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Trade Promotions of Consumer Durables

Abstract:

We study trade promotions for durable goods such as automobiles in which the manufacturers provide special incentives to their dealers for exceeding specific sales targets. We develop a theoretical model of consumer, retailer and manufacturer behavior based on observations about key aspects of the automobile market. Our analysis provides interesting insights about inter-temporal effects of trade promotions as well as the effect of product durability on the promotion strategies of manufacturers. For example, manufacturers of more durable products benefit more from running trade promotions and give deeper discounts. When we test these theoretical results, we find empirical support.
INTRODUCTION

Toyota Camry offered performance-based trade promotion to its dealers for 293 days over a two-year period in 1996-1997. During the same period, Chevy Lumina did not offer this type promotion at all to its dealers. Both cars are top sellers in the mid-range market segment. Why do we observe these different emphases on trade promotions? More generally, what is the economic reasoning behind trade promotions for a durable such as cars?

The specific type of trade promotion that we study is one in which the manufacturer sets a sales quantity target for the dealer and offers a per-unit reward (or reduced wholesale price) for exceeding the specified sales level. Our interest in such trade promotions within the automobile industry is driven by two factors. First, the automobile industry and trade promotions within the industry represent a significant part of our economy. Thus, retail sales associated with this product category exceeds $30 billion dollars per year in the United States alone while trade promotions (i.e., discounts given to dealers) often are in the excess of $1000 per automobile sold. Second, in contrast to the consumer packaged goods (CPG) industry, where trade discounts are normally based on wholesale sales/volume, the vast majority of trade promotions in the automobile industry are based on retail sales performance.\(^1\) This difference is due in large part to the fact that unlike CPG manufacturers, auto manufacturers can verify easily that their dealer has sold a specified quantity of cars above some fixed level (normally the dealer’s quota). Since manufacturers can verify performance, automobile dealers have no incentive to forward buy just to receive a discount. This is in contrast to most CPG trade promotions where forward buying is a common practice.

\(^1\) We acknowledge that some CPG trade promotions are performance based. These promotions are often referred to as scan-backs. See Neslin (2002) for a brief review of trade promotion literature.
We believe these differences in how trade promotions are implemented call for a more
detailed study of trade promotions within a durable product market. Thus, even though there are
numerous theories and empirical studies on trade promotions, none capture the institutional
aspects associated with a durable industry having an active secondary market and the existence
of performance-based promotions. Instead, most trade promotion models assume a) the retailers
can forward buy, b) the manufacturers are not able to easily monitor the retailer’s sales and c) the
product being sold is a non durable and thus there is no secondary market for the used product.
In addition, most trade promotion models are aimed at situations where the retailer is able to
carry multiple products produced by competing manufacturers. In contrast, automobiles (and
many other durables) are sold through franchise dealers who can carry and display only one
manufacturer’s line.  

Relationship With Prior Literature

Our work is related to three broad streams of research, these being trade promotions,
channel coordination and the marketing of durable goods. We briefly review each next.

Research on trade promotions has traditionally focused on pass through rates and forward
buying related issues. For example, Curhan and Kopp 1986, Walters 1989, Murry and Heide
1998, and Tyagi 1999 study factors affecting the likelihood that the retailer passes through the
trade deal to the consumer. Lal, Little and Vilas-Boas (1996) analyze manufacturers’ incentives
to offer trade promotions in the presence of pass through and forward buying problems. Kim
and Staelin (1999) show that manufacturers find it in their best interest to provide mass
merchandisers trade discounts even when they know the retailers will not pass through all of the

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2 Some auto dealers sell multiple brands. However, in the vast majority of cases, either these multiple brands are
brands that are considered non-competing by the manufacturers or these brands are sold by different retail entities
trade promotion to consumers. Lal and Villas-Boas (1998) study trade promotions with multi-
product retailers, and characterize the dependence among promotions of different brands. Rao,
Arjunji and Murthi (1995) show that for grocery products, trade promotions across
manufacturers may be independent. In a paper dealing with performance-based trade promotions
in a CPG market, Bell and Dreze (2003) analyze scan-back trade deals and study the effect of
scan-backs on excess ordering by retailers and the level of sales increase resulting from scan-
backs. Each of the above papers studies CPG markets and pass through rates. In contrast, we
are interested in a type of promotion found in durable goods markets where retailers are
rewarded for achieving specific sales targets. Moreover, our primary interest is in inter-firm
differences, especially those concerned with product durability.

