Conservatism, Optimal Disclosure Policy, and the Timeliness of Financial Reports

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In this paper we study the relation between the properties of firms’ financial reporting systems and the incentives for managers to pre-empt the financial reports by volunteering private information as it is obtained. We show that preemptive disclosures will be made only if the accounting system is not too conservative. The key implication of these findings is that while the degree of conservatism in a financial reporting regime may contribute to finding less of a relation between positive returns and earnings than between negative returns and earnings, this relation may well exist absent any conservatism in the financial reporting regime.

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1. Introduction

In this paper we model the relation between the bias in a company's financial reports and the optimal incentives provided for the manager to engage in voluntary disclosure activities. The key result is that, all else constant, the more conservative the bias in a financial reporting system the less value there is to making information public on a more timely basis through voluntary disclosures. We then demonstrate how this result should manifest itself in the empirical relation between accounting numbers and contemporaneous returns. Specifically, we show that in a regression of earnings on returns, the hypothesized slope is greater for negative than for positive price changes. Most significantly, this is true precisely because some firms are voluntarily disclosing while others are not and holds even if the financial reports for all firms have only liberal biases.

Our main motivation for pursuing this study is to emphasize the importance of establishing theoretical links between the properties of mandatory financial reports and the amount of information provided by voluntary disclosures. Modeling the link between mandatory and voluntary reporting is important for three reasons. First, empirical studies that attempt to document the economic effects of changes in financial reporting uniformly ignore the indirect effects of such changes on other sources of information. Omitting this (potentially) correlated variable makes interpretation of those studies' results problematic. However, without models of the indirect information effects, there is little that the empirical researcher can do to overcome this problem.

This brings us to our second general reason for pursing this study. That is that there is surprisingly little theoretical guidance as to how changing the properties of financial statements might change the other sources of information. Disclosure theories almost always view financial reporting as the unbiased realization of some underlying economic variable and

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1 Technically, the result is established for weakly negative and weakly positive returns.
2 See Antle, Demski and Ryan [1994].
rarely allow for the endogenous creation or dissemination of other information. As a result, most of these studies lack the scope necessary to examine the indirect information effects of changes in financial reporting. Instead they focus on the direct price and volume reactions to changes in the precision of what is modeled as the market’s only public information, the financial report itself.

The lack of studies which focus on the relationship between mandatory and voluntary disclosures is consistent with the view that financial reporting serves as a primary source of information to capital markets. That is, the market’s reaction to the change in the financial report itself is viewed as the economically interesting measure, while the indirect effects of financial reporting are not regarded as first order and are thus ignored. However, transaction based financial reporting is arguably more useful as a provider of confirmatory information than as a primary source of timely information and, within this view, the indirect effects are by definition the first order effects. We seek to provide insights into the relation between accounting numbers and prices based on the view that the transaction based model of financial reporting has probably retained its stature in a competitive marketplace by playing its most useful role.

Our third and final motivation for pursuing this study is the recent interest in assessing the degree of conservatism in financial statements from the observed relations of positive and negative price reactions to earnings. This approach is primarily initiated by Basu (1997) who’s key finding is that, in a regression of earnings on returns, the relation is stronger for negative than it is for positive returns. This is taken as evidence that financial statements are on average a more timely source of bad news than they are of good news and, in turn, that financial statements are conservative. Given the intuitive appeal of the idea in Basu (1997), it is hardly surprising that a number of subsequent papers have employed the Basu (1997) technology to

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3 A notable exception is Diamond [1985].
4 Arguably this is borne out in the results of empirical research as early as Ball and Brown [1968].
5 For examples see Gigler and Hemmer [1998] and the discussion of that paper in Dye [1998].
order financial reports and financial reporting regimes in terms of conservatism and/or
timeliness. Examples are Ball, Kothari, and Robin (1999) in a study ranking different national
reporting regimes in terms of conservatism, and Bushman, Chen, Engel, and Smith (1999)
who use the approach to rank financial statements within a reporting regime in terms of
timeliness.

Empirical measures of "accounting timeliness" are based on the contemporaneous
and/or lagged relation between (changes in) accounting constructs and changes in market
prices. Another type of timeliness, which becomes important when trying to quantify the
timeliness of accounting numbers in this way, is the timeliness of market prices themselves.
Under strong form market efficiency, lags between the time an event occurs and the time the
same event is reflected in market prices is always zero. "Price timeliness" in markets that are
strong form efficient is thus a moot issue to the empirical design. However, if markets are
strong form efficient, prices always reflect all available information independent of the
properties of the information system employed by the firm. Obviously, then earnings and book
value would be irrelevant.6

Under the (at least from an accounting point of view) more realistic perspective of
semi-strong form market efficiency, the "timeliness" of financial reports may have real
implications for the "timeliness" of market prices. Understanding this relationship is therefore
crucial when trying to empirically assess the timeliness of financial reports from market prices.
In semi-strong form efficient markets, value relevant events that give rise to accounting
metrics may not be directly observable to parties external to the firm. Rather, in a semi-strong
form efficient market, information about economic events may be incorporated in share prices
as a result of voluntary disclosures made by the firm, analyst reports, or, in some cases, as a
result of the release of the firm's financial report.7 While information may be reflected in

6Certainly if one takes a valuation perspective of financial reporting.
7Some events get incorporated into price directly because they are publicly observable. Examples include a
change in oil prices or interest rates.
prices ahead of the time it is reflected in the financial reports, in markets that are semi-strong form efficient, the potential exists that it is not.

When "price timeliness" is determined by the lag between the occurrence of an event and the first public report of the event (whatever form this report might take), one determinant of the relation between the timeliness of earnings and the timeliness of returns is the extent to which managers engage in providing timely voluntary disclosures. If firm specific differences in disclosure policy are unrelated to the properties of the accounting regime, such differences simply add noise to empirical timeliness measures. The point we make formally in this paper, however, is that the optimal policy guiding a manager’s incentives to engage in timely voluntary disclosure is directly determined by the level of bias in the financial reports. This, in turn, suggests a need for controlling for firms’ disclosure policies in studies linking properties of financial reports, such as bias, to changes in market prices.

Our paper unfolds as follows. First, in section 2, we present our model. Section 3 then establishes the relation between the nature of bias in the reporting entity’s accounting system and the optimal disclosure policy determined by the principal. In section 4 we demonstrate that while the earnings/returns relation implied by our model is identical to that documented by Basu (1997), this relation is present even when there is no conservatism in the financial reporting regime. Section 5 provides a brief summary.

2. Model

Consider a setting where a risk neutral principal hires a risk and effort averse agent to supply unobservable effort. The actions taken by the agent positively impact the likelihood of a desirable future event. We think of events such as improvements in a production process, improved standing of the company brand, developing a new product, and gaining access to new markets. Such events result in a subsequent set of transactions which are recorded in an
accounting system and, at a predetermined time, aggregated and released to the public as a part of the company’s financial report.

