Products vs. Advertising: Media Competition and the Relative Source of Firm Profits*

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Abstract

In this paper we ask when will a firm earn more profits from selling the attention of its customers to advertisers than from selling the underlying product itself. In other words, when will a firm become an advertising “medium”? We investigate this decision as a function of the intensity and nature of competition. We show that regardless of inherent product value to customers, when the firm faces high within-industry competition it will always earn more profits from the product market; implying that firms cannot “advertise their way out” of intense competition. However, for products of moderate inherent value, we find that the product model is more attractive when competition is at either extreme (very high or low) but the advertising model is more attractive when competition is in the middle range. This results in an inverse-U pattern for relative source of profits as a function of within-industry competition. We also look at the level of competition between-media and identify conditions for firm profits in one industry to increase as a result of heightened competition with another industry. Moreover, we show that as two media are more substitutable (hence competing more head-to-head for advertising campaigns) their source of profits will diverge. In addition, the paper considers the impact of the disutility created by advertising for the product consumer. Interestingly, we find that low levels of consumer disutility may actually increase the proportion of profits from advertising as compared to products.

Keywords: Media, Circulation Industries, Business Model Selection, Competitive Strategy, Game Theory
1 Introduction

The (Phoenix’s) decision to give away copies .... came partly in response to the hordes of Internet sites giving information away for free. It hopes free copies will boost its readership, and thus its advertising revenue. - The Boston Herald on the decision to eliminate the $1.50 newsstand price for the Boston Phoenix, an alternative newsweekly (2002).

“We have to condition them (web users) otherwise... They have to either pay with their attention or pay with their pocket book.” - Patrick Hurley, Senior Vice President of business operations, salon.com (2001).

By their very nature, some products\(^\text{1}\) attract the attention of their customers during use. Viewers of Monday Night Football spend about three hours every week watching and listening to ABC television’s production. To consume the content of the Boston Globe, readers must look at the pages of the newspaper. Visitors to Broadway theaters spend two hours sitting in a dark room, paying attention to little else but the music, dialog and storyline behind the play being produced on stage. In each of these cases, the “product” cannot be used without attracting the awareness and cognitive focus of the customer. Firms that produce such attention-generating products have the option of participating in the advertising market, which is essentially a market for the attention of consumers. These firms must decide to what extent their profits will come principally from advertising as opposed to product sales.

The observed answer to this question differs greatly across industries. Television and radio derive effectively all of their revenue from the advertising market. At the other extreme, live theater has typically not participated directly in the advertising market. In between, newspapers earn on average 80% of revenues from advertising while this number in magazines has historically ranged from 10% to 70%. In this paper, we investigate the forces behind this decision on source of profitability and offer a theory for when – and to what extent – a firm will become an advertising medium.

Interestingly, recent years have seen significant flux regarding this decision. The U.S. movie

\(^{1}\)Throughout this paper, we’ll use the term “product” to refer to both products and services.
theater has traditionally been an “advertising-free” zone. However, this began to change in the mid-90’s as more and more theaters ran specially-produced ads before the feature presentation. As of 2001, nearly 2/3 of the 37,000 screens had some form of advertising on them. This number does not include the rise in “product placements” in the films themselves. The economics of the magazine business have changed dramatically as well. In the past two decades, we have seen a general shift from subscription-driven models to more advertising-driven models. It is particularly interesting to compare the trends in two media, alternative newsweeklies and the Web. The best-known examples of the former are probably the Village Voice in New York and the Boston Phoenix. When the Phoenix decided to give its papers away for free in January 2001, it was the last holdout among the major newsweeklies. The Voice had eliminated its newsstand price several years earlier. As the quote above suggests, one significant motivating factor for this decision was the increased competition these papers were facing from free sources. On the other hand, several dot-coms, particularly those providing content, present a very different example. Many such firms – theStreet.com and salon.com, for example – began by giving the product away for free in hopes of earning advertising revenues. In fact, some companies – AllAdvantage.com and iWon.com, for example – even paid customers for their attention. However, we have recently witnessed a shift toward pay-for-content models. According to a 2002 Forrester Research report, 75% of Internet executives surveyed felt that the proportion of their revenues coming from subscription fees would increase within two years. About half of them felt that they would earn more than 10% of their revenues from subscribers within that time frame. Yahoo! alone is expected to earn over 40% of its revenues by year-end 2002 from “non-advertising” sources, as compared with 23% the previous year. Thus, on the one hand we’re seeing some newspapers shifting their focus toward becoming more advertising-driven, while on the other hand, we’re seeing dot-coms shifting their focus away from advertising revenues toward becoming more product-driven.

We would expect that the optimal proportion of a firm’s profits coming from advertising as opposed to products should depend in part on the relative value that the product provides to


customers as compared with the value that the customers’ attention provides to advertisers. For example, we might assume that a Broadway production would offer more value to a viewer than a 60-minute television drama. Since customers might be willing to pay more for the Broadway production, we’d expect that, all else equal, theater owners would be less inclined to become an advertising medium. Another important dimension of the problem is the role of competition. Competition will limit the firm’s ability to earn profits in both the product and advertising market. However, the nature of competition in these two markets is not the same. While product markets are relatively independent from one another, and hence face direct competition mainly from other firms within their respective industries, there is substantial cross-industry competition for advertising. That is, while *Time* magazine doesn’t directly compete with NBC for readers, it certainly does so for advertisers.\(^4\) Thus, competition may have a different impact on product markets and advertising markets, even though the same firms compete in both.

Note also that the relationship between the advertising market and product market exhibits an asymmetric dependence. Selling a customer’s attention to an advertiser is contingent on first selling a product to that customer, but the reverse is not true.\(^5\) This relationship has two principal economic implications which we refer to as “subsidization” and “output-dominance.” These forces work in opposite directions and drive the core results in this paper.

On the one hand, the product business is shown to *subsidize* the advertising business in the sense that the firm optimally produces more products than it would if it were not able to sell advertising space. Increasing product sales to customers to benefit from advertising, of course, depresses product profits. The implication of this subsidization is that, as competition increases and the firm scales back product output, the subsidy declines. This has a (partial) positive impact on product profits and a negative impact on advertising profits. On the other hand, it is clear that if prices fall by $1 in each market, it will generally have a bigger negative impact on the product

\(^4\)While there could be some interdependence in the product market (for example, viewing much more television could potentially take away from time to read magazines), any dollar spent in one medium is directly taken from another in the media plan/buy given an advertiser’s budget (see, for example, O’Guinn et al. (2000)).

\(^5\)Our characterization of firms that sell to consumers in a product market and then sell their attention to advertisers, is consistent with the definition of a “circulation industry” by Chaudhri (1998). It is also similar to the characterization of a “cross-market network effect” by Chen and Xie (2002).
market than on the advertising market since the output in the former is always dominant. This stems from the fact that the number of advertising impressions can never be “high” without high product quantities, though product quantities are not limited by advertising impressions.\textsuperscript{6} So, as competition increases total industry production increases and prices fall, and this is likely to hurt the product market more than the advertising market given that output is high in the former to begin with. We show that the relative impact of competition on advertising profits and product profits is driven primarily by the interplay between these two forces, moderated by the inherent value of the product to customers.

Not surprisingly, when the inherent value of the product is high, the firm earns more profits from product sales than from advertising. In contrast, when this inherent product value is very low, advertising profits are initially higher than product profits. As competitive intensity increases, however, the firm earns more profits from the product market than from the advertising market even for products with arbitrarily low inherent value. The reason for this is that the effect of less subsidization causes ad profits to decline faster than product profits. The decline in product output has a negative effect on ad profits but a marginally positive effect on product profits, as the firm moves closer to the optimal output level it would choose if it didn’t overproduce to benefit from the ad market.

When the inherent value of the product is at moderate levels, we get the intriguing result that there is an inverse-U shaped relationship between the competitiveness of an industry and the difference between equilibrium profits that come from advertising and product sales. Hence, for these products, it is only at intermediate levels of competition that we are likely to observe the emergence of true advertising media, where advertising represents the predominant source of profits. This pattern emerges because at low levels of competition, firms make most of their profits from product sales, but the effect of output-dominance (as explained above) tends to make advertising become more and more profitable as competition increases. However, as competition becomes “high enough,” the much lower incentive to subsidize takes over and product profits relatively benefit, producing the inverted-U shape. We point out that these results combined suggest that

\textsuperscript{6}Some Broadway plays are seen by millions of patrons who are not shown a single ad. However, the Super Bowl cannot charge advertisers for millions of GRPs (gross rating points) without delivering millions of viewers.
when within-industry competitive intensity is high enough, equilibrium profits are always higher in the product market than in the advertising market. This is true for any product. Firms cannot “advertise their way out” of intense competition.

We also examine how the level of competition between media⁷ for advertising dollars affects overall profitability. We find that an increase in the substitutability of the media (from the advertiser’s perspective) may actually increase total profits for some media. Moreover, as the media compete more intensely with each other due to increased substitutability, their equilibrium business models diverge - some of them derive more of their revenue from advertising while others derive less. We use the model, as well, to examine the total number of advertising impressions all customers in the product market are exposed to (or the level of advertising “clutter”). We find that advertising clutter is increasing in the number of firms within a medium, but only if inter-media competition is high. If inter-media competition is low, the aggregate level of advertising impressions is decreasing in the number of firms. To the extent that marketers or regulators seek to monitor the aggregate level of consumer exposure to advertising for purposes of gauging overall clutter, these findings highlight the importance of explicitly considering the impact of competition both within and between media.

We extend the model to incorporate the realistic possibility that advertising creates disutility for consumers during product consumption. This issue has become especially acute for Internet content providers with the advent of intrusive advertising formats. For example, iVillage.com has announced plans to decrease significantly its use of the “pop-up” ad format on its web site after consumer feedback indicated high levels of annoyance. In this context, our analysis reveals that as the marginal disutility to consumers from advertising initially increases, profits generated from advertising actually increase while those from selling products to customers decrease.

