Forward Buying Without Trade Promotions: Dynamic Channel Coordination

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Abstract

When firms use independent intermediaries to distribute their products, they have to design contracts that will coordinate the channel and align manufacturer and retailer incentives. In this paper, we propose that allowing the retailer to forward buy and hold inventory for the future can move the channel closer to a coordinated setting even in the absence of trade promotions. This is a surprising result of our analysis especially in light of most academic research that suggests that forward buying hurts manufacturers’ profits and that forward buying arises only in the presence of trade promotions. Using a two-period model in which the first period can be seen as the current period and the second period can be seen as the future, we demonstrate the channel coordination role of forward buying in three different channel structures. In the first channel structure, one manufacturer sells through a single retailer and we find that under certain conditions, both parties can be better-off by the retailer’s forward buying. In the second channel structure, two competing manufacturers sell through a common retailer and forward buying becomes more likely than in the manufacturer monopoly case. In particular, the retailer gains more from forward buying and the manufacturers gain less as the competition between the manufacturers becomes more intense. On the other hand, in the third channel structure, two competing retailers purchase from a single manufacturer and forward buying becomes less likely than in the retailer monopoly case. Across all these channel structures, this paper argues that forward buying need not always be a problem for manufacturers because, under certain conditions, it can play an important role in coordinating the channel.
1. Introduction

When firms use independent intermediaries to distribute their products, the problem of coordinating channels can rear its ugly head because manufacturers’ and retailers’ incentives are not aligned. As a result, the focus of a very large literature in marketing is on developing contractual and non-contractual solutions to mitigate the inefficiencies that arise from the lack of alignment in channel settings. The objective of this paper is to examine another non-contractual solution, namely forward buying, that can move the channel closer to a coordinated setting. In particular, we analyze the manner in which forward buying can potentially align manufacturer and retailer incentives with a special emphasis on competitive settings.

Forward buying is a practice by which retailers purchase additional units in a current time period, hold these additional units in inventory and then sell them in subsequent periods. The traditional rationale put forward for this practice is that retailers buy extra units to take advantage of temporarily low wholesale prices that arise from trade promotions. Indeed, in the marketing literature, there are no non-trade promotion related reasons for retailers to forward buy. However, researchers are less clear about whether forward buying helps or hurts manufacturer profits (see, for example, Blattberg and Neslin 1990). For example, if there is no overall increase in demand, then all the manufacturers have achieved is to sell some units at a lower price, a practice that helps the retailer at the expense of the manufacturer. As a result, there has been a significant push from manufacturers toward using scan backs to tie wholesale price discounts to contemporaneous sales at the retail level, i.e., manufacturers are trying to solve a coordination problem associated with forward buying. Considering the joint problems of channel coordination and forward buying begs the following questions: Can there be alternate explanations for forward buying that have nothing to do with trade deals or the traditional
reasons why retailers hold inventory? Does forward buying exacerbate or alleviate the channel coordination problem? How does retailer and manufacturer competition affect the optimality of forward buying and its impact on the channel outcomes? These are the issues we address in this paper by developing a multi-period model in which manufacturers sell their goods through intermediaries.

The essential problem in channel coordination is that retailers and manufacturers have incentives that differ – what’s optimal for one need not be optimal for the other. For example, a manufacturer would prefer the retailer to sell a higher quantity than the retailer chooses to sell, a retailer would prefer to provide a lower level of service than manufacturers’ would like them to provide, and so on. These conflicting incentives increase channel inefficiencies and lower the overall profits of the channel. As a result, researchers in this area have focused on understanding the nature of these inefficiencies and on designing contractual and non-contractual mechanisms that align the manufacturers and retailers incentives, such that retailers choose the appropriate price, service, promotional spending, or any other marketing variable (e.g., Jeuland and Shugan 1983, McGuire and Staelin 1983, Coughlan and Wernerfelt 1985, Moorthy 1987, Lal 1990, Tyagi 1999, Lee and Staelin 1997, Iyer 1998, Bruce et al 2005, Cui, Raju and Zhang 2006a, among others). Similar problems have also been analyzed in the analysis of supply chain problems (see, for example, Bernstein and Federguen 2005, Cachon 2004, Cachon and Lariviere 2005). The economics literature on double marginalization (Spengler 1950), downstream moral hazard (Holmstrom 1979) and bilateral moral hazard (e.g. Holmstrom 1982, Bhattacharya and Lafontaine 1995) also deals with vertical relations where incentives are misaligned.

Although the literature on channel coordination can design mechanisms to address misaligned incentives, the solutions are often very complex contracts that can be too difficult to
implement by practitioners. This suggests that a simple contract coupled with non-contractual arrangements might be easier to implement and also be more appealing to firms. Furthermore, a vast majority of channel coordination papers tend to be static in nature, whereas the problem we are posing occurs over time. Therefore, there is a need to better understand channel coordination issues in dynamic contexts. In this paper, we try to fill in these gaps by showing that the simple practice of allowing retailers to forward buy has the potential to move the channel closer to a coordinated level in a dynamic setting, even with a very simple wholesale price contract.

Within the marketing literature, the practice of forward buying by retailers has been linked directly to the presence of trade promotions, which are temporary price discounts offered by manufacturers to their distribution channel members (e.g., Ailawadi, Kopalle and Neslin 2004). Academic researchers have argued that it is the combination of temporary price reductions coupled with competition for customers that makes forward buying so attractive for retailers, but it is not at all clear whether this is good for the manufacturer (e.g., see Blattberg and Neslin 1990). In an exception to the more common negative view of forward buying, Lal, Little and Villas Boas (1996) show in a model of trade promotion that when manufacturers can write performance-contingent contracts, then retailers can take advantage of periodic trade deals and forward buying can be profitable for both parties. This result arises because of the competition for a “switching” segment of the market. It is important to note that in the absence of a switching segment, there would be no trade deals offered by the manufacturers and, hence, no forward buying by retailers.

Forward buying has also been studied in the operations management literature within the context of firms’ inventory decisions in a variety of models (see Lee and Nahmias (1993) and Zipkin (2000) for reviews). In these papers, inventory emerges as a mechanism to deal with
demand or supply uncertainty or as a tradeoff between ordering and holding costs. However, inventory's role in channel coordination has not been a prominent issue in this literature. An exception to this is a recent working paper by Anand, Anupindi and Bassok (2003) that shows that holding inventory on the retailer’s part is always optimal when it comes to coordinating a simple manufacturer-retailer channel with a two-part contract. In contrast, we look at more complex and competitive channel structures and find that holding inventory at the retailer level is optimal only under certain conditions.

The objectives of this paper are to examine how forward buying by retailers can affect channel coordination across a variety of competitive scenarios. Therefore, we develop a simple model that specifically excludes the standard reasons advanced by researchers for why retailers would forward buy and hold inventory. Thus, we assume a market in which there is no uncertainty about demand and supply, no production lead time, no ordering or set-up costs and manufacturers offer a single price with no temporary price deals. Within this framework, we analyze three channel structures: (1) A single manufacturer sells to a single retailer; (2) Two competing manufacturers sell through a single, common retailer; and (3) a single manufacturer sells to two competing retailers. In all these structures, we allow the retailer(s) to forward buy.

In the single manufacturer – single retailer case, we find conditions under which both the retailer and the manufacturer are better off with forward buying. In addition, there are conditions under which no forward buying occurs and conditions under which forward buying is profitable for the retailer but not for the manufacturer. The principal reason for forward buying to occur in our model is that the presence of inventoried goods in subsequent periods forces the manufacturer to lower its wholesale price that, in turn, leads to a reduction in the retail price.
The net effect is that compared to the case where there is no forward buying, the retailer sells more units and earns higher profits over the two-period horizon when it is able to forward buy.