We limit the agent’s effort choice to be either high or low so that \( e \in \{ e^h, e^l \} \), where \( e^h > e^l \). The effect of the agent’s effort is to increase the probability of achieving output of higher economic value. The realized output is denominated in true economic income, a construct observable only to the agent. We denote this output, \( x \), and restrict our attention to a binary setting where its value can be either high or low. Thus we have \( x \in \{ x^H, x^L \} \), where \( x^H > x^L \). The expected effect of the agent’s effort on the value of output is determined by \( pr(x|e) \), where

\[
1 > pr(x^H|e^h) > pr(x^H|e^l) > 0. \tag{1}
\]

To economize on notation, hereafter we will denote \( pr(x^H|e^h) \) by \( P^h \) and \( pr(x^H|e^l) \) by \( P^l \). We assume that \( (x^H - x^L) \times (P^h - P^l) \) is sufficiently high so that it is always optimal to induce the high effort.

To complete the linkage between effort, output, transactions, and information, we now introduce the properties of the firm’s audited financial statements. We maintain our binary structure by assuming that accounting earnings that result from a particular economic event, \( x \), can also be either high or low, and represent the subsequent financial statement information about \( x \) by \( y \in \{ y^H, y^L \} \), where \( y^H > y^L \). The properties of the firm’s financial reporting system are captured by the probability that high accounting earnings result when true economic income is high, \( \lambda^H \), and that low accounting earnings will result in cases where true economic income is indeed low, \( \lambda^L \). We require that \( \lambda^H \geq 1 - \lambda^H \) and that \( \lambda^L \geq 1 - \lambda^L \) so accounting earnings always have a reasonable interpretation. The relationships between \( e \) and \( x \) and between \( e \) and \( y \) are summarized in Figure 1.

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8We will follow the convention of labeling choice variables with lower case superscripts and random variables with upper case superscripts.
Given our statistical structure, the information most useful in inducing the agent to take the high effort is $x$. But since $x$ is private to the agent and therefore not directly verifiable, the best the principal can do is to rely on the agent’s communication of $x$, denoted $\hat{x}$. In order to exploit this information in the contract the principal must get the agent to truthfully communicate it before the public realization of the verifiable signal, $y$. Communicating $\hat{x}$ before realization of $y$ is required because the risk in the realization of $y$ will be used to induce the agent’s truthful communication of $x$. Delaying disclosure is costly in that $\hat{x}$ then becomes useless for contracting.

We assume that there are exogenous incremental costs associated with writing and adopting a *truthful communication contract* (i.e., a contract which is dependent on $\hat{x}$) that are not incurred in using a *no-communication contract* (i.e., a contract which is not dependent on $\hat{x}$). Communication contracts may be more costly to implement because they are necessarily more complex than no-communication contracts. For instance, a communication contract has more contingencies than a no-communication contract and requires collecting and disseminating more information and at an earlier time. This may in turn require additional organizational activities, like engaging in a budgeting process. We represent the incremental cost of contracting on $\hat{x}$ by $C \geq 0$. Since delayed disclosure carries its own cost, early communication of $\hat{x}$ will only be desirable when the benefit from using this piece of information in the contract exceeds the cost.

Finally, we assume that the two parties to the contract are of the standard type. That is, the principal is risk neutral with preferences specified over his consumption opportunities at

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9This follows from Holmström [1979] since $x$ here is a sufficient statistic for $y$ with respect to $e$.

10It is important to note that we do not view the cost of using a communication contract as arising from making the communication public. In that case it would be better for the contract to make use of private communication and our analysis would not apply to public voluntary disclosures. If our model were to allow for such incremental costs of public communication it would also have to incorporate some sort of potential benefit to communicating publicly.

11For sufficient conditions for when it is optimal to have an agent share post decision information when there are no exogenous costs to communication, see Dye (1983).
the end of the contracting horizon. The risk averse agent has a utility function additively separable in utility for consumption opportunities provided by compensation received at the end of the contracting horizon and the effort exerted over the contracting horizon. Letting \( s(\cdot) \) denote the compensation received by the agent, which can be a function of all signals both parties have access to, either directly or as a result of communication, his utility function is given by

\[
H(s(\cdot), e) = U(s(\cdot)) - V(e) .
\]

(2)
\( U'(\cdot) > 0 \) and \( U''(\cdot) < 0 \) by risk aversion, and \( V(\cdot) > 0 \) by effort aversion. Without loss of generality we will normalize \( V(e^l) = 0 \) and \( V(e^h) = v \). Finally, we use \( G(\cdot) \) to represent the inverse of the agent’s utility for compensation so that \( s(\cdot) \equiv G(U(s(\cdot))) \).

3. Analysis

The principal is faced with a choice regarding the type of contract to offer the agent. He can choose to provide the agent with a contract that gives the agent a financial incentive to truthfully report his private information before he learns the exact earnings realization or he can offer a contract that does not encourage the agent to engage in this type of communication. We start our investigation by specifying the two programs that, when solved, respectively yield the best contract without early disclosure and the best contract that induces the agent to reveal \( x \) at the time it is learned. The contractual benefit of requiring early disclosure of \( x \) is simply the difference in the expected payments between these two contracts. Comparing this amount to the cost of early disclosure, \( C \), determines the type of contract the principal should choose.

When the agent is not asked to make timely disclosures, the optimal use of the financial reports for the purpose of encouraging the agent to choose the high effort level can be obtained as the solution to the following "no-communication" program:

\[
\min_{s(y)} P^h[\lambda^H s(y^H) + (1 - \lambda^H) s(y^L)] + (1 - P^h)[\lambda^L s(y^L) + (1 - \lambda^L) s(y^H)]
\]

(3C)
s.t.
\[
P^h[\lambda^H U(s(y^H)) + (1 - \lambda^H)U(s(y^L))] + (1 - P^h)[\lambda^L U(s(y^L)) + (1 - \lambda^L)U(s(y^H))] - v \geq U \quad (IR)
\]
\[
P^h[\lambda^H U(s(y^H)) + (1 - \lambda^H)U(s(y^L))] + (1 - P^h)[\lambda^L U(s(y^L)) + (1 - \lambda^L)U(s(y^H))] - v
\geq P^L[\lambda^H U(s(y^H)) + (1 - \lambda^H)U(s(y^L))] + (1 - P^L)[\lambda^L U(s(y^L)) + (1 - \lambda^L)U(s(y^H))] \quad (IC')
\]

The principal’s objective in NC is to minimize the expected compensation to the agent given that the contract is acceptable to the agent, IR, and that it provides incentives for the agent to work hard, IC'. The solution to this program is given in Lemma 1.