The rest of the paper is organized as follows. In the next section we relate our work to the relevant literature. In Section 3 we develop a parsimonious model of media competition where firms can generate revenue in both a product market and an advertising market. We present the core

⁷We use the terms intra-medium and within-medium competition to refer to competition a firm faces from other firms in its own medium. We use the terms inter-media and between-media competition to refer to competition a firm faces from other firms in a separate medium. We also use the words medium and industry interchangeably throughout the paper.
results on advertising profits versus product profits as a function of within-industry competition. In addition, we consider the impact of between-industry competition and the implications of competitive intensity on total consumer exposure to advertising. In Section 4, we extend the basic model to explicitly consider the impact of the disutility that advertising may create for consumers. The paper concludes with a summary of the results and a discussion of managerial implications.

2 Relevant Literature

The extant literature relevant to our analysis can roughly be categorized into three broad areas: (1) research that has empirically analyzed the nature of media firms’ business model decision, (2) other analytical models of media, and (3) general models of multi-product firms in which there exists some relationship or “externality” between the different products. We relate our work to each of these streams in turn.

Several empirical papers document differences in revenue source for a particular medium over time. Data presented in Sumner (2001) suggests, for instance, that the predominant fraction of magazine revenue has shifted from circulation (charging customers) to advertising between 1980 and 1998. Several reasons are put forward for this shift, including the possibility that the inherent value of magazines to advertisers (relative to other media such as network television) has increased and that consumers find magazine advertising less annoying and more “satisfying” than television advertising. We control for both of these factors in solving for the firm’s optimal decision. Hall (1976) analyzes the demise of mass circulation magazines in the late 60’s. His findings suggest that magazine publishers followed a strategy of increasing editorial content pages when increasing the volume of advertising pages (the latter tended to increase when circulation increased). Using a systems dynamics methodology, he shows that the combined effect of increasing circulation followed by increasing ad page volume and decreasing marginal advertising rates can lead to reduced overall profitability. Similar relationships are shown empirically in Krishnan and Soley (1987), who present evidence that firms have become more conscious of this potential negative dynamic and correct for it. Limited empirical work considers the relationship between multiple media. Liebowitz (1982) examined the impact of cable networks’ entry on television broadcasters’ revenues and advertising rates in Canada. He found that, overall, broadcasters revenues slightly increased as a result of
cable operators’ entry while their advertising rates remained largely unchanged. To the extent that
cable operators have exclusive territories while broadcasters have overlapping networks, this result
is consistent with our finding (in Section 3.2) that the entry of a new but less competitive medium
may result in higher profits for the incumbent medium.\textsuperscript{8}

There have also been a limited number of theoretical investigations into questions specifically
related to media, but quite different in focus from the question we ask here. Chaudhri (1998)
dresses the issue of media monopolies. In particular, he shows that the traditional view that
a monopolist will restrict output, to the detriment of overall welfare, is not necessarily true in
circulation industries. He shows that the output of a monopolist in the product market will be
higher due to the existence of a perfectly competitive (i.e., price taking) advertising market as
compared with a case in which an advertising market is not present. This basic result is consistent
with our model. He also identifies conditions under which the product is priced above or below
marginal cost. Since his model, by design, specifies the medium as a monopoly, it does not allow
for an explicit analysis of the impact of competitive behavior. Moreover, he does not address the
relative profitability of the advertising business and the product business which forms the core of
the present analysis. Chen and Xie (2002) study a duopoly where cross-market network effects
(see footnote 5) are present. Their paper focuses on how the size of an \textit{ex-ante} loyal customer base
with which an incumbent firm is endowed affects product pricing and profitability. Interestingly,
they show that, in some cases, the firm not possessing a loyal base (an entrant) may earn higher
profits than the incumbent. Their analysis has implications for incumbent source of profits as a
function of the size of the exogenous loyal segment. Masson et al. (1990) look at the relationship
between advertising and products with a specific focus on the disutility caused by an increase in the
relative proportion of advertising vs. content. They model the impact on the consumer’s media
consumption decision so that an increase in advertising minutes will cause some consumers to switch
(or be diverted) to other firms. Their model yields the counter-intuitive result that as competition
within the medium increases, prices in the ad market may actually rise if this “diversion effect”
is large enough. The specification of our model allows us to analyze the impact of advertising
\textsuperscript{8}The particular media chosen for the study may exhibit important idiosyncrasies related to regulation and the
relationship between American and Canadian media markets. This may limit the generalizability of the results in

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disutility (see Section 4). However, we do not assume an \textit{ex-ante} diversion effect and thus our results are driven by very different phenomena. Specifically, our model draws heavily on the fact that different media (radio and television, for example) compete for the same advertising dollars and on the interdependence between the product and advertising markets.\footnote{Masson et al. (1990) focus on a single-medium context. While they do extend their model in an appendix to allow the firm to charge consumers a subscription price in addition to generating revenue by selling to advertisers, their goal in this extension is to re-affirm that if the audience diversion effect is large enough, increased competition will raise ad prices.} Dukes and Gal-Or (2003) model the media firm’s decision as to whether or not to enter into an exclusive contract with an advertiser. Such a contract precludes the television station, for example, from accepting ads from other firms in the same category. To address this, they model (again, in a single-medium context) the interaction between consumer decisions (which stations to listen to and which products to buy), media decisions (how much to charge for advertising) and manufacturer decisions (how much to pay for advertising). Their analysis yields the key insight that exclusive advertising contracts may benefit manufacturers by softening price competition at the product level. This occurs because the overall amount of (informative) advertising declines with such an arrangement. This is likely to be true when the firms in the medium are highly differentiated. Finally, Dukes (2002) addresses the question of the optimal level of advertising in society. He shows that the more differentiated the stations (for example, the more different the programming across radio stations), the more excessive the aggregate level of advertising. This model is also useful in explaining why some product categories (specifically, those with a low level of differentiation) may be characterized by a higher level of advertising than others. It is important to note that, given their focus, both Dukes and Gal-Or (2003) and Dukes (2002) limit their analyses to contexts in which the firms charge only for advertising but not for content. Thus, these models are not able to address the question of relative profitability of products and advertising in a general way.\footnote{See also Steiner (1952) and Beebe (1977) for early models of intra-medium competition.}

Finally, there have been a number of recent papers modeling competition among firms producing several products that are somehow interrelated. While these papers do not explicitly address “media questions,” they are nonetheless relevant to our analysis as there are some similarities in the modeling approach. Van Alstyne and Parker (2002) analyze a context with an exogenous
“internetwork externality.” While the model does not directly relate to media, it might predict the attractiveness of the product relative to the advertising business as a function of the level of disutility associated with advertising. In their model, it would seem that more disutility decreases the relative attractiveness of the advertising model. However, as we show in Section 4, a complete analysis of the competitive forces at work might yield the opposite prediction. At a more general level, Cabral and Villas-Boas (2003) identify conditions for a multiproduct firm to be worse off when there is a direct positive interdependence (either in the demand or cost structure) between the different products it sells. This occurs because strategic complementarity in prices can in some cases cause firms to price more aggressively as the degree of interdependence increases. Hence, the negative strategic effect arising from competition more than outweighs the positive direct effect arising from the interdependence. They also show that the main findings of Strauss (1999) are a special case. Our paper differs in that the primary focus is on the relative source of firm profits from each market rather than on total profits. In addition, our set-up differs from theirs in that, consistent with most media contexts, firms set quantities rather than prices (see footnote 14). Furthermore, the dependence between the two markets in our model is not entirely isomorphic to the conditions necessary to obtain their results. Lastly, we depart from the approach taken by each of these papers in that we model the relationship between products and advertising at a more structural level. This affords us several advantages in answering our primary question of interest. First, we believe that it accurately captures the specific context (media) in which we’re interested. Moreover, it allows for the impact of each business on the other to vary as a function of all of the parameters of the model. Thus, we endogenize this relationship rather than specify it exogenously.

In sum, neither the theoretical nor empirical literatures have offered a general answer to the question of where the firm’s profits should optimally come from. We consider this to be the primary contribution of this paper. Moreover, we believe that our modeling approach – in particular our explicit consideration of both intra- and inter-media competition – represents a significant contribution. This is important because revenue sources from consumers and advertisers are strongly linked within the firm but they are characterized by entirely different competitive environments. There have also been no attempts to our knowledge to explicitly examine the joint roles of inherent product value and advertising disutility in this context.
3 Model

We model competition within and between two “industries” \( m \in A, B \). By “industry,” we think of newspapers, magazines or television, for example. Let industry \( m \) consist of \( N_m \) firms. So, \( N_m \) measures the competitiveness \textit{within} each respective industry (we denote the number of firms in the rival industry as \( N_{-m} \)). For example, if industry \( m \) is broadcast television in the US, then the firms within the industry would include ABC, CBS and NBC. If the industry is general news magazines, then firms within the industry would include \textit{Time} and \textit{Newsweek}. Each firm in the model competes in two markets: (1) a “product market” in which each firm sells its product to interested customers, and (2) an “advertising market” in which it sells the attention of these customers to advertisers. Product market competition is assumed to take place only within industry, while advertising market competition takes place both within and between industries.\(^{11}\)

In the product market, firms face the standard linear (inverse) demand:

\[
p_m = v_m - \sum_{i=1}^{N_m} x_{mi},
\]

where \( x_{mi} \) represents the quantity chosen by firm \( i \) in industry \( m \), and \( v_m \) represents the inherent value of the products offered by firms in industry \( m \). So, while ABC and NBC are product-market substitutes and salon.com and theStreet.com are product-market substitutes, customer demand for watching an ABC show has no impact on that of customer demand to visit the salon.com website.