In the competitive case, we allow two manufacturers to sell through a common retailer and find that compared to the previous bilateral monopoly case, forward buying becomes even more likely. Importantly, the retailer benefits because each manufacturer reduces wholesale price in response to the retailer’s forward buying not only of its own product but also of the competing manufacturer’s product. When we introduce competition at the retailer level and allow a single manufacturer to sell through two competing retailers, we still find the presence of forward buying. Interestingly, we find that retailers are more likely to pass through a greater part of the reduction in wholesale price. We also find that there is a free-riding problem – forward buying by one retailer results in a lower wholesale price for both retailers. As the retailer competition becomes more intense, we find that the incidence of forward buying goes down.

The remainder of this paper is organized as follows. In the next section, we lay out the basic model and detail our assumptions. In Section 3, we analyze forward buying within the three different channel structures. We conclude the paper in Section 4.

2. Model

We develop the simplest possible model that can capture the interactions between manufacturers and retailers and also allow for the possibility of forward buying by the retailers. Recall that the typical reasons put forward to explain the presence of forward buying or carrying inventory are: temporary price reductions offered by the manufacturer, demand or supply uncertainty, supply lead times, and retailer ordering costs. Our model specifically rules out the aforementioned reasons – thus, there is no temporary price cut, no uncertainty, no lead times and
no ordering costs. As a result, this setting allows us to see whether there is an alternate explanation for forward buying and whether this practice is optimal for manufacturers and retailers.

We begin our analysis with a base model in which one manufacturer sells its products through a single retailer. Subsequently, we examine two duopoly cases, one with two competing manufacturers selling through a single retailer, and the other with a single manufacturer selling through two competing retailers. The remainder of this section describes the base model with a single manufacturer selling through a single retailer. In the subsequent sections, we describe the embellishment to the basic structure that leads to the two duopoly cases we consider.

To fully appreciate the role of forward buying, we incorporate time in our analysis by studying a two-period structure. Consumer demand for the product is given by

\[ d_t = \tau - p_t, \]

where \( t = 1, 2 \) denotes time period, \( p_t \) denotes the retailer’s price in period \( t \), \( \tau \) is the market potential and \( d_t \) is the demand in period \( t \). In each period, the manufacturer offers the retailer the opportunity to purchase goods at an announced wholesale price, \( w_t \). In order to capture the retailer’s forward buying decisions, we make a distinction between the retailer’s ordering and selling decisions. Specifically, we allow the retailer to make two simultaneous decisions in each period: the quantity of units to order from the manufacturer and the retail price to charge to consumers.\(^1\) The forward buying quantity or the inventory in period \( t \) \((t=1,2)\), \( I_t \), is the difference between the quantity \( q_t \) that the retailer orders from the manufacturer and the quantity \( d_t \) that consumers demand at the price chosen by the retailer: \( I_t = q_t - d_t \geq 0 \). If the retailer

\(^1\) We can generate qualitatively identical results when the retailers’ decisions are made sequentially. However, the algebra is more tedious and the intuition is less clear in that case.
carries any inventory, it incurs a holding cost of \( h > 0 \) per unit. Units carried in inventory do not deteriorate or perish and can be sold in the subsequent period as new goods. We assume that the manufacturer faces constant marginal costs and set this marginal cost to zero. We assume that the both parties face the same discount factor, \( \rho (\rho \in (0,1)) \).

In each period there are two stages. In the first stage, the manufacturer makes its wholesale price decision and in the second stage the retailer makes its order quantity and retail price decisions. Thus, the four stages of the game are as follows:

- **Stage 1**: The manufacturer sets the first period wholesale price, \( w_1 \).
- **Stage 2**: The retailer chooses the first period order quantity, \( q_1 \) and the first period retail price, \( p_1 \).
- **Stage 3**: The manufacturer sets the second period wholesale price, \( w_2 \).
- **Stage 4**: The retailer chooses the second period order quantity, \( q_2 \) and the second period retail price, \( p_2 \).

We adopt the notion of subgame perfect Nash equilibrium and solve the game backward, starting from Stage 4.

### 3. Analysis

We begin our analysis with the base case, the simplest possible channel structure that allows us to derive insights on how retailer-manufacturer interactions are affected by the retailer’s forward buying strategy. We subsequently add competition at each level of the channel to understand how product market competition modifies the results from the base case.
3.1 One Manufacturer-One Retailer Channel

We begin with the analysis of retailer’s second period (stage 4) decisions. At this stage, the retailer has to make two decisions: the quantity to order from the manufacturer and the retail price to charge consumers. In period 2, the retailer has \( I_1 \geq 0 \) units in the inventory and hence the maximum number of units that it can sell is \( q_2 + I_1 \). At this stage, there is no reason to have any unsold units at the end of the period. Therefore, because demand is given by \( d_2 = \tau - p_2 \), the actual sales at price \( p_2 \) is going to be the smaller of two quantities: \( \tau - p_2 \) and \( q_2 + I_1 \). Thus, the retailer’s optimization problem is given by

\[
\max_{q_2, p_2} \pi_2^R = p_2 \left( \min\{\tau - p_2, q_2 + I_1\} \right) - w_2 q_2 ,
\]

where \( \pi_2^R \) is retailer R’s profits in period 2. It is easy to see that at a given price \( p_2 \), the retailer’s optimal ordering quantity is \( q_2^* = \tau - p_2 - I_1 \). Ordering a quantity lower than \( q_2^* \) results in unmet demand and ordering a quantity greater than \( q_2^* \) results in unsold units that bring no additional benefits. Therefore, the retailer’s optimization problem reduces to

\[
\max_{p_2} \pi_2^R = p_2 (\tau - p_2) - w_2 (\tau - p_2 - I_1) .
\]

This optimization problem yields \( p_2^* = \frac{\tau + w_2}{2} \), and the optimal ordering quantity can be expressed as:

\[
q_2^* = \frac{\tau - w_2}{2} - I_1 .
\]

Thus, the retailer’s optimal ordering quantity is decreasing in the level of inventory carried from period 1. In other words, as the retailer forward buys more units in period one, it needs to order fewer units in period two.
Anticipating the above retailer decisions, the manufacturer chooses its second period wholesale price to maximize its second period profit. The manufacturer’s profit function at this stage is given by \( \pi^M_2 = w_2 q^*_2 = w_2 \left( \frac{\tau - w_2 - 2I_1}{2} \right) \). This leads to the optimal wholesale price,

\[ w^*_2 = \frac{\tau - 2I_1}{2} = \frac{\tau}{2} - I_1. \]

This makes clear that as the retailer forward buys more units in period 1, it decrease its optimal ordering quantity in the second period, thus leading to a weaker demand for the manufacturer’s product in the second period. Therefore, as the first-period’s forward buying increases, the optimal wholesale price in the second period declines.

We next analyze the first period decisions, starting with the retailer’s decisions in Stage 2. The retailer’s profit in the first period is \( \pi^R_1 = p_1 (\tau - p_1) - w_1 q_1 - h I_1 \).\(^2\) The retailer chooses \( q_1 \) and \( p_1 \) to maximize the discounted sum of its profits over the two-period horizon,

\[ \Pi^R = \pi^R_1 + \rho \pi^R_2. \]

The optimal values of \( q_1 \) and \( p_1 \) are given by

\[ p^*_1 = \frac{\tau + w_1}{2}, \quad \text{and} \quad \]  
\[ q^*_1 = \frac{\rho(6\tau - 3w_1) - 4(h + w_1)}{6\rho}. \]  

(5) \quad (6)

The manufacturer chooses the first period wholesale price to maximize the discounted sum of its profit, \( \Pi^M = \pi^M_1 + \rho \pi^M_2 = w_1 q^*_1 + \rho \pi^M_2 \). This leads to the optimal first-period wholesale price, \( \bar{w}_1 = \frac{9\rho \tau - 2h}{8 + 9\rho} \). Note that the first period wholesale price and order quantity decrease with the per unit holding cost, \( h \). To understand this finding, consider the case where \( h \)

\(^2\) We only consider the parameter values for which \( I_1 \geq 0 \).
decreases by a small amount. This decrease in $h$ makes it less “costly” to forward buy and the retailer increases its order quantity. Furthermore, the decrease in $h$ also leads the manufacturer to increase its wholesale price. However, it is important to note that in response to a lower $h$, the increase in the ordering quantity is higher than the increase in wholesale price.

