The Dynamics of Competitive Drug Detailing

Qiang Liu *
The Johnson School at Cornell University
Job Market Paper
October 2007

Abstract

The pharmaceutical industry spends more funds on drug detailing than on any other marketing instrument. Similar to the effect of advertising, the impact of detailing expenditures spills over beyond the current period. We expect forward-looking firms to observe this carry-over effect and adopt a dynamic approach to determine optimal detailing levels by maximizing their long-term profits. We develop a structural model of dynamic oligopoly competition to analyze firms’ scheduling of detailing over time and to estimate detailing costs consistent with such scheduling. The model is estimated using a recently developed two-stage method. In the first stage, physician-level demand is estimated in a hierarchical Bayesian framework. Further, using a semi-parametric approach, we estimate physician-level policy functions that describe each firm’s observed detailing actions, also in a hierarchical Bayesian framework. In the second stage, costs of detailing are estimated assuming that the observed detailing data are consistent with a Markov perfect Nash equilibrium. Our estimated demand model shows evidence of supersaturation at high levels of detailing stock. The estimated policy functions show that the optimal detailing level decreases in own detailing stock. Further, firms escalate detailing in their rivals’ detailing stock when their rivals have low detailing stock, but cut back effort when their rivals have high detailing stock. Our estimates of detailing costs capture all economically relevant information from firms’ decisions on detailing, including both decisions to detail and decisions not to detail. They are substantially larger than extant industry estimates which are only based on accounting information.

*This document contains the first chapter of my thesis at the Johnson School, Cornell University. I would foremost like to thank my advisor, Sachin Gupta, for his guidance and encouragement. Special thanks are due to Sriram Venkataraman, Vrinda Kadiyali, Vithala R. Rao, and Martin T. Wells for their valuable comments and suggestions. This paper has also benefited from discussions with Patrick Bajari and conference participants at the 2006 INFORMS Marketing Science conference. All remaining errors are mine. Correspondence: Qiang Liu, 401 Sage Hall, Johnson Graduate School of Management, Cornell University, Ithaca, New York 14853. Email: QL32@cornell.edu.
1 Introduction

Although the pharmaceutical industry is mainly driven by innovation, it spends an enormous amount of funds on marketing. Indeed, for the top 10 pharmaceutical companies, selling, general, and administrative expenses ranks above research and development spend as the largest single expense.\(^1\) Among various marketing resources expended, detailing - personal selling through representatives - together with related sample distribution is the single largest expenditure. In 2003, the U.S. pharmaceutical industry spent $7.13 billion on detailing, together with $13.04 billion on samples, which is far more than expenditures on other marketing instruments, e.g., $3.23 billion on direct-to-consumer advertising and $0.48 billion on medical journal advertising (IMS Health 2004).

Previous studies have shown a positive effect of detailing on physicians’ prescription behavior. With a comprehensive demand specification, we confirm the positive effect of detailing while allowing for a supersaturation effect.\(^2\) Similar to the effect of advertising, the effect of drug detailing can last for more than one period. This carry-over effect implies that the decision of how much to detail needs to be studied using a dynamic model rather than a static model. Even though researchers have recognized this carry-over effect, previous studies on firms’ detailing behavior have mainly assumed that firms are myopic decision makers in that they only optimize their current profits. In contrast, a forward-looking firm can observe this carry-over effect of detailing and take it into account in making optimal detailing decisions. Based on this assumption and the understanding that competing firms almost invariably detail to the same physicians, we specify a dynamic structural model of oligopoly competition at the individual physician level to explain the scheduling of detailing by competing drugs. Furthermore, we estimate the detailing cost structure consistent with such scheduling.

In marketing and economics, the computational and analytical difficulty of incorporating

---


\(^2\)Supersaturation occurs if excessive marketing effort causes a negative response. An excessive number of visits by sales persons may have a negative effect on sales (see Leeflang et al. 2001).
information from a dynamic equilibrium into a structural parameter estimation algorithm has prohibited researchers from estimating dynamic structural models of decision-making and equilibrium.\textsuperscript{3} The development of two-stage estimation approaches for dynamic models has eased the computational burden substantially and opened the door to more realistic industry competition models. We estimate our proposed model using such an approach, in which it is possible to estimate the dynamic structural model without explicitly solving for the equilibrium. In the first stage, physician-level demand is estimated in a hierarchical Bayesian framework. Further, using a semi-parametric approach, we estimate physician-level policy functions that describe each firm’s observed detailing actions, also in a hierarchical Bayesian framework. In the second stage, the supply side parameters - costs of detailing - are estimated assuming that observed detailing decisions are consistent with a Markov perfect Nash equilibrium (MPNE). Our estimates of detailing costs capture all economically relevant information from firms’ decisions on detailing. They are substantially larger than extant industry estimates which are only based on accounting information. With the estimated physician-level detailing costs, we study the implication of heterogeneity in detailing costs for the quantification of the predicted benefits from targeted detailing.

The rest of this paper is organized as follows: In the next section, we present a brief review of the extant literature. We describe the data in section 3. We specify the competitive detailing model in section 4 and discuss the estimation strategy in section 5. Empirical results are presented and discussed in section 6. We conclude and present a discussion of the limitations of our study and directions for future research in section 7.

2 Literature Review

There is a vast literature on the effect of drug detailing on sales in medical science, economics and marketing. In an extensive and integrative review of detailing studies, Manchanda and

\textsuperscript{3}For example, the computational burden of solving Markov perfect equilibrium makes the nested fixed-point estimator of Rust (1987) impractical for empirical work of dynamic models.
Honka (2005) summarize that detailing has a strong and lasting positive effect on physicians’ prescription behavior. In particular, marketing scholars have detected such positive effects using disaggregate market data (Gönül et al. 2001, Mizik and Jacobson 2004, Manchanda et al. 2004a), though the small magnitudes of the effects in their studies indicate that physicians are “tough sells” (Mizik and Jacobson 2004). Diminishing return to detailing and supersaturation effect of detailing have been reported by Gönül et al. (2001) with a homogeneous response model as well. Manchanda et al. (2004a) find a negative effect of competitive detailing which is about one-fifth that of own detailing. Recent studies find significant heterogeneity in physicians’ response to detailing (Manchanda et al. 2004b, Narayanan and Manchanda 2006).

Marketing scholars have also investigated firms’ detailing decision process. Manchanda et al. (2004b) estimate a negative binomial detailing response model and account for the possible endogeneity of detailing by simultaneously specifying a model of detailing level to each physician as a function of the physician’s responsiveness to detailing. Using Bayesian methods, Dong et al. (2006) estimate a prescription response model and a strategic profit-maximizing detailing equation at the physician level. The response model shows that detailing has carry-over effects. However, for computational tractability, firms are assumed to be myopic.

Because few studies on the dynamics of detailing have been conducted, and detailing is akin to advertising and other marketing communication instruments, we review a broader field, the dynamics of marketing communication. The effects of marketing communication have been characterized as persuasive effects and informative effects (Ackerberg 2003, Narayanan et al. 2005). Either persuasive effects or informative effects may spill over to more than one period and therefore introduce dynamics to the study of marketing communication competition. Erickson (1995) and Erickson (2002) study the dynamics of competitive advertising using differential games in which analytic solutions of the equilibrium advertising strategies are obtained. Dubé and Manchanda (2005) investigate differences in the dynamics
of advertising across geographic markets empirically using a linear-quadratic game involving forward-looking competing firms, which also yields an analytical solution. Analytic solutions of the equilibrium in those games, however, can only be obtained for demand systems that are very restrictive for empirical applications. With a more flexible game specification, Dubé et al. (2005) empirically investigate how competing firms should optimally allocate their advertising over time, where the advertising effect is modeled by an augmented goodwill stock function and the equilibrium concept is Markov perfect. Doganoglu and Klapper (2006) study a similar problem but with a different goodwill production function and without solving for the MPNE. Using the Kalman filter, Sriram and Kalwani (2007) study the optimal levels of advertising and promotion budgets in dynamic markets with brand equity as a mediating variable.

To investigate the effect of detailing on physicians’ prescription writing behavior, our study uses a flexible hierarchical probit model on the demand side. The model allows detailing to exhibit both a diminishing marginal effect and a supersaturation effect. On the supply side, our study relates to Dong et al. (2006) in the sense that both studies investigate firms’ strategic behavior at the physician level using a structural modeling approach and estimate structural parameters, namely marginal costs of detailing. Both studies also address the targeted detailing issue with estimated drug demand and detailing costs. Our study complements Dong et al. (2006) in that our study is based on what we believe is a more appealing assumption: forward-looking firms recognize that the effect of detailing persists and take this carry-over effect into consideration in their detailing decisions. Based on this assumption, we adopt a dynamic oligopoly competition model to analyze the scheduling of detailing. In addition, our study assumes that competing firms possess private information about their costs of detailing and allows for more flexibility in detailing policy. We believe that our flexible yet realistic model specifications enable us to better understand the true nature of competition among pharmaceutical firms.