The channel coordination literature has focused on ways to alleviate the adverse effects
on total channel profits of having the retailer and the manufacturer making independent
marketing mix decisions. A specific instance of this problem, the double marginalization
problem, occurs when the retailer chooses retail prices independent of the manufacturer and the
retail prices end up being too high (Spengler 1950). Some papers in this literature (e.g., Jeuland
and Shugan 1983) examine contractual remedies while others (e.g., McGuire and Staelin 1983,
Trivedi 1998) study structural remedies. Probably the most relevant of these remedies to our
situation of interest is quantity discounts (Jeuland and Shugan 1983, McGuire and Staelin 1986).
Jeuland and Shugan show it is possible for a monopolist manufacturer to design a quantity
discount schedule for its monopolist retailer to perfectly alleviate the double marginalization
problem. However, when McGuire and Staelin (1986) extended this formulation to a situation
where there are two competing manufacturers, they find it is not always optimal for these
competing manufacturers to use quantity discounts (or in fact any two-part tariff). This same

(i.e., the dealer owns multiple retail outlets).

3
conclusion is reached by Ingene and Perry (2000) when they analyzed a monopoly manufacturer distributing its products via competing retailers. These latter two studies provide a cautionary warning not to directly apply the intuition coming from the quantity discount literature to understand the use of performance based trade promotions in a competitive durable product industry.

Two other channel coordination research that have some relevance to our problem use either a principal-agent formulation (see, for example, Lal 1990) or rely on quantity forcing. However, the former literature does not deal with durable goods and product durability related issues and the latter does not allow voluntary participation on part of the retailer.

The third related literature, the durable goods literature, focuses on the time consistency problem (Coase 1972, Stokey 1981, Bulow 1982). A small number of papers in this stream (e.g., Purohit and Staelin 1994, Purohit 1997, Desai, et al 2004) analyzes marketing channel related issues for durable goods. Our formulation is conceptually similar to the models used in these papers and to the models used in Desai and Purohit (1998) and Desai (2001), since all stress that consumers anticipate future events and this anticipation can influence the outcomes in the current period. However, none of these papers have studied trade promotions.

In summary we know from the channel coordination literature that quantity discounts are sometimes useful in solving the double marginalization problem, since they get the retailers to buy more than they would have if they were operating independently. Conversely we know from the durable goods literature that increasing first period sales (with trade promotions) can adversely affect short and long term prices and profits via the effects these increases sales have on the used product market. More generally, although numerous studies have looked at related
The goal of this paper is to understand why differences across manufacturers, specifically differences in product durability, affect trade promotion decisions. We do this by first developing a model of consumer behavior that captures the key underlying factors of a durable market. Then using game theoretic analyses and our model of consumer behavior, we derive equilibrium prices, quantities and trade discounts as a function of product durability for a market where manufacturers distribute their horizontally and vertically differentiated durables through private franchised dealers. We find that unlike McGuire and Staelin (1986), no matter how intense the inter-brand competition, manufacturers will give trade promotions if there are no administrative costs associated with the promotion. We also find that manufacturers selling products that hold their intrinsic value longer benefit more from running performance-based trade promotions than those selling less durable products. Moreover they give deeper discounts.

We also subject some of our analytic results to empirical tests using a unique proprietary database covering the period 1992-1997.

**MODEL DEVELOPMENT**

**Overview of Approach**

We want our model to capture the key observed aspects of the automobile market. This leads us to assume a market composed of multiple manufacturers each of whom distributes their cars through exclusive, but competing, franchised dealerships. By including multiple players at both channel levels, we allow for competition in both the retail and wholesale markets. We make this assumption since the results of prior analyses differ depending on whether or not there is competition in the model. We assume that the manufacturers position their offering to appeal
to those consumers who particularly value their offering thereby creating real and perceived product differences. We also assume that the cars they sell, like most durables, deteriorate over time and have finite lives. Moreover, this rate of deterioration can differ across manufacturers.