We now discuss if the retailer does any forward buying in equilibrium.

**Proposition 1**: Retailer forward buying occurs if and only if $0 \leq h < h_1 = \frac{v\rho(9\rho - 4)}{8 + 12\rho}$. When forward buying occurs, the optimal forward buying quantity is $I_1^* = \frac{9\rho^2\tau - 8h - 4\rho(\tau + 3h)}{2\rho(8 + 9\rho)}$.

Proposition 1 highlights an important result: when the holding cost is not too high, the retailer orders more units than it plans to sell in the first period, holds the additional units in inventory and sells them in the second period. This happens in our model in the absence of all the typical reasons for a retailer to carry inventory, namely, demand or supply uncertainty, supply lead times or high ordering costs. The reason for the retailer to forward buy in the first period is that when the retailer purchases these additional units and carries them in inventory, it needs to order fewer units from the manufacturer in the second period. In other words, the inventory in period 2, gives the retailer a strategic advantage that leads the manufacturer to charge a lower wholesale price in period 2. Note that even though forward buying in period 1 clearly has a benefit in period 2, it also has additional holding costs in period 1. In addition, the retailer purchases a higher quantity from the manufacturer, and this may lead the manufacturer to charge a higher wholesale price in the first period. These additional costs in the first period can offset the second period benefits of forward buying to the retailer. Therefore, forward buying is
not always optimal for the retailer but is optimal when the holding costs are sufficiently low,

\[ 0 \leq h < h_{r1}. \]

As the above discussion describes, forward buying results in the retailer increasing \( q_1 \) which may lead the manufacturer to increase \( w_1 \) which can possibly offset the benefits of a lower \( w_2 \). However, it turns out that the equilibrium \( w_1 \) may not go up much with forward buying and may, in fact, decline with forward buying. The reason is that any change in \( w_1 \) affects the retailer's choice of \( p_1 \) and \( q_1 \) which in turn affect not only the retailer's first period profit but also its second period profit through the inventory that is carried forward. In other words, a change in \( w_1 \) has a direct effect on the retailer's first period profit and a strategic effect on the retailer's second period profit. To understand this more clearly, consider the retailer’s optimal quantity ordered in period 1 in the case when it can forward buy as well as the case when it can not forward buy. The optimal quantities with forward buying (\( q_{1*} \)) and without forward buying (\( q_{1**} \)) are given by:

\[
q_{1*} = \frac{(6\tau - 4h) - (3\rho + 4)w_1}{6\rho}, \text{ and } \quad (7)
\]

\[
q_{1**} = \frac{\tau - w_1}{2}. \quad (8)
\]

From Equations (7) and (8), it is clear that an increase in \( w_1 \) lowers \( q_{1*} \) more than it lowers \( q_{1**} \). Therefore, if the retailer forward buys in period 1, not only is it able to get a more favorable wholesale price in period 2, it may also not have to pay for an offsetting increase in the wholesale price in period 1. Thus, the retailer may find forward buying optimal under conditions where the holding cost is sufficiently low.
The foregoing discussion raises the issue of how forward buying may affect the manufacturer. This leads to the following proposition:

**Proposition 2:** When \( 0 < h < h_{m1} < h_{r1} \), where \( h_{m1} = \frac{\tau [2\rho - (1 - \rho)\sqrt{\rho(8 + 9\rho)}]}{4(1 + \rho)} \), the manufacturer is better-off with the retailer's forward buying.

When the retailer engages in forward buying, the manufacturer faces weaker demand from the retailer in period 2 and its profits in the second period are hurt. However, the second period decline in the manufacturer's profit is offset by the retailer’s purchase of a greater quantity in the first period and, in certain cases, a higher wholesale price in the first period. Proposition 2 is important in that it shows that the retailer's forward buying can also have a positive impact on the manufacturer. Therefore, even when the manufacturer is able to prevent forward buying by the retailer, it may choose not to do so. However, when \( 0 < h_{m1} < h < h_{r1} \), the manufacturer's profit is reduced by forward buying whereas the retailer’s profit increases with forward buying.\(^3\)

In the next section, we allow for competition between manufacturers and examine its impact on the profitability of forward buying.

### 3.2 Two Manufacturers-One Retailer Channel

We now examine the effect of manufacturer competition on the incidence and profitability of forward buying. We consider two symmetric manufacturers selling to a single retailer. We modify our demand function as follows.

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\(^3\) Retailer's forward buying can also upset the manufacturer's production cycles and have additional negative impact on the manufacturer's profit that are not modeled here.
where \( i \) and \( j \) denote the two manufacturers, \( \phi \) is the parameter representing the intensity of competition between the two manufacturers, and \( t \) \((t=1,2)\) denotes time period. These demand functions are based on the quadratic utility function developed by Shubik and Levitan (1980) and are consistent with the demand function used in the previous section (see Equation (1)). An appealing property of this formulation is that the total demand does not change as a consequence of adding an additional manufacturer. In other words, Equations (1) and (9) do not represent a shift in demand in spite of the fact that the manufacturer’s products are differentiated. This ensures that if we observe forward buying in this framework, it is not because of an expansion of demand that may arise from two manufacturers being in the market.

We assume that the two manufacturers are symmetrical in all respects and move simultaneously to choose their wholesale prices. The other aspects of the model are the same as before. Because we solve the model in a manner that is similar to the procedure used in the previous section, we do not present all the details.

In the second period, the retailer maximizes profits by choosing the optimal quantity to order and retail price to charge. This yields:

\[
\begin{align*}
q_{i2}^* &= \frac{\tau - (1 + \phi)w_{2i} + \phi w_{2j} - 4I_{2i}}{4}, \\
p_{i2}^* &= \frac{\tau + w_{2i}}{2},
\end{align*}
\]

(10)

The retailer's decisions for Manufacturer \( j \) are symmetrically defined. The two manufacturers maximize their period 2 profits by simultaneously choosing their optimal wholesale prices. This yields:
As in the one manufacturer-one retailer case, each manufacturer's optimal wholesale price declines with the retailer’s inventory levels and this effect gets stronger with the competition between the two manufacturers. Importantly, each manufacturer's wholesale price declines even when the retailer has inventory of the competing manufacturer's product. This occurs because consumers view the two products as being partial substitutes. Thus, forward buying by the retailer has a more negative impact on a manufacturer profit when the manufacturer faces competition from another manufacturer. It is also easy to see that this negative effect becomes stronger as the competition between manufacturers becomes more intense.