To the best of our knowledge, this is the first study of the dynamics of competitive
detailing. Compared with previous empirical studies of the dynamics of marketing communication, our work is different in the following ways: First, most previous studies do not estimate supply side structural parameters (typically, cost parameters). For instance, Dubé et al. (2005) estimate a demand model first and then plug the estimated demand parameters into a supply structure where a dynamic programming problem is solved assuming exogenous cost information from industry reports. Cost estimates in industry reports, however, often only represent accounting information and do not convey all economically relevant information that determines agents’ behaviors. Similarly, Sriram and Kalwani (2007) focuses on estimating a flexible demand model and looking for optimal strategy with given cost information. Unlike previous studies, we estimate detailing costs from observed firms strategic behaviors and capture all economically relevant information that implied by firms’ decisions. Second, the demand system and detailing equation in our study are both at the individual physician level, whereas Dubé et al. (2005) and Doganoglu and Klapper (2006) model demand and firm decisions only at the market level.

3 Data

The data used for the empirical analysis in this study are from a group of prescription drugs known as statins (or HMG-CoA reductase inhibitors). Statins are used to lower cholesterol levels in people at risk for cardiovascular disease because of hypercholesterolemia. These drugs work in patients’ livers to block a substance needed to make cholesterol, and also to help the body reabsorb cholesterol that has accumulated as plaque on their arterial walls. Statins sales surpassed $15.5 billion in 2004, making them the biggest-selling drugs in the United States.4 Our data contain four major statins: Lipitor produced by Pfizer, Zocor by Merck, Pravachol by Bristol-Myers Squibb (BMS), and Crestor by AstraZeneca. In addition to these four drugs, our data include prescriptions for “non-drug treatment” that are commonly written by physicians. Non-drug treatment methods include eating healthy,

Table 1: Frequency distribution of physician-month level detailing visits

<table>
<thead>
<tr>
<th>Drug</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>≥4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lipitor</td>
<td>4641</td>
<td>1269</td>
<td>372</td>
<td>110</td>
<td>40</td>
</tr>
<tr>
<td>Zocor</td>
<td>4347</td>
<td>1306</td>
<td>506</td>
<td>180</td>
<td>93</td>
</tr>
<tr>
<td>Pravachol</td>
<td>5079</td>
<td>961</td>
<td>288</td>
<td>80</td>
<td>24</td>
</tr>
<tr>
<td>Crestor</td>
<td>1406</td>
<td>681</td>
<td>339</td>
<td>153</td>
<td>101</td>
</tr>
</tbody>
</table>

1 Detailing data of Crestor are available for only 10 months.

Table 2: New written prescriptions per physician-month

<table>
<thead>
<tr>
<th>Drug</th>
<th>Mean</th>
<th>Variance</th>
<th>Share(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lipitor</td>
<td>0.5673</td>
<td>1.1325</td>
<td>32.21</td>
</tr>
<tr>
<td>Zocor</td>
<td>0.3209</td>
<td>0.7071</td>
<td>18.40</td>
</tr>
<tr>
<td>Pravachol</td>
<td>0.2337</td>
<td>0.6826</td>
<td>13.40</td>
</tr>
<tr>
<td>Crestor</td>
<td>0.4384</td>
<td>1.4012</td>
<td>22.30</td>
</tr>
<tr>
<td>Non-drug Treatment</td>
<td>0.4649</td>
<td>1.0689</td>
<td>26.51</td>
</tr>
</tbody>
</table>

1 We report average monthly market share. For Crestor, the average is over 10 months.

quitting smoking, increasing physical activity, moderating alcohol intake and maintaining an ideal body weight.

The data are collected by a market research firm from a panel of 2,768 physicians in the United States. The panel consists of a representative sample of the universe of physicians, balanced across geographic regions, physician specialties, and prescription volumes. Each physician reports the number of detailing calls received and new prescriptions written for each drug. We have access to monthly data for the period June 2002 to May 2004. Since Crestor was introduced in August 2003, we only observe details and prescriptions of Crestor for 10 months.

In our study, we randomly sample 268 physicians from the panel. We report descriptive statistics for the number of details and prescriptions in Table 1 and Table 2 respectively. Table 1 shows that, adjusting for its late entry into the market, Crestor has the largest number of detailing visits, followed by Zocor, Lipitor, and Pravachol. Table 2 shows that Lipitor is the market leader among the four drugs. Despite its newness, Crestor has the second-largest market share.
4 Model

In this section, we present the details of our modeling framework. We start with the demand specification based on a model of prescriptions written by physicians. In the model of prescriptions, detailing increases the value of a drug via a detailing stock which depreciates over time but is replenished by further detailing visits. On the supply side, we consider J pharmaceutical firms, each of which produces one statin drug. The J firms employ detailing as an instrument to compete for a share of each physician’s prescriptions over an infinite time horizon.\(^\text{5}\) For each firm, all economically relevant conditions in month t at physician p are summarized by a vector of state variables, \(s_{pt} \in \mathcal{S}\). Given the state \(s_{pt}\), firms behave strategically and make detailing decisions simultaneously in each period. Equilibrium is reached when each firm maximizes its expected present discounted value, given expectations about the evolution of competition. We assume that both demand and competition are independent across physicians.

4.1 Model of Prescriptions

Prescription variable \(y_{pjt} = 1\) if physician p chooses prescription alternative j; \(y_{pjt} = 0\) otherwise. To model physicians’ prescriptions, we assume that physicians write prescriptions exclusively to maximize the utility of their patients based on their professional judgments. Physicians’ preferences for maximizing patients’ utility is due to a sense of professional integrity and obligation, a desire to maintain their reputation, and a fear of malpractice suits. Specifically, when a patient visits physician p at time t, the physician chooses the prescription alternative j (i.e., \(y_{pjt} = 1\)) that provides the greatest utility for her patient, with the latent utility defined as

\[
U_{pjt} = \alpha_{pj} + \beta_{p}g_{pjt} - 1(g_{pjt} > \gamma_{p})[\delta_{p} \cdot (g_{pjt} - \gamma_{p})^2] + \varepsilon_{pjt} \quad (1)
\]

\[\varepsilon_{pt} \sim N(\bar{\varepsilon}, \Sigma) \quad p = 1, \cdots P, t = 1, \cdots, T\]

\(^\text{5}\)The role of price in physicians’ prescription choices is discussed subsequently.
\[
\theta_p = (\alpha_{p1}, \ldots, \alpha_{pJ}, \beta_p, \gamma_p, \delta_p, g_{p11}, g_{p21}, g_{p31})' \sim MVN(\theta, \Sigma_\theta)
\] (2)

where \(\alpha_{pj}\) is physician \(p\)'s intrinsic utility to prescribe drug \(j\), and this intrinsic utility is related to physician \(p\)'s propensity to favor a certain set of drug features (e.g., drug efficacy, lack of side effects) and to the particular characteristics of her pool of patients; \(g_{pjt}\) is the detailing stock built by firm \(j\) at physician \(p\) through detailing; since we do not observe detailing visits before June 2002, the detailing stock of the three existing drugs in the initial period are estimated as physician-level parameters. We also include in the model an “outside good” that is defined as a prescription for non-drug treatment. Let the utility of the outside option \(U_{p0t}\) be given as \(\varepsilon_{p0t}\). \(\varepsilon_{pt}\) is a physician and time-specific i.i.d. normally distributed random shock vector with mean vector \(\vec{0}\) and variance-covariance matrix \(\Sigma\). With these assumptions, the demand function is specified as a multinomial probit model formulated at the individual physician level.

The impact of detailing stock on physician \(p\)'s prescriptions is determined by three parameters: main effect \((\beta_p)\), latent threshold \((\gamma_p)\), and penalty effect \((\delta_p)\). When the detailing stock at physician \(p\) is below her latent threshold, the main effect, which is linear, captures all the impact of detailing stock on her utility. When the detailing stock exceeds physician \(p\)'s threshold, the impact of detailing stock on her utility includes both the main effect and a penalty effect. The penalty effect captures diminishing marginal effects of detailing and possible super-saturation effects of detailing. An attractive feature of this specification is that it nests two other response models. When \(\gamma_p = 0\), the resulting response model includes a linear term (main effect) and a quadratic term (penalty effect). When \(\gamma_p \rightarrow \infty\), the resulting response model includes only a linear term (main effect).

An often used modeling alternative is to specify a logarithmic transformation of detailing stock. However, there are some disadvantages to such a specification. First, the logarithmic transformation imposes global concavity, which is potentially restrictive. Dubé et al. (2005) have shown that a globally concave specification of marketing communication ef-
fect in the demand function is not consistent with often-observed pulsing strategies. Our threshold penalty specification does not introduce unnecessary concavity. Second, unlike our proposed threshold penalty specification, a logarithmic transformation cannot capture the super-saturation effect.

We assume that drug price does not affect physicians’ prescriptions for the following reasons: First, much of the drug cost is not paid by patients because over 98% patients in our data are covered by insurance.\(^6\) Second, according to the empirical findings of Gönül et al. (2001), physicians have limited price sensitivity. Finally, survey studies like Reichert et al. (2000) have shown that physicians lack accurate knowledge about actual patient costs and insurance coverage of drugs.

Previous studies have shown that detailing visits by pharmaceutical sales representatives influence physicians’ prescription decisions not only in the current period but also in future periods. We model this carry-over effect of detailing on demand via a detailing stock that results from the current detailing and depreciated detailing stock from the previous period. Following the standard Nerlove-Arrow form (Nerlove and Arrow 1965), we assume an exponential decay process to the detailing-stock building function as follows:

\[
g_{pjt} = d_{pjt} + \lambda_j g_{pjt-1} \quad 0 < \lambda_j < 1
\]  

where \(d_{pjt}\) is the number of detailing visits by firm \(j\) to physician \(p\) at time \(t\), \(\lambda_j\) is a firm-specific retention rate that captures the detailing stock carried over from the previous period.