Lemma 1. The optimal no-communication contract is given by
\[
U(s(y^H)) = U + v - \frac{v P^h}{P^h - P^L} + \frac{v \lambda^L}{(P^h - P^L)(\lambda^H + \lambda^L - 1)}
\]
\[
U(s(y^L)) = U + v - \frac{v P^h}{P^h - P^L} - \frac{v (1 - \lambda^L)}{(P^h - P^L)(\lambda^H + \lambda^L - 1)}.
\]

If the principal wants to provide the agent with an incentive to reveal x truthfully before the realization of y, the optimal contract that also ensures that the agent chooses the high effort level is the solution to the following "truthful communication" program:

\[
\begin{align*}
\min_{s(x,y)} & \quad P^h[\lambda^H s(x^H, y^H) + (1 - \lambda^H) s(x^H, y^L)] + (1 - P^h)[\lambda^L s(x^L, y^L) + (1 - \lambda^L) s(x^L, y^H)]
\text{s.t.} & \quad P^h[\lambda^H U(s(x^H, y^H)) + (1 - \lambda^H) U(s(x^H, y^L))] + (1 - P^h)[\lambda^L U(s(x^L, y^L)) + (1 - \lambda^L) U(s(x^L, y^H))] - v \geq U \quad (IR)
\end{align*}
\]
\[
\begin{align*}
& \quad P^h[\lambda^H U(s(x^H, y^H)) + (1 - \lambda^H) U(s(x^H, y^L))] + (1 - P^h)[\lambda^L U(s(x^L, y^L)) + (1 - \lambda^L) U(s(x^L, y^H))] - v
\geq P^L[\lambda^H U(s(x^H, y^H)) + (1 - \lambda^H) U(s(x^H, y^L))] + (1 - P^L)[\lambda^L U(s(x^L, y^L)) + (1 - \lambda^L) U(s(x^L, y^H))] \quad (IC')
\end{align*}
\]
\[
\begin{align*}
\lambda^H U(s(x^H, y^H)) + (1 - \lambda^H) U(s(x^H, y^L)) & \geq \lambda^H U(s(x^L, y^H)) + (1 - \lambda^H) U(s(x^L, y^L)) \quad (TT^H)
\lambda^L U(s(x^L, y^L)) + (1 - \lambda^L) U(s(x^L, y^H)) & \geq \lambda^L U(s(x^H, y^L)) + (1 - \lambda^L) U(s(x^H, y^H)) \quad (TT^L)
\end{align*}
\]
Program $TC$ differs from program $NC$ in several respects. First, the agent’s communication of $x$ becomes available for contracting. In a better world, where the principal could observe $x$ directly, he would not use $y$ in the contract at all. But when only the agent observes $x$ directly, getting him to tell the truth requires keeping $y$, the only hard evidence on the agent’s information, in the contract. As a reflection of this, $TC$ has two additional constraints, designed to ensure that the agent is never better off by lying: $TT^H$ in case $x^H$ is observed and $TT^L$ for when the agent observes $x^L$. The solution to program $TC$ is given in our second Lemma.

Lemma 2. The optimal truthful communication contract is given by

$$U(s(\tilde{x}^H, y^H)) = U + v - \frac{\nu P^h}{\bar{P}_N - \bar{P}_F} + \frac{\nu \lambda^L}{(\lambda^H + \lambda^L - 1)}$$

$$U(s(\tilde{x}^H, y^L)) = U + v - \frac{\nu P^h}{\bar{P}_N - \bar{P}_F} - \frac{\nu (1 - \lambda^L)}{(\lambda^H - \lambda^L)(\lambda^H + \lambda^L - 1)}$$

$$U(s(\tilde{x}^L, y^H)) = U(s(\tilde{x}^L, y^L)) = U + v - \frac{\nu P^h}{\bar{P}_N - \bar{P}_F}$$

A comparison of Lemmas 1 and 2 illustrates some of the interesting properties of the optimal truthful communication contract and allows us to calculate the contractual benefit of early disclosure. First note that when the agent announces that he has observed $x^H$ he receives the same contract as he would have under the no-communication contract. That is, conditional on observing and reporting $x^H$, the agent is paid $s(y^H)$ if accounting earnings are high and $s(y^L)$ if they turn out to be low. Another important property of the optimal communication contract is that when the agent observes and reports $x^L$ he is paid $s(x^L)$ (the conditional payment under the optimal contract when $x$ is directly observable) regardless of the realization of $y$. Hence, the benefit of the optimal communication contract over the no-communication

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$^{12}$Recall that $x$ is a sufficient statistic for $y$ with respect to $e$.

$^{13}$See also Berg et. al, 1990.
contract is that, when economic earnings are low, the agent is paid as though economic earnings were directly verifiable. So the contractual benefit of early disclosure takes on the simple form given in Proposition 1.

Proposition 1. The expected reduction in compensation cost associated with using the communication contract instead of the no-communications contract is given as:

$$\Delta S = E[G(U(s(y))|x^L] - G(E[U(s(y))|x^L]) \geq 0.$$ 

Proposition 1 shows that there is always a weak contractual benefit of early disclosure. This is because when \( \widehat{x} = x^L \) the communication contract pays the agent an amount that does not depend on the subsequent realization of \( y \) and yet provides the agent with the exact same level of (expected) utility as does the risky no-communication contract when \( x^L \) is realized. The inequality in Proposition 1 then follows from the convexity of the risk-averse agent’s inverse utility function, \( G(\cdot) \).

Based on Lemmas 1 and 2 and Proposition 1, we are now in a position to address the main issue of our analysis. We focus specifically on how the amount and type of potential bias in the financial reporting system affects the contractual benefit of having access to \( x \) and, therefore, the equilibrium amount of information publicly available at a given time. Recall that the reporting system is characterized by the probability that high economic earnings results in high accounting earnings, \( \lambda^H \), and that low economic earnings will produce low accounting earnings, \( \lambda^L \). A noiseless accounting system (i.e., one which produces no errors) would necessarily have the property that \( \lambda^H = \lambda^L = 1 \). Bias is present in any system for which \( \lambda^H \) and \( \lambda^L \) are not both equal to one. Proposition 2 provides a preliminary result on the relation between bias in a firm’s financial reporting system and its optimal disclosure policy which serves as a useful first step toward understanding the relation more generally.
Proposition 2. The expected cost of the no-communication contract is strictly higher than the expected cost of the communication contract if and only if \( \lambda^L < 1 \). If \( \lambda^L = 1 \), the expected cost of the two contracts are identical.

To see why having some error in classifying low economic earnings is necessary to create a value of communication, recall that the benefit of having \( x \) communicated early is in shielding the agent from the risk imposed by noise in the accounting system whenever \( x^L \) is realized. However, when \( \lambda^L = 1 \), there is no additional risk introduced by the accounting system when \( x = x^L \) since \( x^L \) is always followed by \( y^L \). That there is strict contractual value to communication as soon as \( \lambda^L < 0 \) follows from the same argument. When \( \lambda^L < 0 \), \( x^L \) is not always followed by \( y^L \). So the optimal no-communication contract exposes the agent to risk when \( x = x^L \) but the optimal communication contract does not.

The implication of Proposition 2 is that with \( C > 0 \) it is never optimal for the principal to use the communication contract if the accounting system has \( \lambda^L = 1 \), even if \( \lambda^H < 1 \). In contrast, as soon as there is some potential that the accounting system will represent a low outcome as a high, there exists values of \( C > 0 \) for which the principal would indeed choose to use the communication contract. This is true for all values of \( \lambda^H \), including when \( \lambda^L < \lambda^H = 1 \).