We use a parsimonious representation of competition by modeling two competing manufacturers, each selling one differentiated product through exclusive, but competing franchised dealers. As in Bulow (1982), we capture the fact that cars deteriorate by postulating a two-period model.³ We assume that a car purchased in the first period is ‘new’ and only becomes used in the second period. Moreover, by the end of the second period, this used car is assumed to deteriorate enough that the car’s economic value is zero. Finally, we assume there are no used cars at the beginning of the first period.⁴

Our overarching goal is to investigate the impact of trade promotions on manufacturers’ and retailers’ prices and profits over a wide range of product quality and market condition situations. We do this via a series of game theoretic analyses. Such analyses require the specification of demand functions for the available products for both periods. Moreover, we want these demand functions to reflect a market that reflects the aggregate effects of number of different buyer behavior patterns. We choose to represent a market where some consumers buy new cars in each period (trading in the old car when they buy the new car), some buy a new car and then hold on to until its economic life is zero, some only buy used cars and some never buy either a new or used car. In order to specify a logically consistent set of demand functions representing such a market, we derive these demand functions by assuming consumers exhibit

³ The reader might ask why we only use two periods when in fact the automobile market clearly exists for longer than these two periods. However, our main interest is in determining the effects of a performance based trade promotions on this durable market. Any long term effects of a trade promotion offered in the first period can be captured in the second period. Thus, adding more periods should not change our overall general insights.

⁴ Adding a supply of used cars in the first period should not affect our results since these used cars, by assumption, would have no economic life by the second period and thus not compete in the new car market in the second period.
utility maximizing behavior. This requires us to make a series of assumptions about how the consumers value the available products and the heterogeneity of the level of this valuation across the population of consumers. We then analyze consumers’ optimal choices for a given set of prices and this analysis gives us the demand for products for a given set of prices. We use this route rather than assuming reduced-form demand functions to ensure that we are holding fixed the underlying situation (e.g., same number of potential buyers, choice behavior, etc.) when we explore different situations by varying the underlying model parameters.

Model Structure

We postulate two consumer segments: a high-valuation segment and a low-valuation segment. The high-valuation segment contains consumers who have greater need (perceived or real) and thus have a greater willingness-to-pay for the product than the consumers in the other segment. There are $n_H$ consumers in the high-valuation segment and $n_L$ consumers in the low-valuation segment. Without loss of generality we normalize the number of consumers by setting $n_H + n_L = 1$. We next assume that the consumers within each segment are distributed uniformly on a [0,1] line segment as in Hotelling (1929) and the consumer’s location on the line segment describes the consumer’s ideal point. We assume the two manufacturers’ products are located at the two extremes of the Hotelling line (see Figure 1). We refer to the manufacturer on the left extreme as Manufacturer $a$ and the manufacturer on the right extreme as Manufacturer $b$. Next we define a lack-of-fit (transportation cost) parameter, $k$, so that the consumer incurs a lack-of-fit disutility of $k x$ when buying Manufacturer $a$’s product and $k (1-x)$ when buying $b$’s product. This way we capture the differences among consumers in their preferences for the two
manufacturers’ products. Low values of \( k \) imply greater substitutability between the two products, i.e., a more competitive market.

Since the product is a durable, we make the common assumption that consumers value it for the services it provides over multiple periods. We define consumer utility on a per-period basis and allow consumers to make a purchase or sell decision each period. A consumer in valuation segment \( i \) (\( i = H, L \)) located at a distance \( x \) from Manufacturer \( a \) gets a per-period gross utility of \( \phi_j \theta_i - kd_j \) from using Manufacturer \( j \)'s car where \( \phi_j = 1 \) if the car is new, and \( \phi_j = \gamma_j (0 < \gamma_j \leq 1) \) if the car is old, and \( d_j \) is the distance between the consumer and Manufacturer \( j \) (\( d_a = x \) and \( d_b = l - x \)). Note that the first term captures the consumer’s valuation for the ideal new car (\( \theta \)) appropriately degraded if it is used (e.g., \( \gamma \)), and the second term reflects the disutility associated with the available car not meeting the consumer’s ideal specifications. The parameter \( \gamma_j \) represents durability or the lack of physical deterioration of the car from usage, i.e., how well the car holds up after usage. In order to allow for differences in the durability between the two manufacturers, we let \( \gamma_a \geq \gamma_b \).

We obtain a consumer’s net utility (i.e., perceived value) by subtracting the relevant prices from the above gross utility. We denote new car prices in the first and second periods by \( p_{1j} \) and \( p_{2j} \) respectively, and used car prices by \( r_{2uj} \) where \( j (j = a, b) \) denotes the manufacturer. The price of a new car, \( p_{1j} \), reflects the two period value of the car and thus is made up of two components: the first is the notional price that reflects the value of the service offered by the car in the first period itself, and the second is the market value of the car the second period, i.e., the used car price, \( r_{2uj} \). In the tradition of the durable goods literature (cf., Bulow 1982), we refer to
the first component as the first period rental price and denote it by \( r_{ij} \) where \( p_{ij} = r_{ij} + r_{2uj} \).