Given the model of prescription choices, we can define the demand for drug \(j\) (i.e., number of new prescriptions written) generated by physician \(p\) at time \(t\), \(Q_{pjt}\), as follows:

\[
Q_{pjt} = M_{pt} s_{pjt}
\]

\(^6\)29\% of patients in our data are covered by Medicare. Even though Medicare did not cover drug expenditures before 2006, more than 90\% of Medicare beneficiaries have supplemental insurance coverage. Source: MedPAC analysis of 2000 Medicare Current Beneficiary Survey, Cost and Use file.
where $M_{pt}$ is the expected number of patients with high cholesterol who visit physician $p$ at time $t$, i.e., the size of the market firms face at physician $p$. $S_{pjt}$ is the market share of firm $j$ with physician $p$ at time $t$, which is the probability of physician $p$ prescribing brand $j$ at time $t$. This probability is given by the probit model,

$$S_{pjt} = \int 1(U_{pjt} > U_{pkt} \forall k \neq j)\varphi(\varepsilon_{pt})d\varepsilon_{pt} \quad (5)$$

### 4.2 Model of Detailing Decisions

On the supply side, firms make detailing decisions with respect to each physician by maximizing their long-term profit from that physician. In month $t$, firm $j$’s profit from physician $p$ is given as

$$\pi_{pj}(d_{pt}, s_{pt}, \nu_{pjt}, dmc_{pj}, \varsigma_{pj}) = Q_{pjt}(d_{pt}, s_{pt}) \ margin_j - (dmc_{pj} - \nu_{pjt}) \ d_{pjt} \quad (6)$$

where $\text{margin}_j$ is the margin of drug $j$, $dmc_{pj}$ is firm $j$’s constant marginal cost of detailing to physician $p$, and $s_{pt}$ is a vector of state variables which summarizes firms’ common economic environment. Specifically $s_{pt}$ includes the four firms’ detailing stock with physician $p$, as well as exogenous state variable vector $\Gamma_t$. That is, $s_{pt} = (\Gamma_t, g_{p1t-1}, \cdots, g_{pJt-1})$. We use two exogenous state variables - Zetia introduction dummy and Crestor introduction dummy. Both drugs entered the market during the period that our data are observed and therefore might change the economic environment of the other three drugs. Zetia is a non-statin cholesterol reduction drug that is used alone or along with a statin. Given our focus on statins, we do not include Zetia in the set of competing drugs that are modeled. However, its entry may have influence on the drugs modeled.

$\nu_{pjt}$ is a private shock that is drawn independently over time from a normal distribution with mean 0 and standard deviation $\varsigma_{pj}$. Notice that with our specification in equation (6), a non-zero mean of $\nu_{pjt}$ is not identified due to the presence of $dmc_{pj}$. Although firm $j$
itself can observe $\nu_{pjt}$ before choosing its action, competitors and researchers cannot observe the realization of this shock, but only have knowledge of the distribution of $\nu_{pjt}$. Here, the private shock mainly accounts for variations in the marginal cost of detailing that are not captured by $dmc_{pj}$. For instance, after a successful presentation to a physician, it may take less effort for a sales representative to arrange the next meeting with that physician in the future. A higher value of $\nu_{pjt}$ implies a lower total marginal cost of detailing, leading to higher profits.

We model the J firms’ competition in detailing to physician p as an infinite horizon game played each month. At the beginning of each month, firms observe the state variables $s_{pt}$ and realizations of their private shocks, and then make simultaneous detailing decisions. Firm j’s optimal detailing decision is made by maximizing its expected discounted profits over the entire horizon.

$$\max_{d_{pj}} E\left[ \sum_{\tau=t}^{\infty} r^{\tau-t} \pi_{pj}(d_{pt}, s_{pt}, \nu_{pjt}) \mid s_{pt} \right]$$

(7)

where $r$ is the monthly discount rate assumed to be common across firms.

4.3 Equilibrium

To analyze the equilibrium, we follow the standard practice in the dynamic oligopoly literature and focus only on MPNE. In a MPNE, each firm’s behavior only depends on the current state and its realized private shock. As demand and competition are both assumed to be independent across physicians, we drop the physician subscript to simplify notation in this subsection. A Markov strategy for firm j is a mapping from state and private shock spaces into action space $\sigma_j : S \times V_j \rightarrow D_j$ and a strategy profile is a vector $\sigma = (\sigma_1, \ldots, \sigma_J)$. Here, $s_t \in S$ and $\nu_{jt} \in V_j$. If behavior is given by a Markov strategy profile $\sigma$, firm j’s expected long term profits given a state $s$ (before the realization of its private shock) can be written
in recursive form as:

\[
V_j(s|\sigma) = E_\nu[\pi_j(\sigma(s, \nu), s, \nu_j) + r \int V_j(s'|\sigma) dP(s'|\sigma(s, \nu), s)|s]
\]

(8)

Here, \(V_j\) is the expected profits at the beginning of a period before the realization of private shocks. \(P(s'|\sigma(s, \nu), s)\) is a transition probability function of state variables. It governs the evolution of state variables from time \(t\) to \(t+1\) because of firms’ actions at time \(t\). In our model, the detailing-stock building function in (3) determines the change of state variables and plays the role of a deterministic state transition function.

For a strategy profile \(\sigma\) to be a MPNE, each firm \(j\) must prefer strategy \(\sigma_j\) to all other Markov strategies \(\sigma_j'\), given its opponents’ strategy profile \(\sigma_{-j}\). Specifically, \(\sigma\) is a MPNE if

\[
V_j(s|\sigma_j, \sigma_{-j}) \geq V_j(s|\sigma_j', \sigma_{-j})
\]

\[
= E_\nu[\pi_j(\sigma_j'(s, \nu_j), \sigma_{-j}(s, \nu_{-j}), s, \nu_j) + r \int V_j(s'|\sigma_j', \sigma_{-j}) dP(s'|\sigma_j', \sigma_{-j}, s)|s]
\]

(9)

for all \(j, s\), and any alternative Markov strategy \(\sigma'\).

We can verify that profit function 6 has the property of increasing differences (or supermodularity), i.e., if \(d'_j \geq d_j\) and \(\nu'_j \geq \nu_j\),

\[
\pi_j(d'_j, \nu'_j, s_j) - \pi_j(d_j, \nu'_j, s_j) \geq \pi_j(d'_j, \nu_j, s_j) - \pi_j(d_j, \nu_j, s_j)
\]

(10)

This property means that the incremental gain to choosing a higher level of detailing (i.e., \(d'_j\) rather than \(d_j\)) is greater when \(\nu_j\) is higher. By Topkis’ Theorem (Topkis 1998), the increasing differences mean that firms’ optimal detailing policy \(\sigma(s, \nu_j)\) (i.e., a MPNE for detailing decision) will be increasing or weakly increasing in \(\nu_j\). This monotonicity of policy function in private shocks facilitates estimation of the policy function.
5 Estimation Strategy

The goal of this paper is to empirically estimate detailing costs as well as demand functions and detailing policy functions in the statins market. To achieve this goal, we follow the two-stage estimation procedure laid out in Bajari et al. (2007) and Bajari and Hong (2005). The following subsections describe the estimation strategy in detail.

5.1 Overview

Two-stage estimation for dynamic models stemmed from work by Hotz and Miller (1993) and Hotz et al. (1994), which proposes this approach for a single-agent, discrete choice dynamic problem. Since then, two-stage estimation has been extended to multi-agent discrete choice and continuous action dynamic problems in recent work by Aguirregabiria and Mira (2007), Pakes et al. (2007), and Bajari et al. (2007). The basic idea of two-stage estimation is described by Bajari et al. (2007) as follows: First, in an equilibrium model, firms are assumed to have correct beliefs about their economic environment and the actions of other agents. As a result, we can empirically recover the firms’ equilibrium beliefs by estimating the probability distribution for their observed actions. Second, in an equilibrium model, firms are assumed to maximize expected discounted profits given their beliefs. The conditions for optimality can be represented as a system of inequalities that require each firm’s observed decisions at each state be weakly preferred to any feasible alternatives. The dynamic model’s structural parameters are finally estimated as the solution to this system of inequalities.