We specify the price \textit{per customer impression} \( q_m \) an advertiser pays as follows:

\[
q_m = w_m - \sum_{i=1}^{N_m} y_{mi} - \gamma \sum_{j=1}^{N_{-m}} y_{-mj},
\]

where \( y_{mi} \) is the number of advertising campaigns sold by firm \( i \) in media type \( m \). For example, this would be the number of ads appearing in a given issue of the \textit{Boston Globe} or the number of commercials on an NBC show. The inherent value of a customer impression to the advertiser in medium \( m \) is given by \( w_m \). This might capture, for example, the attractiveness of the audience

\(^{11}\)In a separate analysis (available from the authors) we show that our main results hold even if there is competition between industries in the product market as well. Note also that we assume advertisers are drawn from a set outside of the \((N_m + N_{-m})\) firms in the model.
reached by the medium or the level of attention generated. Once again, subscript \((-m)\) refers to the “other” medium,\(^{12}\) and \(\gamma\) measures the substitutability between media types, i.e. to what extent advertiser demand for placing ads in medium \(-m\) affects demand in medium \(m\).

In our formulation, each campaign provides the advertiser with access to the firm’s \(x_{mi}\) customers, and total payment by an advertiser to firm \(i\) for running the campaign is then \(q_m \cdot x_{mi}\). This rate reflects the fact that the more customers will be exposed to the ad, the more the advertiser is willing to pay for running the campaign. This characterization is consistent with the observed practice in most media. For example, online advertising payment is typically rendered on a CPM (cost per thousand impressions) basis. In television, advertisers pay per GRP (gross rating point) and in radio on a cost per point basis.\(^{13}\) We point out that the specification in Equation \((2)\) assumes that the price for a customer impression is not itself an \textit{ex-ante} function of the number of customers delivered. Besides matching the observed practice, as explained above, this specification is also attractive in that it implies that advertiser payment (linearly) increases in the number of customer impressions. This characterization is also consistent with Chen and Xie (2002) and Dukes (2002), though in both of these papers the authors assume that price-per customer impression is exogenously given.

Given \((1)\) and \((2)\) and assuming, without loss of generality, zero marginal cost of production in both markets, the profit function for firm \(i\) in industry \(m \in \{A, B\}\) is the sum of profits generated in the product and advertising markets:

\[
\Pi_{mi} = p_m x_{mi} + q_m y_{mi} x_{mi} = \left[ v_m - \sum_{i'=1}^{N_m} x_{i'm} \right] x_{mi} + \left[ w_m - \sum_{i'=1}^{N_m} y_{i'm} - \gamma \sum_{j=1}^{N-m} y_{mj} \right] y_{mi} x_{mi}. \tag{3}
\]

Each firm selects quantities \(x_{mi}\) and \(y_{mi}\) in a standard Cournot fashion.\(^{14}\) Solving for the equilib-

\(^{12}\)So, for example, in the profit function for firms of media type \(A\), \(-m\) refers to firms of media type \(B\).

\(^{13}\)In these schemes, “one point” corresponds to 1 percent of the target audience reached through the medium.

In the newspaper and magazine industries, it is customary to charge an advertising rate for the amount of space allocated to the ad (full page, column lines of classified ad etc.). However, it is commonplace for the media buyer to translate this rate to an effective CPM (see O’Guinn et al. (2000), pp. 453-454). Moreover, several studies (Thompson (1989), Chen and Xie (2002)) have clearly shown that such rates are strongly and positively correlated with overall circulation (note that ‘advertising rate’ would correspond in our model to the term \(q_m \cdot x_{mi}\)).

\(^{14}\)The specification of the model as a Cournot game is a natural one for several reasons. On one hand, firms in these industries tend to think in terms of the underlying quantities. Magazine publishers explicitly consider how
rium yields:

\[ y^*_m = \frac{C_m}{K}, \]

\[ x^*_m = \frac{1}{(N_m + 1)^2} \left[ v_m + \left( \frac{C_m}{K} \right)^2 \right] = \frac{v_m + (y^*_m)^2}{(N_m + 1)^2}, \]

\[ \Pi^*_m = \frac{1}{(N_m + 1)^2} \left[ v_m + \left( \frac{C_m}{K} \right)^2 \right] = (x^*_m)^2, \]

where \( K \equiv 1 + N_m + N_{-m} + N_m N_{-m} (1 - \gamma^2) \) and \( C_m \equiv w_m + N_{-m} (w_{-m} - \gamma w_m) \). Note that the equilibrium is unique and symmetric within industry. Thus, we drop the subscripts \( i \) and \( j \) that denote individual firms unless this distinction is necessary. We require that prices in both markets be non-negative.\(^{15}\) It is easy to verify that prices in the advertising market are \( q^*_m = \frac{C_m}{K} \). Since \( K > 0 \), a necessary and sufficient condition for non-negative advertising prices is:

\[ C_m \geq 0 \iff \frac{w_m}{w_{-m}} > \frac{\gamma N_{-m}}{N_m + 1}, \quad m \in \{A, B\}. \] \(^{(5)}\)

For the condition in (5) to hold for both media, we simply need the relative intrinsic value of a customer impression in each medium to be “not too different” (since the RHS of (5) is always strictly less than 1). In the product market, \( p^*_m = \frac{v_m - N_m (C_m/K)^2}{N_m + 1} \) so a sufficient condition for many pages of advertising to have in each issue, radio broadcasters consider how many “stop sets” to run per hour and the managers of websites decide how many banners and/or pop-ups to serve (Gotham (2002)). The dependence of ad revenue on the underlying number of customers from the product market cause media firms to consider output as the relevant strategic variable; Newspaper publishers tend to think in terms of circulation, Internet sites in terms of page views and visits and radio broadcasters in terms of number of listeners. Moreover, the Cournot model is generally considered to be a reasonable way to model competition when the firms have capacity constraints (Kreps and Scheinkman, 1983). This characterization clearly holds with respect to both of the markets in which our firms compete. For example, in the product market there is significant fixed cost associated with launching a new magazine, newspaper, radio station or television station. Similarly, in the advertising market, there are clear constraints on the number of ads a firm can run during a program and the number of banner ads a website can serve up in a reasonable time. Finally, this approach is consistent with Masson et al. (1990) which utilizes a Cournot specification as well as with Dukes (2002) which specifies the media firm as choosing the amount of advertising it will run.

\(^{15}\)Note that if we included variable costs, the company might actually be selling products at an operational loss.
non-negative product prices is that \( v_m \) be high enough.\(^{16}\) Given these results, \( \Pi_m^a^* \) and \( \Pi_m^p^* \), the firms’ equilibrium profits in the advertising and product markets respectively can be written as:

\[
\Pi_m^a^* = q_m^* y_m^* x_m^* = y_m^* x_m^* = \left( \frac{C_m}{K} \right)^2 \left[ \frac{v_m + \left( \frac{C_m}{K} \right)^2}{N_m + 1} \right],
\]

\[
\Pi_m^p^* = p_m^* x_m^* = \left[ \frac{v_m - N_m \left( \frac{C_m}{K} \right)^2}{N_m + 1} \right] \left[ \frac{v_m + \left( \frac{C_m}{K} \right)^2}{N_m + 1} \right].
\]

It is interesting to compare the results of this model to a baseline model in which the firms are not able to act as advertising media.\(^{17}\) In this case, the firms maximize the following profit function:

\[
\bar{\Pi}_m = \bar{p}_m \bar{x}_m = \left[ v_m - \sum_{i=1}^{N_m} \bar{x}_{m,i} \right] \bar{x}_m.
\]

The standard Cournot equilibrium solution with only a product market is characterized by:

\[
\bar{x}_m^* = \bar{p}_m = \frac{v_m}{N_m + 1},
\]

\[
\bar{\Pi}_m^* = \left( \frac{v_m}{N_m + 1} \right)^2.
\]

Before presenting our results, we highlight several observations that will aid in providing intuition for the main findings.

**Remark 1 (Subsidization)** Compared to the case in which it cannot sell advertising, the firm always (a) produces strictly more products and (b) sets product prices that are strictly lower.

These are clear from a comparison of Equations (4) and (8). We also see that total profits are strictly higher when the firm has the option of being an advertising medium (this is consistent with previous work on circulation industries, e.g. Chaudhri (1998)). However, an important implication of this “overproduction” is that product profits are lower than they would be if the firm could not advertise:

\[
\Pi_m^p^* - \bar{\Pi}_m^* = \left( \frac{C_m}{K(N_m + 1)} \right)^2 \left( v_m - N_m \left( v_m + \left( \frac{C_m}{K} \right)^2 \right) \right) < 0. \tag{10}
\]

\(^{16}\)Specifically, we assume that \( v_m \geq N_m \left( \frac{C_m}{K} \right)^2 \) throughout.

\(^{17}\)This might be for technical reasons (for example, it is difficult for services firms to sell the attention of their clients to advertisers), or because the value of a customer impression to the advertiser is negligible. Another reason could be related to regulatory constraints.
One interpretation of (10) is that firms’ product business optimally subsidizes their advertising business. It is important to point out that since all firms in the medium have an incentive to overproduce, there is a strong combined negative effect of subsidization on product price (and hence product profits). As we discuss below, this subsidy is a drain on product profits in favor of the advertising business.

**Remark 2 (Output-Dominance)** High advertising sales implies high product sales. However, the reverse is not true.

This can be seen in the following inequality:

\[ x_m^* - (y_m^*)^2 = p_m^* \geq 0. \]  \hspace{1cm} (11)

Since prices are constrained to be non-negative, high \( y_m^* \) necessarily means that \( x_m^* \) is high. This is related to Remark 1 in that the extent of the firm’s “overproduction” of products is increasing in \( (y_m^*)^2 \).

