalizes some of the ideas in Holmstrom (1999) critical of GH and HM and follows up on the discussion of Bolton and Scharfstein (1998) that a theory of the firm should incorporate the separation of ownership and control. Finally, as briefly explained above, the notion of the corporation and limited liability is important in our theory; it is not simply a legal fiction serving as a “nexus of contracts,” as in Jensen and Meckling (1976).

A second focus of ours is on the idea of authority. Alchian and Demsetz (1972) object to the notion of “authority,” arguing that contracts are voluntary. In our model, agents voluntarily enter into contracts, but there is still a role for “authority.” We seek to explore the meaning of authority, first discussed in this context by Coase, by presenting a model in which the idea arises naturally. Our notion of authority is similar to that of a "coordination effect" introduced in Stole and Zwiebel (1999).

To summarize, we propose a theory of the corporation, as opposed to a theory of the firm.

The paper proceeds as follows. In Section II we describe the model economy. Section III develops the benchmark case of symmetric information and Nash bargaining. Section IV introduces asymmetric information and a simple reduced-form model of bargaining. Section V solves the full model with take-it-or-leave-it offers when there is asymmetric information. In Section VI we analyze the relative social and private efficiency of the two forms of organization. Section VII addresses the issue of why internal transfer prices cannot replicate market prices. Finally, Section VIII concludes. Proofs are in an Appendix.

II. The Model Economy

We consider an economy in which both intermediate goods and final goods are produced. Intermediate goods are used as inputs for the production of final goods. Production of both goods requires labor inputs from agents. Principal(s) own the production technologies and hire agents (to

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6 Other attributes include centralization of management in the board of directors and ready transferability of shares. See Henn and Alexander (1983).
supply effort) in a labor market. We consider two ways of organizing the production of the final good. In one case, hereafter called Non-Integration, production takes place in two firms. Firm one produces the intermediate input good and then sells it to the second firm in a market. The second firm then uses this input to produce the final output good. When production takes place in two separate firms, there are two principals, one (implicitly) owning the technology that produces the intermediate good and the other (implicitly) owning the technology that produces the final good. Each principal hires an agent who supplies the effort for production and the skills to trade with the other firm. In order for the final good to be produced the upstream firm has to buy the intermediate input good from the downstream firm. The agents from each firm meet and bargain to determine a price at which the input good is traded.

In the second case, which we refer to as Integration, all the production takes place in one firm. A single principal owns both technologies and hires both agents so that there is no market for the intermediate input good. “Trade” then occurs inside the firm. There is no “price” associated with the transfer of the good inside the firm. Below, we make clear what this means. In that regard, however, note that in the first case, Non-Integration, property rights to the input good change hands in the market transaction, while in the second case, Integration, there is no transfer of property rights.

Production works as follows. Each agent makes a nonverifiable discrete effort choice, \( e \), with \( e \in \{ e_H, e_L \} \) and \( e_H > e_L \). An agent, called A1, makes an effort choice that determines the quality of the intermediate input good. A second agent, called A2, combines his effort choice with the intermediate input good to make the final good. Because effort choices are discrete there are four possible quality states for the final output: HH, HL, LH, and LL, where the first letter represents the effort choice of A1 and the second letter represents the effort choice of A2. The state HL, for example, represents the outcome where A1 made a high effort and A2 made a low effort. The quality of the final output, and hence its price, depend upon the quality of both the input good (i.e., the effort choice of A1) and the effort choice of A2. Thus, while the quality of the input good only depends on A1’s effort choice, \( e_H \) or \( e_L \), the quality of the final output depends on A2’s effort choice as well as A1’s. Agents have a disutility of effort denoted by \( \Phi(e_i) \equiv \Phi_i \) with \( \Phi_H > \Phi_L \). Finally, similar to previous work, we focus on the ex ante effort choices made by A1 and A2. Namely, we focus on the effort choices made by the two agents before they meet.

The price of the final product is given by \( P_{ij} \) where the first subscript refers to the effort choice of A1 and the second subscript refers to the effort choice of A2, so \( i, j \in \{ H, L \} \). These prices are taken to be exogenous, satisfying the following assumptions:
Assumption 1: \( P_{HH} > P_{HL} = P_{LH} > P_{LL} \) and \( P_{HH} - P_{HL} < P_{LH} - P_{LL} \).

The second property is the usual neoclassical assumption reflecting the concavity of the production function with respect to investment (of effort). The first property allows us to capture two basic ideas in the simplest way possible. One is that the more effort expended the higher is the value of the final product. The other is that prices are noisy in the sense that principals cannot fully disentangle the state of nature from observing the final good price. Namely, the price of the final good does not allow principals to differentiate between the states HL and LH. Without loss of generality we make the following additional assumption:

Assumption 2: \( P_{HH} - 2\Phi_H > P_{HL} - \Phi_H - \Phi_L > P_{LL} - 2\Phi_L \).

Assumption 2 says that the socially efficient investment is for both agents to make a high effort. This will serve as a benchmark to which we can compare other equilibria. Whenever the equilibrium does not entail high effort from both agents we say that the First Best has not been achieved. (Satisfying Assumptions 1 and 2) we will express the final price as:

\[
P_j = A + \Phi_i + \Phi_j + I(i, j)
\]

where \( A > 0 \),

\[
I(i, j) = \begin{cases} 
1 + \varepsilon & \text{for } (i, j) = (H, H) \\
1 & \text{for } (i, j) = (H, L) \text{ or } (L, H) \\
0 & \text{for } (i, j) = (L, L) 
\end{cases}
\]

and \( 0 < \varepsilon < 1 \). In Figure 1 we plot this production function. From the figure it is clear that the marginal profit is always greater than the marginal cost of effort and that this difference is decreasing (i.e., the production function is concave).

We also assume:
**Assumption 3:** Both principals and agents are risk neutral in final wealth and both have limited liability.

For agents limited liability means that the contract offered to an agent has to, at the very least, compensate him for his cost of effort.\(^7\) For the principals limited liability means that if the final output is not produced and no profits are made, they cannot be forced to honor the contracts signed with their agents. While limited liability can be justified for principals on the same grounds that it can be justified for agents, we can also think of limited liability for the principals as corresponding to the notion that equity holders of a firm cannot be made to invest more than their initial equity.

Our economy is one of incomplete contracting. The incompleteness of contracts results from effort not being a verifiable quantity. Principals, however, can write (verifiable) contracts contingent on prices. In general this means that contracts can be written on both the price of the final product as well as the price of the intermediate product (if this price exists). Both principals observe both prices.

In the final goods market prices are not fully revealing (Assumption 1). Our focus is on how the input good price is formed and how information from that market helps resolve the informational problems of the output market. An important difference between the intermediate input market and the final good market in our setting is that the market for the final output is (implicitly) a market in which there are many agents, some informed, bidding for the final good. The market for the intermediate input good is a bilateral market in which there is not necessarily information aggregation. The intermediate input good price will depend on whether A1 and A2 are informed and how they bargain.

When production takes place in two firms, the agents, having been hired and having made their effort choices, search for trading partners at a cost. We assume that the cost of searching is sufficiently high so as to guarantee that the principals prefer that the agents negotiate a price and trade with each other rather than go and search for another partner. Alternatively, one can look at the situation where the two agents make relationship specific investments and hence are better off trading with each other in this bilateral monopoly.\(^9\) Finally, while it is possible to assume that agents either bargain over their own surplus or over the surplus of their firm we make the following assumption:

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\(^7\) This is the standard assumption made in agency models where the contracts can be written on noisy signals of the true effort choice.

\(^8\) We interpret limited liability as an ex-ante constraint on the contracts offered to the agents. Contracts that may result in losses to agents are therefore forbidden. For a more elaborate discussion of limited liability see Sappington (1983).

\(^9\) The notion that the two parties are locked into the relationship and must therefore reach an agreement is a common assumption among papers that deal with the theory of the firm. (See for example GH (1986), HM (1990)). An alternative interpretation is of an investment-specific relationship. Once the investment has been made there is additional value when staying with your partner compared to searching for a new one. In the words of Williamson
**Assumption 4:** *When agents negotiate a price in the intermediate input market, they seek to maximize their firm’s total surplus.*

This assumption is made for simplicity. Although agents choose prices so as to maximize their companies’ surpluses they are still concerned with their own utility levels. An agent will make an effort choice to maximize his compensation based on the wages offered to him under the contract with the principal. Moreover, these wage contracts have to be such that it will be in the interests of the agents to agree to transact with each other at the specified prices. The basic issue concerns how an agent behaves when he negotiates a price on behalf of his principal. A strong form of agency problems would assume that agents bargain for prices that maximize their individual utility levels. We adopt a weaker view, namely, that while agents make unverifiable effort choices they do seek to maximize their firm’s surplus when they bargain. Implicitly, this view is a reduced form for noncontractual institutions and influences that enforce this objective function.

### III. The Benchmark Case: Organizational Choice With Fully Informed Agents

We begin the analysis with a benchmark case in which the agents are fully informed about the quality of the intermediate input good being traded in the market. This means that A2 is costlessly informed about the effort choice of A1 and A1 is costlessly informed about the effort choice of A2. Thus, the quality state of both the input and the output goods are known to the agents when they meet, though effort is not verifiable. In this benchmark setting, we examine the optimal organizational choice and show that Integration into a single firm weakly dominates Non-Integration.

The analysis proceeds by first examining the case of Non-Integration and then considering the case of Integration. The solution to the Integration case of this section will be the same throughout the remaining models of price formation that come later in the paper.

### A. Non-Integration With Fully Informed Agents

The sequence of events is as follows:

1. Principals hire agents in the labor market, signing compensation contracts.

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(1979): “The transactions that I wish to emphasize here, however, are exchanges of the recurring kind. Although large-numbers competition is frequently feasible at the initial award stage for recurring contracts of all kinds, idiosyncratic transactions are ones for which the relationship between buyer and supplier is quickly thereafter transformed into one of bilateral monopoly-on account of the transaction-specific costs…” (p. 241).
A1 and A2 search for trading partners, meet, and bargain. As a result of the bargaining game a price is formed.