Following this approach, in the first stage we estimate firms’ demand functions, policy functions governing firms’ detailing behavior, and state transition functions at the individual physician level. In the second stage, we estimate the marginal cost of detailing ($dmc_{pj}$) and standard deviation of the private shock distribution ($\varsigma_{pj}$) by simulating the behavior of firms, given the first stage estimates and imposing the equilibrium conditions of the MPNE embodied in equation (9).
5.2 First Stage Estimation

Demand & State Transitions  It would be ideal to be able to estimate demand functions for each physician separately. However, this approach is not feasible because we only have a limited number of observations for each physician (maximum of 24 months). Fortunately, the Bayesian hierarchical framework offers a natural way to pool data across physicians and yet get usable physician-specific demand functions as well as overall demand functions while controlling for heterogeneity. Here, the dependent variable is a physician’s prescription decisions \( \{y_{pt}\} \) and the proposed demand model can be written in a hierarchical form as follows:

\[
U_{pt} | y_{pt}, \theta_p, \lambda, \{d_{pt}\}, \Sigma, \tau = 1, \ldots, t \quad \text{Latent utility}
\]

\[
\theta_p | \theta, \Sigma_\theta \quad \text{Heterogeneity}
\]

\[
\lambda | \{U_{pt}\}, \{\theta_p\}, \{d_{pt}\}, \Sigma, t = 1, \ldots, T, p = 1, \ldots, P \quad \text{Detailing-stock retention}
\]

For simplification, we fix \( \Sigma \), the variance-covariance matrix of latent utility, as an identity matrix. This simplification helps us avoid an identification problem in the probit model but introduces an IIA-like restriction at physician level. Bayesian inference for the model parameters involves drawing from a joint posterior distribution of all parameters. However, drawing from a joint posterior distribution for a probit model is problematic because the likelihood function does not have a closed form. Using the data augmentation technique proposed by Tanner and Wong (1987), we treat unknown utilities \( U_{pt} \) as parameters and use Gibbs sampler to obtain draws for them from their own full conditional distributions. Bayesian estimation is carried out by a Markov chain Monte Carlo (MCMC) procedure combining the Gibbs sampler and random walk Metropolis algorithm. The transitions of state variables are known once we have estimates of detailing retention rate from the demand estimation.
**Detailing Policy**  
Function estimation aims to empirically recover the optimal strategies, \( \sigma_j : S \times V_j \rightarrow D_j \) from the observed data. In practice, policy function estimation involves regressing the observed actions on all state variables and private shocks. Estimation of policy functions is challenging because these functions are equilibrium outcomes and a particular parametric form assumed for the policy functions would be unlikely consistent with the primitives of the underlying dynamic game. In other words, a flexible estimation approach is preferred where the functional relationship between \((d_{ptj})\) and \((s_{pt}, \nu_{ptj})\) is mainly determined by the observed data.

To estimate firms’ detailing actions, we adopt an ordered probit model to approximate the distribution of detailing actions at a state. The ordered probit model is a natural structure to model ordered discrete choice data (like detailing actions in this study). With the ordered probit model, we get the distribution of detailing decisions at each state. Recalling that the optimal detailing policy function is non-decreasing in private shocks, we can therefore estimate policy functions by mapping the quantiles of \(\nu\) into detailing levels given by the ordered probit model at each state.

To estimate detailing policy functions as flexibly as possible, we incorporate a Bayesian penalized splines (P-splines) model in the ordered probit regression model. This estimator provides a flexible and easily implemented semi-parametric regression methodology for fitting complex problems involving nonlinear or irregular relationships. Because P-spline can be viewed as the best linear unbiased predictor (BLUP) in a mixed model framework, the procedure is attractive in terms of theory and practice. The estimation of mixed model in Bayesian framework is straightforward because Bayesian framework treats all parameters as random variables and provides exact inference for them by a pooling machinery. This makes Bayesian analysis of P-spline especially attractive due to the equivalence between P-spline and mixed model. Ruppert et al. (2003) provide a comprehensive discussion of P-splines models and their estimation.

Because the data for each physician are very limited (maximum of 24 months), it is not
feasible to estimate a policy function for each physician separately. On the other hand, pooling data across all physicians to estimate a single overall policy function is problematic due to heterogeneity across physicians. Heterogeneity in physicians’ responsiveness to detailing, size of physicians’ patient base, etc. creates different economic environments for firms at different physicians. Optimal firms, therefore, must tailor their policies to the physician-specific economic environment. Taking advantage of the powerful shrinkage ability of hierarchical Bayesian framework, we pool data across physicians and then estimate physician-specific policy functions as well as an overall policy function.

Specifically, we define firm j’s latent value of detailing to physician p at time t \((W_{pjt})\) as

\[
W_{pjt} = f(g_{pjt}, \text{sumg}_{p-jt}, \vartheta) + f_p(g_{pjt}, \text{sumg}_{p-jt}, \vartheta) + \zeta_p \Gamma_t + \epsilon_{pjt}
\]  

\[(11)\]

\[
\epsilon_{pjt} \sim N(0, 1) \quad p = 1, \ldots, P, t = 1, \ldots, T, \quad t' = t - 1
\]

\[
d_{pjt} = i \quad \text{if} \quad \tau_i \leq V_{pjt} < \tau_{i+1} \quad \quad i = 0, 1, 2, 3, 4
\]

where \(\Gamma_t\) is a vector of common exogenous covariates; \(\zeta_p\) is a physician-specific parameter vector associated with \(\Gamma_t\). \(\epsilon_{pjt}\) is normally distributed across p and t for each j. \(\tau_i\) is a latent “cut point” and satisfies \(\tau_i < \tau_{i+1}\), \(\tau_0 = -\infty\), \(\tau_5 = \infty\), and we fix \(\tau_1 = 0\) for identification. \(f(\cdot)\) is a flexible overall function of firm j’s own detailing stock \((g_{pjt})\) and the sum of its competitors’ detailing stock \((\text{sumg}_{p-jt})\); \(f_p(\cdot)\) is the deviation of firm j’s policy curve at physician p from j’s overall detailing policy curve. Both \(f(\cdot)\) and \(f_p(\cdot)\) are modeled as low-rank thin-plate splines as follows.\(^7\)

\[
f(\cdot, \vartheta) = \eta_0 + \eta_1 g_{pjt} + \eta_2 \text{sumg}_{p-jt} + \sum_{k=1}^{K} \mu_{1k} |g_{pjt} - \kappa_k|^3 + \sum_{k=1}^{K} \mu_{2k} |	ext{sumg}_{p-jt} - 
\]

\[
f_p(\cdot, \vartheta_p) = \eta_{p0} + \eta_{p1} g_{pjt} + \eta_{p2} \text{sumg}_{p-jt} + \sum_{k=1}^{K'} \mu_{p1k} |g_{pjt} - \kappa_{pk}|^3
\]

\(^7\)Other spline basis functions, e.g. truncated polynomials, cubic splines, and B-splines, can also be used in P-spline. Here, we focus on low-rank thin-plate splines because they tend to have good numerical properties, which is important for the mixing properties of the MCMC chains.
\[ + \sum_{k=1}^{K'} \mu_{p2k} |\text{sum}_{g_{p-j't'}} - \eta_{pk}|^3 \]  

where \( \vartheta = (\eta_0, \eta_1, \mu_{11}, \ldots, \mu_{1K}, \mu_{21}, \ldots, \mu_{2K})' \); \( \vartheta_p = (\eta_{p0}, \eta_{p1}, \mu_{p11}, \ldots, \mu_{p1K'}, \mu_{p21}, \ldots, \mu_{p2K'})' \); \( \kappa_1 < \kappa_2 < \cdots < \kappa_K \) are overall fixed knots for variable \( g_{p-j't'} \) and \( \iota_1 < \iota_2 < \cdots < \iota_K \) are overall fixed knots for variable \( \text{sum}_{g_{p-j't'}} \); \( \kappa_{pk} \) and \( \iota_{pk} \) are physician-level counterparts defined in the same way. To avoid over-fitting in the estimation, for the overall curve, we minimize the following term:

\[ ||W - f(\cdot, \vartheta)||^2 + \frac{1}{\lambda} \vartheta'D\vartheta \]

where \( D = \begin{bmatrix} 0_{2\times2} & 0_{2\times K} & 0_{2\times K} \\ 0_{K\times2} & \Omega_1 & 0_{K\times K} \\ 0_{K\times2} & 0_{K\times K} & \Omega_2 \end{bmatrix} \)

where the \((l, k)th\) entry of \( \Omega_1 \) is \( |\kappa_l - \kappa_k|^3 \) and the \((l, k)th\) entry of \( \Omega_2 \) is \( |\iota_l - \iota_k|^3 \). Therefore, we penalize on \( \mu_{1k} \) and \( \mu_{2k} \), which are coefficients of \( z_{1k} = |g_{p-j't'} - \kappa_k|^3 \) and \( z_{2k} = |\text{sum}_{g_{p-j't'}} - \eta_{pk}|^3 \) respectively. And \( \Omega_1 \) is the \( K \times K \) thin-plate spline penalty matrix for \( \mu_{1k} \) and \( \Omega_2 \) is the penalty matrix for \( \mu_{2k} \). For the physician-level deviation curve, we also penalize coefficients of \( z_{1pk} = |g_{p-j't'} - \kappa_{pk}|^3 \) and \( z_{2pk} = |\text{sum}_{g_{p-j't'}} - \eta_{pk}|^3 \), and define physician-specific \( K' \times K' \) penalty matrices \( \Omega_{p1} \) and \( \Omega_{p2} \) respectively in the same way as for the overall curve. We illustrate the re-parametrization of our P-splines models as Bayesian mixed models in the Appendix.

5.3 Second Stage Estimates

The first stage provides estimates of demand functions and firms’ equilibrium policy functions. The second stage estimation is concerned with finding supply side structural parameters \( \phi_{pj} = (dmc_{pj}, \varsigma_{pj}) \) that can rationalize these observed policy functions. In other words,
the goal is to estimate costs that are consistent with profit-maximizing observed detailing decisions.