Next we turn to a more general analysis of the effect of reporting bias on the optimal voluntary disclosure policy. For this we require a representation of bias in general and of conservatism in particular. We adopt the statistical definition of bias, which is the difference between the expected value of an estimator and the true value of an underlying parameter. In our context the estimator is accounting earnings, \( y \), and the parameter is true economic earnings, \( x \). Therefore, our accounting system contains the following biases:

\[
b(x^H) = - (1 - \lambda^H)(x^H - x^L) \text{ and } \]
\[
b(x^L) = (1 - \lambda^L)(x^H - x^L). \]
In our setting $b(x^H) < 0$ whenever $\lambda^H \neq 1$ and $b(x^L) > 0$ whenever $\lambda^L \neq 1$. We say that a bias is conservative (liberal) whenever it is less than (greater than) zero and that one accounting system is unambiguously "more conservative" than another if both biases are weakly lower. Intuitively, this is because as $\lambda^L$ increases, the likelihood that the accounting system will report low economic earnings as high decreases. And as $\lambda^H$ decreases, the likelihood that high economic earnings are reported as low accounting earnings increases.

However, changing $\lambda^L$ and $\lambda^H$ may affect the level of risk introduced into the principal-agent relation as well as the amount of bias in the financial reports. Both properties have implications for the value of admitting timely communication. But since we are interested in isolating the role of accounting bias in facilitating communication, we need to restrict attention to changes in $\lambda^L$ and $\lambda^H$ that do not change the level of risk by changing the amount of noise in the financial reporting system. But this presents an interesting problem. How does one change bias without changing risk?

In its most abstract, risk is the property of a gamble that a so called “risk-averse” individual will pay to avoid. The risk associated with a gamble cannot in general be assessed simply from the properties of the gamble itself, because the notion of risk is individual specific. However, the amount a risk-averse individual requires to accept a gamble, the risk-premium, can be quantified. Equivalently, for any individual, all gambles can be ranked by the difference between the expected payoff of the gamble and the value the individual places on the gamble, its certainty equivalent. Accordingly, in an agency setting where it is optimal to always induce the same level of effort, as in ours, changing the bias in the accounting system while keeping the level of risk constant is equivalent to changing $\lambda^L$ and $\lambda^H$ such that the expected compensation cost, which consists of compensation for effort and the risk-premium required by the agent, remains constant. This is the procedure we follow in our comparative statics analysis, giving us Proposition 3, our main result.
Proposition 3. Holding the equilibrium risk-premium in the no-communication contract constant, \( \frac{dA}{d\lambda} < 0 \), and \( \frac{d\lambda}{d\lambda} < 0 \). Thus, the value of communication is strictly decreasing in the conservatism of the financial reporting system.

Proposition 3 confirms what Proposition 2 only hints at. Ceteris Paribus, the more conservative the accounting system, the less contractual value there is to early disclosure. This is because keeping the level of risk constant in the benchmark no-communication setting requires that \( \lambda^H \) is a decreasing function of \( \lambda^L \). Therefore, increasing \( \lambda^L \) while keeping the risk constant unambiguously increases the conservatism of the financial reporting system. The final part of the link is provided by the result that the risk-sharing benefit to timely disclosures is also strictly decreasing in \( \lambda^L \). As this benefit shrinks it may eventually be swamped by the cost of early disclosure, \( C \). Then only firms for which the accounting standards are sufficiently liberal would adopt a policy of providing timely voluntary disclosures.

The economic intuition behind Proposition 3 can be obtained from Figure 2. Figure 2 portrays the mapping of two optimal no-communication contracts into utility for a specific agent. The two optimal contracts are generated from two different accounting systems that result in the same expected wage. The points \( C^L \) and \( C^H \) (\( L^L \) and \( L^H \)) represent the payments associated with \( y^L \) and \( y^H \) respectively for the more conservative (liberal) accounting system.\(^{14}\) Since the principal always induces the high level of effort, holding the equilibrium risk-premium constant is assured if the two accounting systems have the same expected compensation cost. And since both contracts are optimal they also provide the same expected utility. Graphically, this means that the solid lines connecting \( C^L \) with \( C^H \) and \( L^L \) with \( L^H \) must intersect on the dashed horizontal line representing the agent’s required expected utility from compensation, \( U + v \). The horizontal distance between the point \( FB \) and the point of intersection, \( SB_{NC} \), is the risk-premium.\(^{15}\)

\(^{14}\)As can be verified from the proof of proposition 3, both \( s(y^H) \) and \( s(y^L) \) are increasing in the degree of conservatism. Thus \( C^L > L^L \) and \( C^H > L^H \) in Figure 2 along both dimensions.

\(^{15}\)\( FB \) stands for first-best and \( SB_{NC} \) for second-best, no-communication.
Recall that if a communication contract is used the optimal contract when \( x^H \) is realized is identical to the optimal no-communication contract. Therefore, the points \( C^L \) and \( C^H \) (\( \mathcal{L}^L \) and \( \mathcal{L}^H \)) in Figure 2 also represent the optimal communication contract payments associated with \( y^L \) and \( y^H \) for the more conservative (liberal) accounting system conditional on the agent reporting \( \tilde{x}^H \). By Lemma 2, \( U(s(\tilde{x}^L, y)) \) is constant in \( y \) and independent of \( \lambda^L \) and \( \lambda^H \). So when the agent reports \( \tilde{x}^L \) under a communication contract, he receives the compensation/utility combination indicated by the point \( \tilde{x}^L \) under either accounting system. The amount of compensation associated with \( \tilde{x}^L \) is identified by \( I \) on the horizontal axis.

It can be verified from Lemma 1 that \( E[U(s(y))|x^L] = U + v - \frac{y^p^h}{p^h - p^l} \) and is therefore equal to \( U(s(\tilde{x}^L, y)) \). So the agent’s expected utility conditional on \( x^L \) is independent of accounting systems and whether the contract uses communication. Under a no-communication contract, this level of expected utility is a convex combination of the levels of utility associated with \( C^L \) and \( C^H \) (\( \mathcal{L}^L \) and \( \mathcal{L}^H \)) for the more conservative (liberal) accounting system. And since the expected utility conditional on \( x^L \) under an optimal communication contract is the same as an optimal no-communication contract, the expected compensation conditional on \( x^L \) for each of the two accounting systems is found where the solid lines connecting \( C^L \) with \( C^H \) and \( \mathcal{L}^L \) with \( \mathcal{L}^H \) intersect with the dashed horizontal line identifying \( E[U(s(y))|x^L] = U(s(\tilde{x}^L, y)) = U + v - \frac{y^p^h}{p^h - p^l} \). For the more conservative (liberal) accounting system the expected compensation conditional on \( x^L \) is identified by \( II (III) \) on the horizontal axis.