4. Trade of the input good takes place at the negotiated price.

5. A2 sells the final product in the output market and agents receive their compensation.

As was assumed in Assumption 4, agents Nash bargain over the total surplus of their companies. In this case then the prices in the market for the input good will correspond to the “quality” states, as follows:

<table>
<thead>
<tr>
<th>State</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>HH</td>
<td>( P_H )</td>
</tr>
<tr>
<td>LH</td>
<td>( P_M )</td>
</tr>
<tr>
<td>HL</td>
<td>( P_M )</td>
</tr>
<tr>
<td>LL</td>
<td>( P_L )</td>
</tr>
</tbody>
</table>

Using the Nash bargaining solution, and assuming both agents have an outside option of zero, we have that \( P_H \), for example, is the solution to \( \max_p (P_{HH} - p)^{\alpha} (p - 0)^{1-\alpha} \) where for the rest of the paper we take the case of symmetric bargaining power, \( \alpha = 0.5 \). Notice that the assumption of Nash bargaining means that the prices of the intermediate input good have the same informational content as do the prices of the final output. In other words there is a one-to-one correspondence between the input price and the final output price. This observation is enough to tell us that the allocative role of prices in the input market is not particularly important.

A.1 Optimal Contracts: The Input Producer

We first solve for the optimal contract that P1 offers A1. The optimal contract can either induce a high effort from A1 or a low effort. Contracts will be contingent on observed input and final good prices.\(^{10}\) An important feature of the optimization problem facing P1 is the dependence of her firm’s profits on the decision made by the second firm (i.e., A2’s effort choice). In order to specify P1’s problem we need to define her belief regarding the effort choice of the agent outside of her authority, A2. Let \( q \) be the probability assessment of P1 that A2 will make a high effort choice. Of course A2’s effort choice depends on his contract with P2. Given the belief \( q \), limited liability, and the price formation

\(^{10}\) While one can imagine P1 only being able to write a contract contingent on the price of the intermediate good, we focus on the case where P1 can contract on both the intermediate good price and the final output price. In Appendix C we consider the case where P1 cannot contract on the final output price. There we show that the main difference in the equilibrium is that the equilibrium with less information given to P1 is actually more efficient.
mechanism, P1 has to compare her profits from inducing a high effort to her profits from inducing a low effort from A1. We derive the optimal contract under each alternative and obtain a condition on q under which the optimal contract induces high effort.

Let $W^K_{ij}$ be the wage paid to A1 by P1 when prices $P_{ij}$ and $P_k$ are reported. $P_{ij}$ is the final output price while $P_k$ is the intermediate input price. When inducing a high effort from A1 the maximization problem for P1 is to choose wages to:

$$\max_{(w^H_{ij})} q(P_H - W^H_{HH}) + (1-q)(P_M - W^M_{HL})$$

Subject to:

$$W^H_{HH} \geq \Phi_H$$
$$W^M_{HL} \geq \Phi_H$$
$$W^M_{LH} \geq \Phi_H$$
$$W^L_{LL} \geq \Phi_L$$

$$qW^H_{HH} + (1-q)(W^M_{HL}) - \Phi_H \geq qW^M_{HL} + (1-q)(W^L_{LL}) - \Phi_L$$

The objective function describes the expected profits to P1 net of the wages she pays. The first four constraints reflect limited liability and the last one is the usual incentive compatibility constraint.

**Lemma 1:** The solution to this problem is: $W^H_{HH} = \Phi_H + \Delta\Phi$, $W^M_{HL} = \Phi_H$, $W^L_{LL} = \Phi_L$, where $\Delta\Phi \equiv \Phi_H - \Phi_L$.

**Proof:** See Appendix A.

Note that these contracts are optimal for any value of q. Even if q=1 it is still the case that $W^H_{HH} = \Phi_H + \Delta\Phi$. The reason is that while P1 believes that A2 made a high effort there is no verifiable way for her to prove that $P_M$ corresponds to the state LH and not to the state HL. Thus, P1 must still offer her agent a contract that satisfies the limited liability constraint, $W^M_{LH} \geq \Phi_H$. This means that the IC constraint can only be satisfied if $W^H_{HH} = \Phi_H + \Delta\Phi$.

If the principal decides to induce her agent to make a low effort choice she maximizes:
\[
\max_{\{W_i^j\}} \quad q(P_M - W_{HL}^M) + (1 - q)(P_L - W_{LL}^L)
\]
Subject to:
\[
W_{HH}^H \geq \Phi_H \\
W_{HL}^M \geq \Phi_H \\
W_{LM}^M \geq \Phi_H \\
W_{LL}^L \geq \Phi_L \\
qW_{HL}^M + (1 - q)W_{LL}^L - \Phi_L \geq qW_{HH}^H + (1 - q)W_{HL}^M - \Phi_H
\]

**Lemma 2:** The solution to this problem is: \( W_{HH}^H = W_{HL}^M = \Phi_H, W_{LL}^L = \Phi_L \).

**Proof:** Obvious.

### A.2 Optimal Contracts: The Output Producer

Now consider P2. Like P1, P2 can write contracts contingent on both the price of the intermediate input good and the price of the final output. Let \( b \) denote the probability assessment of P2 that A1 will produce a high quality input good. When deciding to induce a high effort from A2, P2 chooses the compensation contract to solve:

\[
\max_{\{W_i^j\}} \quad b(P_{HH}^H - P_H - W_{HH}^H) + (1 - b)(P_{HL}^M - P_M - W_{HL}^M)
\]
Subject to:
\[
W_{HH}^H \geq \Phi_H \\
W_{HL}^M \geq \Phi_H \\
W_{LM}^M \geq \Phi_H \\
W_{LL}^L \geq \Phi_L \\
BW_{HH}^H + (1 - b)W_{HL}^M - \Phi_L \geq BW_{HL}^M + (1 - b)W_{LL}^L - \Phi_L
\]

**Lemma 3:** The solution to this problem is: \( W_{HH}^H = \Phi_H + \Delta \Phi, W_{HL}^M = W_{LL}^L = \Phi_H, W_{LL}^L = \Phi_L \).

**Proof:** See Appendix A.

If P2 decides to induce a low effort from her agent, she solves:
\[
\max_{W^L} b(P_{HL} - P_{LM} - W^M_{HL}) + (1-b)(P_{LL} - P_{L} - W^L_{LL})
\]

Subject to:

\[
\begin{align*}
W^H_{HH} & \geq \Phi_H \\
W^M_{HL} & \geq \Phi_H \\
W^L_{LL} & \geq \Phi_L 
\end{align*}
\]

\[
bW^M_{HL} + (1-b)W^L_{LL} - \Phi_L \geq bW^H_{HH} + (1-b)W^M_{HL} - \Phi_H
\]

**Lemma 4:** The solution is trivially: \( W^H_{HH} = \Phi_H, W^M_{HL} = \Phi_H, W^L_{LL} = \Phi_L \).

**Proof:** Obvious.

An equilibrium is: (1) a set of contracts designed by the principals to maximize their respective expected utilities; (2) effort choices by each agent such that their expected utilities are maximized given the contracts; and (3) correct beliefs, held by each player, regarding the choices of the other players. Combining the above conditions results in a Nash equilibrium. While we do not need to solve for the actual equilibrium \( b \) and \( q \) for the analysis of this section we give the proposition describing the equilibrium, Proposition B1, and its proof in Appendix B.

**B. Integration**

In the case of Integration, all production takes place within a single firm in which a single principal owns both technologies and hires both agents. One principal then has the authority to write the contracts of both agents. However, the input market does not exist and so agents do not need to agree on an input price in order for property rights to the input good to be transferred from one division (agent) to another. The single principal though may ask the agents to report signals of quality. We call such signals “transfer prices” and analyze them in Section VII where we will show that these transfer prices cannot replicate the information in market prices.

To solve for the equilibrium we need to find the optimal contracts written by the principal and the effort choices made by the two agents. Since the compensation of each agent depends on the effort made by the other agent too, each agent has to have a belief about what the other agent will do. As before we let \( q \) denote the probability assessment that A1 has regarding the chance that A2 will make a high effort choice. Similarly, let \( b \) denote the probability assessment that A2 has regarding the chance that A1 will
make a high effort choice. Note that the single principal has the authority to optimally choose both contracts, which in equilibrium implies that she chooses both of these probabilities. This is so because in equilibrium the principal’s contract choices and the beliefs, \( b \) and \( q \), must be consistent with the actions taken by the agents. Given the beliefs \( b \) and \( q \), the principal solves:

\[
\max_{\{w\}} \{qb(P_{HH} - 2W_{HH}) + [q(1-b) + b(1-q)](P_{HL} - 2W_{HL}) + (1-q)(1-b)(P_{LL} - 2W_{LL})\}
\]

Subject to:

\[
\begin{align*}
W_{HH} &\geq \Phi_H \\
W_{HL} &\geq \Phi_H \\
W_{LL} &\geq \Phi_L \\
b(W_{HH} - \Phi_H) + (1-b)(W_{HL} - \Phi_H) &\geq b(W_{HL} - \Phi_L) + (1-b)(W_{LL} - \Phi_L) \\
q(W_{HH} - \Phi_H) + (1-q)(W_{HL} - \Phi_H) &\geq q(W_{HL} - \Phi_L) + (1-q)(W_{LL} - \Phi_L)
\end{align*}
\]

The objective function is simply the expected value of the principal’s profits. The first three constraints are limited liability constraints while the last two are incentive compatibility constraints for \( A1 \) and \( A2 \).

**Lemma 5:** The optimal contracts are given by:

i. For \( \{q, b\} \neq 0 \): \( W_{HH} = \Phi_H + \Delta \Phi \); \( W_{HL} = \Phi_H \); \( W_{LL} = \Phi_L \).

ii. Else:\( W_{HH} = \Phi_H \); \( W_{HL} = \Phi_H \); \( W_{LL} = \Phi_L \).

**Proof:** See Appendix A.

This leads to the following equilibrium:

**Proposition 1:** The equilibrium consists of the contracts specified in Lemma 5 and the following beliefs \( q^* \) and \( b^* \) that depend on the marginal cost of effort \( \Delta \Phi \equiv \Phi_H - \Phi_L \) as follows:

i. For \( \Delta \Phi \leq \varepsilon \): \( q^* = b^* = 1 \).

ii. For \( \varepsilon < \Delta \Phi \leq 1 \): \( q^* = 0, b^* = 1 \); \( q^* = 1, b^* = 0 \).

iii. For \( 1 < \Delta \Phi \): \( q^* = b^* = 0 \).