Once again, we drop the physician subscript to simplify notation in this subsection. Suppose that firm \( j \) and its competitors follow Markov strategy \( \sigma \), the value function of firm \( j \) at state \( s \) is given as

\[
V_j(s|\sigma; \phi_j) = E[\sum_{t=0}^{\infty} r^t \pi_j(\sigma(s_t, \nu_t), s_t, \nu_j; \phi_j)|s_0 = s; \phi_j]
\] (13)

Notice from profit function (6) that the profit \( \pi_j \) is linear in the supply side parameters \( dmc_j \) and \( \varsigma_j \) if we define \( \nu_j = \varsigma_j \omega_j \) where \( \omega_j \sim N(0, 1) \). As a consequence, firm \( j \)'s value function \( V_j \) can be further expressed as a linear function of \( \phi_j \):

\[
V_j(s|\sigma; \phi_j) = M_j(s|\sigma) + W_j(s|\sigma) \cdot \phi_j
\] (14)

where

\[
M_j(s|\sigma) = E[\sum_{t=0}^{\infty} r^t Q_{jt}(\sigma(s_t, \nu), s_t) \text{ margin}_j|s_0 = s]
\] (15)

\[
W_j(s|\sigma) = E[\sum_{t=0}^{\infty} r^t \sigma_j(s_t, \nu)|s_0 = s]
\] (16)

The linearity of the unknown parameters significantly reduces the computation burden in the second stage estimation because we do not have to simulate separate value function paths for each set of parameters. With the linearity, the optimality conditions (9) can be written as

\[
M_j(s|\sigma_j, \sigma_{-j}) + W_j(s|\sigma_j, \sigma_{-j}) \cdot \phi_j \geq M_j(s|\sigma'_j, \sigma_{-j}) + W_j(s|\sigma'_j, \sigma_{-j}) \cdot \phi_j
\] (17)

Let \( X \in \chi \) index the equilibrium conditions, so that each \( X \) denotes a particular deviation
Define a new function $g(X; \phi_j)$:

$$g(X, \phi_j) = M_j(s|\sigma_j', \sigma_{-j}) - M_j(s|\sigma_j, \sigma_{-j}) + [W_j(s|\sigma_j', \sigma_{-j}) - W_j(s|\sigma_j, \sigma_{-j})] \cdot \phi_j$$  \hspace{1cm} (18)

Here $g(X, \phi_j) \geq 0$ means that strategy $\sigma_j'$ is a profitable deviation from the optimal policy. On the other hand, the optimality condition (17) is satisfied if and only if $g(X, \phi_j) \leq 0$. As a consequence, if the observed detailing policy is the firm’s optimal policy, upon plugging the true detailing cost parameters into the profit function, the calculated function value of $g(X, \phi_j)$ should be less than or equal to zero. The intuition for second stage estimation therefore comes from this fact: if we can find parameters such that chances of profitable deviations from the optimal policies are minimized, those parameters then will be as close as possible to the true cost of detailing.

Formally, suppose $H$ is a distribution over the set $\chi$ of inequalities that is chosen by the researcher. We define the function:

$$Q(\phi_j) = \int 1(g(X; \phi_j) > 0) \cdot g(X; \phi_j) dH(X)$$ \hspace{1cm} (19)

The true parameters, $\phi_j^0$, satisfy:

$$Q(\phi_j^0) = 0 = \min_{\phi_j \in \Theta} Q(\phi_j)$$ \hspace{1cm} (20)

Therefore, the estimator of $\phi_j$ is formed by minimizing the sample analog of $Q(\phi_j)$. Specifically, the estimation proceeds in two steps. In the first step, we construct the sample analog of $Q(\phi_j)$. Let $\{x_k\}_{k=1,...,n}$ be a set of inequalities from $\chi$ randomly drawn from the distribution $H$. These i.i.d. draws are created by drawing states at random and adding normally distributed noise to the estimated optimal policy. The resulting alternative policies $\sigma_j'$ represent slight perturbation of the estimated optimal policy. For each chosen inequality, $x_k$, we use a forward simulation procedure to construct empirical analogues of $W_j$ and $M_j$. The
resulting empirical counterpart to \( g(X, \phi_j) \) is then denoted by \( \tilde{g}(x, \phi_j) \). The sample analog of \( Q(\phi_j) \) is given:

\[
Q_n(\phi_j) = \frac{1}{n_t} \sum_{k=1}^{n_t} [1(\tilde{g}(x_k; \phi_j) > 0) \cdot \tilde{g}(x_k; \phi_j)]
\] (21)

In the second step, we use an optimization procedure to minimize this objective function and get the estimator. That is

\[
\hat{\phi}_j = \arg \min_{\phi_j \in \Theta} Q_n(\phi_j)
\] (22)

### 6 Results and Discussion

In this section, we present and discuss the results from our empirical estimation, first from the demand model, then from the policy functions, and finally from the supply model.

#### 6.1 Demand Model

We use a program written in C to carry out the MCMC procedure for the demand model estimation. Parameter inference is based on 40,000 iterations after a burn-in period of 40,000 iterations of the MCMC. We report the estimates of demand parameters as well as their 5% Bayesian probability intervals in Table 3.

The estimates of detailing-stock retention rates for all drugs are above 0.70, which indicates that the detailing stock of all firms exhibit strong carry-over effects. The detailing stock of Crestor lasts much longer in physicians’ minds than those of other competing drugs. This is not surprising for two reasons: (1) Crestor is a new drug and there are more opportunities for sales representatives to create goodwill in physicians’ minds or to deliver valuable information to physicians and (2) sales representatives from AstraZeneca (manufacturer of Crestor) are more effective. Evidence from industry indicates that AstraZeneca leads in
detail minutes per call and percent calls rated excellent or good.\textsuperscript{8}

Lipitor has the largest estimated intrinsic utility which is consistent with its biggest market share among the four drugs. The impact of detailing stock on prescriptions is determined by an estimated positive main effect of detailing stock on prescriptions $\beta$, an estimated positive threshold $\gamma$ and an estimated positive penalty effect $\delta$. To illustrate the implied effect of detailing stock on latent prescription utility, we plot in Figure 1 the relationship between detailing stock and the deterministic component of utility. Our estimates confirm results from previous studies that detailing stock has a positive effect on physicians’ prescriptions. The estimates also suggest that detailing stock has a diminishing marginal effect beyond a threshold level. What is even more interesting is that increases in detailing stock can result in a reduction in the likelihood of physicians’ prescriptions. This supersaturation effect of detailing stock indicates that excessive detailing may have adverse effects on physicians’ prescriptions. This negative effect on physicians’ prescription happens possibly because (1) physicians may get frustrated or bored by repetitive educational information and promotion stimulus from sales representatives, (2) physicians develop suspicions about the motives and tactics used by sales representatives.

To assess the importance of including a latent threshold and a penalty effect in the utility function, we compare the fit statistics of the proposed demand model ($M_0$) with two alternative models - a model in which detailing stock enters the utility function linearly ($M_1$) and a model in which detailing stock enters the utility function in the logarithmic form ($M_2$). We report the log of integrated likelihood of the three models and the log of Bayes factor $BF_{M_0}/$. in Table 4. According to a guideline slightly modified from Jeffreys (1961), the evidence for model $M_0$ is barely worth mentioning if log of $BF_{M_0}/$ is 0 to 1, positive if 1 to 2.5, strong if 2.5 to 5, and decisive if greater than 5. The results show decisive evidence that the proposed demand model performs better than the two alternative models.

In addition to the population mean estimates, the estimated deviations of the distribution\textsuperscript{8}IMS Health Integrated Promotional Services, 12 months ending July 03.
Table 3: Posterior Means of Parameters in Demand Model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Population Mean</th>
<th>Population Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Main effect</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.1360</td>
<td>0.0501</td>
</tr>
<tr>
<td>(0.11, 0.16)$^1$</td>
<td>(0.04, 0.06)</td>
<td></td>
</tr>
<tr>
<td><strong>Threshold</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>3.9353</td>
<td>0.7722</td>
</tr>
<tr>
<td>(3.76, 4.08)</td>
<td>(0.56, 0.97)</td>
<td></td>
</tr>
<tr>
<td><strong>Penalty effect</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.0137</td>
<td>0.0003</td>
</tr>
<tr>
<td>(0.02, 0.01)</td>
<td>(2e$^-3, 4e^-3$)</td>
<td></td>
</tr>
</tbody>
</table>

**Initial detailing stock$^2$**

<table>
<thead>
<tr>
<th>Drug</th>
<th>$g_{10}$</th>
<th>Population Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lipitor</td>
<td>2.3281</td>
<td>1.1701</td>
</tr>
<tr>
<td></td>
<td>(2.07, 2.59)</td>
<td>(0.70, 1.64)</td>
</tr>
<tr>
<td>Zocor</td>
<td>1.9086</td>
<td>1.2805</td>
</tr>
<tr>
<td></td>
<td>(1.70, 2.14)</td>
<td>(0.83, 1.83)</td>
</tr>
<tr>
<td>Pravachol</td>
<td>1.7292</td>
<td>1.7669</td>
</tr>
<tr>
<td></td>
<td>(1.55, 1.92)</td>
<td>(1.13, 2.32)</td>
</tr>
<tr>
<td>Crestor</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Detailing-stock retention rate$^3$**

<table>
<thead>
<tr>
<th>Drug</th>
<th>$\lambda_1$</th>
<th>Population Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lipitor</td>
<td>0.7804</td>
<td>(0.73, 0.81)</td>
</tr>
<tr>
<td>Zocor</td>
<td>0.7050</td>
<td>(0.66, 0.74)</td>
</tr>
<tr>
<td>Pravachol</td>
<td>0.8587</td>
<td>(0.84, 0.88)</td>
</tr>
<tr>
<td>Crestor</td>
<td>0.9225</td>
<td>(0.87, 0.96)</td>
</tr>
</tbody>
</table>