We also point out that \( p_m^* \) is increasing in both \( \gamma \) and \( N_m \). The reason for this is clear: higher value of both of these parameters implies more inter-media competition, which makes the advertising market less attractive and less worth subsidizing. This results in lower levels of both \( y_m^* \) and \( x_m^* \); the latter effect has a positive impact on \( p_m^* \). On the other hand, when there is little effective inter-media competition (either \( \gamma \) or \( N_m \) are low), then there is a great deal of subsidization since the advertising market becomes very attractive. It can also easily be verified that when \( v_m \) is not too low (or \( \gamma \) and \( N_m \) high enough), price in the product market is decreasing in \( N_m \). This implies from (11) that:

\[ \frac{\partial p_m^*}{\partial N_m} \leq 0 \Leftrightarrow \frac{\partial x_m^*}{\partial N_m} \leq \frac{\partial (y_m^*)^2}{\partial N_m} < 0. \]  \hspace{1cm} (12)

Thus, output in the product market is more sensitive to intra-medium competition than advertising output is in this case. Furthermore, it is straightforward to show that equilibrium product output in our model is more sensitive to \( N_m \), as compared to the benchmark case (7)-(9) where linkage with an advertising market is not possible, i.e., \( \left| \frac{\partial x_m^*}{\partial N_m} \right| < \left| \frac{\partial y_m^*}{\partial N_m} \right| \). \(^{18}\)

\(^{18}\)The same is not true for the advertising market. The equilibrium number of campaigns offered by each firm \( (y_m^*) \) would be the same if we assumed a benchmark scenario, whereby the number of customer impressions is fixed.
3.1 Advertising Profits vs. Product Profits: The Impact of Within-Industry Competition

We now turn to our main inquiry which is an understanding of what drives relative profitability in the two markets. We define the difference between advertising profits and product profits in industry $m$ as $\Delta_m \equiv \Pi^a_m - \Pi^p_m$. We’re interested here in the impact of competitive intensity, as measured by the number of firms in the industry ($N_m$), on $\Delta_m$.\(^{19}\) The following Proposition offers insight into this issue and shows that the inherent value of the product ($v_m$) plays an important moderating role. (All proofs are contained in the Appendix).

**Proposition 1** (a) There always exists a $v_m$ and a $N_m$ such that for all $v_m < v_m$, when $N_m < N_m$, $\Delta_m > 0$ and when $N_m \geq N_m$, $\Delta_m < 0$. (b) If $\gamma$ and $N_{-m}$ are high enough, then there also exists a $\bar{v}_m > v_m$ and a pair $\{N_m, \bar{N}_m\}$ such that for $v_m < v_m < \bar{v}_m$, when $N_m \in (N_m, \bar{N}_m)$, $\Delta_m > 0$ and $\Delta_m < 0$ for $N_m \notin (N_m, \bar{N}_m)$. (c) For all other values of $v_m$ that don’t satisfy (a) or (b) above, $\Delta_m < 0$ for all $N_m$.

A graphical representation of Proposition 1 is shown in Figure 1. The first thing to notice is that in all cases, $\Delta_m < 0$ when $N_m$ gets high. The advertising market never yields more profits than the product market in highly competitive industries. This is true even when $w_m$ is high so that ads are likely to be very effective at attracting the attention of customers and will thus be of great interest to advertisers. High levels of competition hurt the advertising business more than the product business. The second point to notice, as expected, is that when the inherent value of the product is high the advertising market is never attractive, and when the inherent value of the product is low the advertising market tends to generate more profits (if competition is not too intense). However, when $v_m$ is moderate, the relative profitability of the advertising model as compared with the product model is characterized by an inverted-U shape over a certain range.

\(^{19}\)It is natural in the context of our model to measure the intensity of competition within the medium as a function of $N_m$. As Tirole (1988) (pp. 222) states: “...consider symmetric firms...The only reasonable measures of concentration are then equivalent to the number of firms in the industry.” See also Seade (1980) for a justification of differentiating with respect to the number of firms to study the effects of entry. We also point out that in an alternative specification one could have a fixed cost of operation that determines the number of firms active in equilibrium. Then, one could study changes in the fixed cost (that serves as a measure of barriers to entry) and obtain qualitatively similar results.
Figure 1: Relative Profitability of Selling Advertising vs. Products

The core intuition behind the results can be understood by focusing on the interplay between the two economic forces introduced above: output-dominance and subsidization. To see how these forces relate, note that the “direct” effects, i.e., holding firm choices constant, of competition on firm profits in the two markets are:

$$\frac{\partial \Pi^a_m}{\partial N_m} = -x_m^* \cdot y_m^2, \quad \frac{\partial \Pi^p_m}{\partial N_m} = -x_m^{*2}. \quad (13)$$

From Remark 2, we know that the direct effect is more pronounced on products than it is on advertising. Specifically,

$$\frac{\partial \Pi^a_m}{\partial N_m} - \frac{\partial \Pi^p_m}{\partial N_m} = -x_m^* \cdot y_m^2 + x_m^{*2}$$

$$= x_m^* \cdot (x_m^* - y_m^{*2}) > 0. \quad (14)$$

Thus, output-dominance – the fact that the firm will not have high advertising output without having high product output – results in positive pressure on $\Delta_m$. That is, *ceteris paribus*, $\Delta_m$ will be increasing in $N_m$. Yet another key factor to consider is the impact of more competition on
product output choice and what this means for profits in each market:20

\[ \frac{\partial \Pi^a_m}{\partial x^*_m} \cdot \frac{\partial x^*_m}{\partial N_m} \left( \frac{\partial \Pi^p_m}{\partial x^*_m} + (N_m - 1) \frac{\partial \Pi^p_m}{\partial x^*_m} \right) \cdot \frac{\partial x^*_m}{\partial N_m} \quad j \neq i. \] (15)

The first expression in (15) is a cross-market strategic effect related to how a change in product output impacts advertising profits, while the second expression is the impact of the change in all firms’ product output choice on product profits.21 Looking at the net effect of these terms on \( \Delta_m \) we get:

\[ \frac{\partial \Pi^a_m}{\partial x^*_m} \cdot \frac{\partial x^*_m}{\partial N_m} \left( \frac{\partial \Pi^p_m}{\partial x^*_m} + (N_m - 1) \frac{\partial \Pi^p_m}{\partial x^*_m} \right) \cdot \frac{\partial x^*_m}{\partial N_m} =
\]

\[ y^*_m \cdot \frac{\partial x^*_m}{\partial N_m} - (y^*_m - (N_m - 1)x^*_m) \cdot \frac{\partial x^*_m}{\partial N_m} =
\]

\[ (2y^*_m + (N_m - 1)x^*_m) \cdot \frac{\partial x^*_m}{\partial N_m} < 0. \] (16)

Thus, subsidization has the opposite impact from output-dominance on \( \Delta_m \) with increased competition (recall that \( \frac{\partial x^*_m}{\partial N_m} < 0 \)). Specifically, the degree of subsidization declines with more intense competition and advertising profits suffer while product profits benefit. To summarize, output-dominance drives \( \Delta_m \) up while subsidization pulls it down. We now turn to the three cases of Proposition 1 and explain their intuition in terms of output-dominance and subsidization.

---

20It is useful to decompose the impact of a parameter into “direct effects” and “strategic effects” (Tirole (1988), pp. 324-325). The former capture the impact of changes in the parameter holding firm decisions constant. The latter capture the impact of changes in firm decisions induced by changes in the parameter. Clearly, the expression in (14) is the difference in direct effects in the two markets, while the quantities in (15) represent a central subset of the strategic effects of competition on \( \Delta_m \) relevant in media markets. These strategic terms are related to the subsidization effect of product output discussed in connection with Remark 1. Note that it is less instructive to look at strategic effects related to changes in \( y^*_m \) because these do not embody much added insight beyond a benchmark model with no endogenous dependence on the product market (see also footnote 18). For completeness, we provide in the Appendix a full decomposition of the direct and strategic effects.

21Note also that in the above \( \frac{\partial \Pi^p_m}{\partial x^*_m} \) is not zero. This should not be confused as a contradiction of the envelope theorem which says that the derivative of the total profit function for firm \( i \) with respect to \( x^*_m \) should be zero at the interior optimum. This is because here we are looking at the derivative of product profits only: \( \Pi^p_m \) and not of total profits: \( \Pi_m = \Pi^p_m + \Pi^a_m \) (it is obviously the latter that is optimized in our case).
3.1.1 Low-Value Products: case (a)

When $v_m$ is low, $p_m^*$ is low and products are mainly produced in order to attract customers whose attention is sold in the advertising market. More importantly, $x_m^*$ is “close to” $y_m^2$. As such, the impact of output-dominance is negligible, and subsidization is the key driving factor. Hence, $\Delta_m$ decreases in $N_m$ and becomes negative at some point. It is particularly noteworthy that, in this case, even for low $v_m$ the product model contributes more profits when competition is high enough.

3.1.2 High-Value Products: case (c)

When $v_m$ is high, the opposite is the case. Here, the subsidization effect becomes negligible. To see this, note that $|\partial x_m^*/\partial N_m|$ is strictly decreasing in $v_m$. Higher-valued products are less sensitive to competition. Moreover, since $x_m^*$ is increasing in $v_m$ and $y_m^*$ is not a function of $v_m$, the impact of output-dominance (which is proportional to $x_m^{s2}$) grows. Thus, $\Delta_m$ is strictly increasing in $N_m$ in this region. However, it always remains negative. When the product is of high value, product output is principally for profitable sale in the product market. Thus, product prices are very high relative to advertising prices.

3.1.3 Moderate-Value Products: case (b)

Here we face an interplay between the impacts of output-dominance and subsidization, resulting in the inverted-U shape shown in Figure 1. To understand why this occurs, observe that at low $N_m$ outputs per-firm are high. Hence, output-dominance (14) initially prevails over subsidization (16). As $N_m$ increases, both effects are decreasing in absolute value, however the former declines more rapidly than the latter. In particular, since product output is more sensitive to competition than advertising output (see (12)), $x_m^* \cdot (x_m^* - y_m^{s2})$ gets very small as $N_m$ gets large. At the same time the existence of a term proportional to $N_m$ in $(2y_m^{s2} + (N_m - 1)x_m^*)\partial x_m^*/\partial N_m$, which originates from the fact that all firms in the medium are overproducing, mitigates the decline of subsidization. This results in subsidization having greater impact at higher $N_m$, yielding the inverted-U shape.