Finally, we assume that the discount factor is one.

**Consumer Choice and Product Demand**

Consumers in each period have the option of buying Manufacturer a’s new car, Manufacturer b’s new car or not buying any car at all. In addition, consumers in period 2 also have the choice of selling or buying the first period new car (hereafter referred to as a used car). Since each consumer buys at most one car a period, the number of consumers who find it optimal to buy a given manufacturer’s new or used car at the prevailing prices determines the demand for that new or used car in that specific period. This leads us to determine for all consumers their optimal choice (i.e., whether to buy a car or not and if so which brand and type) for a given set of prices. We provide technical details of this analysis in the Appendix. Here we list the additional assumptions we need to make about our parameter set to insure that our demand functions reflect a market that possesses our desired diverse set of behaviors, e.g., downward sloping industry demand, some consumers buying new cars each period, some buying new cars and then holding on to them, some buying only used cars and some not buying any car, new or used. Specifically, we need to assume that the willingness-to-pay parameter, \( \theta_L \), is large enough so that some low-valuation consumers find it best to buy a new car in the first period, but not large enough to preclude some of the low-valuation consumers finding it best to only buy used cars and still others finding it best not buy any car at all. This assumption ensures that the industry-level demand for the new cars in the first period and the industry-level demand for the used cars in the second period can vary with the relevant prices and are not exogenously fixed. We also assume that \( \theta_H \) is high enough so that all high-valuation consumers find it optimal to buy a new car in
the first period. This assumption ensures that there is inter-brand competition in the first period. In the absence of this assumption, some of the high-valuation consumers in the middle of the $[0, I]$ line segment may remain inactive in period 1 and then each manufacturer would enjoy a local monopoly in period 1. We also assume that $\theta_H$ is high enough so that all high-valuation consumers find it optimal to trade in their current car and buy a new car in period 2. High-valuation consumers also have the option of holding on to the currently owned cars. However, the difference in utilities provided by these two options is independent of the consumer’s location, $x$. Therefore, in our setup either all high-valuation consumers will buy a new car in period 2 or no high-valuation consumers will buy a new car in period 2. Our assumption about $\theta_H$ ensures the former because the latter results in zero demand for new cars in period 2.\footnote{We have analyzed another model formulation in which the lack-of-fit disutility, $k$, is lower for used cars than for new cars thereby allowing some high valuation consumers to find it best to hold on to their currently owned car in period 2. This formulation results in a more complex model but still supports our major results. For simplicity and parsimony we have opted for the current model.}

Finally since the cars traded in by the high valuation consumers have to be purchased, i.e., the market needs to clear, we need to assume the number of low-valuation consumers is enough greater than the number of high-valuation consumers that the market will absorb all the used cars and still have some consumers remaining who do not own a car. If we don’t make this assumption then some of the high-valuation consumers will not be able to employ their optimal strategy because of their inability to sell their used cars. In addition, any increase in the first-period new car sales would be exactly equal to a decrease in the second-period used car sales since the market would be fully covered. The net of all these (reasonably innocuous) parameter assumptions is the existence of eight possible groups of consumers with respect to their two period purchase behavior. We graphically show the location of these groups in Figure 2.
We show in the Appendix that with this set of reasonably non-restrictive parameter assumptions, a formal analysis of the consumer choices leads the following linear in price demand functions.

\[(2) \quad q_{1a} = \frac{(2 - n_H)(2(1 - n_H)\theta_L + kn_H) + (4 - n_H)(1 - n_H)(\gamma_a \theta_L - p_{1a}) - (1 - n_H)(\gamma_a \theta_L - p_{1a})}{4k(2 - n_H)},
\]

\[q_{1b} = \frac{(2 - n_H)(2(1 - n_H)\theta_L + kn_H) + (4 - n_H)(1 - n_H)(\gamma_b \theta_L - p_{1b}) - (1 - n_H)(\gamma_b \theta_L - p_{1b})}{4k(2 - n_H)},
\]

\[(3) \quad q_{2a} = \frac{n_H(k - p_{2a} + p_{2b})}{2k}, \quad q_{2b} = \frac{n_H(k + p_{2a} - p_{2b})}{2k};
\]

where \(q_{jt}\) is the \(t^{th}\) period \((t = 1, 2)\) demand for brand \(j\) \((j = a, b)\) and \(\chi_1 = \frac{kn_H + 2\theta_L - 2n_H \theta_L}{2k}\). Note that the underlying parameters of our model uniquely define the intercepts and own and cross price sensitivities for each demand function.