**Intrinsic utility$^4$**

<table>
<thead>
<tr>
<th>Drug</th>
<th>$\alpha_1$</th>
<th>Population Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lipitor</td>
<td>$-0.1840$</td>
<td>0.4873</td>
</tr>
<tr>
<td></td>
<td>$(-0.27, -0.10)$</td>
<td>(0.40, 0.59)</td>
</tr>
<tr>
<td>Zocor</td>
<td>$-0.7156$</td>
<td>0.5712</td>
</tr>
<tr>
<td></td>
<td>$(-0.81, -0.62)$</td>
<td>(0.47, 0.68)</td>
</tr>
<tr>
<td>Pravachol</td>
<td>$-1.1444$</td>
<td>0.8542</td>
</tr>
<tr>
<td></td>
<td>$(-1.25, -1.03)$</td>
<td>(0.65, 1.08)</td>
</tr>
<tr>
<td>Crestor</td>
<td>$-0.8812$</td>
<td>0.9817</td>
</tr>
<tr>
<td></td>
<td>$(-1.00, -0.76)$</td>
<td>(0.80, 1.19)</td>
</tr>
</tbody>
</table>

---

1. (5%, 95%) posterior percentile listed in parentheses.
2. Crestor’s initial detailing stock is assumed to be zero because Crestor is newly launched; for other drugs, the reported population deviation is for log($g_{ij1}$).
3. Detailing-stock retention rates are homogeneous across physicians.
4. Intrinsic utility of non-drug treatment is set to 0 for identification.
Figure 1: Detailing-stock effect

<table>
<thead>
<tr>
<th>Model</th>
<th>Log of Integrated Likelihood</th>
<th>Log of $BF_{M_0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_0$</td>
<td>-12324.27</td>
<td>0</td>
</tr>
<tr>
<td>$M_1$</td>
<td>-12420.88</td>
<td>96.61</td>
</tr>
<tr>
<td>$M_2$</td>
<td>-12346.00</td>
<td>21.73</td>
</tr>
</tbody>
</table>

of physician-level parameters are also displayed in Table 3. These estimates represent the heterogeneity among physicians. We can see that physicians show substantial heterogeneity in their responsiveness to detailing stock as well as in the intrinsic utility of drugs. This heterogeneity supports our introduction of Bayesian hierarchical structure to pool data across physicians.

6.2 Detailing Policy

As discussed in section 5, we use a hierarchical Bayesian ordered probit model with P-splines to estimate firms’ detailing policy at the physician level. We use a MCMC procedure carried out by a program written in C for the detailing policy estimation. Parameter inference is based on 40,000 iterations after a burn-in period of 40,000 iterations of the MCMC.
Table 5: Parameter Estimates of Exogenous State Variables in Policy Functions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Lipitor</th>
<th>Zocor</th>
<th>Pravachol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crestor introduction dummy</td>
<td>0.1716</td>
<td>0.2720</td>
<td>−0.2110</td>
</tr>
<tr>
<td></td>
<td>(0.06, 0.29)</td>
<td>(0.17, 0.38)</td>
<td>(−0.34, −0.10)</td>
</tr>
<tr>
<td>Zetia introduction dummy</td>
<td>−0.3879</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(−0.52, −0.27)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1 (5%, 95%) posterior percentile listed in parentheses.

In the estimation of detailing policy functions of Lipitor, Zocor, and Pravachol, we use 10 knots for the overall curves and 5 knots for the physician-specific deviation. Since we only observe 10 months of Crestor sales, we used 10 knots for the overall curve and 3 knots for the physician-specific deviation. Alternative numbers of knots were also used to check robustness. The results turn out to be fairly insensitive to the number of knots.

In Table 5, we report the estimated coefficients of exogenous state variables in the detailing policy functions. The results suggest that drugs are heterogeneous in their response to new drug entry. Both Lipitor and Zocor respond to the entry of Crestor by increasing their detailing visits to physicians, while Pravachol responds by decreasing its number of detailing visits to physicians. The estimated coefficients of Zetia introduction dummy for Lipitor and Pravachol both are insignificant and, therefore, removed from policy functions. This suggests that both Lipitor and Pravachol do not respond in detailing effort to Zetia’s entry. The negative coefficient of Zetia introduction dummy for Zocor indicates that Merck (manufacturer of Zocor) reduced its detailing effort for Zocor after the introduction of Zetia.

It is useful to note that Zetia is jointly marketed by Schering-Plough and Merck, and Zetia is promoted for use together with Zocor. Therefore, the substitution of marketing efforts between these two drugs can possibly explain the decrease of detailing efforts for Zocor after the introduction of Zetia.

Because the estimation of the relationship between covariates and the dependent variable is a superposition of several piecewise polynomials, interpreting the P-spline coefficients is not insightful. Instead, to illustrate the dependence between detailing decisions and state
variables given by our ordered probit model with P-splines, we plot in Figure 2 for each of
the four drugs the predicted latent value of detailing with respect to its own detailing stock
and competitors’ detailing stock.

As shown in Figure 2, the latent value of detailing decreases as a firm’s own detailing stock increases. In other words, firms tend to direct less detailing to a physician if they already have a high detailing stock at that physician. This result is consistent with the finding of Dubé et al. (2005) that the optimal advertising level decreases in own goodwill. An interesting aspect of our result is that as own detailing stock increases, firms reduce their
detailing at an accelerating rate.

A firm’s detailing response to competitors’ detailing stock is more complex. When competitors’ detailing stock is low, a firm’s latent value of detailing increases in its competitors’ detailing stock at a diminishing rate. When competitors’ detailing stock exceeds a threshold, a firm’s detailing decreases at an accelerating rate. The intuition behind this asymmetric response is as follows: when competitors’ detailing stock is not high, a firm tends to increase its detailing effort to steal market. However, when competitors’ detailing stock is very high, using detailing to gain market share becomes an unattractive strategy possibly because of the diminishing return to detailing.

Figure 2 (right-lower panel) shows that Crestor’s latent value for detailing does not decrease until there is a very high level of competitors’ detailing stock. This suggests that Crestor has a more aggressive detailing strategy than its three competitors. It responds strongly to competitors’ detailing stock unless their presence or reputation is very strong. As a comer new to the market, an aggressive detailing strategy can help Crestor achieve market share in a short time. The higher carry-over effect of detailing-stock for Crestor (see Table 3) also suggests that it is worthwhile for Crestor to be aggressive in detailing.

Our finding of a non-monotonic response of firms’ detailing decisions to competitors’ detailing stock supports our choice of a highly flexible, semi-parametric specification for the policy functions. To further validate our proposed policy function specification ($M_0$), we compare the fit statistics with an alternative parametric policy function ($M_1$) in which detailing stock enters the latent value function linearly. Thus, $M_1$ is a hierarchical Bayesian ordered probit model with utility linear in own and competitive detailing stock. We report the log of integrated likelihood of the two models and the log of Bayes factor $BF_{M_0}$. in Table 6. Based on Jeffreys (1961) guidelines, our results indicate that the evidence in favor of $M_0$ is decisive for Lipitor and Crestor, strong for Zocor, and positive for Pravachol. To assess predictive validity, we calculate Mean Absolute Error (MAE) in holdout sample as shown in Table 6. Results indicate that $M_0$ outperforms $M_1$ in holdout sample prediction.
too. In summary, the estimated policy functions show that latent value of detailing is a nonlinear, non-monotonic function of own detailing stock and competitors’ detailing stock. A parametric linear specification of detailing policy function gives misleading results.

### 6.3 Detailing Costs

Analysis of detailing costs provides vital information for understanding, planning, controlling, and evaluating firms’ detailing performance. In spite of its importance, the analysis of detailing costs is mainly based on accounting information that does not capture all economically relevant information. In addition, the estimates of detailing costs based on accounting information often do not reflect differences in detailing costs across physicians. In practice, visits to different physicians may take sales representatives different degrees of effort because of the substantial differences between physicians in their attitudes toward detailing, the extent to which they are time constrained, etc.

As an example of currently available estimates, Neslin (2001) reported that the average cost per detailing call in 1999 for office based physicians was $138. His estimate includes fixed costs associated with keeping a sales representative in the field (salary, car allowance, travel, bonus, etc.), but none of the variable costs (samples, marketing materials, etc.).

---

Table 6: Policy Model Fit Statistics

<table>
<thead>
<tr>
<th>Model</th>
<th>Log of Integrated Likelihood</th>
<th>Log of BF (<em>{M_0}/</em>{M_1})</th>
<th>MAE in Holdout Sample(^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lipitor (_M_0)</td>
<td>-4337.40</td>
<td>0</td>
<td>0.5279</td>
</tr>
<tr>
<td>Lipitor (_M_1)</td>
<td>-4352.90</td>
<td>15.50</td>
<td>0.5647</td>
</tr>
<tr>
<td>Zocor (_M_0)</td>
<td>-4772.15</td>
<td>0</td>
<td>0.6008</td>
</tr>
<tr>
<td>Zocor (_M_1)</td>
<td>-4775.56</td>
<td>3.41</td>
<td>0.6197</td>
</tr>
<tr>
<td>Pravachol (_M_0)</td>
<td>-3383.77</td>
<td>0</td>
<td>0.4455</td>
</tr>
<tr>
<td>Pravachol (_M_1)</td>
<td>-3385.77</td>
<td>2.00</td>
<td>0.4701</td>
</tr>
<tr>
<td>Crestor(^1) (_M_0)</td>
<td>-2526.52</td>
<td>0</td>
<td>0.4701</td>
</tr>
<tr>
<td>Crestor(^1) (_M_1)</td>
<td>-2571.74</td>
<td>45.22</td>
<td>0.4701</td>
</tr>
</tbody>
</table>

\(^1\) Holdout analysis is not performed due to limited data.