As stated in the Proposition, we need for $\gamma$ and $N_m$ to be high enough to yield the pattern above. When these parameters are small (when inter-media competition is not very intense), then $p_m^*$ is small. This is an intuitive implication since a decline in the intensity of competition from
the other medium makes the advertising business more attractive. Since \( p^*_m \) vanishes, it must also be the case that \( \frac{\partial x^*_m}{\partial N_m} \) approaches \( \frac{\partial y^2_m}{\partial N_m} \). In this case, the curve of \( |x^*_m \cdot (x^*_m - y^2_m)| \) as a function of \( N_m \) is always below that of \( |(2y^2_m + (N_m - 1)x^*_m) \cdot \frac{\partial x^*_m}{\partial N_m}| \). As the two decreasing curves do not cross-over, the inverted-U shape cannot occur.

In sum, we find that the advertising model is relatively attractive under very specific conditions. First, we need for competition within the medium to not be too intense. Second, as expected, we need the inherent value of the product to neither be too low nor too high. When this value is very low, the advertising model is relatively attractive for low levels of competition. It is only when the value is moderate, that we get the intriguing result that the advertising model is attractive when the within-industry competitive intensity is neither too low nor too high.

These results might help explain the apparent contradiction in the revenue model dynamics discussed in the Introduction. There, we noted the difference between the alternative newsweekly category as compared with the dot-com content category. Both seemed to be facing more competition. However, in the former, we observed an increased focus on advertising profits while in the latter the opposite was true. It seems reasonable to assume that the alternative newsweeklies (such as the Boston Phoenix) originally faced a relatively low degree of intra-medium competition as compared with dot-coms (such as salon.com). Thus, if we assume that both of them deliver “moderate” value, an increase in competitive intensity would explain a relative increase in advertising revenues at the newsweeklies and a decrease in advertising revenues at dot-coms.

### 3.2 Media Substitutability

The traditional model of intra-industry competition generally suggests that the lower the differentiation, or the higher the substitutability, between firms the lower are firm profits. The classic example of this result is found in standard Hotelling models. In these models, firm profits are strictly decreasing with lower transportation costs, which is a measure of substitutability. However, this is not necessarily the case in the inter-industry context that we model here (i.e., where two separate industries compete). Specifically, as the next Proposition shows, it may be the case that greater substitutability (higher \( \gamma \)) is beneficial for one industry but not the other. This will be shown to be due to the impact of substitutability when the two industries are asymmetric.
Proposition 2  \( K' \equiv 1 + N_m + N_{-m} + N_m N_{-m} (1 + \gamma^2). \) If
\[
2\gamma N_m (N_m + 1) w_m > w_{-m} K'
\]
then overall firm profits in industry \( m \) \((-m)\) are increasing (decreasing) in the degree of inter-media substitutability. Otherwise, overall firm profits in industry \( m \) are decreasing in inter-media substitutability.

So, greater substitutability, i.e. higher values of \( \gamma \), may actually help some firms. Note, by symmetry, that Proposition 2 implies that if \( 2\gamma N_{-m} (N_m + 1) w_{-m} > w_m K' \), then profits in industry \(-m\) are increasing in media substitutability and profits in industry \( m \) are decreasing. This gives rise to the following Corollary:

Corollary 1  For any given set of parameters, at most one of the two industries can have profits that are increasing in the degree of inter-media substitutability.

The intuition behind these results lies in the fact that an increase in substitutability has two primary effects on a firm in industry \( m \). First, for any given level of output \( y_{-m} \), this implies lower prices. Thus, \textit{ceteris paribus}, this leads to lower choices of \( y_m \). However, the same forces are at work in industry \(-m\). If the latter respond by choosing a lower \( y_{-m} \), then it may in fact be optimal for firms in industry \( m \) to select higher \( y_m \). That is, the two industries are facing two changes to their output (and therefore pricing) decision: a negative direct impact on price as a result of higher \( \gamma \) and a potential positive strategic impact on price if the other industry drops output. As we show in the proof to the Proposition (see Appendix), when condition (17) holds, then an increase in \( \gamma \) results in higher \( y_m^* \) and \( q_{m}^* \). As suggested by (4), \( x_m^* \) will also be higher. Thus, firms in industry \( m \) produce more products and more campaigns and sell the latter at a higher price. This results in higher profits.

The key factor that drives this result – and the reason why we don’t observe it in standard intra-industry competition settings – is that when industries \( m \) and \(-m\) are asymmetric \((N_m \neq N_{-m})\) they respond differently to the level of media substitutability.\(^{22}\) More specifically, Equation (17)\(^{22}\) To see this, assume that \( N_m = N_{-m} \equiv N \) and that \( w_m = w_{-m} \equiv w \). Then, Equation (17) becomes \( 2\gamma N (N + 1) w > w K' \). But, then, it should hold for both industries \( m \) and \(-m\) which contradicts Corollary 1. Thus, it is the asymmetry that is driving the result.
holds only when $N_m$ is high relative to $N_{-m}$ and $w_m$ is high relative to $w_{-m}$. The latter is fairly straightforward: media that attract more-valuable customers may benefit from an increase in inter-media substitutability. However, the former may be more surprising: the more competitive medium may benefit from an increase in inter-media substitutability. When $N_m \gg N_{-m}$, a firm in industry $m$ is competing directly with a lot of firms in its own industry and indirectly with a very small number of firms in the other industry. Now, if we make the media more similar in the eyes of an advertiser (letting $\gamma$ increase), then the firms in industry $m$ now face an increased threat from very few firms while those in industry $-m$ face a threat from a large number of firms. This asymmetry makes industry $-m$ more vulnerable to this change in $\gamma$.

Another necessary condition for this result is that $\gamma$ not be too small. Clearly, as $\gamma$ approaches zero, (17) cannot hold for any set of parameters. This is because when $\gamma$ is low, the two industries are already significantly isolated from each other as advertising media. When this is the case, the fact that firms in industry $-m$ may adjust their output levels in the face of an increase in substitutability will not have much of an effect on the output decisions made by firms in industry $m$. Hence, we require a sufficient level of “connectedness” between the industries for the results above to hold.

Finally, recall that $x_m^* > \bar{x}_m^*$. The firms “overproduce” products to subsidize the advertising business (see Remark 1). Given this, whenever $x_m^*$ increases, it must always be true that product profits $\Pi_m^p$ decline. This is formalized in the following Proposition.

**Corollary 2** When (17) holds, industry $m$ product profits decrease in the degree of inter-media substitutability while advertising profits increase.

This result suggests that, when the competitive intensity between industries is sufficiently asymmetric – that is, when $N_m \gg N_{-m}$ – then the relative proportion of advertising and product profits in the two industries will move in opposite directions as a function of inter-media substitutability. Specifically, an increase in substitutability will lead to higher advertising profits and lower product profits by industry $m$. The opposite will be true of industry $-m$. Thus, more inter-media substitutability may lead to a greater disparity across industries in terms of their source of profits.
3.3 Aggregate Exposure to Advertising

In previous sections we showed that firms may derive more or less profits from advertising depending, in part, on the intensity of intra- and inter-medium competition \( (N_m, N_{-m}) \) and on the degree of inter-medium substitutability \( (\gamma) \). Given the model specification, we are also able to analyze the impact of competition on the “total amount” of advertising consumers are exposed to as they consume the product. Marketers often use total consumer exposure to advertising as a measure of “clutter” (Dolan (2000)). Given that advertisers are interested in the degree of clutter when formulating their media plan (O’Guinn et al. (2000)), it is useful to understand the source of this important phenomenon. Specifically, should we expect more intra-medium competition to increase clutter, and to what extent does the answer depend on the level of inter-media competition?

We define the following:

\[
X_m \equiv N_m \cdot x_m, \\
Y_m \equiv X_m \cdot y_m.
\]  

So, \( X_m \) is the industry-level aggregate output of products and \( Y_m \) is the analogous volume of ad impressions. In our model, \( Y_m \) represents clutter. The following Proposition focuses on the impact of within-industry competition \( (N_m) \) on clutter in industry \( m \). The impact is shown to be moderated by the level of inter-media competition and substitutability \( (N_{-m} \text{ and } \gamma) \).

**Proposition 3** The total number of advertising impressions produced in industry \( m \) is increasing in \( N_m \) if and only if \( N_{-m} \text{ and } \gamma \) are high enough. Otherwise, the total number of advertising impressions is decreasing in \( N_m \).

The result is surprising as it suggests that, in some regions of the parameter space (i.e., when \( N_{-m} \text{ and } \gamma \) are “low”), an increase in the number of firms in a medium may actually decrease the aggregate number of impressions produced by all firms in that medium. To understand the intuition behind the result, first consider what happens when \( \gamma \) and \( N_{-m} \) approach 0. In this case, we can drop the subscript \( m \) for clarity. Differentiating the total number of advertising impressions with respect to the number of firms in the medium yields:

\[
\frac{\partial Y}{\partial N} = -\frac{w}{(N+1)^5} \left[ (N - 1) (N + 1)^2 v + (3N - 1) w^2 \right],
\]  

(20)
which is negative for all positive values of \( N \). Hence, when \( \gamma \) and \( N_m \) are low, \textit{increasing the number of firms decreases the overall amount of advertising produced}. This is a striking result and reinforces the idea that advertising can in some cases be extremely sensitive to competition. To see this, it is instrumental to compare the “competitive elasticities” of product and advertising volume, respectively: 

\[
\frac{\partial x}{\partial N}/x = -\frac{(N + 1)^2 v + 3w^2}{(N + 1)^3 v + (N + 1)w^2},
\]

\[
\frac{\partial xy}{\partial N}/xy = -\frac{2(N + 1)^2 v + 4w^2}{(N + 1)^3 v + (N + 1)w^2}.
\]

Both of the competitive elasticities are negative, but comparing the numerators, clearly, the advertising competitive elasticity is strictly higher (in absolute value). Industry-wide advertising volume is thus more sensitive to competition than is product volume in this context. This is because the output of advertising impressions is a \textit{compound outcome} of the number of campaigns \( y \) and the number of customers \( x \). Since both of these decrease in \( N \), advertising volumes are particularly sensitive to competition and, thus, total industry advertising volume may actually decline as competition increases.\(^{23}\)

As stated in the Proposition, however, when \( \gamma \) and \( N_m \) are high enough, we find that increased within-medium competition yields \textit{more} aggregate impressions. When \( \gamma \) and \( N_m \) were small, increases in \( N_m \) were dominated by the compounded effect of declines in \( x_m \) and \( y_m \). When \( \gamma \) and \( N_m \) are high, advertising (and product) output is far less sensitive to increased competition within the medium.\(^{24}\) Thus, decreases in \( x_m \) and \( y_m \) are dominated by the linear increase in \( N_m \).