Because the benefit of communication arises exclusively from eliminating the risk in the agent’s contract when \( x^L \) is realized, the ex-ante value of communication can be calculated directly as \( (1 - P^h) [II - I] - C \) in the more conservative case and \( (1 - P^h) [III - I] - C \) in the more liberal case. The terms in square brackets are the expected compensation cost savings due to improved risk sharing when \( x^L \) is realized and \( 1 - P^l \) is the probability of that realization. That \( [III - I] > [II - I] \) reflects the result established in Proposition 3. The
principal’s choice of a disclosure policy in the end depends on a comparison of the magnitude of the costs and benefits associated with early disclosures. The more liberal the financial reporting system is, the more likely it is that this comparison falls out in favor of early disclosure.

Proposition 3 is premised on the assumption that the properties of a firm’s financial reporting system are the result of laws and standards and thus, unlike the voluntary disclosures, not at the individual firm’s discretion. However, while standards and laws inevitably restrict discretion, firms typically have some ability to affect properties such as conservatism in their financial reporting systems. Because Proposition 3 holds constant the agency cost in the no-communication contract it also reveals how firms would use limited discretion in choosing the bias in their accounting systems. Specifically, since the value of communication, and therefore the agency cost associated with a communication contract, is decreasing in conservatism, all firms would choose to be as liberal as possible. However, firms that cannot pass the threshold over which communication becomes optimal are indifferent as to the nature of the bias in their financial reporting systems. Accordingly, the result that disclosing firms have less conservative financial reporting systems also holds when firms’ have limited discretion to determine the properties of their financial reporting systems.

While the intuition behind the specific results obtained here is pretty clear, on a more general level the relation between the size and direction of accounting biases and the amount of alternative information available to market may seem less intuitive. One could argue that when accounting is conservative managers may be more inclined to engage in voluntary disclosure activities to ensure that good news known to them also becomes available to market participants -- despite the conservative accounting systems inability to reflect such good news, at least without significant delay. Even so, our results, which suggest the opposite, are consistent with a couple of stylized facts.
First, when contrasting different accounting regimes, a country such as Germany is typically viewed as having a more conservative financial reporting environment than the United States. What is noteworthy in the context of the preceding analysis is that German firms also are perceived as being more reluctant to volunteer information than their US counterparts. Second, our results are also consistent with what appears to be a systematic effort on the part of firms engaged in preemptive voluntary disclosure activities to generate positive earnings surprises. In our model only firms with sufficiently liberal accounting systems make such preemptive disclosures. What is interesting then is that due to the very nature of a liberal accounting system, inconsistencies between a voluntary disclosure and a subsequent financial report will more likely be in the form of a positive earnings surprise. In our model such patterns do not imply that managers are engaged in some sort of earnings management.

4. Implications for the Earnings/Returns Relation

Having established a formal link between the biases in a company’s financial reporting system and incentives for the agent to convey value relevant information on a timely basis, we now turn our attention to providing implications of our results for the empirical relation between accounting data and share price changes. We are interested in how the relation between conservatism and voluntary disclosure manifests itself in the following regression

\[ y = \alpha + \beta \Delta P + \epsilon, \]  

where the dependent variable, \( y \), is the accounting earnings and the independent variable, \( \Delta P \), is the stock price change (or return) measured over the same period as \( y \). Specifically, we are interested in how the relation between conservatism and disclosure incentives may account for differences in the regression coefficients for positive and negative market price changes. Let \( \beta^+ \) and \( \beta^- \) represent the predicted value of \( \beta \) in equation (3) for non-negative and non-positive returns respectively.
Even though all firms within a given accounting jurisdiction are subjected to a specific set of accounting standards, the standards may have different implications for the bias in different firms’ financial reports. That is, the accounting standards within a given jurisdiction may be quite conservative for some firms but quite liberal for others. In our model, these differences can be described in terms of the distributions characterized by $\lambda^H$ and $\lambda^L$.

Proposition 3 predicts that firms which engage in timely voluntary disclosure will have less conservative financial reporting systems than firms which do not. As a result, there should be a stronger contemporaneous relation between market returns and accounting earnings when the financial reporting regime is sufficiently conservative because the less conservative firms will pre-empt their financial reports by providing timely voluntary disclosures. For the less conservative firms, "returns lead earnings" as a result of their preemptive voluntary disclosure. For the more conservative firms "earnings lead returns". Let $\bar{\lambda}^H_D$ ($\bar{\lambda}^L_D$) and $\bar{\lambda}^H_N$ ($\bar{\lambda}^L_N$) be the average values of $\lambda^H$ ($\lambda^L$) for the disclosing and the non-disclosing firms respectively. By the argument just made $\bar{\lambda}^H_D \geq \bar{\lambda}^H_N$ and $\bar{\lambda}^L_D \leq \bar{\lambda}^L_N$. In order to generate predictions on the regression coefficients we require that, given any accounting jurisdiction, there are a large number of firms for which the standards are sufficiently liberal to induce voluntary disclosures and that there are also a large number of firms such that application of the standards is conservative enough to favor nondisclosure.

Crucial to Basu’s (1997) design is the assumption that the measured return captures all the economic events reflected in the associated earnings. Then returns are a measure of the underlying economic events and earnings is a noisy representation of those events. Basu’s prediction that $\beta^+ < \beta^-$ then follows from what is effectively a hypothesized "scaling difference" inherent to conservative accounting systems. The point we wish to make is that if

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16 It could be argued that US GAAP is conservative for firms heavily involved in R&D but quite liberal for firms that compensate their employees with employee stock options.

17 Basu (1997) measures returns over the year starting nine months prior to the earnings release and ending three months after to try and capture all preemptive disclosures as well as give the market time to impound any new information contained in earnings.
the returns measure does not capture all preemptive disclosures reflected in the associated earnings, then the effect of the bias in the accounting system on the firm’s voluntary disclosure policy will generate the same prediction on the slope coefficients even without a "scaling difference".\(^\text{18}\)

To demonstrate this we remove any potential scale difference between good and bad news from our model by assuming that the distribution of economic events (i.e., returns) is symmetric and that earnings scale good and bad news equivalently. Specifically, we assume that \(x^H = -x^L \equiv x\), that \(P^h = \frac{1}{2}\), and that \(y^H = -y^L = x\). This results in a prediction of \(\beta^+ = \beta^-\) when the returns window captures all preemptive disclosures. Furthermore, in order that the measured return capture all the economic events, \(x\), underlying earnings, \(y\), we assume that \(x\) is made public in some way before the close of the returns window. Even so, if a preemptive voluntary disclosure is made outside the returns window, our model also predicts that \(\beta^+ < \beta^-\) without the scaling difference hypothesized in Basu (1997). Proposition 4 provides our model’s predictions.

Proposition 4. In any sample of firms where some firms make preemptive disclosures outside the returns window and some firms do not, the regression coefficient in equation (3) is strictly greater for non-positive price changes than for non-negative price changes. That is, \(\beta^- > \beta^+\).