The retailer's optimal first period decision for Manufacturer i's products are given below (its decisions for Manufacturer j's products are symmetrically given).

\[
p^*_i = \frac{\tau + w_{i1}}{2}, \quad q^*_i = \frac{\tau}{2} + \frac{-h(2 + \phi)^2 (3 + 4\phi) + \chi_j w_{j1} - \chi_i w_{i1}}{4(3 + 2\phi)(3 + 4\phi)\rho}
\]

where \(\chi_i = (1 + \phi)[12 + 9\rho + \phi(24 + 11\phi + 18\rho + 8\phi\rho)]\)

and \(\chi_j = \phi[8 + 9\rho + \phi(16 + 7\phi + 18\rho + 8\phi\rho)].\)

As in the monopoly manufacturer case, a manufacturer's choice of first-period wholesale price not only affects the retailer's first-period decisions but also its second-period decisions. Furthermore, with forward buying, the retailer will order a higher total quantity than it would if there were no forward buying. However, because of the substitutability between the two products, any increase in one manufacturer's wholesale price will lead the retailer to shift demand toward the competing manufacturer. As a result, the competition between the two manufacturers limits each manufacturer's ability to increase its first period wholesale price even when the retailer engages in forward buying.
The manufacturers’ optimal wholesale prices in the first period are given by

\[ w_{ij}^* = w_{ji}^* = \frac{2\tau(3 + 2\phi)^2(3 + 4\phi)\rho - h(2 + \phi)(6 + 9\phi - 2\phi^3)}{48 + 156\phi + 186\phi^2 + 99\phi^3 + 20\phi^4 + (2 + \phi)(3 + 2\phi)^2(3 + 4\phi)\rho} \] (13)

This leads to the following Proposition:

**Proposition 3:** The retailer will find forward buying from two competing manufacturers optimal when

\[ 0 \leq h < \frac{\tau\rho((2 + \phi)(3 + 2\phi)^2(3 + 4\phi)\rho - 24 - 60\phi - 40\phi^2 - \phi^3 + 4\phi^4)}{(2 + \phi)^2(((2 + \phi)(3 + 2\phi)(3 + 4\phi)\rho + 12 + 36\phi + 35\phi^2 + 11\phi^3)} \]

As in the manufacturer-monopoly case, the retailer finds it optimal to buy more than what it needs in period 1, so that it can get lower wholesale prices in period 2. The main difference here is that forward buying of either manufacturer's product lowers the wholesale price of both manufacturers in the second period. Therefore, forward buying in the first period provides greater second-period benefits to the retailer when there is competition between the manufacturers. Essentially, forward buying of either product allows the retailer to play the manufacturers off each other.

The above discussion also indicates that the manufacturers may have less to gain from the retailer’s forward buying when there is competition between the manufacturers. We can derive the conditions for the manufacturers to be better-off with the retailer's forward buying. However, intractable algebra prevents us from fully characterizing the interaction between the degree of manufacturer competition and forward buying. Therefore, we assume \( \rho = 1 \) for the following proposition.4

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4 We have not been able to discover a value of \( \rho \) for which the Proposition 4 is not true. However, we are not able to prove the proposition analytically for a general value of \( \rho \).
Proposition 4: When $\rho = 1$, the manufacturers' profits are higher with the retailers forward buying when $0 \leq h < h_{m_2}$ where $h_{m_2}$ is as defined in Equation (14). Manufacturers' profits are less likely to increase with forward buying as the competition between them becomes more intense.

$$h_{m_2} = \tau \frac{\phi}{(2 + \phi)} \frac{2(2 + \phi)(306 + \beta_j) - \phi \sqrt{2(102 + \beta_j)(72 + \beta_k)}}{(2 + \phi)^2 (1224 + \beta_j)}$$

(14)

where $\beta_i = \phi[6120 + \phi[12402 + \phi[13134 + \phi[7752 + \phi(2442 + 323\phi)]]]]$,

$\beta_j = \phi[1377 + \phi[2349 + \phi[1839 + \phi[585 - \phi(5 + 28\phi)]]]]$, $\beta_k = \phi[327 + \phi[378 + \phi(191 + 36\phi)]]$ and $\beta_l = \phi[318 + \phi[490 + \phi(310 + 69\phi)]]$.

It is possible for the competing manufacturers to be better-off with the retailer's forward buying. However, the parameter space for which this is true shrinks as the competition between manufacturers becomes more intense, i.e., $\phi$ increases. This is due to two reasons. First, the retailer's inventory from either manufacturer forces both manufacturers to reduce their second period wholesale prices. This effect gets stronger as the competition between manufacturers becomes more intense (see Equation 11). Second, each manufacturer faces a demand curve in the first period that is more sensitive to its first period wholesale price. Therefore, each manufacturer has a more limited ability to increase its first period wholesale price, even though the retailer buys more in that period.

We now examine the effect of retailer competition on the equilibrium outcomes.
3.3 Two Retailers and One Manufacturer

We modify our model to allow for two retailers who sell a product from a single manufacturer. We assume that the two retailers A and B are symmetric and differentiated from each other and their demand functions are given by:

\[
\begin{align*}
\frac{d_{Ai}}{2} &= \left(\tau - p_{Ai} + \theta(p_{Bi} - p_{Ai})\right) \\
\frac{d_{Bi}}{2} &= \left(\tau - p_{Bi} + \theta(p_{Ai} - p_{Bi})\right)
\end{align*}
\]

where the parameter \(\theta\) represents the intensity of competition between the two competitors. As in the case with two manufacturers, note that the demand formulation holds fixed the total demand for the manufacturer’s product. In other words, adding a retailer does not expand the market size compared to the base case in Section 3.1 or the two manufacturer case in Section 3.2. Because both retailers are symmetrical, we assume that the manufacturer cannot discriminate between them and charges them the same wholesale price. The sequence of events is the same as before except that the two retailers make their price and ordering quantity decisions simultaneously. Therefore, we only report important parts of the analysis and delegate the details to the appendix.

The second period optimal price and ordering quantity for Retailer A are as follows (Retailer B’s decisions are symmetrically defined).

\[
\begin{align*}
p^*_{A2} &= \frac{\tau + w_2(1 + \theta)}{2 + \theta}, \\
q^*_{A2} &= \frac{(\tau - w_2)(1 + \theta) - 2(2 + \theta)I_{A1}}{2(2 + \theta)}
\end{align*}
\]

As in the previous two cases, if the retailer carries inventory from the previous period, it buys less in the second period. In addition, holding all else fixed, the retail competition forces each retailer to charge a lower price and order a greater quantity. It is important to note that each
retailer’s optimal ordering quantity is affected only by its own and not its competitor’s inventory.

A more important effect of the retail competition is that each retailer’s price responds more to the wholesale price charged by the manufacturer. More formally, \( \frac{\partial p^*_2}{\partial w_2} = \frac{1 + \theta}{2 + \theta} > 0 \) and

\[
\frac{\partial^2 p^*_2}{\partial w_2 \partial \theta} = \frac{1}{(2 + \theta)^2} > 0.
\]

We know from Section 3.1 that the main benefit of forward buying to the retailer is that it enjoys a reduction in the second period wholesale price. But \( \frac{\partial^2 p^*_2}{\partial w_2 \partial \theta} > 0 \) indicates that with competition, a greater part of any reduction in the second period wholesale price will get passed on to the customers and therefore, the retailer may have less to gain from such reductions in wholesale price.\(^5\)

In order to better understand why retailers may have less to gain from wholesale price reductions, consider how the retailer’s second period profit is affected by changes in the second period wholesale price. In particular, \( \frac{\partial \pi^*_2}{\partial w_2} = \frac{(\tau - w_2)(1 + \theta) + 2I_{A_2}(2 + \theta)^2}{(2 + \theta)^2} < 0 \) and

\[
\frac{\partial^2 \pi^*_2}{\partial w_2 \partial \theta} = \frac{(\tau - w_2)\theta}{(2 + \theta)^3} > 0.
\]

In other words, the retailer’s second period profits increase with a decline in the second period wholesale price, but this change becomes smaller as the competition between the retailers increases.

The manufacturer’s optimal wholesale price in the second period is given by

\[
w^*_2 = \frac{\tau(1 + \theta) - (I_{A_2} + I_{B_2})(2 + \theta)}{2(1 + \theta)}.
\]

Equation (17) shows that the second period wholesale price is reduced when either retailer carries inventory from the previous period. Therefore, even if a single retailer carried inventory

\(^5\) Desai (2000) observes a similar effect in a single period model.
from the first period, the manufacturer reduces the second period wholesale price which ends up benefiting both retailers.

Given the optimal prices and quantities in period 2, we can now determine the retailer A's first period price and ordering quantity decisions (given below). Retailer B's decisions are symmetric.

\[
p_{A1}^* = \frac{\tau(10 + 6\theta + \rho \theta^2) + 2(1 + \theta)[w_i(5 + 2\theta) - \theta h]}{20 + 22\theta + 6\theta^2} \tag{18}
\]

\[
q_{A1}^* = \frac{2(1 + \theta)\rho[5\tau(2 + \theta) + \theta h - w_i(5 + 2\theta)] - \tau \theta^2 \rho^2 - 8(h + w_i)(1 + \theta)(2 + \theta)}{4\rho(2 + \theta)(5 + 3\theta)} \tag{19}
\]

As in the second period, the retail price here becomes more sensitive to changes in the wholesale price as the competition between the retailers becomes more intense. The manufacturer’s first-period wholesale price is given in Table 3.