**Proof:** To find the equilibrium we need to solve for the optimal \( q^* \) and \( b^* \). Given the optimal contracts, the principal’s objective function is:
\[ qb(P_{HH} - 2\Phi_H - 2\Delta\Phi) + [q(1-b) + b(1-q)](P_{HL} - 2\Phi_H) + (1-q)(1-b)A. \]

The principal will choose \( q^* \) and \( b^* \) depending on the relation between \( P_{HH} - 2\Phi_H - 2\Delta\Phi \) (both agents make a high effort), \( P_{HL} - 2\Phi_H \) (one agent makes a high effort), and \( A \) (both agents make a low effort). These are equivalent to, \( A + 1 + \epsilon - 2\Delta\Phi \), \( A + 1 - \Delta\Phi \), and \( A \) respectively. Then depending on \( \Delta\Phi \), the principal will choose either \( q^*=b^*=1 \), \( q^*=b^*=0 \), or \( q^*=1 \) and \( b^*=0 \).

Recall our assumption that the First Best solution is to make a high effort, i.e., that

\[ P_{HH} - 2\Phi_H > P_{HL} - \Phi_H - \Phi_L > P_{LL} - 2\Phi_L. \]

In comparison, from the equilibrium conditions stated in Proposition 1, both agents make a high effort if and only if:

\[ P_{HH} - 4\Phi_H + 2\Phi_L > P_{HL} - 2\Phi_H, \tag{1} \]

while one agent makes a high effort if and only if:

\[ P_{HL} - 2\Phi_H > P_{LL} - 2\Phi_L. \tag{2} \]

Given our definition of the price function \( P_{ij} \), (1) and (2) are equivalent to:

\[ A + (1 - \Delta\Phi) + (\epsilon - \Delta\Phi) > A + (1 - \Delta\Phi) \tag{1'} \]

and

\[ A + (1 - \Delta\Phi) > A. \tag{2'} \]

If (2') holds then the principal will induce only one agent to make a high effort choice. If (1') holds too, then the principal will induce both agents to make a high effort choice.

Recall that \( \Delta\Phi \) is the marginal cost of effort to the agents. The marginal product of the first high effort made is \( 1 + \Delta\Phi \), while \( \epsilon + \Delta\Phi \) is the marginal product of the second high effort made.

Depending on the relation between \( \Delta\Phi \), \( 1 \) and \( \epsilon \) the principal either induces high effort from both agents, from one agent, or decides not to induce any effort from either agent. Notice that this equilibrium does not always result in First Best efficiency. Unlike an unconstrained social planner, the principal does not necessarily induce both agents to make high effort choices even though she can control the actions of both (through the contracts she writes). The reason for this deviation from the efficient outcome is that limited liability and the ability of agents to hide behind each other allows them to extract some of the surplus from the principal.\(^{11}\)

\(^{11}\) This problem is the “moral hazard in teams” problem analyzed in Holmstrom (1982).
C. Which Hand: Visible or Invisible?

Having solved for the optimal contracts for each organizational form we can now prove that the principals prefer Integration over Non-Integration.

**Proposition 2:** When input prices are formed by fully informed agents based on Nash bargaining, principals always (weakly) prefer to integrate into a single firm.

**Proof:** While the proposition can be shown by direct calculation of the total profits under each organizational form, it is also clear through inspection of the contracts. Looking at the optimal contracts above in the case of Integration, note that for any b and q chosen in the equilibrium, the contracts are the same as those in the case of Non-Integration. Thus, the combined profit of the principals can only be higher in the case of Non-Integration if they are able to choose a better combination of q and b. But, in the case of Integration one principal has the authority to write both contracts, choosing both b and q, while in the Non-Integration case P1 chooses b and P2 chooses q. It is clear then that Integration will provide the principal with a weakly dominating profit.

The result in Proposition 2 should not be surprising. Intuitively, separating production into two firms has two affects. The first is the introduction of a coordination problem for the two principals who now compete with each other. The second is the introduction of the input price, which may potentially provide information to each principal regarding the effort choice of her agent. Since, in this case, the informational content of the input price is the same as that of the final output price, the separation into two firms only creates a coordination problem for the principals (the choices of q and b are now done separately) without giving them any countervailing informational benefits regarding whether the true state is HL or LH. In other words the intermediate input price is redundant in terms of the information it provides to principals. Thus, the intermediate input price is serving no allocational role.

IV. Costly Information Production by Both Principals and Agents

The above benchmark case assumed that agents were costlessly informed about the quality of the intermediate input when it was sold in the market. We saw that, in equilibrium, there was no informational content to the input prices beyond that already conveyed in the output prices. In this section we take a first step towards a different scenario in which the input markets prices will eventually have an informational role. Realistically, the quality of complicated input goods is not easily determined and so, in this section, we assume costly information production by agents, in addition to principals.
Specifically, we assume that A2’s ability to detect the quality of the input good depends on his effort choice. In other words, it is costly for A2, the input buyer, to determine whether A1 has made a high effort. This means that if A2 makes a low effort (and hence does not produce information) he is uninformed when he buys the input and this adversely affects the quality of the final output. On the other hand, if A2 makes a high effort, then he is informed about the quality of the input, which has a positive affect on the quality of the final output. Thus, effort affects the quality of the final output, but also determines whether A2 can judge the quality of the input good.  

A. Input Market

Our assumption regarding information production is:

**Assumption 5:** A2 is informed about A1’s effort choice (and hence about the quality of the input good) only when he makes a high effort. If he makes a low effort choice, he is uninformed.

The interpretation is this: When A2 makes a high effort we assume he develops the skills and knowledge to identify the quality of the intermediate good which then allows him to adjust production so as to maximize the quality of the final output good. However, when A2 makes a low effort he remains uninformed regarding the quality of the input good and that results in a final output of lower quality.

If A2 makes a high effort choice, then when A1 and A2 meet, there is full information so that they form the price as before, that is, according to Nash bargaining. However, when A1 and A2 meet and A2 has made a low effort, A2 is uninformed and so there is asymmetric information between the two parties. Hence they cannot Nash bargain. In this section, we do not completely specify a bargaining game for price determination when A2 is uninformed. Rather, we adopt the minimal assumption that:

**Assumption 6:** When A2 is uninformed the agents agree on a single price $P$, where $P < P_M < P_H$.

The assumption specifies a price that a rational A2 will demand. The logic is this. When A2 makes a low effort, the best case scenario for him is that the final output is worth $P_H$ (which corresponds to him paying $P_M$), thus he will not be willing to pay more than this upper bound as this will reveal that he did not make a high effort. How much below $P_M$ the price $P$ is, we do not say, as this would depend on relative bargaining power. Under Assumption 6, the input price/quality schedule takes the form:

---

12 While it is also possible to think of A1, as well as A2, as being either informed or uninformed, we analyze the simpler case in which A1 is always informed about the potential use of his product.
Note that observing the input price together with the output price allows one to fully separate the states HL and LH. Taken together, the input price and the output price are now fully informative. Recall that when observing the output price alone principals cannot distinguish between the states HL and LH. Now having the input price too allows them to separate between these two states since in the state HL the input price is $P$ while in the state LH it is $P_M$.

The input price and the final output price, together, will reveal the true (quality) state. But, that is only true if the agents, who have to agree on the input price when they bargain, actually do behave according to the above price/quality schedule. If agents behave strategically to maximize their compensation, then this may not be true.

Given the quality state of the input good, prices are chosen to maximize the firm's surplus. However, in order for agents to agree on these prices at the corresponding states it must be the case that their contracts give them the incentives to do so. In particular, agents may offer their counterparty a different price in some of the states if this price results in higher compensation to them and higher profits for the counterparty. That is, agents may find it mutually beneficial to manipulate the input price, by choosing a price that is inconsistent with the true quality state. For this reason, although the prices specified above will provide additional information, it will come at a cost to each of the principals. Namely, principals will have to pay their agents in order for them to transact at those given prices in the given states. We will show that, as a result of this cost, principals will still prefer to organize into a single firm, Integration. Forming prices in this way results in input market prices that do not provide principals with any ‘free information’.

**B. Optimal Contracts: The Input Producer**

As before we first solve for the optimal contract that P1 offers A1. However, since agents care about their wages rather than their company's profits, managers need to offer contracts that are increasing in the firms’ profits so that managers will choose the prices that maximize the firms’ profits. This will require additional constraints on the contracts.

When inducing a high effort from A1 the maximization problem for P1 is to choose wages to:
The first four constraints reflect limited liability while the last one is the usual incentive compatibility constraint. The interesting constraint is the fifth one, \( W_{LH}^M \geq W_{LH}^P \). Since the principal cares about her profits, she is better off receiving the higher price \( P_m \) rather than the price of \( P \) since \( P_m > P \). Her agent, however, does not care about his principal's profits, but rather about his own salary. Since receiving a price of \( P_m \) reveals that he has made a low effort and since the other agent will always agree to pay a lower price than is specified in the mechanism, the agent will only sell the input for the higher price if his salary provides him with sufficient compensation. Thus, it is costly to enforce this constraint. In this sense the prices, although informative, do not provide the principal with any free information. Henceforth, we will call this constraint the ‘non-manipulation’ constraint.

**Lemma 6:** The solution to this problem is: \( W_{HH}^H = \Phi_H + \Delta \Phi \), \( W_{LH}^M = W_{HL}^P = \Phi_H \), \( W_{LL}^P = \Phi_L \).

**Proof:** See Appendix A.

If the principal decides to induce her agent to make a low effort choice she then maximizes:

\[
\max_{\{w_{ij}^p\}} q(P_m - W_{LH}^M) + (1 - q)(P - W_{LL}^P)
\]

Subject to:

\[
\begin{align*}
W_{HH}^H & \geq \Phi_H \\
W_{HL}^P & \geq \Phi_H \\
W_{LH}^M & \geq \Phi_L \\
W_{LL}^P & \geq \Phi_L \\
W_{LH}^M & \geq W_{LH}^P \\
qW_{HH}^H + (1 - q)W_{HL}^P - \Phi_H & \geq qW_{LH}^M + (1 - q)W_{LL}^P - \Phi_L
\end{align*}
\]
Lemma 7: The solution to this problem is: \( W_{LH}^M = \Phi_H, W_{LL}^P = \Phi_L \).

Proof: Obvious.

Again, while limited liability would have been satisfied if \( W_{LH}^M = \Phi_L \), A1 would never sell at the high price of \( P_M \) if it meant he would receive a low salary. For this reason P1 has to add the additional non-manipulation constraint to her optimization problem. The non-manipulation constraint results in P1 having to pay her agent the same amount she would have paid him had she not known the true state. To reiterate, it is in this sense that prices are informative, but at a cost.