Before using these derived demand functions we feel it is beneficial to summarize what we have done so far. Our goal is to adequately represent two-period demand for competing durables that are differentiated both vertically (on the quality/deterioration dimension) and horizontally (on the taste dimension), where consumers are heterogeneous on these two dimensions. We also want these demand functions to reflect the fact that consumers exhibit different types of buying patterns, e.g., some consumers replace the automobile before it wears out, others keep their initial new purchase until it no longer has economic value and still others only buy used cars. Conversely we want to rule out “uninteresting” cases, i.e., situations where there are no new car sales in the second period, there is an upward sloping industry demand curve, there are not enough used car buyers to clear the market for cars that are traded in, etc. This required us to put some (innocuous) restrictions on our parameter set. However, by starting from a set of basic buyer behavior assumptions we are assured our formulation is logically
consistent, the demand coefficients are tied to our parameters and the demand functions represent observable purchase behavior. In addition our model captures both intra-firm and inter-firm competition and allows us to analyze not only the current but also the future effects of offering performance-based discounts to retailers. To the best of our knowledge, ours is the first work to develop a set of dynamic demand functions with all of these complexities.

We next use these (linear) demand functions to determine equilibrium prices, quantities and profits for the manufacturers and the retailers under different strategies.

**ANALYSIS**

**Overview of Analysis**

In the interest of readability, we describe here the rules of the games and briefly explain how we solve each game. We provide more technical details in the Appendix.

Given our interest in the future effects of trade promotions, we limit our attention to the situation where a manufacturer only can give a promotion in the first period. We do this so that we can assess the effect of first period promotion on the second period used market. We look at four games: (i) Neither manufacturer offers a trade promotion (NN), (ii) only Manufacturer $a$ offers a trade promotion (PN), (iii) only Manufacturer $b$ offers a trade promotion (NP) and (iv) both manufacturers offer trade promotions (PP). Although our main interest is in comparing outcomes of the NN and PP games, we need to analyze the PN and NP games to be able to analyze the PP game and to establish if the PP game is an equilibrium. In all four games we employ backward induction, i.e., solve the retailer/manufacturer game beginning with the last stage of the game, although the sequence of events and decision variables differ across games.

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6 In our notation, P denotes promotion in the first period and N denotes no promotion. We use the first letter for Manufacturer $a$ and the second letter for Manufacturer $b$. 

In the NN game, we begin by using our derived second-period demand functions to determine the two retailers’ optimal second-period retail prices as functions of second-period wholesale prices. The two manufacturers’ optimal second-period wholesale prices are then derived assuming Stackelberg price leadership within a channel. The first-period analysis proceeds in a similar way: we use the derived first-period demand functions found in Equation 2 to solve the retailers’ and the manufacturers’ maximization problems in that order. In this way, first period wholesale prices not only affect first period retail prices but also second period used car prices. Thus, there is a specific link between first period actions and second period outcomes.

In the PN game, the second-period analysis proceeds as in the NN game. However, the first period is different because of the trade promotion offered by Manufacturer \(a\). This trade promotion offers a quantity target and an associated wholesale price that the dealer qualifies for if it achieves the quantity target. In choosing the quantity target and the associated wholesale price, Manufacturer \(a\) ensures voluntary participation by its dealer. That is, Manufacturer \(a\) chooses optimal wholesale prices and quantity target subject to the constraint that its dealer gets at least \(\pi^{NN*}_a\) profit under the promotion where \(\pi^{NN*}_a\) is dealer \(a\)’s profit in the NN game. If dealer \(a\) does not accept the promotion, the PN games becomes the NN game. In effect, the menu of wholesale prices that the manufacturer offers is

\[
W_{1a} = \begin{cases} 
W^{PN*}_{1a} & \text{if } q_{1a} \geq q^{PN*}_{1a} \\
W^{NN*}_{1a} & \text{otherwise}
\end{cases}
\]

where \(q^{PN*}_{1a}\) is the sales quantity target set by manufacturer \(a\) in the PN game.
The NP game is identical to the PN game except that Manufacturer b rather than Manufacturer a offers the trade promotion. In this game, Manufacturer b has to ensure that its dealer gets at least $\pi_{b}^{NN^*}$ profit where $\pi_{b}^{NN^*}$ is dealer b’s profit in the NN game.