As Table 3 shows, the retailers may engage in forward buying even in the case of retail competition. In addition, the extent of forward buying is influenced by the intensity of competition. In particular, each retailer forward buys less as the retail competition becomes more intense. This is essentially due to two reasons. First, because of the free riding problem between the two retailers, forward buying by one retailer would result in a lower wholesale price in the second period, which would benefit not only the forward buying retailer but also the other retailer. Thus, forward buying creates a free riding incentive for each retailer. Because the two retailers compete for customers, this free riding hurts even more as the competition becomes more intense. And second, the main benefit of forward buying, i.e., reduced second period wholesale price, becomes less valuable as the competition heats up. Any reduction in the second period wholesale price is more likely to be passed on to the final customers in the form of a retail price reduction, without adding to the retailer's profit margin. As discussed earlier, each retailer's
second period profit is affected less by the second period wholesale price when the competition is more intense.

A corollary of the above result is that there also are conditions under which a retailer would do forward buying in a less competitive situation but would not do so in a more competitive situation. This suggests that competition among retailers can be exacerbated by forward buying. Compared to the case of no forward buying, the presence of forward buying in a highly competitive market leads the retailers to lower retail prices in both periods and earn lower profits.

In the absence of retail competition, when a retailer engages in forward buying, a positive forward buying quantity is sufficient to conclude that the retailer makes more profit with this strategy. However, that is not the case in a competitive situation: a prisoners' dilemma situation may compel the retailers to forward buy even if the strategy leads to an equilibrium that gives both retailers less profits. Our analysis confirms this conjecture and we find that the two retailers may be worse-off with forward buying but still do forward buying for competitive reasons. Although we can identify the conditions under which such prisoners' dilemma situations will arise, intractable algebra prevents us from developing comparative statics with respect to the model parameters. Propositions 5 and 6 summarize the above discussion.

**Proposition 5:** The equilibrium forward buying quantity decreases as the competition between retailers increases.
**Proposition 6:** When $0 < h_{t3}$ both retailers do forward buying. However, when $0 < h_{t3}$ and $0 < h_{r4} < h_{r3} < h_{r5}$, both retailers are worse-off with forward buying compared to the outcome when neither retailer does forward buying.

We conclude this section by noting that as in the previous two cases, the manufacturer in this case can also be better-off with the retailers' forward buying for some values of the parameters. We prove this result in the Appendix.

4. **Summary and Conclusions**

The objective of this paper was to examine the effects of forward buying by retailers on the profits of the retailers and manufacturers. As has been well documented, retailers’ forward buying can have substantial negative effects on manufacturers' production operations and logistics (see for example, Blattberg and Lavin, 1987, Blattberg and Neslin 1990). Our focus is on discovering some new positive effects of retailers’ forward buying on retail and wholesale prices and on profits enjoyed by various channel members.

We develop a two period model in which the first period can be seen as the current period and the second period can be seen as the future. We begin our analysis with a simple one manufacturer-one retailer channel and find that both parties can be better-off by the retailer’s forward buying. Holding all else constant, forward buying in the current period reduces the retailer's requirements in the future period, leading to lower wholesale prices and consequently lower retail prices. Although forward buying results in a higher purchase order from the retailer to the manufacturer, the manufacturer's ability to benefit from it is limited. This is because forward buying results in the manufacturer facing a steeper demand curve in the current period which limits its ability to increase its current wholesale price. These benefits to the retailer are
partly offset by the cost of holding the inventory. Interestingly, the manufacturer may also be better off if the increase in its total sales offset the reduction in wholesale prices.

As noted earlier, manufacturers are increasingly turning to scanbacks that allow them to offer a lower wholesale price only on the units sold during the promotional period. Thus, any units not sold during the promotional period (i.e., they are inventoried) are charged the subsequent higher price. Implicitly, this suggests that forward buying is inherently bad for the manufacturer. Our analysis suggests that a move toward eliminating forward buying need not be optimal. Indeed we find that under specific conditions, the manufacturer is better off with forward buying, because forward buying increases the total number of units sold even when there is no shift in the demand for the product.

An interesting implication of our work is that forward buying plays a heretofore undiscovered role in coordinating the channel. It is well-established that a linear price contract, such as a wholesale price charged by the manufacturer, leads to inefficiencies because of double marginalization (Spengler 1950). The net effect is that quantities are lower and prices are higher at the consumer level. In this paper, we show that under specific conditions, forward buying can improve manufacturer and retailer profits. Essentially, forward buying leads to a decrease in the average wholesale price charged by the manufacturer over the two periods and an increase in the total quantity sold, thus bringing the solution closer to that of an integrated channel. The retailer benefits because the presence of goods in its inventory allow it to get a favorable price from the manufacturer in period 2.

We build further on this insight to study how retail and manufacturer level competition affects the outcomes. We find that when two competing manufacturers are selling to a common retailer, forward buying becomes more likely than in the manufacturer monopoly case. In
addition, the retailer gains more from forward buying and the manufacturers gain less as the
competition between the manufacturers becomes more intense. This is due to the fact the
competition between manufacturers further limits the manufacturers' ability to increase
wholesale prices in the first period in response to higher demand from the retailer. On the other
hand, when two competing retailers purchase from a single manufacturer, forward buying
becomes less likely than in the retailer monopoly case. The reason for this is that a reduction in
the wholesale price is less valuable to retailers facing retail competition. The product market
competition forces them to pass on a part of the second period wholesale price reduction to the
consumers. In addition, the two competing retailers face a free riding problem. Any forward
buying by one retailer reduces the wholesale price for both retailers, thus one retailer can benefit
by letting the other retailer incur the cost of forward buying. We also find that sometimes
retailers do forward buying because they may be in a prisoners' dilemma situation. This result
provides an interesting contrast to the belief that although manufacturers are hurt by retailer's
forward buying, they allow this practice because of competitive pressures. Interestingly, our
results show that depending on parameter values, forward buying can alleviate or exacerbate the
effects of product market competition for retailers as well as manufacturers.

By choice, our model is relatively simple – we want to control for known explanations for
retailers' forward buying and inventory holding. The lack of demand or supply uncertainty, lack
of ordering cost, lack of trade promotion are the aspects that we intentionally avoided to develop
the new insights described in the paper. In order to maintain tractability, we also had to make
some simplifying assumptions. In particular, the choice of linear demand function was made for
this reason. However, Lee and Staelin (1997) have shown that in many channel models, the
nature of strategic interaction (strategic substitutability versus strategic complementarity), rather
than the specific demand function determines the equilibrium outcomes. In spite of using a relatively simple demand function, we were not able to analyze a model in which both retailers and manufacturers faced competition. However, our analysis does provide insights about the individual effect of competition at each level. In other words, although we are unable to say much about the interaction between two types of competition, we are able to describe their main effects. Finally, we acknowledge that having only two periods may seem as a limiting assumption. Dynamic models often have to make this assumption for tractability reasons (see, for example, Hauser, Simester and Wemerefelt 1994). We have also analyzed a more general n-period version of the one manufacturer-one retailer model and have found that depending on the level of holding costs, the retailers will carry inventory in some periods.