Finally, the following Corollary applies the insights developed for Proposition 3 to develop an understanding of economy-wide clutter, \( Y_m + Y_{-m} \):

\(^{23}\)This result may have interesting welfare implications as well. For example, if one holds the view that an advertising impression is, on net, always a social “good,” (see, for example Stegeman (1991) and Dukes (2002)), then this suggests that unlimited competition within a medium may not be socially optimal. If the specific medium in question is somewhat unique in terms of the value it offers advertisers (i.e., \( \gamma \) is low), then higher levels of competition may decrease the number of impressions produced.

\(^{24}\)To see this mathematically, note that 

\[
\frac{\partial^2 y}{\partial N_m \partial N_{-m}} \xrightarrow{\gamma \rightarrow 1} K_1 \cdot [w_m + w_{-m} + (w_m - w_{-m})(N_{-m} - N_m)]
\]

which, by the assumption that \( w_m \) and \( w_{-m} \) are not “too different,” is positive. Thus, the negative slope of \( y_m \) in \( N_m \) flattens out at high \( \gamma \) and \( N_{-m} \).
Corollary 3  A necessary condition for economy-wide advertising clutter to increase as the number of media firms increases is that $\gamma$ is large enough.

We interpret this as suggesting that the advertising clutter that we seem to observe is due in large part to the proliferation of relatively substitutable media. The simple increase in the number of firms within medium does not, on its own, result in increased clutter.

4 Advertising Disutility

Thus far, we have assumed that firms choose the number of ad campaigns to which each customer is exposed ($y_{mi}$) so as to maximize total profits. We have not, however, considered the potentially-negative impact that advertising can have on customers while they are consuming the product. It is often the case that customers find exposure to high levels of advertising to be distracting and annoying, even though the ads may have informational value (Sumner, 2001; Dukes and Gal-Or, 2003). For example, consumers’ annoyance with TV advertising is one principal driver of growth for products developed by TiVo and Microsoft that allow viewers to skip television ads (Wathieu, 2001). Similarly, it is becoming increasingly popular for ISPs as well as third-party developers to offer software that disables certain types of online ads (such as pop-ups). This issue has become especially acute with the advent of increasingly intrusive “rich media” online advertising formats.

Given that advertising has the potential to create customer disutility, which can vary by medium (Sumner, 2001), it would seem important to incorporate this negative aspect into our model and investigate its impact. In order to do so, we need the product demand function to reflect lower utility to consumers as the number of campaigns they have to endure goes up. Formally, we modify (1) as follows:

\begin{equation}
    p_{mi} = v_m - \alpha_m y_{mi} - \sum_{i=1}^{N_m} x_{mi},
\end{equation}

where $0 \leq \alpha_m < \pi_m$ is the advertising disutility parameter in industry $m$. Thus, the higher $\alpha_m$, the more a customer is “annoyed” with having to be exposed to each ad campaign. The expression $\overline{a_m} = w_m - \gamma N_{-m} (w_{-m} - a_{-m})/(N_{-m} + 1)$ is an upper bound on $a_m$ to ensure that $y_{mi} \geq 0$. Our assumption that $\alpha_m \geq 0$ implies that we do not consider situations where consumption of the product or service is secondary to the individual’s desire to view ads. This may arise, for example, with viewership of the Super-Bowl (where companies
for price in the advertising market remains the same as in (2). We’d like to understand how firms’ actions and profits in each market are affected by the extent to which consumers experience disutility from advertising as they consume the product. For convenience, we define \( \hat{K}_m \equiv (N_m^2 + 1)(N_m - 1) - \gamma^2 N_m N_m (N_m + 1) \). Solving for the equilibrium in the same fashion as we did in Section 3, we obtain:

**Proposition 4** The equilibrium output of firms in industry \( m \), both in the product and advertising markets, decreases as the level of advertising disutility \( \alpha_m \) increases. Furthermore, as \( \alpha_m \) increases, prices in the advertising market increase while prices in the product market initially decrease (whenever \( \hat{K}_m > 0 \)) and subsequently increase.

Advertising disutility per-ad \( \alpha_m \) has two effects on industry \( m \) firms in the product market. First, it directly lowers the “inherent value” generated to consumers from product consumption (much like a decrease in \( v_m \) itself). Holding number of ad campaigns constant, this effect clearly lowers product price. However, an increase in \( \alpha_m \) also makes each firm want to reduce the number of advertising campaigns \( y_m \). This, in turn, lowers the incentive to “overproduce” products in order to subsidize the advertising market. This causes \( x_m^* \) to decrease, but tends to increase prices. For small values of \( \alpha_m \) (and when \( \hat{K}_m > 0 \)) the inherent value effect dominates and prices in the product market decrease, but for larger values of \( \alpha_m \) less subsidization dominates yielding a U-shaped pattern (if instead \( \hat{K}_m \leq 0 \), which only occurs if \( N_m \ll N_m \) and \( \gamma \to 1 \), the lowered subsidization effect dominates throughout, and prices will be monotonically increasing in the disutility level). Given the lowered number of advertising campaigns allotted, the price to each advertiser is strictly increasing as the disutility parameter increases. In particular, the equilibrium solution for \( y_m^* \) is similar to that in (4), except that \( C_m \) is now replaced with \( \hat{C}_m = (w_m - \alpha_m) + N_m ((w_m - \alpha_m) - \gamma (w_m - \alpha_m)) \). Thus, interestingly, the impact of advertising disutility in the product market on equilibrium advertising output is equivalent to a decrease in the inherent value of a customer impression to the advertiser.

Next, we examine the impact of advertising disutility on the profits in each market. Not surprisingly, the negative feedback of advertising on product demand reduces total profits \( (\Pi_m^*) \).
However, the impact on profitability in each of the separate markets can exhibit a strikingly different pattern, as stated in the following Corollary.

**Corollary 4** When the degree of inter-media substitutability \((\gamma)\) is not too close to 1, then as the level of advertising disutility in industry \(m\) increases, advertising profits in that industry initially increase and subsequently decrease while product profits initially decrease and subsequently increase.

The intuition behind the above Corollary follows from the results of Proposition 4. Consider first the advertising market. Here output \((y^*_m)\) monotonically decreases as advertising disutility \(\left(\alpha_m\right)\) increases. The resulting increase in ad prices initially brings firms in industry \(m\) closer to joint maximum achievable profits (unless \(\gamma\) is very close to 1). However, when \(\alpha_m\) is high, the lowered levels of \(y^*_m\) cause \(\Pi^*_m\) to decrease, yielding an overall inverse U-shaped pattern. In the product market the reverse pattern occurs. Initially, the “inherent value” effect discussed above tends to lower product price while the subsidization effect lowers output, causing product profits to fall as advertising disutility increases. However, at higher levels of marginal disutility, the reduced level of output allows prices to increase (while at the same time the sharp decrease in \(y^*_m\) softens the negative “inherent value” effect). This leads to a U-shaped pattern of profits in the product market. A direct implication of the above Corollary is that the difference between advertising profits and product profits \((\Delta_m)\) is initially increasing in the degree of disutility. As such, and somewhat counterintuitively, small amounts of disutility may actually make the advertising model more attractive relative to the product model.

These results have several interesting managerial implications. Consider, for example, the recent licensing agreement between Yahoo! and four rich media advertising technology firms.\(^{26}\) The ad formats these four firms enable have been noted by several industry analysts to be intrusive and have the potential to cause customer dissatisfaction with the host site - corresponding in our model to an increase in \(\alpha_m\). In this respect, it may be the case that Yahoo’s profits from advertising will increase relative to the context in which they served only static banner ads (consistent with Corollary 4). By contrast, iVillage Inc. conducted internal research revealing that over 90% of

\(^{26}\)Source: *The Wall Street Journal*, July 15, 2002. The four firms are: Eyeblaster, Unicast, Point Roll and Eyewonder. Rich media capabilities enable the advertiser to serve dynamic, eye-grabbing ads that, for example, can float across the screen, play streaming video, or ‘come to life’ as the mouse rolls over them.
its readers listed pop-up ads as the most frustrating online advertising method. This has led the company to publicly announce intentions to stop or limit serving this specific ad format in the future.27

5 Conclusion

In this paper we have presented an analysis of competition within and between media. The core question that we have posed is whether and when will a firm earn more money from the sale of advertising than from the sale of the underlying product itself. In investigating this decision, we have focused on the intensity of competition faced by the firm, taking into account the inherent value of the product to consumers. Not surprisingly, when the inherent value of the product is high (low), firms in that industry tend to earn profits mainly from the product (advertising) market. When the inherent value of the product is moderate, the firm will again optimally earn more from the sale of products if the within-industry competitive intensity is either very high or very low. It is only in the middle range of competition that the advertising model can be more attractive. The intuition we offer for the behavior of relative profitability source is different at each end of the competitive spectrum. When there is a lot of competition, both advertising profits and product profits suffer. However, the decrease in product output per-firm reduces the extent to which the product business subsidizes the advertising business and has a (partial) positive impact on product profits. Moreover, it has a negative impact on advertising profits since this leads to fewer customers whose attention the firm can sell. This results in advertising profits that fall more precipitously than product profits. On the other hand, when competition is fairly low, the advertising model may become more and more attractive as competition grows. This is because the primary effect in this region comes via the price drops that occur as new firms enter. Since product output is much higher relative to (the square of) advertising output, these price declines have a bigger (negative) impact on product profits than on advertising profits. Thus, the advertising business becomes more attractive.