In the PP game, the second period analysis proceeds as in the other games. However, the first period analysis is different because now each manufacturer offers a trade promotion by specifying a quantity target and a wholesale price. If dealer $a(b)$ does not accept the promotion, the PP game becomes NP (PN) game. Therefore, Manufacturer a has to ensure that its dealer gets at least $\pi_{a}^{NP^*}$ profit from the trade promotion scheme where $\pi_{a}^{NP^*}$ is the dealer’s profit in the NP game in which only Manufacturer b offers a trade promotion. Similarly, Manufacturer b has to ensure that its dealer gets at least $\pi_{b}^{PN^*}$ where $\pi_{b}^{PN^*}$ is the dealer’s profit in the PN game in which only Manufacturer a promotes. Manufacturer a’s menu of wholesale prices is as follows.

$$w_{ia} = \begin{cases} w_{ia}^{pp*} & \text{if } q_{ia} \geq q_{ia}^{pp*} \\ w_{ia}^{np*} & \text{otherwise} \end{cases}$$

where $q_{ia}^{pp*}$ is the sales quantity target set by manufacturer a in the PP game. Manufacturer b’s menu of wholesale prices is

$$w_{ib} = \begin{cases} w_{ib}^{pp*} & \text{if } q_{ib} \geq q_{ib}^{pp*} \\ w_{ib}^{np*} & \text{otherwise} \end{cases}$$

where $q_{ib}^{pp*}$ is the sales quantity target set by manufacturer b in the PP game.

Solving the four games is straightforward although the math gets complicated quickly. Still, we are able to obtain closed form solutions for all prices, quantities and profits as a function of our model parameters. This allows us to compare solutions across all four games for all sets of feasible parameters. Also, since we derived our demand functions from underlying buyer behavior, we are able to compare solutions across varying sets of parameter values and be
assured we are holding fixed other aspects of our model, i.e., we can interpret comparative statics results. We discuss some of our most interesting finding next.

**Key Results**

Our main interest is in understanding the differences between PP and NN games and how these differences vary by our quality parameter, $\gamma_i$. However, before we describe these quality related differences, we note that over the total range of feasible parameter values we find the equilibrium solution is for both firms always to offer trade promotions.

The reason behind this finding is the net influence of two opposing forces. First, as conjectured earlier we find the negative effect of lower second period used car prices (and thus lower first period new car retail prices) when both manufacturers offer trade promotions. This decrease in used car prices is not due to an increase in the number of used cars being traded in (this number does not vary in our model between the NN and PP games), but instead to the fact that the increased sales in the first period come from some of those consumers who would have otherwise bought used cars in period 2. Consequently the marginal consumer deciding between buying a used car and not purchasing a car at all when trade promotions are given is located farther from the end points. This in turn means that this marginal consumer has a lower gross utility thereby leading to a lower second period used car market clearing price.

Acting in the opposite direction is the influence of the financial incentives to the retailers to increase the quantity sold. This allows the manufacturers to partially mitigate the double marginalization problem and better coordinate the channels. We note, however, that trade promotions only partially solve this problem since the quantity sold in the PP case, although higher than in the NN case, is still lower than the sales quantity found in the integrated case.
However, the net result of these two different effects is that manufacturers have higher profits under PP than NN and this result holds for all feasible parameter values.

In coming to this conclusion we note that our analysis does not include any financial costs of administering the trade promotion. If such costs are included, then depending on the magnitude of these costs, the equilibrium may involve one or both firms not offering trade promotions. We return to this issue soon.

We now compare PP and NN outcomes and describe how they are affected by the durability of the two competitors’ products.

**Proposition 1:** The depth of trade promotion offered by a manufacturer \((w_{ij}^{NN*} - w_{ij}^{PP*})\) increases as the manufacturer’s product durability increases.