There are several interesting avenues for extending our research. One possibility is to examine how trade promotions affect these strategic reasons to do forward buying. Another potential possibility is to allow the manufacturers to charge quantity discounts or quantity premia. This can allow the manufacturers to reward or penalize forward buying as necessary. Another factor that could affect our results and therefore merit further investigation is the high-end or low-end positioning of the retailers and manufacturers. Finally, it would be interesting to extend our reasoning in an empirical setting and directly explore the link between forward buying and the extent of product competition. This strikes us as an important avenue for further research.
Bibliography


Appendix

Proof of Proposition 1

From Table 1, \( q_i^* = \frac{\tau \rho (4 + 9 \rho) - 2h(4 + 5 \rho)}{2 \rho (8 + 9 \rho)} \) and \( d_i^* = \frac{4 \tau + h}{8 + 9 \rho} \). Therefore,

\[
I_1^* = q_i^* - d_i^* = \frac{\tau \rho (9 \rho - 4) - 4h(2 + 3 \rho)}{2 \rho (8 + 9 \rho)},
\]

which is positive if and only if \( h < h_{r_1} = \frac{\tau \rho (4 + 9 \rho)}{8 + 12 \rho} \). \( \square \)

Proof of Proposition 2

The manufacturer’s profit with the retailer’s forward buying is given by

\[
\prod^M = \frac{\tau \rho (-4h + 9 \tau \rho) + 4h^2 (1 + \rho)}{2 \rho (8 + 9 \rho)}
\]

and the manufacturer’s profit without the retailer’s forward buying is given by

\[
\pi^M = \frac{\tau^2 (1 + \rho)}{8}, \quad \prod^M - \pi^M = \frac{\tau^2 [\rho (19 - 9 \rho) - 8] + 16h^2 (1 + \rho) - 16 \tau \rho h}{8 \rho (8 + 9 \rho)} > 0
\]

iff \( h < h_{m1} = \frac{\tau (2 \rho - (1 - \rho) \sqrt{\rho (8 + 9 \rho)})}{4(1 + \rho)} \). \( \square \)

Proof of Proposition 3

From Table 2, the equilibrium level of inventory, \( I_{r1} \) is given by

\[
\tau \rho [\rho (2 + \phi)(3 + 2 \phi)(3 + 4 \phi) + 4 \phi^2 - 24 - 60 \phi - 40 \phi^2 - \phi^3] - h(2 + \phi)[12 + 36 \phi + 35 \phi^2 + 11 \phi^3 + \rho (2 + \phi)(3 + 2 \phi)(3 + 4 \phi)]
\]

\[
4 \rho [48 + 156 \phi + 186 \phi^2 + 99 \phi^3 + 20 \phi^4 + \rho (2 + \phi)(3 + 2 \phi)^2 (3 + 4 \phi)]
\]

which is positive iff \( h < h_{r2} = \frac{\tau \rho [(2 + \phi)(3 + 2 \phi)^2 (3 + 4 \phi) \rho - 24 - 60 \phi - 40 \phi^2 - \phi^3 + 4 \phi^4]}{(2 + \phi)^2 [(2 + \phi)(3 + 2 \phi)(3 + 4 \phi) \rho + 12 + 36 \phi + 35 \phi^2 + 11 \phi^3}] \). \( \square \)

Proof of Proposition 4

Let Manufacturer profits with the retailer’s forward buying be \( \prod^M_i \) and its profits without the retailer’s forward buying be \( \pi^M_i \). It can be shown that the two roots of \( \prod^M_i - \pi^M_i = 0 \) are \( h_{m2} \).
and \( h_{m3} \) where \( h_{m2} = r \frac{2(2 + \phi)(306 + \beta_j) - \phi \sqrt{2(102 + \beta_j)(72 + \beta_k)}}{(2 + \phi)^2(1224 + \beta_i)} \) and

\[
h_{m3} = r \frac{2(2 + \phi)(306 + \beta_j) + \phi \sqrt{2(102 + \beta_j)(72 + \beta_k)}}{(2 + \phi)^2(1224 + \beta_i)}
\]

\[
\beta_i = \phi[6120 + \phi[12402 + \phi[13134 + \phi[7752 + \phi(2442 + 323\phi)]]]]
\]

\[
\beta_j = \phi[1377 + \phi[2349 + \phi[1839 + \phi[585 - \phi(5 + 28\phi)]]]] \quad \beta_i = \phi[327 + \phi[378 + \phi(191 + 36\phi)]] \quad \beta_k = \phi[318 + \phi[490 + \phi(310 + 69\phi)]]
\]

Since \( h_{m2} < h_{m3} \), \( \Pi_i^M - \pi_i^M > 0 \) when \( 0 < h < h_{m2} \) or \( 0 < h_{m3} < h \). However, it can be verified that \( h_{m3} > h_{r2} \). Therefore, for any value of \( h > h_{m3} \), the retailer will not engage in forward buying. Therefore, \( 0 < h_{m3} < h \) is ruled out and \( \Pi_i^M - \pi_i^M > 0 \) is possible only when \( 0 < h < h_{m2} < h_{r2} \). With \( \rho = 1 \),

\[
h_{m2} - h_{r2} = r \left[ \frac{2(2 + \phi)(306 + \beta_j) - \phi \sqrt{2(102 + \beta_j)(72 + \beta_k)}}{(2 + \phi)^2(1224 + \beta_i)} \right] - \frac{(1 + \phi)^2[30 + \phi(51 + 20\phi)]}{(2 + \phi)^2[30 + \phi[81 + \phi(69 + 19\phi)]]}
\]

The expression inside the brackets above is a function of a single parameter \( \phi \). We can verify that this term is negative for any \( \phi > 0 \) so that \( h_{r2} > h_{m2} \). Therefore, the manufacturers are better-off with the retailer's forward buying when \( h < h_{m2} < h_{r2} \). When \( h < h_{r2} < h_{m2} \), the retailer is better-off with forward buying but the manufacturers are not. \( \square \)

Next, we show that the manufacturers’ profits are less likely to increase with the retailer's forward buying as \( \phi \) increases. Note that \( \frac{h_{m2}}{r} = \frac{2(2 + \phi)(306 + \beta_j) - \phi \sqrt{2(102 + \beta_j)(72 + \beta_k)}}{(2 + \phi)^2(1224 + \beta_i)} \) is a function only of a single parameter, \( \phi \). It can be shown that for values of \( 0 < \phi \)
<0.7859; \frac{h_{m2}}{\tau} is positive and decreases with \( \phi \) and that for values of \( 0.7859 < \phi \); \( \frac{h_{m2}}{\tau} \) is negative and the manufacturers are always worse-off with retailers forward buying.

Proof of Proposition 5

From Table 2, Retailer A’s optimal inventory is given by:

\[
I^*_A = \frac{\tau\rho[25\rho + \theta^3(5\rho - 6) + 9\theta(5\rho - 4) - 16 - 26\theta^2(1 - \rho)] - 2h(1 + \theta)(2 + \theta)^2(4 + 5\rho)}{2\rho(2 + \theta)[24 + 25\rho + \theta(28 + 25\rho + \theta(8 + 6\rho))]]
\]

\[
\frac{\partial I^*_A}{\partial \theta} = \frac{1}{2\rho(2 + \theta)[24 + 25\rho + \theta(28 + 25\rho + \theta(8 + 6\rho))]]^2 \{\tau\rho(-8(2 + \theta)^2[14 + \theta(20 + 7\theta)] - 2(2 + \theta)\rho[110 + \theta[225 + \theta(140 + 27\theta)]] + \rho^2[375 + \theta[750 + \theta[585 + \theta(210 + 29\theta)]]] - 2h(2 + \theta)^2(4 + 5\rho)[4(2 + \theta)^2 + \rho[25 + \theta(26 + 7\theta)]}\}.
\]

\[
\frac{\partial I^*_A}{\partial \theta} = -\frac{(4 + 5\rho)[4(2 + \theta)^2 + \rho[25 + \theta(26 + 7\theta)]]}{\rho[24 + 25\rho + \theta(28 + 25\rho + \theta(8 + 6\rho))]]}
\]

is negative for any \( 0 < \rho \leq 1 \) and \( 0 \leq \theta \).