C. Optimal Contracts: The Output Producer

When deciding to induce a high effort from A2, P2 chooses the compensation contract to solve:

\[
\max_{(W^*_P)} b(P_{HH} - P_H - W_{HH}^H) + (1 - b)(P_{LH} - P_M - W_{LH}^M)
\]

Subject to:

\[
W_{HH}^H \geq \Phi_H \\
W_{LH}^M \geq \Phi_H \\
W_{HL}^P \geq \Phi_H \\
W_{LL}^P \geq \Phi_L \\
W_{HL}^P \geq W_{LH}^M \\
bW_{HH}^H + (1 - b)W_{LH}^M - \Phi_H \geq bW_{HL}^P + (1 - b)W_{LL}^P - \Phi_L
\]

Note the non-manipulation constraint: \( W_{HL}^P \geq W_{LH}^M \). While prices are fully revealing, because of the agency problem and the fact that the agent cares only about his compensation, he is better off buying the input at the higher price, which reflects that he made a high effort. Since the first firm’s (P1’s) agent will
obviously be willing to accept a higher price, the principal, P2, has to offer her agent a set of contracts that assures her that her agent will buy the input at the lowest price possible. With the non-manipulation constraint, prices, although fully revealing, do not save the principal any money.

**Lemma 8:** The solution to this problem is: \( W^H_{HH} = \Phi_H + \Delta \Phi, \quad W^P_{HL} = W^M_{LH} = \Phi_H, \quad W^P_{LL} = \Phi_L. \)

**Proof:** See Appendix A.

If P2 decides to induce a low effort from her agent, she solves:

\[
\max_{\{W_{ij}\}} b(P_{HL} - P - W^P_{HL}) + (1 - b)(P_{LL} - P - W^P_{LL})
\]

Subject to:

\[
\begin{align*}
W^H_{HH} &\geq \Phi_H \\
W^M_{LH} &\geq \Phi_H \\
W^P_{HL} &\geq \Phi_L \\
W^P_{LL} &\geq \Phi_L \\
W^P_{HL} &\geq W^M_{LH} \\
\end{align*}
\]

\[
bW^M_{HL} + (1-b)W^P_{LL} - \Phi_L \geq bW^H_{HH} + (1-b)W^M_{HL} - \Phi_H
\]

**Lemma 9:** The solution is trivially: \( W^P_{HL} = \Phi_H, \quad W^P_{LL} = \Phi_L. \)

As before, an equilibrium is: (1) a set of contracts designed by the principals to maximize their respective expected utilities; (2) effort choices by each agent such that their expected utilities are maximized given the contracts; and (3) correct beliefs, held by each player, regarding the choices of the other players. Combining the above conditions results in equilibrium. This is a set of conditions on the exogenous parameters that lead to different outcomes for \( b \) and \( q \). For the analysis in this section we do not need to derive the equilibrium \( q^* \) and \( b^* \). For that reason we leave the calculation of the equilibrium to Proposition B2 in Appendix B.

**D. Which Hand: Visible or Invisible?**

Having solved for the optimal contracts we can now prove that the principals will always prefer to organize production in a single firm.
**Proposition 3:** When agents choose a single price $P$ whenever $A_2$ makes a low effort, and is therefore uninformed about the quality of the input good, then the principals always (weakly) prefer to integrate into a single firm.

**Proof:** As was the case before, this result can be directly checked by calculation the profits for each case. But, one can easily verify that the contracts given to agents are the same in the case of Integration and Non-Integration. But, the integrated organization has the advantage of coordination. Namely, in the case of Integration one principal chooses both $q^*$ and $b^*$ while in the case of Non-Integration $P_1$ chooses $b^*$ while $P_2$ chooses $q^*$, non-cooperatively.

Interestingly, there is no benefit from the information in input prices. The reason is that while this information allows the principals to relax the limited liability constraints and hence reduce the cost of contracting, there are additional non-manipulation constraints that must be satisfied. So, although prices allow the principals to identify the state in which their agent made a low effort the non-manipulation constraints force them to pay fully for that information.

To summarize, with costly information production by $A_2$, agents must still form a price when $A_2$ is uninformed. When there is a single price that the agents agree to in this situation, principals must take account of the strategic behavior of agents. But, this is costly for principals. Producing information, by paying for non-manipulation by the agents, does not allow principals to gain any ‘free’ information from prices. The market has no advantage over the firm.

**V. Costly Information Production and the Take-It–Or-Leave-It Game**

In this section we maintain the above assumptions about costly information production and specify a simple bargaining game to be played when $A_2$ is uninformed, by virtue of having made a low effort. The bargaining game dramatically alters the role of the input price. Specifically, it will allow for prices that give principals some information for which they do not have to pay.

**A. Overview**

The previous section simply specified that when $A_2$ was uninformed trade occurred at one price, $P$. In this section we will be more specific about the price formation process. Instead of Assumption 6, we now assume:
**Assumption 7:** When the buyer, A2, is uninformed, the two agents’ bargain by playing a take-it-or-leave-it game where each agent gets to make the offer with equal probability.\textsuperscript{13}

When the game is take-it-or-leave-it we assume that each agent will suggest a price that extracts the entire surplus from the other firm (subject to the constraint that the other firm will have sufficient funds to pay its agent).\textsuperscript{14} The price/quality schedule takes the following form:\textsuperscript{15}

<table>
<thead>
<tr>
<th>State</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>$HH$</td>
<td>$P_H$</td>
</tr>
<tr>
<td>$LH$</td>
<td>$P_M$</td>
</tr>
<tr>
<td>$HL &amp; LL$</td>
<td>$A_1$ offers $\bar{P}$ w.p. 0.5 $A_2$ offers $P$ w.p. 0.5</td>
</tr>
</tbody>
</table>

$\bar{P}$ is the maximum price A2 is willing to pay A1 subject to the condition that his firm’s profits be sufficient to pay him his wage. Since A2 cannot distinguish between the states HL and LL, $\bar{P}$ is paid in both states. Thus, we get that: $\bar{P} = P_{LL} - \Phi_L$. When observing $\bar{P}$, P2 understands that A2 made a low effort and hence need only pay him his disutility of effort $\Phi_L$. The above price then assures that if the state is LL, P2 gets zero net profits. Similarly, $P$ is the lowest price that gives P1 zero profits after paying A1 his promised compensation. Thus, $P = \Phi_H$.\textsuperscript{16} We naturally assume that: $P \leq P_M$ and $P_H \leq \bar{P}$.

Are the input prices providing valuable information to the principals? Recall that the only information the principals need, in order to eliminate the agency costs (of not being able to contract directly on the agents’ investment decision), is whether the state is HL or LH. Observing the input price allows principals to separate the two states since different input prices are negotiated in each state. Thus, input prices help reduce the principals’ cost of inducing effort. However, unlike the previous case we will show that the principals can get some of this information for free due to the competition between the two agents and the form of the bargaining game.

\textsuperscript{13} There is some literature on bargaining under asymmetric information, which gives support to this choice of specification. For example, Samuelson (1984) shows that when there is asymmetric information, similar to ours, between a buyer and a seller the optimal mechanism is for each to make a take-it-or-leave-it offer.

\textsuperscript{14} The results remain the same if we assume that the take-it-or-leave-it price extracts the entire surplus.

\textsuperscript{15} We will show that in equilibrium agents will trade and it will be in their interest to do so.

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B. Optimal Contracts: The Input Producer

When inducing a high effort the maximization problem for P1 is to choose wages for A1 to:

\[
\max_{W_{HH}^1} q(P_H - W_{HH}^1) + (1 - q)(\frac{P - W_{HL}^1}{2} + \frac{P - W_{LL}^1}{2})
\]

Subject to:

\[
\begin{align*}
W_{HH}^1 & \geq \Phi_H \\
W_{HL}^1 & \geq \Phi_H \\
W_{LL}^1 & \geq \Phi_H \\
W_{HL}^1 & \geq \Phi_L \\
W_{LL}^1 & \geq \Phi_L \\
W_{HL}^1 & \geq W_{HL}^M \\
W_{HL}^1 & \geq W_{HL}^H \\
qW_{HH}^1 + (1 - q)(\frac{W_{HL}^1 + W_{LL}^1}{2}) - \Phi_H & \geq qW_{HL}^1 + (1 - q)(\frac{W_{LL}^1 + W_{LL}^1}{2}) - \Phi_L
\end{align*}
\]

The first five constraints reflect limited liability. The next two constraints are the non-manipulation constraints. The last constraint is the incentive compatibility constraint.

**Lemma 10:** The solution to this problem is:

\[
W_{HH}^1 = \Phi_H + \Delta \Phi, \quad W_{HL}^1 = W_{HL}^M = W_{HL}^H = \Phi_H, \quad W_{LL}^1 = W_{LL}^H = \Phi_L.
\]

**Proof:** See Appendix A.

Note that the incentive compatibility constraint binds, giving the agent some rents when \(P_H\) is received. The rents are due to the agents’ ability to agree to trade at the lower price \(P\) when the negotiated price is

\[16\text{ As will be seen shortly } \Phi_H \text{ is the optimal wage that A1 gets when he is offered } P.\]
$P_L$. Since $P$ is negotiated in both HL and LL states, the non-manipulation constraint allows $A1$ to extract the rents. For this solution the profit of the principal is:

$$qP_H + \frac{1-q}{2}(\bar{P} + P) - \Phi_H - q\Delta\Phi.$$  

(3)

If the principal is interested in inducing a low effort choice by the agent, he then maximizes:

$$\max_{(\bar{W}_P)} q(P_M - W_{HL}^M) + (1-q)(\frac{\bar{P} - W_{LL}^P}{2} + \frac{P - W_{LL}^P}{2})$$

Subject to:

$$W_{HH}^H \geq \Phi_H$$
$$W_{HL}^P \geq \Phi_H$$
$$W_{HL}^P \geq \Phi_H$$
$$W_{LL}^P \geq \Phi_L$$
$$W_{LL}^P \geq \Phi_L$$
$$W_{HL}^M \geq W_{HL}^P$$
$$W_{HL}^P \geq W_{HL}^H$$

$$qW_{HL}^M + (1-q)(\frac{W_{HL}^P + W_{LL}^P}{2}) - \Phi_L \geq qW_{HH}^H + (1-q)(\frac{W_{HL}^P + W_{LL}^P}{2}) - \Phi_H$$

Lemma 11: The solution to this problem is:

$$W_{HH}^H = W_{HL}^P = W_{HL}^M = W_{HL}^P = \Phi_H, \quad W_{LL}^P = W_{LL}^P = \Phi_L.$$ 

Proof: Obvious.