This is a comparative statics result and thus is a within firm finding. The underlying economic reasons for it are two-fold. First, as we noted earlier, a performance based trade promotion is a mechanism for the manufacturers to increase the quantity sold by the independent dealer thereby partially solving the double marginalization problem. Second, holding all else constant, as the durability of a product increases, the manufacturer faces a greater reduction in new car sales from the level of sales preferred by the manufacturer i.e., there exists a more severe double marginalization problem. More formally, \(q_{ij}^{*} - q_{ij}^{NN*} > 0\) and \(\frac{\partial(q_{ij}^{*} - q_{ij}^{NN*})}{\partial \gamma_j} > 0\), where \(q_{ij}^{*}\) denotes Manufacturer j’s \((j = a, b)\) optimal sales quantity in period 1 when both manufacturers are integrated. Therefore, it is in the manufacturer’s best self interest to provide more (deeper) incentives to its dealer to sell more units.
Proposition 1 result also suggests that if the two manufacturers have different levels of product durability, their double marginalization problems may differ in severity. Proposition 2 addresses this question directly.

**Proposition 2:** The manufacturer selling the more durable product offers a greater depth of trade promotion and gains more profit from the trade promotion than the manufacturer with lower product durability.

Proposition 2 is a between firm statement and the intuition for it is similar to the intuition for Proposition 1: holding all else constant, higher product durability results in a more severe double marginalization problem for the manufacturer. To illustrate this, we compare the first period sales quantities in the NN case with those for integrated manufacturers. We find that manufacturer $a$ (i.e., the higher durability manufacturer) suffers a greater loss in sales due to double marginalization inherent in decentralization. In particular, remembering that $\gamma_a > \gamma_b$, we find that

\[
(q_{1a}^* - q_{1a}^{NN*}) - (q_{1b}^* - q_{1b}^{NN*}) = \frac{2(\gamma_a - \gamma_b)(8 - 5n_H)(2 - 3n_H + n_H^2)\theta_L}{k(32 - 22n_H + n_H^2)(8 - 7n_H + n_H^2)} > 0,
\]

a condition that holds over the total range of feasible parameters. As a result, manufacturer $a$ has more to gain by offering a trade promotion. Moreover, Manufacturer $a$’s retailer has a more attractive default option than the other retailer. If he does not participate in the trade promotion, he can earn a higher profit than the other retailer can earn by not participating in trade promotion. In other words, the high durability manufacturer faces a tighter voluntary participation constraint than the low durability manufacturer. Consequently, we find the depth of trade promotion, $w_{ij}^{NN*} - w_{ij}^{PP*}$ is greater for the manufacturer with higher product durability.
Our discussion so far has assumed the administrative costs of running trade promotions are negligible. When trade promotions costs are substantial, there is an interesting corollary to Proposition 2. It’s easy to see that when the administrative costs are extremely high they can outweigh the benefits of trade promotions for both manufacturers. For a lower range of promotion costs, the manufacturer with higher product durability may be the only manufacturer who offers trade promotion. For the other manufacturer, who gains less from the trade promotion, the administrative costs of running the trade promotion can make this action too costly. This leads to the following corollary.

**Corollary 1:** If there are fixed costs associated with administering trade promotions and these costs are independent of the durability of the cars, one of the following outcomes will occur.

(i) Both manufacturers will run trade promotions,

(ii) Only the high-durability manufacturer will run trade promotion,

(iii) Neither manufacture will run trade promotion.

The net result of this corollary is that since we anticipate there will be times “in the real world” when the administrative costs will preclude only the low-durability manufacturer from running the trade promotion, we would expect to find empirically that low-durability manufacturers use trade promotions less often. Thus far our focus has been on manufacturer profits. Proposition 3 further describes how trade promotions affect the retail market of the two manufacturers differentially.

**Proposition 3:** When both manufacturers offer trade promotions: (a) the percent decline in same period retail price due to trade promotions, (b) the percent increase in the same period sales due
to trade promotions, and (c) the percent decline in next period used car prices due to trade
promotions, are all greater for the manufacturer with higher product durability.

As we discussed earlier, trade promotions result in an increase in first period sales, a
decrease in first-period retail prices and a decrease in second-period used car prices. Proposition
3 shows that all of these effects are higher for the higher durability manufacturer.