Next we show that the value of \( \frac{\partial I^*_A}{\partial \theta} (h = 0) \) is negative and therefore for any positive value of h,

\[
\frac{\partial I^*_A}{\partial \theta} \quad \text{is negative.}
\]

\[
\frac{\partial I^*_A}{\partial \theta} \mid_{h=0} = \frac{\tau\rho(-8(2 + \theta)^2[14 + \theta(20 + 7\theta)] - 2(2 + \theta)\rho[110 + \theta[225 + \theta(140 + 27\theta)]] + \rho^2[375 + \theta[750 + \theta[585 + \theta(210 + 29\theta)]]]}{2\rho(2 + \theta)[24 + 25\rho + \theta(28 + 25\rho + \theta(8 + 6\rho))]]^2
\]

It is easy to see that the denominator is positive. It is also can be verified that the numerator is negative for values of \( 0 \leq \theta \) and \( \rho_1 < 0 < \rho < 1 \leq \rho_2 \) where

\[
\rho_1 = \frac{1}{375 + \theta[750 + \theta[585 + \theta(210 + 29\theta)]]} \{220 + \theta[560 + \theta[505 + \theta(194 + 27\theta)]] - \sqrt{(2 + \theta)^3[27050 + \theta[83225 + \theta[102360 + \theta[62850 + \theta(19254 + 2353\theta)]]]]}\}. 
\]
Thus, \( \frac{\partial f^*_{41}(h)}{\partial \theta} < 0 \) for any \( h > 0 \) and the forward buying quantity decreases with the level of competition.

\[ \rho_2 = \frac{1}{375 + \theta[750 + \theta(585 + \theta(210 + 29\theta))]} \bigg\{220 + \theta[560 + \theta(505 + \theta(194 + 27\theta))] - \\
\sqrt{(2 + \theta)^3[27050 + \theta[83225 + \theta[102360 + \theta[62850 + \theta(19254 + 2353\theta)]]]]} \bigg\}. \]

Thus, \( \frac{\partial f^*_{41}(h)}{\partial \theta} < 0 \) for any \( h > 0 \) and the forward buying quantity decreases with the level of competition.

**Proof of Proposition 6**

The optimal forward buying quantity is,

\[ f^*_{41} = \frac{\rho[25\rho + \theta^3(5\rho - 6) + 9\theta(5\rho - 4) - 16 - 26\theta^2(1 - \rho)] - 2h(1 + \theta)(2 + \theta)^2(4 + 5\rho)}{2\rho(2 + \theta)[24 + 25\rho + \theta[28 + 25\rho + \theta(8 + 6\rho)]]} , \]

which is positive for

\[ 0 \leq h < h_{r_3} = \rho \tau \frac{\rho[25 + \theta[45 + \theta(26 + 5\theta)]] - 2(1 + \theta)(2 + \theta)(4 + 3\theta)}{2(1 + \theta)(2 + \theta)^2(4 + 5\rho)} . \]

Thus both retailers forward buy for values of \( 0 < h < h_{r_3} \).

Next, we compare the profits of a retailer with and without forward buying. We find that the retailers are better-off with forward buying for values of \( 0 < h < h_{r_4} < h_{r_5} \) or \( 0 < h_{r_4} < h_{r_5} < h \), but the retailers are worse off with forward buying for values of \( 0 < h_{r_4} < h < h_{r_5} \). It can be shown that \( h_{r_5} > h_{r_3} \) and thus the retailers would forward buying only when \( 0 < h < h_{r_3} < h_{r_5} \). For \( 0 < h < h_{r_3} < h_{r_4} \) the retailers are better off with forward buying. However, when \( h_{r_4} < h < h_{r_3} \) and \( 0 < h < h_r \) the retailers are forward buying but are worse-off.
\( h_{r_4} \) and \( h_{r_5} \) are given by;

\[
h_{r_4} = \frac{\rho^r}{2(1 + \theta)[64(2 + \theta)^5 + \gamma_1]} \quad \{ -2\sqrt{\sigma_1[24 + 25 \rho + \theta(28 + 25 \rho + \theta(8 + 6 \rho))] - 3(1 + \theta)(150 + \theta \gamma_2) - 2 \rho^2(2500 + \theta \gamma_3) + \theta^3 \rho^3(5 + 2 \theta)[15 + \theta(18 + 5 \theta)] \} \},
\]

\[
h_{r_5} = \frac{\rho^r}{-2(1 + \theta)[64(2 + \theta)^5 + \gamma_1]} \quad \{ 2\sqrt{\sigma_1[24 + 25 \rho + \theta(28 + 25 \rho + \theta(8 + 6 \rho))]^2 - 3(1 + \theta)(150 + \theta \gamma_2) - 2 \rho^2(2500 + \theta \gamma_3) + \theta^3 \rho^3(5 + 2 \theta)[15 + \theta(18 + 5 \theta)] \} \}
\]

and

\[
\gamma_1 = 16(2 + \theta)^2[81 + \theta[122 + \theta(59 + 8 \theta)] + 4 \rho^2(2 + \theta)[400 + \theta[790 + \theta[554 + \theta(154 + 135 \theta)]] - \theta^2 \rho^3(5 + 2 \theta)[15 + \theta(18 + 5 \theta)]
\]

\[
\gamma_2 = 364 + \theta[361 + \theta[197 + \theta(59 + 7 \theta)]
\]

\[
\gamma_3 = 8100 + \theta[10760 + \theta[7451 + \theta[2790 + \theta(521 + 36 \theta)]],
\]

\[
\sigma_1 = -192(1 + \theta)^3(2 + \theta)^5 + 16 \rho(1 + \theta)(2 + \theta)^5(45 + \theta \gamma_4) + 4 \rho^2(1 + \theta)(2 + \theta)(4 + \theta)(114 + \theta \gamma_5) + \rho^3(400 + \theta \gamma_6) + \theta^2 \rho^4(3 + 2 \theta)(5 + 2 \theta)[15 + \theta(18 + 5 \theta)],
\]

\[
\gamma_4 = 127 + \theta[154 + \theta[86 + \theta(19 + \theta)]
\]

\[
\gamma_5 = 356 + \theta[430 + \theta(227 + 43 \theta)],
\]

\[
\gamma_6 = 1840 + \theta[3909 + \theta[4402 + \theta(2593 + 2 \theta[351 + 5 \theta(4 - \theta)]]]]
\]

It can be shown that \( h_{r_4} < h_{r_4} \) and that \( h_{r_4} < h_{r_5} \). However note that \( h_{r_3} \) can be larger or smaller than \( h_{r_4} \). □
<table>
<thead>
<tr>
<th>Condition</th>
<th>( 0 \leq h &lt; h_{t_1} = \frac{\tau \rho(-4 + 9 \rho)}{8 + 12 \rho} )</th>
<th>( h \geq h_{t_1} \geq 0 )</th>
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<td>( w_1 )</td>
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<tr>
<td>( d_1 )</td>
<td>( \frac{4\tau + h}{8 + 9 \rho} )</td>
<td>( \frac{\tau}{4} )</td>
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<tr>
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<td>( I_1 )</td>
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Table 2: Analysis of two manufacturers and a single retailer channel