Note that the agent still gets some rents when he makes a low effort choice and $A2$ makes a high effort choice. Again, the reason is that the non-manipulation constraint binds. From Lemma 11 we see that the principal’s profit is:

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17 Although these four prices are the only ones that will appear in equilibrium we have to specify the compensation for any reported price. Clearly, $A1$ is indifferent between reporting the appropriate specified price if for any other
\[ qP_M + \frac{1-q}{2} (\overline{P} + P) - q\Phi_H - (1-q)\Phi_L. \] 

(4)

**Proposition 4:** Let \( \Delta P = P_H - P_M \). The principal, \( P_1 \), writes a contract that induces high effort from his agent, \( A_1 \), if and only if: 
\[ q \geq \frac{\Delta \Phi}{\Delta P}. \]

**Proof:** Compare the profits in (3) and (4) above.

Proposition 4 demonstrates the externality that arises for \( P_1 \) due to the price formation in the input market and the inability to contract with \( A_2 \). The condition says that \( P_1 \) will induce a high effort from \( A_1 \) only if she believes that the chance, \( q \), that \( P_2 \) will induce a high effort from \( A_2 \) is sufficiently high. If, on the other hand, she believes that \( A_2 \) will make a low effort, then she is better off inducing a low effort from her agent too. Why is this true? From the point of view of \( P_1 \), if \( P_2 \) induces a high effort from \( A_2 \), then \( P_1 \)'s marginal benefit from inducing a high effort from \( A_1 \) (gross of wages) is \( P_H - P_M \). If, however, \( P_2 \) does not induce \( A_2 \) to make a high effort, then the marginal benefit to \( P_1 \) from inducing a high effort from \( A_1 \) is zero. This is simply because when \( A_2 \) makes a low effort the price is formed via take-it-or-leave-it offers and, hence, does not reflect \( A_1 \)'s effort choice. Since \( P_1 \) also considers the additional contracting cost of inducing high effort, she will only do so if she believes that the probability that \( A_2 \) made a high effort is sufficiently high. As will be seen in the next section this intuition does not hold for \( P_2 \).

**C. Optimal Contracts: The Output Producer**

Now consider \( P_2 \). Like \( P_1 \), \( P_2 \) can write contracts contingent on both the price of the intermediate input good and the price of the final output. Let \( b \) denote the probability assessment of \( P_2 \) that \( A_1 \) will produce a high quality input good. When deciding to induce a high effort from \( A_2 \), \( P_2 \) chooses the compensation contract to solve:

\[
\max_{(w_{ij}^*)} \quad b(P_{HH} - P_H - W_{HH}^H) + (1-b)(P_{HL} - P_M - W_{HL}^M)
\]

Subject to:

price \( X \), \( P_1 \) specifies a payment of \( W(P_{HL}, X) = \Phi_L \) and \( W(P_{HH}, X) = W(P_{HH}, X) = \Phi_H \). Also, if negotiations breakdown, principals’ limited liability binds and agents get nothing.
\[ W_{HH}^H \geq \Phi_H \]
\[ W_{HL}^M \geq \Phi_H \]
\[ W_{HL}^P \geq \Phi_L \]
\[ W_{LL}^P \geq \Phi_L \]
\[ W_{PL}^P \geq \Phi_L \]
\[ W_{PL}^L \geq \Phi_L \]
\[ W_{PL}^M \geq W_{HL}^M \]
\[ bW_{HH}^H + (1 - b)W_{HL}^M - \Phi_H \geq b\left(\frac{W_{HL}^P + W_{HL}^L}{2}\right) + (1 - b)\left(\frac{W_{LL}^P + W_{LL}^L}{2}\right) - \Phi_L \]

As before, in addition to the standard incentive compatibility constraint and the limited liability constraints we have the non-manipulation constraint that assures P2 that A2 will offer to pay the lower price \( P \) rather than \( P_M \) when he gets to make the take-it-or-leave-it offer.

**Lemma 12:** The solution to this problem is:

\[ W_{HH}^H = \Phi_H + \frac{\Delta \Phi}{2}, \quad W_{LL}^P = W_{LL}^{p} = \Phi_L, \quad W_{HL}^M = W_{HL}^{p} = \Phi_H \]

**Proof:** See Appendix A.

In this case the profit of the principal is:

\[ b(P_{HH}^H - P_H - \Phi_H - \frac{\Delta \Phi}{2}) + (1 - b)(P_{HL}^L - P_M - \Phi_H). \]  \hspace{1cm} (5)

If P2 decides to induce a low effort from his agent he solves:

\[ \max_{(W_{PL}^P)} b(P_{HL}^L - \frac{\overline{P} + W_{HL}^P}{2} - \frac{P + W_{PL}^P}{2}) + (1 - b)(P_{LL}^L - \frac{\overline{P} + W_{LL}^L}{2} - \frac{P + W_{PL}^L}{2}) \]

Subject to:
Lemma 13: The solution is trivially: $W_{hh}^p \geq \Phi_h$, $W_{hl}^p \geq \Phi_L$, $W_{lh}^p \geq \Phi_L$, $W_{ll}^p \geq W_{ll}^m$, $W_{lh}^l \geq \Phi_h$.

\[ b \left( \frac{W_{hl}^p + W_{lh}^p}{2} \right) + \frac{1-b}{2} \left( \frac{W_{ll}^p + W_{ll}^p}{2} \right) - \Phi_h \geq b W_{hh}^l + (1-b)W_{hl}^m - \Phi_L \]

Proposition 5: $P_2$ induces $A_2$ to make a high effort if and only if:

\[
\frac{1 - \varepsilon}{1 + \varepsilon + P_H - P_L} 
\]

which by definition of the prices is equivalent to: $b \leq \frac{1}{2 - \varepsilon + \Delta \Phi}$.

Proof: Compare the profits of the principal from (5) and (6).

Proposition 5 shows the externality faced by $P_2$ due to $A_1$’s independent effort choice decision. In this case, $P_2$ chooses to induce high effort from her agent only if she believes that the probability, $b$, of $A_1$ making a high effort is sufficiently low. $P_2$ does not find it optimal to induce $A_2$ to make a high effort if $A_1$ is making a high effort too. Intuitively, since the profits to $P_1$ are a cost to $P_2$ it is clear that if $P_1$ gets higher profits when both $A_1$ and $A_2$ make high efforts (see Proposition 4) then $P_2$ will have higher costs in that case. To complete the argument one must demonstrate that these higher marginal costs outweigh the marginal profit to $P_2$ from the final price net of the marginal contracting cost of inducing high effort from $A_2$. While this is done formally in the proof of Proposition 5 the logic behind it is that the input price gives $P_1$ “too much” of the surplus from the added effort of $A_2$. 

D. Equilibrium

With Propositions 4 and 5 in hand we are ready to find the Nash equilibrium of the economy. Recall that an equilibrium is: (1) a set of contracts designed by the principals to maximize their respective expected utilities; (2) effort choices by each agent such that their expected utilities are maximized given the contracts; and (3) correct beliefs, held by each player, regarding the choices of the other players. Intuitively, Proposition 4 says that P1 will only induce a high effort if the chance that P2 induces a high effort is sufficiently high. Similarly, Proposition 5 says that P2 will induce a high effort from her agent as long as the probability that P1 will also do so is low. For this reason we get the following mixed strategy equilibrium.

**Proposition 6:** The Nash Equilibrium is given by the contracts specified in Propositions 4 and 5 and by the following beliefs, $q^*$ and $b^*$:

1. For $\Delta \Phi \leq \varepsilon$  
   
   $$q^* = \frac{\Delta \Phi}{\Delta P} \quad \text{and} \quad b^* = \frac{1}{2 - \varepsilon + \Delta \Phi}.$$  

2. For $\Delta \Phi > \varepsilon$  
   
   $$q^* = 1 \quad \text{and} \quad b^* = 0.$$

**Proof:** To find an equilibrium we first need to demonstrate that it is optimal for agents to agree on a price and trade. Then we need to find the optimal contracts written by principals to maximize their profits, the optimal effort choices made by the agents to maximize their utility and, finally, beliefs that are consistent with the actual actions taken. We first show that it is indeed optimal for agents to agree on a price and trade the input good. Assume, by contradiction, that the agents bargain as specified by the mechanism, but that one agent decides to break off negotiations. Since it is too costly for each agent to search for a different counterparty, each agent goes back to his principal, reports that the negotiations broke down, and asks for his compensation. Since each principal has limited liability, the only way she can pay her agent is if trade takes place. Thus, the limited liability of the principals binds and the agents get nothing. Knowing that the principals can credibly claim they have limited liability when no trade occurs, agents will never find it in their interest to permanently disagree on a price at which to trade. Thus, in the market, agents always agree on a price according to the specified mechanism. The rest of the proof is left to Appendix A.

Integration is not the dominant organizational form, as in the previous cases. Now, the input price is serving an important allocative role because it supplies some ‘free’ information to principals.
Recall that the only information the principals need, in order to eliminate the agency costs (of not being able to contract directly on the agents’ investment decision), is whether the state is HL or LH. The take-it-or-leave-it offer game means that the non-manipulation constraints only need to be enforced by each principal half of the time. The other half of the time, the other principal is enforcing the non-manipulation constraint. Each principal is, thus, supplying some information to his counterpart at no cost.

To see this recall that $P_2$ is the price that $A_2$ is willing to pay $A_1$ when $A_2$ is uninformed and $A_1$ gets to make the take-it-or-leave-it offer. Since $A_2$ cannot distinguish between the states HL and LL, $P_2$ is paid in both states. The price $P_2$ is the observed price because $P_2$ enforces the non-manipulation constraint imposed on $A_2$. But, this benefits $P_1$. Observing $P_2$, $P_1$ learns that $A_2$ made a low effort and this information, combined with the final price allows him to distinguish HL from LL and hence learn the effort choice of $A_1$. $P_1$ did not pay for this information. Similarly, $P_1$ is the price that $A_1$ offers $A_2$ when $A_2$ is uninformed and $A_1$ gets to make the take-it-or-leave-it offer. This price is paid because $P_1$ enforces the non-manipulation constraint imposed on $A_1$. Again, observing $P_1$, $P_2$ knows that $A_2$ did not make a high effort. $P_2$ learns this information because $P_1$ paid to enforce the non-manipulation constraint on $A_1$.

The introduction of the take-it-or-leave-it offer game allows the two principals to coordinate to share the burden of producing information and, consequently, to reduce the cost of information production. The price now contains information that is not paid for completely by each party. This is the source of its value.