\(^2\) We use Mean Absolute Error (MAE) instead of Root Mean Squared Error (RMSE) because RMSE is more sensitive to extreme values. Both \(_M_0\) and \(_M_1\) are estimated using the first 18 months of data for each physician, and the number of details in the remaining six months is predicted.
Our estimates of the marginal cost of detailing \( dmc_{pj} \) are obtained under the assumption that observed levels of detailing represent profit-maximizing detailing equilibrium of competing firms. Thus, all relevant economic information is included in the estimates. Some questions are pertinent: How should our estimates be interpreted? What are the components of the marginal cost of a detailing visit estimated using our approach?

Consider a multi-drug firm that can flexibly increase or decrease the size of its total sales force effort, as well as the sales force effort allocated to any given drug and physician. Flexibility in overall sales force effort can be achieved, for example, by outsourcing (i.e., by employing a contract sales force). In determining the optimal sales force effort (in our case, we consider number of detailing visits to be the measure of effort) to apply to a given drug and physician, what is the economic cost of one detailing visit? We believe the relevant cost for this firm is the allocated full cost of a sales representative’s time. This includes salary, commissions, benefits, travel costs, and any other out-of-pocket costs incurred to visit the physician. It is important to note that while costs such as sales representatives’ salary and benefits are ordinarily considered fixed and hence not part of marginal cost, the foregoing discussion implies that our estimates of the marginal cost of a detailing visit do include these components.

Additionally, since drug samples are often left behind by representatives at physician offices along with detailing visits, the estimated cost of a detailing visit includes the economic cost of such samples. Besides the manufacturing cost, we also value the samples at their opportunity costs, namely, the manufacturer margin on the foregone sales. Forgone sales are estimated assuming that a firm is able to sell a sample as a regular unit with a probability equal to its expected market share if it does not distribute that sample.

Finally, my estimates also reflect costs of detailing that would be incurred by the firms in those periods when firms actually do not undertake any detailing. In those periods, firms make decisions not to detail based on their prediction of the marginal cost of detailing and the return to detailing. Because optimal detailing is increasing in private shock, I expect
that the costs of detailing estimated only from periods when firms have detailing would be smaller than the costs of detailing inferred from periods when firms have no detailing. My estimates of marginal costs of detailing is the average over both kinds of periods and capture all economically relevant information from both decisions to detail and decisions not to detail through private shocks. Therefore, they are expected to be larger than estimates derived from out-of-pocket costs (even if I supplement the opportunity cost of sampling) which can only capture cost information from decisions to detail.

For each of the four drugs, we estimate marginal costs of detailing to a “representative” physician as well as standard deviation of the corresponding private shock distribution. We use the population means of the estimated demand model and detailing policy functions to predict firms’ demand and detailing policy at this “representative” physician. We standardize both the market size (number of patient visits) and margin at this representative physician as 1. With these specifications, detailing cost parameters for this representative physician are estimated using the approach laid out in section 5.3. For each of the four drugs, point estimates were obtained using 200 simulation paths over a lifetime of 500 months each. Each simulation path was replicated 200 times and averaged to obtain expected values. We fixed the monthly discount factor at 0.99 in this process.

The estimates of marginal costs of detailing and standard deviations of private shock distributions are reported in Table 7 in the left panel labeled “forward-looking.” Because the asymptotic distribution of the estimator is determined by the first stage sampling error adjusted for its effect on the second state estimate, we use a bootstrap procedure to estimate standard errors for parameter estimates. As shown in Table 7, the estimates of standard deviations of private shock distributions indicate that four drugs have relatively similar variability in marginal costs of detailing.

To get a more interpretable measure of detailing cost, we go further to compute the unstandardized dollar value of detailing marginal cost by approximating the market size and
<table>
<thead>
<tr>
<th>Drugs</th>
<th>Forward-looking dmc</th>
<th>$ value of dmc</th>
<th>$ value of dmc</th>
<th>MDD</th>
<th>Myopic dmc</th>
<th>$ value of dmc</th>
<th>MDD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lipitor</td>
<td>0.2521</td>
<td>333-444</td>
<td>0.0674</td>
<td>0.0021</td>
<td>0.2276</td>
<td>0.0400</td>
<td>0.0533</td>
</tr>
<tr>
<td>Zocor</td>
<td>0.1444</td>
<td>243-324</td>
<td>0.0539</td>
<td>0.0024</td>
<td>0.1373</td>
<td>0.0414</td>
<td>0.0591</td>
</tr>
<tr>
<td>Pravachol</td>
<td>0.2056</td>
<td>292-390</td>
<td>0.0633</td>
<td>0.0017</td>
<td>0.1884</td>
<td>0.0456</td>
<td>0.0347</td>
</tr>
<tr>
<td>Crestor</td>
<td>0.4178</td>
<td>427-569</td>
<td>0.0527</td>
<td>0.0021</td>
<td>0.3543</td>
<td>0.0267</td>
<td>0.0866</td>
</tr>
</tbody>
</table>

1 Standard errors of parameter estimates are listed in parentheses.
2 MDD is the minimum deviation distance in the estimation process. For comparison, we standardized MDD using the simulated life-time value.

margin. Together with an approximate renewal rate, we use the median number of patient visits per month and median days of therapy in the data to approximate the market size at the representative physician. Further, we use retail price and industry average manufacturer mark-up to approximate the firms’ margin per prescription for each of the four drugs. We also assume a common filling rate for the written prescriptions for these four drugs. We display the results in Table 7 in the column labeled “$ value of dmc”. The results indicate that the four firms have different marginal costs of detailing at this representative physician.

For comparison, we adjust Neslin's (2001) estimate of $138 for the detailing cost in 1999 to $147 in 2002 based on growth in the CPI. We supplement this estimate with the opportunity cost and manufacturing costs of samples as follows. We take the expected market share to be 0.25 in our four-drug case. The retail value of samples per call is inferred from the industry reported ratio of detailing expenditure to retail value of sampling (IMS Health 2004). Once

---

9Here, we approximate the average renewal rate using the ratio of volume of new written prescriptions to volume of renewal written prescriptions and an assumed drug switching probability during the lifetime of a patient. For illustration we assume two values for this switching probability - 0 as the lower bound, and 0.5 as the upper bound. The discounted value is calculated by allocating renewal prescriptions according to a statin therapy duration curve provided by Cardinal et al. (2006).


11A study in General Practice indicates that up to 25% patients do not take Statins as prescribed. Howell et al. (2004). Similar filling rate of Statins is suggested by Benner et al. (2002).
again, industry reported data are used to approximate firms’ mark-up and marginal costs of production.\textsuperscript{10} These assumptions result in Neslin’s estimate being raised to $240 per detailing visit. Compared with our estimates of dollar value of $dmc$, this number is still considerably smaller. This is not surprising because the adjusted Neslin estimate still does not reflect cost information from decisions not to detail. The analog of adjusted Neslin’s $240 estimate in our study should be the average of \((dmc_{pj} - \nu_{pjt})\) under optimal detailing policy. Using the estimated detailing policy, we approximate the average dollar value of \((dmc_{pj} - \nu_{pjt})\) over physicians and time for each of the four drugs, the results turn out to be: Lipitor $204 - $272, Zocor $115 - $153, Pravachol $126 - $168, and Crestor $360 - $480. The average value of \((dmc_{pj} - \nu_{pjt})\) over the four drugs, physicians and time is $235 - $313, which is consistent with the adjusted value of Neslin’s estimate. We want to emphasize here that this estimate of marginal costs of detailing only applies to the observed optimal detailing scenario, and cannot reflect the cost structure change along with the change of detailing policy and therefore is not suitable for policy simulation study.

Among the four drugs, Crestor has the largest marginal cost of detailing. Because of Crestor’s newness, AstraZeneca (manufacturer of Crestor) needs to give physicians more samples to induce them to adopt Crestor. Also, sales representative from Crestor may need to invest more time to establish a relationship and rapport with physicians and possibly AstraZeneca invests more on selling and has a higher quality sales force. Another possible reason is that Crestor may have a much lower filling rate compared with other three drugs and the average filling rate used in our calculation, therefore, may overestimate the marginal costs of detailing for Crestor.\textsuperscript{12}

As a benchmark for comparison, we also estimate costs of detailing by applying the

\textsuperscript{12}From a report by ImpactRx, Inc., No Margin for Error, September, 2005, we can see that, in California, Crestor’s market share in new \textit{dispensed} prescriptions is significantly smaller than that in new \textit{written} prescription, which suggests that Crestor’s written prescriptions may have a much lower filling rate. This is also intuitively reasonable because Crestor is a new drug and patients are concerned with its side effects. An adjusted filling rate of 60\% for Crestor is obtained based on the discrepancy between market shares based on dispensed versus written prescriptions and average filling rate for all Statins in previous studies; this results in much smaller estimates of Crestor’s total marginal costs of detailing \((dmc_{p4} - \nu_{p4t})\) under the optimal detailing policy scenario – $288 - $384.
optimality condition that firms only optimize their current profits. These estimates are obtained by following the same procedure as described above, but with 0 discount rate. The parameter estimates are listed in Table 7 in the right panel labeled “Myopic.” Comparing results with the forward-looking model, we see that the estimated marginal costs of detailing from the myopic model are considerably smaller. The assumption that firms are forward-looking is attractive not only for theoretical reasons, but also for the following two empirical observations. First, as shown in Table 7, standardized minimum deviation distances under the forward-looking assumption are smaller than those under the myopic assumption. This suggests that the forward-looking assumption is more consistent with the observed data. Second, at the estimated myopic costs, firms will have no incentive to detail since they would not cover their detailing cost. This happens because the observed number of details in the data is considerably larger than might be consistent with a myopic firm.