Proposition 4 shows how including the effects of conservatism on the optimal disclosure policy yields the exact coefficient differences predicted and documented by Basu (1997). The more important message, however, is that finding this pattern is not necessarily evidence of conservatism in financial reporting; the pattern is present here even when all firms experience a liberal bias, that is when \(\bar{\lambda}_N^H\) and \(\bar{\lambda}_D^H\) are greater than \(\bar{\lambda}_N^L\) and \(\bar{\lambda}_D^L\) respectively. In our model the difference in slope coefficients is generated by the presence of disclosing firms,

\(^\text{18}\)Basu (1997) recognizes the importance of controlling for whether or not earnings is the market’s source of information by also running a short window "event study". There he predicts that the magnitude of the earnings response coefficient is greater for good news than for bad news.
not a difference in the way the accounting system reports good verses bad news for non-disclosing firms. Firms that make preemptive disclosures outside of the returns window will experience smaller changes in price within the window than firms that do not make preemptive disclosures. Furthermore, since these firms have more liberal accounting systems, their earnings will be closer in expectation to those of firms with good news than to those of firms with bad news. The slope coefficients take on the predicted signs because the difference in expected earnings divided by change in price between disclosing firms and non-disclosing firms with bad news is larger than the same difference for disclosing firms and non-disclosing firms with good news.\textsuperscript{19}

5. Conclusion

In this paper we have analyzed the relation between the amount of bias in a financial reporting system and the incentives given to managers to either pre-empt the financial reports by volunteering private information or to withhold such information until the financial statement is made public. We show that when there are costs associated with early voluntary disclosures, such disclosures can only be optimal if the firms accounting system is not too conservative.

The key implication of this finding is that even though the degree of conservatism in a financial reporting regime may contribute to finding less of a relation between positive returns and earnings than between negative returns and earnings, this relation may well exist absent any conservatism in the financial reporting regime. Thus, finding the pattern documented by Basu (1997) and others cannot be taken as evidence that the regime in question has a conservative bias without having provided sufficient controls for differences in disclosure activities across firms and over time.

\textsuperscript{19}This holds even if $x$ doesn't become public also for the non-disclosing firms at the time the financial report is released. If that was assumed to be the case, the only change would be that the price reaction for the non-disclosing firms at the time the financial report is released would be based on $E[x|y]$ rather than on $x$ directly.
Figure 1. Probabilistic relation between effort and feasible economic and accounting outcomes.

- $e \in \{e^h, e^l\}$ is productive effort
- $x^H$ is an event of High economic value
- $x^L$ is an event of Low economic value
- $y^H$ is a High earnings realization
- $y^L$ is a Low earnings realization
- $p^h$ is the probability for $x^H$ if the agent supplies a high level of effort
- $p^l$ is the probability for $x^H$ if the agent supplies a low level of effort
- $\lambda^H$ is the probability that $x^H$ is followed by $y^H$
- $\lambda^L$ is the probability that $x^L$ is followed by $y^L$
Figure 2. The effect of communication on equilibrium compensation costs

$U + v$ is the agent’s minimum utility requirement from compensation.

$E[U|x^H]$ is the agent’s expected utility conditional on observing $x^H$.

$E[U|x^L]$ is the agent’s expected utility conditional on observing $x^L$.

$FB$ identifies the agent’s First-best utility/compensation.

$SB_{NC}$ identifies the agent’s second-best expected utility/compensation in the benchmark no-communication setting.

$L^H, L^L$ are utility/compensation combinations associated with $y^H$ and $y^L$ respectively for a relatively liberal accounting system.

$C^H, C^L$ are utility/compensation combinations associated with $y^H$ and $y^L$ respectively for a relatively conservative accounting system.

$\tilde{x}^L$ is the utility/compensation combinations for a communication contract when the agent reports $\tilde{x}^L$ under either accounting system.
REFERENCES


APPENDIX

Proof of Lemma 1. By standard arguments, both the IR and IC constraints are binding.
The optimal solution characterized in lemma 1 therefore can be obtained simply by solving
these two equations for the two unknown contract points. \( Q.E.D. \)

Proof of Lemma 2. The proof consists primarily of five claims.

claim (i): If \( TT^H \) is slack then \( U(s(x^L, y^L)) = U(s(x^L, y^H)) \).
proof: Suppose \( TT^H \) is slack and \( U(s(x^L, y^L)) < U(s(x^L, y^H)) \). Then lower \( U(s(x^L, y^H)) \)
by \( \delta \) and increase \( U(s(x^L, y^L)) \) by \( \frac{\delta}{\lambda} \). This variation is neutral to the IR, IC, and \( TT^L \)
constraints and for sufficiently small \( \delta \) does not violate the (slack) \( TT^H \) constraint. Therefore,
this variation is feasible. Furthermore, because the agent is risk averse, this variation reduces
the expected wage, contradicting the hypothesis that the optimal contract has \( TT^H \) slack and
\( U(s(x^L, y^L)) < U(s(x^L, y^H)) \).

Suppose \( TT^H \) is slack and \( U(s(x^L, y^L)) > U(s(x^L, y^H)) \). Then lower \( U(s(x^L, y^H)) \)
by \( \delta \) and increase \( U(s(x^L, y^L)) \) by \( \frac{\delta}{\lambda} \). Again, this variation is neutral to the IR, IC, and \( TT^L \)
constraints and does not violate (slack) \( TT^H \) for sufficiently small \( \delta \). This contradicts the
hypothesis that the optimal contract has \( TT^H \) slack and \( U(s(x^L, y^L)) > U(s(x^L, y^H)) \),
completing the proof of the claim.

claim (ii): If \( TT^L \) is slack then \( U(s(x^H, y^L)) = U(s(x^H, y^H)) \).
proof: Suppose \( TT^L \) is slack and \( U(s(x^H, y^L)) > U(s(x^H, y^H)) \). Then lower \( U(s(x^H, y^L)) \)
by \( \delta \) and increase \( U(s(x^H, y^H)) \) by \( \frac{\delta}{\lambda} \). This variation is neutral to the IR, IC, and \( TT^H \)
constraints and for sufficiently small \( \delta \) does not violate the (slack) \( TT^L \) constraint. Therefore,
this variation is feasible. Furthermore, because the agent is risk averse, this variation reduces
the expected wage, contradicting the hypothesis that the optimal contract has \( TT^L \) slack and
\( U(s(x^H, y^L)) > U(s(x^H, y^H)) \).