While the intermediate input price is valuable, there is a cost to organizing production in two firms. Notice from Proposition 6 that, while the equilibrium $q^*$ and $b^*$ depend on exogenous parameter values, for all parameter values the equilibrium does not result in First Best effort levels. Under our assumptions, an unconstrained social planner would choose $q^* = b^* = 1$, i.e., high effort levels would always be induced. The equilibrium, however, is not First Best for two interrelated reasons. First, there is a coordination failure in that $P_1$ and $P_2$ cannot meet and commit to induce high effort levels from their agents. We say that $P_1$ does not have the authority to write $A_2$’s compensation contract because $A_2$ has been hired by $P_2$. The coordination failure matters because, second, when $A_1$ makes a high effort choice, $P_2$ does not want to induce high effort from $A_2$. This is a result of the price formation mechanism.

The input price is a valuable signal because it supplies information useful for contracting, but the market does not allow principals to coordinate since each principal only has the authority to contract with his own agent. Thus, the trade-off determining organizational form is between the value of information...
from prices when there is a market versus the authority to coordinate when the market is closed via Integration.

VI. The Efficiency of the Visible versus the Invisible Hand

Having identified the trade-off between the two organizational forms, we now consider the choice of organizational form: Integration versus Non-Integration. We analyze which form is socially efficient, which form the principals privately prefer and, finally, which form is privately preferred by the agents. The choice of whether to vertically integrate will depend on balancing the informational gains resulting from having the market for the intermediate good against the costs of principals not having the authority to coordinate contracts in this case.

We consider constrained social efficiency, that is, the socially efficient organizational form chosen by a social planner. This corresponds to the organizational choice that results in the largest total surplus. Namely, we look for the organizational form that maximizes the combined surplus of both the two agents and the two principals. The following proposition gives a summary of the socially efficient structure for the different parameter values of the model.

**Proposition 7 (Constrained Social Efficiency):** The constrained socially efficient organizational form is:

i. For $\Delta \Phi \leq \varepsilon$ : The efficient organizational form is Integration.

ii. For $\varepsilon < \Delta \Phi \leq 1$ : Both Integration and Non-Integration have the same level of efficiency.

iii. For $1 < \Delta \Phi$ : The efficient organizational form is Non-Integration.

**Proof:** See Appendix A.

To understand the proposition recall that in the Non-Integration case, the first-best outcome is never achieved, i.e., $q^*$ and $b^*$ are never both equal to one (see Proposition 6). However, with two firms it is also the case that $q^*$ and $b^*$ are never both equal to zero. $P_1$ wants to induce a high effort choice only if she believes that the chance of $P_2$ making a high effort is sufficiently high. But, if $P_2$ believes that there is a good chance that $P_1$ will induce a high effort from $A_1$, then $P_2$ does not want to induce a high effort from $A_2$. This is the logic of the mixed strategy equilibrium discussed above. The integrated organizational form is different in that both $q^*$ and $b^*$ may be equal to one, or both may be zero, because a single principal chooses both contracts. Therefore, since the equilibrium $q^*$ and $b^*$ determine social efficiency, the socially efficient outcome depends on whether, in the case of Integration, the principal chooses contracts resulting in $q^*=b^*=1$ or $q^*=b^*=0$. 

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From Lemma 5 it is clear that, in the case of Integration, the marginal cost of inducing an additional agent to make a high effort is $2\Delta\Phi$. The marginal benefit is $P_{II} - P_{II} = 1 + \Delta\Phi$ for the first agent and $P_{II} - P_{II} = \varepsilon + \Delta\Phi$ for the second agent. Since the principal is the one who makes the choice of contracts, she simply compares the marginal costs and benefits. Therefore, when $\Delta\Phi$ is greater than 1 she chooses to induce low efforts from both agents, which results in the lowest efficiency level. Hence Non-Integration is more efficient. When $\Delta\Phi$ is less than $\varepsilon$ she chooses to induce high effort from both agents, which is first best. Hence Integration is more efficient. Finally, when $\varepsilon < \Delta\Phi \leq 1$ neither organizational form achieves first best. In the case of Integration the principal chooses to induce high effort from only one agent. With Non-Integration, $q^*=1$ while $b^*=0$ so the result is the same, namely, only one agent makes a high effort choice. Clearly, the social efficiency of the two forms is then the same.

Privately, principals and agents view the trade-off between the informational properties of the market for the intermediate good against the authority to coordinate differently than the social planner. Thus, the choice of organization is not necessarily the same.

**Proposition 8 (Principals):** When principals choose the optimal organizational form their choices are given by:

i. For $\Delta\Phi \leq \varepsilon$ : $\exists \varepsilon^*$ such that for $\Delta\Phi \leq \varepsilon^*$ principals choose Integration and for $\Delta\Phi > \varepsilon^*$ they choose Non-Integration.

ii. For $\varepsilon < \Delta\Phi \leq 1$: Principals are indifferent between Integration and Non-Integration.

iii. For $1 < \Delta\Phi$ : Principals choose Integration.

**Proof:** See Appendix A.

Figure 2 illustrates the benefits of each organizational form to the principals. The figure plots the profits to the principals as a function of the marginal cost of effort for each of the two organizational forms of production. In the Figure we assume that $A = 5$ and that $\varepsilon = 0.8$. With Integration the principal gets control over the contracts of both agents but she loses the information revealed in the intermediate input price. Authority over the contracts is most valuable in the two extreme cases when the marginal cost of inducing effort, $\Delta\Phi$, is very high or very low. When $\Delta\Phi$ is very low the principal uses her authority to induce both agents to make a high effort. On the other hand when $\Delta\Phi$ is very high she uses her authority to induce low effort from both since this is optimal for her. In the intermediate case, when
is not too high and not too low, the cost of losing the intermediate input price dominates that of having authority.

Agents’ organizational choices are given by:

**Proposition 9 (Agents):** When agents choose the optimal organizational form their choices are given by:

i. For $\Delta \Phi \leq \varepsilon$ : Agents choose Integration.

ii. For $\varepsilon < \Delta \Phi \leq 1$ : Agents are indifferent between Integration and Non-Integration.

iii. For $1 < \Delta \Phi$ : Agents choose Non-Integration.

**Proof:** See Appendix A.

Proposition 9 says that agents always make the socially efficient organizational choice while, from Proposition 8, we saw that principals do not always make the socially efficient decision. In particular, when the principal controlling the single firm chooses not to induce either agent to make a high effort, agents are clearly better off with Non-Integration. With two firms, agents can use the contract externality to extract some surplus from principals, so they are better off. The social planner prefers this organizational form because some high effort is induced in the mixed strategy equilibrium.

Propositions 7, 8, and 9 raise the issue of whether it is principals or agents who make the organizational form decision. Depending on who has that decision-making power, there may be mergers that increase shareholder value (when principal/shareholders get the power to make the decision) or mergers that decrease shareholders value but increase efficiency (when agent/managers have control). For example, in the case where $\varepsilon^* \leq \Delta \Phi \leq \varepsilon$, agents choose Integration whereas principals choose Non-Integration. Note that while a merger here will lead to a loss to shareholders, it still increases efficiency. The notion of efficiency we have in mind is not that of shareholder wealth but rather of the combined wealth of shareholders and the agent. The agent can be thought of as all the employees of the company.

**VII. Transfer Prices, Market Prices, and Property Rights**

When there is a single firm, Integration, the first agent, A1, transfers the intermediate input good to the second agent, A2, inside the firm. There is no “market” or associated “price” because there is no transfer of ownership. Nevertheless, there could be a number(s) called a “transfer price(s)” that is associated with transfer of the good from A1 to A2 inside the firm. In other words, the principal could ask the agents to send signals or reports to her about the quality of the input good. So far, we have not considered the possibility that such “transfer prices” could replicate the information content of the

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intermediate market prices. To understand the difference between the organizational forms, it is important to ask what is special about information produced in the market as compared to information produced internally.

The information that the principal wants to obtain regards whether the true state is HL or LH. Since the principal would use this information to reduce the cost of contracting, agents will not want to replicate this information. Consequently, the principal will have to ‘force’ agents to report these states by enforcing constraints. The main issue concerns whether the principal can credibly force agents to replicate the market price internally without paying them to do so. A transfer price can replicate the market input price only if the principal does not have to pay for all of the information about the states HL and LH.

The principal sees the final price, but this price does not differentiate between states HL and LH. She needs more information. Imagine the following mechanism. Suppose the single principal controlling the firm requires agents to “agree on a price” before A1 gives the input good to A2. Each agent has to report one of the four input prices corresponding to the four quality states (the exact numbers do not matter as long they are distinct and it is understood how they correspond to the quality states). We assume that:

**Assumption 8:** Agents cannot make side payments to each other when they work in the same firm.

Assumption 8 simply guarantees that agents do not make side deals to manipulate prices (which would simply increase the cost of obtaining information from them) or agree to report some new numbers that are not in the schedule.

Compensation for each agent is based on the following contract. If the principal gets two different reports then she does not want trade to occur and each agent gets nothing (here the limited liability of the principal binds because there is no production). If, however, agents report the same price then their compensation will be according to the reported efforts. With these price reports, there are two ways that the principal can obtain information from the agents. First, she can threaten the agents with low compensation payments if they report different signals. Or, secondly, she can offer them high compensation if their signals coincide and hence are informative (since they cannot make side deals to manipulate the signal).

Consider the first possibility of threatening low compensation for contradictory price reports. Since agents have limited liability it is infeasible to threaten them with low pay unless it is the case that

---

the principal's limited liability constraint binds too (namely, if she too has no money). The principal will, in fact, have no money if production does not place. But, a contract under which the principal instructs agents to cease production if their price reports differ is not re-negotiation-proof. If agents disagree on a price and report this to the principal it is never in her interest to follow up on the threat that they should not trade. In this case the principal, who owns both the input good and the final output, has two options. She can either go along with the original contract specifying no trade, in which case she gets nothing or, alternatively, she can renege on the agents’ contract and instruct the agents to proceed to produce the final good, thereby getting a strictly positive profit. Clearly, she will prefer that the agents “trade” and complete production. Knowing this, agents will never agree to report an informative signal since this will reduce their expected compensation.

Now, consider the second possibility, that of offering agents bonuses to induce them to report the true state. This means that each agent $i$’s contract has to be such that $W_i(P_{\text{Final}}, K) \geq W_i(P_{\text{Final}})$, where $K$ is the reported true state (input price). But, then the contracting cost to the principal cannot be reduced by having agents report an input price so that transfer prices within the firm do not reduce the cost of inducing effort and cannot replicate the input market price.