In addition to this analysis of intra-medium competition, we also looked at the nature of inter-medium substitutability. Interestingly, we found that some firms may actually benefit from more

intense competition between media, for example, between radio and television. In particular, the more competitive of the two industries may find its overall profits increase as a result of this increase in media substitutability. Using the same model, we showed that as different media compete more head-to-head for advertising dollars, their source of profits – the extent to which they are primarily advertising-driven or product-driven – will diverge. One will tend more towards an advertising model while the other more towards a product model. This is consistent with the empirical findings of Liebowitz (1982). We also investigated the impact of competition on the total amount of advertising impressions that consumers may be subjected to, and identified conditions for the total number of impressions to actually decrease in the level of within-industry competition. With more firms, fewer overall impressions (or clutter) may actually be produced.

Finally, it is important in any study of advertising media to consider the fact that product consumers may consider intrusive the advertising that they are subjected to. By incorporating this characteristic into our model, we were able to analyze the impact of such disutility or “externality” on the firms’ optimal decisions. Interestingly, we found that in some cases low levels of advertising disutility actually increase the proportion of profits that the firm earns from advertising as compared with products. Essentially, while volumes of both advertising and products decline as a result of more ad disutility, product output decline is reinforced by a decline in product prices as well. This is brought on directly by the decrease in the product’s effective value in the eyes of customers. On the other hand, the decline in advertising volume is offset by the increase in ad prices.

The analysis presented in this paper is relevant for many industries where firms wrestle with the decision of which business model to adopt. The focus has been on understanding the drivers behind the relative source of firm profits, given the ability to leverage demand generated in one market to interested parties in a second market. This issue has recently been the center of much debate at Internet-related companies. Some, like Yahoo! have made a conscious decision to try and boost non-advertising profits by charging for content and services. Others, are still hoping for the advertising model to pan out. Interestingly enough, and in part due to the disutility many consumers experience from the onslaught of annoying Internet advertising formats, salon.com has decided to “let the customer decide”. The company recently began offering consumers the option to join its premium service that provides ad-free content at a cost of $30 per year vs. the option to
earn temporary access to the subscription-based content by having to click through ads. Future research may indeed extend the analysis we have presented by allowing consumers explicit choice between such options. We also point out that though we have focused primarily on media, the central issues addressed in the paper may apply to other industries where greater levels of product use can be leveraged for advertising sales. For example, a bus company charges consumers for the transportation service provided but also has the option of allowing advertisements both in and on the bus. While the latter type of ads would require some modifications to our model (as the consumers purchasing the product are not the same as those exposed to the ads), we believe that the nature of our analysis is relevant in such contexts as well.

Appendix

Proof of Proposition 1:

\[ \Delta_m = \frac{\left[ v_m + \left( \frac{C_m}{K} \right)^2 \right] \left( 2N_m + 1 \right) \left( \frac{C_m}{K} \right)^2 - v_m}{(N_m + 1)^2}. \]  

(A1)

So, we know that \( sgn(\Delta_m) = sgn\left( (2N_m + 1) \left( \frac{C_m}{K} \right)^2 - v_m \right) \). Since \( K \) is linear in \( N_m \) and \( C_m \) is not a function of \( N_m \), we know that there are at most two real values of \( N_m \in \mathbb{R}^+ \) such that \( (2N_m + 1) \left( \frac{C_m}{K} \right)^2 = v_m \). Since \( (2N_m + 1) \left( \frac{C_m}{K} \right)^2 \) vanishes as \( N_m \to \infty \), we know that \( sgn\left( (2N_m + 1) \left( \frac{C_m}{K} \right)^2 - v_m \right) \) is always negative for high values of \( N_m \). When \( N_m = 1 \), \( (2N_m + 1) \left( \frac{C_m}{K} \right)^2 = 3 \left( \frac{C_m}{2+2N_m(2-\gamma^2)} \right)^2 = v_m \).

So, if \( v_m \) is less than this, we know that there exists a \( N_m \) such that \( \Delta_m < 0 \) for all \( N_m > N_m \) and positive otherwise (resulting in case (a) of the Proposition). On the other hand, if \( 3 \left( \frac{C_m}{2+2N_m(2-\gamma^2)} \right)^2 < v_m \), then \( \Delta_m < 0 \) at \( N_m = 1 \). We now determine there exists a “middle” region in which \( \Delta_m > 0 \). By straight differentiation, we find that \( \frac{\partial}{\partial N_m} \left( (2N_m + 1) \left( \frac{C_m}{K} \right)^2 \right) = \frac{2C_m^2}{1+(1-\gamma^2)N_m} \). The term in brackets is positive for all \( N_m < \frac{\gamma^2 N_m}{1+(1-\gamma^2)N_m} \). The first thing to notice is that both \( \gamma \) and \( N_m \) must be large enough for this middle section to exist. In the limit as \( \gamma \to 0 \), \( (2N_m + 1) \left( \frac{C_m}{K} \right)^2 \) is strictly decreasing and thus the inverted U-shape cannot exist. The same is true as \( N_m \) vanishes. Moreover, this derivative implies that \( (2N_m + 1) \left( \frac{C_m}{K} \right)^2 \) is maximized at \( N_m = \frac{\gamma^2 N_m}{1+(1-\gamma^2)N_m} \). This maximum value is \( \frac{C_m^2}{1+N_m(2+1)N_m} > 0 \). Thus, if \( v_m < \frac{C_m^2}{1+N_m(2+1-\gamma)N_m} = \overline{v}_m \), there exists a region in which \( \Delta_m > 0 \). If, on the other hand, \( v_m > \frac{C_m^2}{1+N_m(2+1-\gamma)N_m} \), it is always the case that \( \Delta_m < 0 \) (resulting in case (c) of the Proposition). So, for \( 3 \left( \frac{C_m}{2+2N_m(2-\gamma)} \right)^2 < v_m < \frac{C_m^2}{1+N_m(2+1-\gamma)N_m} \), there exist a pair \( \{N_m, \overline{N}_m\} \) such that \( \Delta_m < 0 \) for \( N_m < \overline{N}_m \) and \( N_m > N_m \), while \( \Delta_m > 0 \) in the middle (resulting in case (b) of the Proposition). Two final details remain. First, we want to show that \( \overline{v}_m > \underline{v}_m \). Note that

\[ \frac{C_m^2}{1+N_m(2+1-\gamma)N_m} - 3 \left( \frac{C_m}{2+2N_m(2-\gamma)} \right)^2 = \frac{C_m^2}{1+N_m(2+1-\gamma)N_m} \cdot \left[ \frac{3+2\gamma^2-2N_m-4\gamma^2(N_m+1)+3(N_m+2)}{N_m(\gamma^2-2)-2} \right] \frac{C_m}{1+N_m(2+1-\gamma)N_m}. \]

Both terms in the denominator are positive. To see that the numerator is positive, note that it is decreasing in \( \gamma \) so that

\[ 3 + N_m \left( 2\gamma^4 N_m - 4\gamma^2 (N_m + 1) + 3(N_m + 2) \right) > 3 + N_m (N_m + 4) > 0. \]

So, the difference is positive, implying that there always exists a \( v_m \in \left( 3 \left( \frac{C_m}{2+2N_m(2-\gamma)} \right)^2, \frac{C_m^2}{1+N_m(2+1-\gamma)N_m} \right) \). To ensure case (a) of the proposition, define \( \overline{v}_m \) such that for all \( v_m > \overline{v}_m \) prices in the product market are strictly positive. Thus, we want to show that there are conditions under which \( \overline{v}_m > \underline{v}_m \) so that the region in which \( v_m < \underline{v}_m \) can still exist with positive prices. Substituting in the expressions for \( \overline{v}_m \) and \( \underline{v}_m \), this
requires that $3 \left( \frac{C_m}{2 + N_{-m}(2 - \gamma)} \right)^2 > N_m (C_m/K)^2$ or $\left( \frac{\sqrt{3}}{2 + N_{-m}(2 - \gamma)} \right)^2 > (\sqrt{N_m}/K)^2$. Since both terms inside the parentheses are positive, we will drop the quadratic. Further, we’ll look at the case in which $N_{-m} = 1$ and $\gamma = 1$. Then, this condition becomes $\frac{\sqrt{3}}{3} = .58 > \frac{\sqrt{N_m}}{2 + N_m}$. The RHS of this inequality reaches a maximum value of .35 at $N_m = 2$, so we’re done.