Suppose \( TT^L \) is slack and \( U(s(x^H, y^L)) < U(s(x^H, y^H)) \). Then lower \( U(s(x^H, y^L)) \)
by \( \delta \) and increase \( U(s(x^H, y^H)) \) by \( \frac{\delta}{\lambda} \). Again, this variation is neutral to the IR, IC, and \( TT^H \)
constraints and does not violate (slack) \( TT^L \) for sufficiently small \( \delta \). This contradicts the
hypothesis that the optimal contract has \( TT^L \) slack and \( U(s(x^H, y^L)) < U(s(x^H, y^H)) \),
completing the proof of the claim.

claim (iii): The solution can not have both constraints \( TT^H \) and \( TT^L \) slack.
proof: If both \( TT^H \) and \( TT^L \) are slack, then by claims (i) and (ii) these constraints can be
written as:

\[
\begin{align*}
U^H - U^L &> -\frac{1}{\lambda^H}(U^H - U^L) \\
U^H - U^L &< -\frac{1}{\lambda^L}(U^H - U^L)
\end{align*}
\]

where, by claims (i) and (ii) respectively,
\[ U^L \equiv U(s(x^L, y^L)) = U(s(x^L, y^H)) \]
\[ U^H \equiv U(s(x^H, y^H)) = U(s(x^H, y^L)) \]

But now \( U^H \geq U^L \) violates \( TT^L \) and \( U^H \leq U^L \) violates \( TT^H \). Therefore a solution can not have both constraints \( TT^H \) and \( TT^L \) slack.

**claim (iv):** The solution to program \( TC \) has constraint \( TT^H \) slack and \( TT^L \) binding.

**proof:** Suppose \( TT^H \) binds. Then claim (iii) implies \( TT^L \) is slack. Furthermore, claim (ii) implies \( U^H \equiv U(s(x^H, y^H)) = U(s(x^H, y^L)) \), so \( IC \) and \( TT^L \) can be written as:

\[
U^H \geq \frac{\nu}{\nu - \rho} + \lambda^L U(s(x^L, y^L)) + (1 - \lambda^L)U(s(x^L, y^H)) \quad IC
\]
\[
U^H < \lambda^L U(s(x^L, y^L)) + (1 - \lambda^L)U(s(x^L, y^H)) \quad TT^H
\]

generating a contradiction and proving \( TT^H \) slack. \( TT^L \) binding follows from claim (iii).

**claim (v):** The \( IR \) and \( IC \) constraints are both binding.

**proof:** \( IR \) binds by the standard argument. If \( IC \) is slack, then \( U(s(x^H, y^H)) \) can be lowered by \( \delta \) and \( U(s(x^L, y^H)) \) can be increased by \( \delta \frac{\lambda^H}{(1 - \rho^H)(1 - \lambda^L)} \) to generate an improvement. Note that this variation is \( IR \) neutral, puts slack into \( TT^L \), and by claim (iv) satisfies \( TT^H \) provided \( \delta \) is sufficiently small.

To complete the proof of lemma 2, solve the three binding constraints; \( IR, IC \) and \( TT^L \), for the three unknowns; \( U^L \equiv U(s(x^L, y^H)) = U(s(x^L, y^L)), U(s(x^H, y^H)) \) and \( U(s(x^H, y^L)) \).

\[ Q.E.D. \]

**Proof of Proposition 1.** Using Lemmas 1 and 2, we have

\[
E[G(U(s(y)))) = P^h [\lambda^h G(U(s(y^H))) + (1 - \lambda^H)G(U(s(y^L)))]
\]
\[
+ (1 - P^h)[(1 - \lambda^L)G(U(s(y^H))) + \lambda^L G(U(s(y^L)))]
\]

and

\[
E[G(U(s(\hat{x}, y)))) = P^h [\lambda^h G(U(s(y^H))) + (1 - \lambda^H)G(U(s(y^L)))]
\]
\[
+ (1 - P^h)G((1 - \lambda^L)U(s(y^H)) + \lambda^L U(s(y^L))).
\]

The difference is the contractual value of communication. \[ Q.E.D. \]
Proof of Proposition 2. From Lemma 1 we have

\[ U(s(y^L)) = \begin{cases} U + v - \frac{v^{P_h}}{p^s - p^L} - \frac{\lambda^L (1 - \lambda^L)}{(p^s - p^L)^2} & \text{for } \lambda^L < 1, \\ U + v - \frac{v^{P_h}}{p^s - p^L} & \text{for } \lambda^L = 1. \end{cases} \]

Then, since by Lemma 1 \( E[U(s(y))|x^L] = U + v - \frac{v^{P_h}}{p^s - p^L} \) and due to the convexity of \( G(\cdot) \),

\[ E[G(U(s(y)))|x^L] = \begin{cases} G(U + v - \frac{v^{P_h}}{p^s - p^L}) = G(U(s(x^L, y))) & \text{for } \lambda^L < 1, \\ G(U + v - \frac{v^{P_h}}{p^s - p^L}) = G(U(s(x^L, y))) & \text{for } \lambda^L = 1. \end{cases} \]

The Proposition 2 now follows from Proposition 1. \( Q.E.D. \)

Proof of Proposition 3. To prove this proposition we rely on the following lemma:

**Lemma A1.** For any risk-averse agent, holding the amount of risk in the optimal no-communication contract constant while changing \( \lambda^L \) requires \( \frac{\lambda^H}{\lambda^L - 1} < \frac{dU}{dx^L} < \frac{\lambda^H - 1}{\lambda^L - 1} < 0. \)

Proof of Lemma A1. From Lemma 1 we have

\[ EU = \pi U(s(y^H)) + (1 - \pi) U(s(y^L)) \]

and

\[ EG = \pi G(U(s(y^H))) + (1 - \pi) G(U(s(y^L))) \]

where

\[ \pi \equiv P^h \lambda^H + (1 - P^h)(1 - \lambda^L). \]

Holding the risk-premium constant while meeting (IR) implies \( \frac{dEU}{dx^L} = \frac{dEG}{dx^L} = 0 \). Thus,

\[ \frac{d\pi}{dx^L} \Delta U + \pi \frac{dU(s(y^H))}{dx^L} + (1 - \pi) \frac{dU(s(y^L))}{dx^L} = 0 \]

and

\[ \frac{d\pi}{dx^L} \Delta G + \pi \frac{G'(U(s(y^H)))}{dx^L} + (1 - \pi) \frac{G'(U(s(y^L)))}{dx^L} = 0 \]

where \( \Delta U \equiv [U(s(y^H)) - U(s(y^L))] \) and \( \Delta G \equiv [G(U(s(y^H))) - G(U(s(y^L)))] \)

Multiplying \( \frac{dEU}{dx^L} \) by \( \Delta U \) and \( \frac{dEG}{dx^L} \) by \( \Delta G \), and subtracting both sides of \( \frac{dEU}{dx^L} \) from both sides of \( \frac{dEG}{dx^L} \), after rearranging terms we obtain

\[ \pi \frac{G'(U(s(y^H)))}{dx^L} \left[ \frac{\Delta G}{\Delta U} - G'(U(s(y^H))) \right] + (1 - \pi) \frac{G'(U(s(y^L)))}{dx^L} \left[ \frac{\Delta G}{\Delta U} - G'(U(s(y^L))) \right] = 0. \quad (A1) \]

By convexity of \( G(\cdot), \left[ \frac{\Delta G}{\Delta U} - G'(U(s(y^H))) \right] < 0 < \left[ \frac{\Delta G}{\Delta U} - G'(U(s(y^L))) \right]. \)