Thus, we have shown:

**Proposition 10:** When the input market is internalized into a single firm by one principal hiring both agents and owning all the assets used for production, the principal cannot increase her profits by inducing the agents to report an informative signal (namely, the true state).

The logic of the proposition relies heavily on the notion of limited liability for the principal or, equivalently (as discussed in the Introduction) on the idea of “incorporation.” And this, in turn, is linked to whether property rights are being transferred in conjunction with reporting a signal or negotiating a price. In the input market, there is competition between the two principals: P1 wants a high input price while P2 wants a low input price. Transfer of property rights to the input only occurs when a bargain is struck and a price is reported. Agents behave strategically, and form a price, because if they do not they are not compensated because principals are incorporated as “firms” (with limited liability). Thus, agents are somewhat schizophrenic. On the one hand, principal-agent pairs are competing, that is, the agents want to strike a bargain with their counterpart. But, on the other hand, each agent is also competing with his own principal because each agent would also like the bargain with the other agent to extract as much as possible from his respective principal.

Inside the firm, while agents cannot explicitly coordinate (by Assumption 8), it is never in their interests to report an informative price because it then allows the principal to lower the cost of
contracting. Agents understand this clearly and there is no countervailing force since principal-agent pairs are not competing. Why is there no countervailing force? The answer is that property rights to the input good are not being transferred inside the firm so the limited liability constraint of the principal is not credible in forcing agents to compete with each other. Since agents understand that inside the firm property rights to the input do not get transferred, they know that not agreeing on a transfer price will not affect production. Hence they do not report a transfer price. In the market, because of the competition between the two firms, agents have to agree on a price even though it reveals information about their marginal efforts. They understand that agreeing on a price is necessary for transferring property rights.

VIII. Discussion

To explain General Motors, a theory of the firm should incorporate (so to speak) the essential features of the modern corporation. These features include: (1) limited liability, that is, the corporation is a separate legal entity; (2) the separation of ownership and control, that is, those running the firm are distinct from those who own residual cash flow and control rights; and (3), asset ownership, that is, the owners (principals) have residual rights of control over the firm’s assets. In this paper we develop a theory in which all three features play critical roles. The theory shows that a defining feature of the corporation is the substitution of authority for informative market prices. The visible hand is sometimes more valuable than the invisible hand.

We start from an agency relationship in which a principal hires an agent. In our model this means that the principal controls the contract of the agent, has ownership rights to the output produced by the agent, and also that the principal has limited liability and hence that this “firm” is a separate legal entity. Given this basic building block we ask the question that is at the heart of any theory of the firm: What is the basic tradeoff between organizing economic activity via external markets and organizing the activity internally, within the boundaries of the firm? We show that the combination of the principals having limited liability and having ownership rights results in internal transfer prices that are uninformative and external prices that are informative to shareholders. We find that the authority the principal has over the agents’ contracts also has economic value. Thus, based on the three characteristics of the modern corporation identified above, a natural tension arises between the value of having authority to coordinate activity within a firm and the value of having informative market prices outside the firm.

How then is the corporation distinguishable from the market? In other words, why can’t the information in market prices be replicated internally with transfer prices? The answer we provide is intuitive. In order to facilitate production, agents need to trade with each other. However, trading at a negotiated price reveals information to their respective principals regarding their marginal efforts and hence reduces their agency rents. Why then do they go ahead and agree on a price when they are in
separate firms, but they do not agree when they are in the same firm? When production is organized in separate firms, agents understand that they will not be compensated unless there is profit. Profit requires trade and trade requires them to agree on a price. Hence, they are forced to reveal some information. Limited liability plays a key role. Since principals have limited liability they cannot be forced to compensate agents when there is no trade.
APPENDIX A

Proof of Lemma 1: We are looking for the minimum cost contracts that satisfy the constraints. Clearly then $W_{LL}^L = \Phi_L$. Furthermore, $W_{HL}^M = W_{IH}^M$ since the two states are indistinguishable from each other. From the first order conditions we get,

$$\frac{d}{dW_{HH}^H} = -q + \lambda_5 q + \lambda_1 = 0$$

$$\frac{d}{dW_{HL}^M} = -(1-q) + \lambda_5 (1-2q) + \lambda_2 = 0$$

where $\lambda_5$ is the Lagrange multiplier for the IC constraint, $\lambda_1$ is the Lagrange multiplier for the limited liability (LL) constraint on $W_{HH}^H$, and $\lambda_2$ is the Lagrange multiplier for the LL constraint on $W_{HL}^M$. We can solve the maximization problem assuming that $\lambda_1 = 0$ and then show that at the optimum the constraint does not bind. From the first equation then we see that $\lambda_5 = 1$ which means that the IC binds. Further, from the second equation we see that this leads to $\lambda_2 = q$. For $q = 0$ clearly the contracts we specified are optimal. For $q \neq 0$ the last condition implies that $W_{IH}^M = \Phi_H$ and the rest of the Lemma then follows.

Proof of Lemma 3: The proof of this Lemma is exactly the same as the proof of Lemma 1. For that reason we omit it.

Proof of Lemma 5: At the optimum $W_{LL} = \Phi_L$. We next prove that both of the incentive compatibility constraints (IC) must bind. Assume to the contrary that they do not; say that the IC constraint for A1 does not bind, then substituting $W_{LL} = \Phi_L$ into IC we have: $bW_{HH}^H + (1-2b)W_{HL}^H > \Phi_H - b\Phi_L$ with a strict inequality. By limited liability $W_{HH}^H$ and $W_{HL}^H$ must be at least $\Phi_H$. The inequality can then be true only if both $W_{HH}^H = \Phi_H$ and $W_{HL}^H = \Phi_H$. Otherwise we can reduce one of the payments without violating any of the constraints and thereby increase the objective function. But if this is the case then the left-hand side must equal $(1-b)\Phi_H$ which is less than $\Phi_H - b\Phi_L$. This violates the IC. A similar argument can be made to show that the second IC constraint binds. Thus both the IC constraints have to bind. Given that the IC constraints bind we will prove that $W_{HL} = \Phi_H$. Assume, to the contrary, that $W_{HL} = \Phi_H + \delta$
where \( \delta > 0 \). The objective function is then:

\[
-qb\left(\frac{2b-1}{b}\right) + (q + b - 2qb)(-\delta) = -b\delta < 0.
\]

Since this is negative we get that at the optimum \( W_{HL} = \Phi_H \). The equilibrium contracts are then

\[
W_{HH} = \Phi_H + \Delta \Phi \text{ and } W_{HL} = \Phi_H.
\]

**Proof of Lemma 6:** First it is easy to see that \( W_{LL}^H = \Phi_L \). Second, as is well known in these type of problems, we can solve assuming the first constraint \( W_{HH}^H \geq \Phi_H \) does not bind and then show that the solution satisfies this assumption. Denoting the LaGrange multipliers by \( \lambda_i \) for constraint \( i = 2 \ldots 6 \) we get the following first order conditions:

\[
\frac{d}{dW_{HH}} = -q + \lambda_1 + \lambda_6 = 0
\]

\[
\frac{d}{dW_{PL}} = -(1 - q) + \lambda_2 - \lambda_5 + \lambda_6 (1 - q) = 0
\]

\[
\frac{d}{dW_{ML}} = \lambda_3 - q \lambda_6 = 0
\]

Since \( \lambda_i = 0 \implies \lambda_6 = 1 \implies \lambda_5 = q \implies \lambda_2 = q \). This means that the IC constraint binds as well as that \( W_{LL}^M = W_{HL}^P = \Phi_H \) and the lemma follows.

**Proof of Lemma 8:** The proof of this Lemma is exactly the same as the proof of Lemma 6. For that reason we omit the proof.

**Proof of Lemma 10:** To minimize the cost of the contract both \( W_{LL}^P \) and \( W_{LI}^P \) must equal \( \Phi_L \). Furthermore, from the different constraints \( W_{HL}^M \geq W_{HL}^P \geq \Phi_H \). The maximization problem is then equivalent to minimizing:

\[
q W_{HH}^H + (1 - q)\left(\frac{W_{HL}^P + W_{HL}^L}{2}\right)
\]

(A1)
Using the IC constraint we see that this is greater than or equal to \( \Phi_H + qW^M_{HL} - q\Phi_L \) which at its minimum equals \( \Phi_H + q\Delta\Phi \). Consequently, if we can find a set of contracts such that (A1) equals \( \Phi_H + q\Delta\Phi \) those contracts will be optimal. It is easy to verify that the contracts specified in Lemma 10 satisfy this condition.

**Proof of Lemma 12:** From limited liability \( W^P_{LL} = W^P_{HL} = W^P_{HL} = \Phi_L \) and from the non-manipulation constraint \( W^P_{HL} = W^M_{HL} \). Thus, the incentive compatibility constraint becomes:

\[
bW^H_{HH} + (1-b)W^M_{HL} \geq \frac{1}{2}bW^M_{HL} + \Phi_h - \frac{b\Phi_L}{2}.
\]

The left-hand side is the expected cost to P2. The minimum of the right hand side is \( \Phi_H + \frac{b\Delta\Phi}{2} \). Thus, any set of contracts that has this expected cost (and satisfies the limited liability constraints) is optimal. It is easy to see that the contract where \( W^M_{HL} = \Phi_h \) and \( W^H_{HH} = \Phi_h + \frac{\Delta\Phi}{2} \) has these features so that it is optimal.

**Proof of Proposition 6:** Since \( \Delta P \equiv P_h - P_L = \frac{\varepsilon + \Delta\Phi}{2} \), \( \Delta\Phi \leq \varepsilon \) implies that \( \frac{\Delta\Phi}{\Delta P} \leq 1 \).

Hence, when \( \Delta\Phi \leq \varepsilon \), both conditions \( q \geq \frac{\Delta\Phi}{\Delta P} \) and \( b \leq \frac{1}{2 + \varepsilon + \Delta\Phi} \) are not trivially satisfied so that the Nash equilibrium is: \( q^* = \frac{\Delta\Phi}{\Delta P} \) and \( b^* = \frac{1}{2 + \varepsilon + \Delta\Phi} \). When \( \Delta\Phi > \varepsilon \), the condition on \( q \) is never satisfied (i.e., \( q > 1 \)) so that P1 always induces low effort. This means that \( b^* = 0 \). This in turn means that the condition for P2 to induce high effort is always satisfied which means that \( q^* = 1 \).