6.4 Targeting

A recent trend in research on targeting is to explicitly account for firms’ strategic behaviors (Shaffer and Zhang 1995, Besanko et al. 2003). As the level of detailing is determined for each individual physician, this context presents a unique opportunity to study targeting at the individual level in the presence of firms’ strategic behavior. For instance, Dong et al. (2006) quantify the benefits of targeted detailing at the individual physician level under firms’ strategic behavior.

Given that we have physician-specific demand estimates and physician-level detailing policy estimates, we can follow the second stage estimation procedure to get firms’ marginal costs of detailing to each physician. With these estimated physician-level marginal costs of detailing, we can quantify the benefits of physician-level targeting i.e., determining detailing levels specifically for each physician. For illustration, we choose one physician - physician P - and use her to illustrate the benefits of targeted detailing. This is done for computational convenience, since the forward simulations for each cost estimation are very time consuming.
We calculate the expected profits to Pfizer from physician P in two scenarios. In the first scenario, Pfizer uses an overall optimal detailing policy based on population estimates for detailing policy functions (“overall detailing policy”) to detail to physician P. In the second scenario (“targeted detailing policy”), Pfizer uses an optimal detailing policy determined for physician P (based on posterior estimates for this physician). In both scenarios, we use the marginal costs of detailing estimated for physician P and we assume that competing firms also follow their optimal detailing policy estimated for physician P. Results are reported in the second column of Table 8 labeled “Detailing cost at P.” The example shows that physician-level targeted detailing to physician P has substantial benefits - a 52.59% increase in profits - compared with non-targeted detailing.

Our focus is to understand the implication of the heterogeneity in marginal cost of detailing across physicians in assessing the benefits of targeted detailing. To assess the benefits from estimating physician-level marginal cost of detailing, we also calculate the incremental profits of targeting using the marginal cost of detailing estimated for a representative physician. Results are shown in the third column of Table 8. We see that ignoring differences in marginal cost of detailing across physicians gives misleading estimates of the benefits of targeting. In particular, it appears that targeting results in reduced profits if we do not account for heterogeneity in detailing costs.

7 Conclusion

This study has introduced a comprehensive approach to understand pharmaceutical firms’ detailing behavior in a dynamic competitive market. Our study involves the following com-
ponents. First, we estimate physician-level demand. From the estimated demand system, we confirm that drug detailing has a positive effect on physicians’ prescription choices and this positive effect spills over into future months. Our demand model allows for a diminishing marginal effect of detailing and a supersaturation effect of detailing. Second, we empirically estimate firms’ detailing policies from the observed data using a hierarchical ordered probit model with P-splines. This flexible semi-parametric approach helps us learn the nonlinear, non-monotonic relationship between the latent value of detailing and own and competitive detailing stock. The policy function estimates show that a firm tends to have more detailing when its own detailing stock is small. Further, the overall detailing policy suggests that a firm increases its detailing efforts in competitors’ detailing efforts when its competitors have small detailing stock but curtails its efforts in competitors’ detailing when its competitors have large detailing stock. Third, using the estimated demand and policy functions, and assuming firms are forward-looking in their detailing decisions, we estimate firms’ marginal costs of detailing. Our estimates of the cost of a detailing visit are substantially higher than extant industry estimates based on accounting information. We show that an alternative assumption that firms are myopic gives unreasonable results.

Our estimates of detailing costs provide vital information for firms to better understand and evaluate their detailing. With the estimated physician-level costs of detailing, we show how heterogeneity in marginal cost of detailing across physicians matters in assessing the benefits of physician-level targeted detailing. With that cost information, firms can also respond optimally to potential changes in their economic environment without following a slow process of trial and error. In addition, our study also provides a way for outsiders of the business to develop an understanding of the cost structure of detailing from available industry data.

We now discuss some of the limitations of our work and future research avenues.

- In the drug demand model, we do not distinguish between the persuasive and informative effects of detailing because of data limitations. An important area for future
research is to model these two effects of detailing separately and incorporate this more sophisticated demand model into the dynamic competition framework. This will allow researchers to understand the different detailing strategies a firm launching a new drug might adopt.

- To model the carry-over effect of detailing, we use a deterministic exponential decay function. It would be helpful to explore other forms of decay processes to deepen our understanding of the carry-over effect of detailing. A nonparametric model of the decay process can be a more flexible way to approach this problem. Similarly, it would be useful to examine a stochastic decay process.

- A potential issue of concern is whether an econometric endogeneity problem biases the estimated effects of detailing stock. Endogeneity occurs when time-varying drug characteristics affect physicians’ prescription choices, and firms’ detailing decisions are also based on these drug characteristics. However, these characteristics are unobserved to the analyst. The unavailability of appropriate instrumental variables prohibits us from investigating this issue in our demand estimation at present. Another potential concern is the simultaneity issue raised by Manchanda et al. (2004b). Specifically, if firms decide on their detailing levels to individual physicians based on knowledge of their behaviors, e.g., the detailing-stock response parameters, then parameter estimates would be biased. This problem can potentially be solved by simultaneously estimating a demand model and a strategic supply model. Of course, this approach may also introduce new problems if the supply side restriction is not correctly specified. Since simultaneous estimation of a demand model and a supply model still remains intractable for dynamic games, we cannot employ this approach in our study. However, in a Bayesian hierarchical framework, detailing-stock response parameters are obtained from a compromise between physician-level data and the prior, which is the population mean of the distribution of those physician-level parameters. Our under-
standing is that bias in the estimates like this, if it exists, can only happen through the population mean. With more informative physician-level data, this problem will be less severe.
Appendix: Reparametrization of Bayesian P-splines

Refer section 5.2 in main paper. Considering the vector $\mu_1, \mu_2, \mu_{p1}$ and $\mu_{p2}$ respectively as a set of random parameters, we can represent the penalized spline ordered probit regression model as a generalized linear mixed model using the reparametrization $a = \Omega_1^{1/2} \mu_1$, $b = \Omega_2^{1/2} \mu_2$, $c_p = \Omega_{p1}^{1/2} \mu_{p1}$, $d_p = \Omega_{p2}^{1/2} \mu_{p2}$ and defining $\Phi = Z_1 \Omega_1^{-1/2}, \Psi = Z_2 \Omega_2^{-1/2}, \Phi_p = Z_{p1} \Omega_{p1}^{-1/2}, \Psi_p = Z_{p2} \Omega_{p2}^{-1/2}$. We refer to Ruppert et al. (2003) and Crainiceanu et al. (2005) for the explanation of the theoretical base for this reparametrization. The reparametrized models are

$$f(g_{pjt}, \text{sum}_{pjt}, \theta) = \eta_0 + \eta_1 g_{pjt} + \eta_2 \text{sum}_{pjt} + \sum_{k=1}^{K} a_k \varphi_{tk} + \sum_{k=1}^{K} b_k \psi_{tk}$$

$$f_p(g_{pjt}, \text{sum}_{pjt}, \theta_p) = \eta_{p0} + \eta_{p1} g_{pjt} + \eta_{p2} \text{sum}_{pjt} + \sum_{k=1}^{K'} c_{pk} \varphi_{ptk} + \sum_{k=1}^{K'} d_{pk} \psi_{ptk}$$

Here $\varphi_{tk}, \psi_{tk}, \varphi_{ptk}, \psi_{ptk}$ are the $(t, k)$th entry of $\Phi, \Psi, \Phi_p, \Psi_p$ respectively. We assume that all random parameters in the above model are mutually independent and,

$$a_k \sim N(0, \varsigma_a^2), \quad k = 1, \cdots, K$$

$$b_k \sim N(0, \varsigma_b^2), \quad k = 1, \cdots, K$$

$$c_{pk} \sim N(0, \varsigma_c^2), \quad p = 1, \cdots, P, k = 1, \cdots, K'$$

$$d_{pk} \sim N(0, \varsigma_d^2), \quad p = 1, \cdots, P, k = 1, \cdots, K'$$

$$\eta_{p0} \sim N(0, \varsigma_{\eta0}^2), \quad p = 1, \cdots, P$$

$$\eta_{p1} \sim N(0, \varsigma_{\eta1}^2), \quad p = 1, \cdots, P$$

$$\eta_{p2} \sim N(0, \varsigma_{\eta2}^2), \quad p = 1, \cdots, P$$

In the hierarchical structure, the prior distributions for all $\sigma^2$s above are given as follows:

$$\varsigma_a^2, \varsigma_b^2, \varsigma_c^2, \varsigma_d^2, \varsigma_{\eta0}^2, \varsigma_{\eta1}^2, \varsigma_{\eta2}^2 \sim Gamma(10^{-5}, 10^{-5})$$

38
References