Thus,

\[ \text{sign}\left( \frac{dU(s(y^H))}{dx^L} \right) = \text{sign}\left( \frac{dU(s(y^L))}{dx^L} \right). \]
Using the expressions in Lemma 1, we have

\[
\frac{dU(s(y^H))}{d\lambda^L} = \frac{v}{P^h - P^L} \left[ \frac{1}{(\lambda^L + \lambda^H - 1)} + \frac{(\frac{d\lambda^H}{d\lambda^L} + 1)(1-\lambda^L)}{(\lambda^L + \lambda^H - 1)^2} \right] = \frac{v}{P^h - P^L} \cdot \frac{\lambda^H + \theta \lambda^H (1-\lambda^L)}{(\lambda^L + \lambda^H - 1)^2}
\]

and

\[
\frac{dU(s(y^L))}{d\lambda^L} = \frac{v}{P^h - P^L} \left[ \frac{1}{(\lambda^L + \lambda^H - 1)} - \frac{(\frac{d\lambda^H}{d\lambda^L} + 1)\lambda^L}{(\lambda^L + \lambda^H - 1)^2} \right] = \frac{v}{P^h - P^L} \cdot \frac{\lambda^H - \theta \lambda^H \lambda^L}{(\lambda^L + \lambda^H - 1)^2}.
\]

which are both continuously differentiable and monotone in \(\frac{d\lambda^H}{d\lambda^L}\) with

\[
\frac{dU(s(y^H))}{d\lambda^L} > 0 \quad (\forall \lambda^H, \lambda^L)
\]

\[
\frac{dU(s(y^L))}{d\lambda^L} > 0 \quad (\forall \lambda^H, \lambda^L)
\]

Thus, \(\text{sign} \left( \frac{dU(s(y^H))}{d\lambda^L} \right) = \text{sign} \left( \frac{dU(s(y^L))}{d\lambda^L} \right)\) requires either

\[
\frac{\lambda^H - 1}{\lambda^L - 1} > \frac{d\lambda^H}{d\lambda^L} > \frac{\lambda^H}{\lambda^L - 1} \quad \text{or} \quad \frac{\lambda^H - 1}{\lambda^L - 1} < \frac{d\lambda^H}{d\lambda^L} < \frac{\lambda^H}{\lambda^L - 1}.
\]

And whenever \(1 \gg \{\lambda^H, \lambda^L\} \gg \frac{1}{2}\) the first relationship is satisfied. \(Q.E.D.\)

Now, since

\[
E[U(x^L)] = (1 - \lambda^L)U(s(y^H)) + \lambda^L U(s(y^L))
\]

and

\[
E[G(x^L)] = (1 - \lambda^L) s(y^H) + \lambda^L s(y^L),
\]

we can write

\[
E[U(x^L)] = \omega EU + (1 - \omega) U(s(y^L))
\]

\[
E[G(x^L)] = \omega EG + (1 - \omega) s(y^L),
\]

where

\[
0 < \omega = \frac{1 - \lambda^L}{\pi} = \frac{1 - \lambda^L}{P^h \lambda^H + (1 - P^h)(1 - \lambda^L)} < 1.
\]

Then,

\[
\frac{dE[G(x^L)]}{d\lambda^L} = \frac{d\omega}{d\lambda^L} \left[ E G - G(U(s(y^L))) \right] + (1 - \omega) G'(U(s(y^L))) \frac{dU(s(y^L))}{d\lambda^L}
\]

\[
= \frac{d\omega}{d\lambda^L} \pi \Delta G + (1 - \omega) G'(U(s(y^L))) \frac{dU(s(y^L))}{d\lambda^L}
\]

and

\[
\frac{dE[U(x^L)]}{d\lambda^L} = \frac{d\omega}{d\lambda^L} \left[ EU - U(s(y^L)) \right] + (1 - \omega) \frac{dU(s(y^L))}{d\lambda^L}
\]

\[
= \frac{d\omega}{d\lambda^L} \pi \Delta U + (1 - \omega) \frac{dU(s(y^L))}{d\lambda^L}.
\]

From lemmas 1 and 2

\[
\frac{dE[U(x^L)]}{d\lambda^L} = 0.
\]

Therefore,

\[
\frac{dE[G(x^L)]}{d\lambda^L} = \frac{dE[G(x^L)]}{d\lambda^L} - \frac{dE[U(x^L)]}{d\lambda^L} G'(U(s(y^L)))
\]

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\[ = \frac{d\omega}{dx} \pi [\Delta G - \Delta U G'(U(s(y^L)))]. \]

Dividing both sides of this equation by \( \pi \Delta U \) gives

\[ \text{sign} \left( \frac{d\omega}{dx} \right) = \text{sign} \left( \frac{dE[G(x^L)]}{dx} \right) \]

By lemma A1, \( \frac{d\omega}{dx} < 0 \), \( \frac{dE[G(x^L)]}{dx} < 0 \) then follows from

\[ \frac{\Delta G}{\Delta U} > G'(U(s(y^L))) \] by the convexity of \( G(\cdot) \).

\[ Q.E.D. \]

Proof of Proposition 4.

\[ \beta^+ = \frac{E(y|\Delta P > 0) - E(y|\Delta P = 0)}{\alpha - \Delta P > 0}, \quad \text{and} \quad \beta^- = \frac{E(y|\Delta P = 0) - E(y|\Delta P < 0)}{\alpha - \Delta P < 0}. \]

Here we have:

\[ E(y|\Delta P > 0) = \lambda^H_N x - (1 - \lambda^H_N) x = (2\lambda^H_N - 1) x, \]

\[ E(y|\Delta P = 0) = \frac{\pi}{2} \left( \lambda^H_D - (1 - \lambda^H_D) + (1 - \lambda^L_D) - \lambda^L_D \right) \]

\[ = \left( \lambda^H_D - \lambda^L_D \right) x, \]

and

\[ E(y|\Delta P < 0) = \left( 1 - \lambda^L_N \right) - \lambda^L_N = \left( 1 - 2\lambda^L_N \right) x. \]

Thus,\[ \beta^+ = \frac{(2\lambda^H_N - 1)x - (\lambda^H_D - \lambda^L_D)x}{x - 0} = 2\lambda^H_N - \left( \lambda^H_D - \lambda^L_D \right) - 1, \]

and

\[ \beta^- = \frac{(\lambda^H_D - \lambda^L_D)x - (1 - 2\lambda^H_N)x}{0 + x} = 2\lambda^L_N - \left( \lambda^H_D - \lambda^L_D \right) - 1. \]

Thus,\[ \beta^+ - \beta^- = 2\left( \lambda^H_N - \lambda^L_N \right) - 2\left( \lambda^H_D - \lambda^L_D \right) < 0 \]

where the inequality follows from \( \lambda^H_N < \lambda^H_D \) and \( \lambda^L_N > \lambda^L_D \) by Proposition 3. The last part of the proposition follows from \( \alpha = E(y|\Delta P = 0) \) here.

\[ Q.E.D. \]