**Proof of Proposition 7:** In the case of Non-Integration the total expected surplus is given by:

\[
q^*b^*(P_{HH} - 2\Phi_h) + [q^*(1-b^*) + b^*(1-q^*)](P_{HL} - \Phi_h - \Phi_L) + (1-q^*)(1-b^*)(P_{LL} - 2\Phi_L).
\]

For \( \Delta\Phi \leq \varepsilon \), \( q^* \) and \( b^* \) are less than 1 and their values are given in Proposition 6. For \( \Delta\Phi > \varepsilon \), \( q^* = 1 \) and \( b^* = 0 \). Comparing this to the surplus in the case of Integration we see that for \( \Delta\Phi \leq \varepsilon \), Integration achieves first best, i.e., \( q^* = b^* = 1 \) and a surplus of \( P_{HH} - 2\Phi_h \), so this is the efficient structure. For \( \varepsilon < \Delta\Phi \leq 1 \), with Non-Integration, the expected surplus is A+1, which is equal to the output in the case
of Integration \( P_{HH} - \Phi_H - \Phi_L = A + 1 \). Thus the efficiency of both economies is the same. Finally, for \( \Delta \Phi > 1 \) with Integration the principal induces low effort from both agents, which results in a surplus of \( P_{LL} - 2\Phi_L = A \). This is less than \( A + 1 \) so here Non-Integration dominates.

**Proof of Proposition 8:** With Integration the principal gets:

i. \( A \) when \( \Delta \Phi \geq 1 \).

ii. \( A + (1 - \Delta \Phi) \) when \( \varepsilon \leq \Delta \Phi < 1 \).

iii. \( A + (1 - \Delta \Phi) + (\varepsilon - \Delta \Phi) \) when \( \Delta \Phi \leq \varepsilon \).

With Non-Integration the expected surplus of P1 and P2 can be calculated by subtracting the agents’ compensation from the total expected output. This yields an expected surplus of:

\[
q^* b^*(P_{HH} - 2\Phi_H - \frac{3}{2} \Delta \Phi) + (1 - q^*)b^*(P_{HL} - \frac{3\Phi_H + \Phi_L}{2}) + (1 - b^*)(P_{HL} - 2\Phi_H) + (1 - b^*)q^*(P_{LL} - 2\Phi_L) \tag{A2}
\]

For \( \Delta \Phi \leq \varepsilon \), \( q^* = \frac{\Delta \Phi}{\Delta P} \) and \( b^* = \frac{1}{2 + \Delta \Phi - \varepsilon} \) while for \( \Delta \Phi > \varepsilon \), \( q^* = 1 \) and \( b^* = 0 \). Calculating the profits and comparing them to the principals’ profits under Integration gives the following:

For \( \Delta \Phi > 1 \), Integration is better since \( P_{HH} - 2\Phi_H = A + 1 - \Delta \Phi \) is less than \( A \). Here the ability of the principal to choose \( q^* = b^* = 0 \) dominates. For \( \varepsilon < \Delta \Phi \leq 1 \), under both economies the principals get a surplus of \( A + (1 - \Delta \Phi) \) so they are indifferent. Finally, when \( \Delta \Phi \leq \varepsilon \), the principal gets \( A + (1 - \Delta \Phi) + (\varepsilon - \Delta \Phi) \) in Integration by inducing both agents to make a high effort. This needs be compared to (A2) above. Substituting in the values of \( q^* \) and \( b^* \) we get that (A2) equals

\[
-\frac{4\Delta \Phi^3 + (-5 + 4\varepsilon)\Delta \Phi^2 + (6 - \varepsilon)\Delta \Phi + 2\varepsilon}{2(\varepsilon + \Delta \Phi)(2 - \varepsilon + \Delta \Phi)}. \tag{1}
\]

The condition for choosing Non-Integration over Integration then translates into:

\[
\frac{-4\Delta \Phi^3 + (-5 + 4\varepsilon)\Delta \Phi^2 + (6 - \varepsilon)\Delta \Phi + 2\varepsilon}{2(\varepsilon + \Delta \Phi)(2 - \varepsilon + \Delta \Phi)} - (1 + \varepsilon) + 2\Delta \Phi \geq 0.
\]

One can see that as \( \Delta \Phi \downarrow 0 \) the expression becomes negative which means that Integration is preferred. However as \( \Delta \Phi \uparrow \varepsilon \) the expression becomes positive which means that Non-Integration is better for the principals.

**Proof of Proposition 9:** The agents’ expected compensation net of their cost of effort under the Non-Integration structure is:
$$qb\left(2\Phi_H + \frac{3}{2}\Delta\Phi - 2\Phi_H\right) + (1-q)b\left(\Phi_H + \frac{\Phi_H + \Phi_L}{2} - \Phi_H - \Phi_L\right) + (1-b)q\left(2\Phi_H - \Phi_H - \Phi_L\right) + 0$$

for $\Delta\Phi \leq \varepsilon$ (which simplifies to: $\Delta\Phi \left(q + \frac{b}{2}\right)$) and $\Delta\Phi$ for $\Delta\Phi > \varepsilon$. Under Integration agents get a net expected value of: $2\Delta\Phi$ for $\Delta\Phi \leq \varepsilon$, $\Delta\Phi$ for $\varepsilon < \Delta\Phi \leq 1$ and 0 for $1 \leq \Delta\Phi$. These values are derived by subtracting the private cost of effort from the optimal contracts. Using the above we compare the two organizational forms (from the agents’ perspective) to get the proposition.
Appendix B

**Proposition B1** The equilibrium $q^*$ and $b^*$ are given by,

i. For $\Delta \Phi \leq \varepsilon : \quad q^* = b^* = 1$.

ii. For $\varepsilon < \Delta \Phi \leq 1 : \quad q^* = b^* = \frac{1 - \Delta \Phi}{1 - \varepsilon}; \quad q^* = 0, b^* = 1; \quad q^* = 1, b^* = 0$.

iii. For $1 < \Delta \Phi : \quad q^* = b^* = 0$.

**Proof of Proposition B1:** We first calculate the profits to P1 and P2 from inducing their agents to make a high effort and then calculate their profits from inducing a low effort from their agents (both under the optimal contracts found in Lemmas 1-4). These calculations lead to the following two conditions:

1. P1 will induce her agent to make a high effort if and only if $q \leq \frac{1 - \Delta \Phi}{1 - \varepsilon}$.

2. P2 will induce her agent to make a high effort if and only if $b \leq \frac{1 - \Delta \Phi}{1 - \varepsilon}$.

Consequently, the Nash equilibrium in mixed strategies is:

i. For $\Delta \Phi \leq \varepsilon : \quad q^* = b^* = 1$.

ii. For $\varepsilon < \Delta \Phi \leq 1 : \quad q^* = b^* = \frac{1 - \Delta \Phi}{1 - \varepsilon}; \quad q^* = 0, b^* = 1; \quad q^* = 1, b^* = 0$.

iii. For $1 < \Delta \Phi : \quad q^* = b^* = 0$.

**Proposition B2:** To calculate the equilibrium $q$ and $b$ we first need to find the profit for each principal (P1 and P2) from inducing their agent to make a high effort and a low effort. Since Lemma's 6-9 give us the optimal contracts in these cases it is a simple algebraic exercise to derive the following conditions for when P1 and P2 will induce their respective agents to make a high effort choice:

\[
q(\varepsilon + \Delta \Phi - 2z) \geq \Delta \Phi - z \quad \text{(b1)}
\]

\[
b \leq \frac{2z - 2}{\varepsilon - \Delta \Phi - 2} \quad \text{(b2)}
\]

where $z \equiv P_M - P$.

Given these two conditions for P1 (b1) and P2 (b2), depending on the relation between the parameters $z, \varepsilon, \Delta \Phi$, the mixed strategy Nash equilibrium $q^*$ and $b^*$ can be found.
Appendix C

When P1 can only write contracts on the intermediate input price we have the following:

**Proposition C1:** P1 writes a contract that induces high effort if and only if:

\[ q \geq \frac{\Delta \Phi}{4\Delta P} + \sqrt{\left(\frac{\Delta \Phi}{4\Delta P}\right)^2 + \frac{\Delta \Phi}{2\Delta P}}. \]

**Proposition C2:** P2 induces A2 to make a high effort if and only if:

\[ b \leq \frac{1 + \frac{\bar{P} + P}{2} - P_L}{1 - \epsilon + P_H - P_L}. \]

**Proposition C3:** The Equilibrium is given by:

1. For \( \Delta \Phi \leq \epsilon \) \( q^* = \frac{\Delta \Phi}{4\Delta P} + \sqrt{\left(\frac{\Delta \Phi}{4\Delta P}\right)^2 + \frac{\Delta \Phi}{2\Delta P}} \) and \( b^* = \frac{1}{1 - \epsilon + 1 - \Delta \Phi} \).

2. For \( \Delta \Phi > \epsilon \) \( q^* = 1 \) and \( b^* = 0 \).

The proofs of these propositions are similar to the proofs of the propositions in the main text and are therefore omitted. From the above three propositions one can verify that:

\[ q^* = \frac{\Delta \Phi}{4\Delta P} + \sqrt{\left(\frac{\Delta \Phi}{4\Delta P}\right)^2 + \frac{\Delta \Phi}{2\Delta P}} \geq \frac{\Delta \Phi}{\Delta P}, \]

so that this equilibrium is (weakly) more efficient than the one studied in the main text where the contract of A1 was also contingent on the final output price.
References


Eccles, Robert and Harrison White, “Price and Authority in Inter-Profit Center Transactions,” American Journal of Sociology 94, 1988, S17-S51.


Table 1: Comparison of GM to Iceland, Luxembourg and Ireland

<table>
<thead>
<tr>
<th>Country/Firm</th>
<th>General Motors</th>
<th>Iceland</th>
<th>Luxembourg</th>
<th>Ireland</th>
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<tr>
<td>GDP/Value Added *+</td>
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<td>7.5</td>
<td>17</td>
<td>69</td>
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<tr>
<td>Labor Force</td>
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<td>148,000</td>
<td>218,000</td>
<td>1,500,000</td>
</tr>
</tbody>
</table>


+Value added is defined as the sum of net income and total employee salary.

Sources: National Accounts of OECD Countries, 1997 Volume 1.
Fortune 500, The Global 500 List.
Figure 1: The Production Function

\[ P_{ij} \]

\[ \Delta \Phi \]

\[ \epsilon + \Delta \Phi \]

\[ 1 + \Delta \Phi \]

\[ \Phi_i + \Phi_j \]
Figure 2: Principals’ Profits: Integration versus Non-Integration

For the above graph $\varepsilon = 0.8$. 