<p>| | | |</p>
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<td>$0 \leq h &lt; h_{r_2} = \frac{\rho(2 + \phi)(3 + 2\phi)^2(3 + 4\phi) + 4\phi^4 - \phi^3 - 40\phi^5 - 60\phi - 24}{(2 + \phi)^2[12 + 36\phi + 35\phi^2 + 11\phi^3 + \rho(2 + \phi)(3 + 2\phi)(3 + 4\phi)]}$</td>
<td>$h \geq h_{r_2} \geq 0$</td>
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<td>$W_{li}$</td>
<td>$\frac{2\tau\rho(3 + 2\phi)^2(3 + 4\phi) - h(2 + \phi)(6 + 9\phi - 2\phi^3)}{48 + 156\phi + 186\phi^2 + 99\phi^3 + 20\phi^4 + \rho(2 + \phi)(3 + 2\phi)^2(3 + 4\phi)}$</td>
<td>$\frac{\tau}{2}$</td>
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<tr>
<td>$W_{i2}$</td>
<td>$(2 + \phi){\rho(3 + 2\phi)(3 + 4\phi)[2\tau + h(2 + \phi)] + h(1 + \phi)[12 + \phi(24 + 11\phi)]}$</td>
<td>$\frac{\tau}{2}$</td>
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<td>$W_{i1^6}$</td>
<td>$\frac{(1 + \phi)}{4\rho[48 + 156\phi + 186\phi^2 + 99\phi^3 + 20\phi^4 + \rho(2 + \phi)(3 + 2\phi)^2(3 + 4\phi)]}$</td>
<td>$\frac{\tau(1 + \phi)}{4(2 + \phi)}$</td>
</tr>
<tr>
<td>$q_{i1}$</td>
<td>$\frac{(1 + \phi)(2 + \phi)[2\tau + h(2 + \phi)] + h(1 + \phi)[12 + \phi(24 + 11\phi)]}{4\rho[48 + 156\phi + 186\phi^2 + 99\phi^3 + 20\phi^4 + \rho(2 + \phi)(3 + 2\phi)^2(3 + 4\phi)]}$</td>
<td>$\frac{\tau(1 + \phi)}{4(2 + \phi)}$</td>
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<tr>
<td>$q_{i2}$</td>
<td>$\frac{(1 + \phi)(2 + \phi)(3 + 4\phi)[2\tau + h(2 + \phi)] + h(1 + \phi)[12 + \phi(24 + 11\phi)]}{4\rho[48 + 156\phi + 186\phi^2 + 99\phi^3 + 20\phi^4 + \rho(2 + \phi)(3 + 2\phi)^2(3 + 4\phi)]}$</td>
<td>$\frac{\tau(1 + \phi)}{4(2 + \phi)}$</td>
</tr>
<tr>
<td>$d_{i1}$</td>
<td>$\frac{\tau[48 + 156\phi + 186\phi^2 + 99\phi^3 + 20\phi^4 + \rho(2 + \phi)(3 + 2\phi)^2(3 + 4\phi)]}{4\rho[48 + 156\phi + 186\phi^2 + 99\phi^3 + 20\phi^4 + \rho(2 + \phi)(3 + 2\phi)(3 + 4\phi)]}$</td>
<td>$\frac{\tau(1 + \phi)}{4(2 + \phi)}$</td>
</tr>
<tr>
<td>$d_{i2}$</td>
<td>$\frac{\tau[12 + 36\phi + 35\phi^2 + 11\phi^3 + \rho(2 + \phi)(3 + 2\phi)(3 + 4\phi)]}{4\rho[48 + 156\phi + 186\phi^2 + 99\phi^3 + 20\phi^4 + \rho(2 + \phi)(3 + 2\phi)^2(3 + 4\phi)]}$</td>
<td>$\frac{\tau(1 + \phi)}{4(2 + \phi)}$</td>
</tr>
<tr>
<td>$p_{i1}$</td>
<td>$\frac{\tau}{2} \frac{2\tau\rho(3 + 2\phi)^2(3 + 4\phi) - h(2 + \phi)(6 + 9\phi - 2\phi^3)}{2[48 + 156\phi + 186\phi^2 + 99\phi^3 + 20\phi^4 + \rho(2 + \phi)(3 + 2\phi)^2(3 + 4\phi)]}$</td>
<td>$\frac{\tau(3 + \phi)}{2(2 + \phi)}$</td>
</tr>
<tr>
<td>$p_{i2}$</td>
<td>$\frac{h(2 + \phi)^2(3 + 2\phi)(3 + 4\phi) + \tau[84 + 9(246 + 4(254 + 5(23 + 4\phi)])] + \tau[2(2 + \phi)(3 + 2\phi)^2(3 + 4\phi) + h(1 + \phi)(2 + \phi)[12 + \phi(24 + 11\phi)]}{2\rho[48 + 156\phi + 186\phi^2 + 99\phi^3 + 20\phi^4 + \rho(2 + \phi)(3 + 2\phi)^2(3 + 4\phi)]}$</td>
<td>$\frac{\tau(3 + \phi)}{2(2 + \phi)}$</td>
</tr>
<tr>
<td>$I_{i1}$</td>
<td>$\frac{\tau\rho(2 + \phi)(3 + 2\phi)(3 + 4\phi) + \phi^4 - 24 - 60\phi - 40\phi^2 - \phi^3 - h(2 + \phi)[12 + 36\phi + 35\phi^2 + 11\phi^3 + \rho(2 + \phi)(3 + 2\phi)(3 + 4\phi)]}{4\rho[48 + 156\phi + 186\phi^2 + 99\phi^3 + 20\phi^4 + \rho(2 + \phi)(3 + 2\phi)^2(3 + 4\phi)]}$</td>
<td>0</td>
</tr>
</tbody>
</table>

6 Decisions related to Manufacturer j are defined symmetrically.
Table 3: Analysis of a single manufacturer and two-retailer channel

<table>
<thead>
<tr>
<th></th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{A1}$</td>
<td>$2(1 + \theta)(2 + \theta)^2(4 + 5\rho) + \tau p[2(1 + \theta)^2(10 + 7\theta) + \theta(5 + \theta(5 + \theta))]\rho]$</td>
</tr>
<tr>
<td>$W_{A2}$</td>
<td>$2h(1 + \theta)(2 + \theta)^2(4 + 5\rho) + \tau p[2(1 + \theta)(2 + \theta)(10 + 7\theta) + \theta(5 + \theta(5 + \theta))]\rho]$</td>
</tr>
<tr>
<td>$Q_{A1}$</td>
<td>$\frac{\tau^2(1 + \theta)(4 + 2\theta)(2 + \theta)(25 + 8\rho) - \theta \rho}{8\rho(2 + \theta)(24 + 25\rho + \theta(28 + 25\rho + \theta(8 + 6\rho)))}$</td>
</tr>
<tr>
<td>$Q_{A2}$</td>
<td>$\frac{\tau(1 + \theta)}{4(2 + \theta)}$</td>
</tr>
<tr>
<td>$D_{A1}$</td>
<td>$\frac{\tau(96 + \rho(2104 + \theta)(24 - 5\rho)(6 + \rho) + 2[16 - \rho(2 + \rho)]] + 2h(1 + \theta)(8 + \theta(5 + 2\theta)(4 + \rho))}{8(2 + \theta)(24 + 25\rho + \theta(28 + 25\rho + \theta(8 + 6\rho)))}$</td>
</tr>
<tr>
<td>$D_{A2}$</td>
<td>$\frac{\tau(1 + \theta)}{4(2 + \theta)}$</td>
</tr>
<tr>
<td>$P_{A1}$</td>
<td>$\frac{\tau(96 + 200\rho + \theta(112 + 298\rho + \theta)[32 + \rho(154 + 5\rho + 2\theta(14 + \rho))]) - 2h(1 + \theta)(2 + \theta)(8 + \theta(5 + 2\theta)(4 + \rho))}{4(2 + \theta)(24 + 25\rho + \theta(28 + 25\rho + \theta(8 + 6\rho)))}$</td>
</tr>
<tr>
<td>$P_{A2}$</td>
<td>$\frac{\tau(3 + \theta)}{2(2 + \theta)}$</td>
</tr>
<tr>
<td>$I_{A1}$</td>
<td>$\frac{\tau^2(1 + \theta)(2 + \theta)(2 + \theta)(4 + 5\rho) + \tau p[44 + 25\rho + \theta(50 + 15\rho + \theta(14 + \rho))]}{4(2 + \theta)(24 + 25\rho + \theta(28 + 25\rho + \theta(8 + 6\rho)))}$</td>
</tr>
</tbody>
</table>

7 Retailer B's decisions are defined symmetrically.