**Complete Delineation of the Direct and Strategic Effects of Within-Industry Competition on Profits.**

Our intuition for the main results in Proposition 1 are related to the differences in direct and subset of relevant strategic effects. For completeness we provide the entire decomposition of firm profits in each market in terms of these two types of effects. In the product market:

$$\frac{d\Pi^*_m}{dN_m} = \frac{\partial \Pi^*_m}{\partial N_m} < 0 + \frac{\partial \Pi^*_m}{\partial x_{mi}} < 0 \cdot \frac{\partial x^*_m}{\partial N_m} + (N_m - 1) \frac{\partial \Pi^*_m}{\partial x_{mj}} < 0 \cdot \frac{\partial x^*_m}{\partial N_m}, \forall j \neq i. \tag{A2}$$

The first term on the RHS of (A2) is the direct effect of within-industry competition on product profits (see also (13)) and is negative. The second term is a strategic effect related to own output change, while the third term is an additional strategic effect arising from the symmetric response of all other firms in industry $m$ to the change in $N_m$. These two latter terms are relevant for capturing the impact of subsidization on product market profits (see also (15)). As for the advertising market:

$$\frac{d\Pi^*_a}{dN_m} = \frac{\partial \Pi^*_a}{\partial N_m} < 0 + \frac{\partial \Pi^*_a}{\partial x_{mi}} > 0 \cdot \frac{\partial x^*_m}{\partial N_m} + (N_m - 1) \frac{\partial \Pi^*_a}{\partial y_{mj}} < 0 \cdot \frac{\partial y^*_m}{\partial N_m} + N_{-m} \frac{\partial \Pi^*_a}{\partial y_{-m}} < 0 \cdot \frac{\partial y^*_m}{\partial N_m}, \forall j \neq i. \tag{A3}$$

The first term on the RHS of (A3) is again the direct effect of within-industry competition on advertising profits (see also (13)) and is negative. The second term is a cross-market strategic effect related to how a decrease in product subsidization impacts advertising profits, and is negative (see also (15)). The third term is a strategic effect arising from the symmetric response of all other firms in industry $m$ to the change in $N_m$, while the fourth term is an additional strategic effect arising from the symmetric response of firms in industry $-m$ to the change in $N_{-m}$. Note that in the advertising market $\frac{\partial \Pi^*_a}{\partial y_{mi}} = 0$ in equilibrium (unlike in the product market). The formal proof of Proposition 1, as given above, of course reflects each of these terms. However, in expressing the intuition behind $\frac{\partial \Delta_m}{\partial N_m}$, it’s useful to focus on the direct effects and a subset of the strategic terms. These best explain the fundamental qualitative behavior of $\frac{\partial \Delta_m}{\partial N_m}$ and the unique aspects of media markets.
Proof of Proposition 2: Begin with straightforward differentiation:

$$\frac{\partial \Pi_m^*}{\partial \gamma} = 4N_mC_m \frac{(2\gamma N_m (N_m + 1) w_m - w_{-m}K') (K^2v_m + C_m^2)}{(N_m + 1)^2 K^5}.$$  \hfill (A4)

Since $K$ is positive and $C_m$ is assumed to be positive, this is positive if Equation (17) holds. Assume that $\frac{\partial \Pi_a}{\partial \gamma} > 0$ and $\frac{\partial \Pi_p}{\partial \gamma} > 0$. The former implies that

$$2\gamma N_m (N_m + 1) w_m - w_{-m}K' > 0$$

which implies that

$$w_m \left[ 2\gamma N_m (N_m + 1) \frac{w_{-m} - K'}{w_m} \right] < w_m \left[ \frac{4\gamma^2 N_m N_{-m} (N_m + 1)(N_m + 1)}{K'} - K' \right] = \frac{w_m}{K'} \left[ 4\gamma^2 N_m N_{-m} (N_m + 1)(N_m + 1) - K'^2 \right] = -\frac{w_m K^2}{K'} < 0,$$

which contradicts the premise that $\frac{\partial \Pi_p}{\partial \gamma} > 0$.

Proof of Corollary 1: Straightforward from above.

Proof of Corollary 2: Again, by differentiation:

$$\frac{\partial \Pi_a}{\partial \gamma} = 2N_mC_m \frac{[2\gamma N_m (N_m + 1) w_m - w_{-m}K''] [K^2v_m + 2C_m^2]}{(N_m + 1) K^5}$$

$$\frac{\partial \Pi_p}{\partial \gamma} = 2N_mC_m \frac{[w_{-m}K'' - 2\gamma N_m (N_m + 1) w_m] [(N_m - 1) K^2v_m + 2N_mC_m^2]}{(N_m + 1)^2 K^5}.$$

Clearly, the signs of the two derivatives are always opposite each other: if advertising profits increase, then product profits decrease and vice versa. So, we’re done. When Equation (17) holds, we know that $\frac{\partial \Pi_a}{\partial \gamma} > 0$ and that, therefore, $\frac{\partial \Pi_p}{\partial \gamma} < 0$.

Proof of Proposition 3: Using the equilibrium values,

$$\frac{\partial Y_m}{\partial N_m} = K' \cdot \{ K^2 \left[ 1 - N_m^2 + N_{-m} \left( 1 - (1 - \gamma^2) N_m^2 \right) \right] v_m +$$

$$C_m^2 \left[ 1 - N_m (2 + 3N_m) + N_{-m} \left( 1 - (1 - \gamma^2) N_m (2 + 3N_m) \right) \right] \}.$$  \hfill (A8)
where \( K'' = \frac{C_m}{(N_m+1)^2K^2} > 0 \). First, note that as \( N_m \to 0 \), \( \frac{\partial Y_m}{\partial N_m} \to K'' \{ K^2 [1 - N_m^2] v_m + C_m [1 - N_m (2 + 3N_m)] \} < 0 \). Similarly, note that as \( \gamma \to 0 \), \( \frac{\partial Y_m}{\partial N_m} \to K'' \{ K^2 [1 - N_m] + N_m \} v_m + C_m [1 - N_m (2 + 3N_m)] [1 + N_m] \} < 0 \). Now, let \( \gamma \to 1 \). Then, \( \frac{\partial Y_m}{\partial N_m} \to K'' \{ K^2 [1 - N_m^2 + N_m] v_m + C_m [1 - N_m (2 + 3N_m) + N_m] \} \), which is positive for high enough \( N_m \).

**Proof of Corollary 3:** By Proposition 3, we know that \( Y_m \) will decrease in \( N_m \) when \( \gamma \) is low. Moreover, \( \frac{\partial y_m}{\partial N_m} = -\frac{\gamma C_m}{K^2} \). Thus, \( Y_m \) is always decreasing in \( N_m \).

**Proof of Proposition 4:** Solving for the equilibrium when prices in the product market are determined as in (22) and prices in the advertising market are as in (2) yields a solution similar to (4) with the exception that we replace \( C_m \) with \( \hat{C}_m = (w_m - \alpha_m) + N_m ((w_m - \alpha_m) - \gamma (w_m - \alpha_m)) \), which has to be positive. Then, differentiating the equilibrium expressions for output in each market in media \( m \) with respect to \( \alpha_m \) we get:

\[
\begin{align*}
\frac{\partial y_m^*}{\partial \alpha_m} &= -\frac{N_m - N_m^m}{(N_m+1)^2} \gamma N_m N_m = -\frac{N_m + 1}{K} < 0 \\
x_m^* &= \frac{v_m + (y_m^*)^2}{(N_m+1)} \frac{\partial x_m^*}{\partial \alpha_m} = 2 \frac{y_m^*}{(N_m+1)} \frac{\partial y_m^*}{\partial \alpha_m} < 0
\end{align*}
\]  \hspace{1cm} (A9)

As for prices in each market:

\[
\begin{align*}
\frac{\partial \hat{p}_m}{\partial \alpha_m} &= N_m \frac{\partial y_m^*}{\partial \alpha_m} - \gamma N_m \frac{\partial y_m^*}{\partial \alpha_m} \frac{N_m N_m (1 - \gamma^2) + N_m}{K} > 0. \\
\frac{\partial \hat{p}_m}{\partial \alpha_m} &= -y_m^* (1 + \frac{2N_m}{(N_m+1) \frac{\partial y_m^*}{\partial \alpha_m}}) - \frac{\partial y_m^*}{\partial \alpha_m} \\
&= \hat{C}_m - \frac{(N_m^2 + 1) (N_m + 1) + \gamma^2 N_m N_m (N_m + 1) + \alpha_m (N_m + 1)}{(N_m+1)^2 K^2}. \\
\end{align*}
\]  \hspace{1cm} (A10)

Letting \( \alpha_m \to 0 \), \( sgn \left[ \frac{\partial \hat{p}_m}{\partial \alpha_m} \right] \to sgn \left[ -(N_m^2 + 1) (N_m + 1) + \gamma^2 N_m N_m (N_m + 1) \right] = sgn \left[ -\hat{K}_m \right]. \)

For greater values of \( \alpha_m \), \( sign \left[ \frac{\partial \hat{p}_m}{\partial \alpha_m} \right] \) is obviously positive (this can most easily seen from the penultimate step in (A10), since \( y_m^* \to 0 \) as \( \alpha_m \) increases, while \( \frac{\partial y_m^*}{\partial \alpha_m} > 0 \) and is constant see (A9)).

**Proof of Corollary 4:** Differentiating product market profits in medium \( m \) (and using the chain rule) we get:

\[
\frac{\partial (x_m p_m^*)}{\partial \alpha_m} = p_m \frac{\partial x_m^*}{\partial \alpha_m} + x_m^* \frac{\partial p_m^*}{\partial \alpha_m}.
\]  \hspace{1cm} (A11)

From the previous proof, the first term on the RHS is always negative while the second term is initially decreasing but is subsequently increasing in the disutility parameter, eclipsing the first term when \( a_m \) is
high enough.\(^{29}\) (To see why the entire derivative will subsequently be increasing in \(a_m\), note that the RHS of (A11) can be written as:

\[y_m^*(\frac{2}{N_m+1} \partial y_m^* (p_m^* - N_m x_m^*) - x_m^*) - a_m x_m^* \partial y_m^*\]

As \(a_m \to a_m\), \(y_m^* \to 0\) while \((-a_m x_m^* \partial y_m^*)\) will remain positive).

Differentiating advertising profits in medium \(m\) and letting \(\gamma \to 0\):

\[
\left[ \frac{\partial (y_m^* x_m^* q_m^*)}{\partial \alpha_m} \right]_{\alpha_m=0} = \frac{w_m((N_m-1)(N_m+1)^2v_m+(N_m-3)w_m^2)}{(N_m+1)^5},
\]

which is positive for \(N_m \geq 2\) (this is trivial for \(N_m \geq 3\) and holds for \(N_m = 2\) from the condition that \(p_m^* \geq 0\)). When \(\alpha_m\) becomes larger, since \(y_m^* \to 0\) rapidly (and \(x_m^*\) is decreasing as well), advertising profits will ultimately decrease.

\(^{29}\)A sufficient condition for this is \(\gamma < \sqrt{\frac{(N_m+1)(N_m-1)}{N_m(N_m-3)w_m^2}}\).
References