Parts (a) and (b) of Proposition 3 result from the fact that the manufacturer with higher
product durability can benefit more from solving its double marginalization problem, and
therefore sets a higher target for its retailer. They are also related to the fact that the high
durability manufacturer offers a deeper cut in the trade promotion to the retailer. Part (c) is
related to Part (b) in that the higher durability manufacturer’s trade promotion results in more
low valuation consumers buying a new car, thereby reducing the pool of potential used car
buyers. This results in a greater reduction in used car prices for the higher durability firm, since
the marginal consumer now has a lower gross utility. Proposition 3 is interesting because it
shows that larger (deeper) trade promotions are actually a signal of “strength” and not
“weakness” in that high durability manufacturers provide the incentive to their dealers to give
greater discounts on their cars. Moreover, it points out that one should not use depreciation
values to measure durability, since the former reflects used car prices that in turn are affected by
the promotional strategies of the two types of firms while the latter represents a physical
characteristic of the car. Propositions 1-3 make it clear that trade promotions help the
manufacturers address the double marginalization problem. We now examine how retailers are
affected by trade promotions.
Proposition 4: Each retailer makes less profit when both manufacturers offer trade promotions (PP case) than when both manufacturers do not use trade promotions (NN case).

Promotion 4 is surprising since retailers participate voluntarily in any trade promotion scheme. Thus, in our model the manufacturers have to ensure that their retailer is not worse off if the retailer takes advantage of the promotion. For example, in the PN game, Manufacturer \(a\) has to ensure that Retailer \(a\) gets \(\pi_{a}^{NN}^{*}\) under the promotion. However, each retailer is adversely affected by the other manufacturer’s trade promotion. That is, \(\pi_{a}^{NP} < \pi_{a}^{NN}^{*}\) and \(\pi_{b}^{PN} < \pi_{b}^{NN}^{*}\).

When both manufacturers offer trade promotions, Manufacturer \(a\) has to ensure that its retailer gets \(\pi_{a}^{NP}^{*}\) profit with the trade promotion. Therefore, in the PP game, Retailer \(a\) gets its guaranteed profit \(\pi_{a}^{NP}^{*}\), which is less than \(\pi_{a}^{NN}^{*}\), its profit in the NN game. The same logic holds for Manufacturer \(b\).

Another way of looking at the Proposition 4 result is that trade promotions, by better coordinating the channels, partially removes the buffering actions of the retailers, thereby making retail competition more intense. This increased competition reduces the retailers’ market power.\(^{7}\)

A very interesting implication of the above result is that one manufacturer’s trade promotion allows the other manufacturer to leave a smaller surplus to the retailer. This does not necessarily mean that one manufacturer’s promotion increases the other manufacturer’s profits because the other manufacturer’s profits are also hurt by higher sales of the competing manufacturer’s product.

\(^{7}\) We thank an anonymous reviewer for suggesting the market power interpretation.
In order to further investigate the effect of one manufacturer’s promotion on the other manufacturer, we examine how the status of one manufacturer’s promotion affects the incremental benefits of promotion that the other manufacturer enjoys. For example, for Manufacturer \( a \), we compare the incremental benefits of promotion when \( b \) does not promote, 
\[
\Pi_a^{PN^*} - \Pi_a^{NN^*},
\]
with the incremental benefits of promotion when \( b \) does promote, 
\[
\Pi_a^{PP^*} - \Pi_a^{NP^*}.
\]

Given the complexity of our model, it’s not possible to fully characterize these effects. However, Corollary 2 shows a surprising possibility.

**Corollary 2:** There exists a set of parameter values for which low-durability manufacturer’s promotion can increase the incremental benefit of promotion for the higher durability manufacturer so that 
\[
\Pi_a^{PP^*} - \Pi_a^{NP^*} > \Pi_a^{PN^*} - \Pi_a^{NN^*}.
\]

Thus, manufacturer \( a \) might see larger increases in profits due to promotion when it’s competitor is already promoting than if manufacturer \( a \) is the only one to promote. We haven’t been able to identify a set of parameters for which a similar result would hold for manufacturer \( b \). However, we also haven’t been able to rule out this possibility analytically.

The above propositions and corollaries follow directly from our model, which, we acknowledge, is an abstraction of the “real world”. Consequently, one might still wonder if these conclusions are observed empirically. With this in mind we next present an analysis of six years of trade promotions within the US automobile market and compare the results of this analysis with some of our assumptions and predictions. After this we discuss the implication and limitations of our theoretical and empirical results in the Summary and Conclusions section.

**EMPIRICAL ANALYSIS**

Two central hypotheses coming from our theoretical development are as follows:

**H1:** Trade promotions are more likely to be held for products with higher durability.
H2: The depth of a trade promotion is higher for products with higher durability.

Hypothesis H1 is based on the result that the firm with the more durable product gains more from the trade promotion. As described in Corollary 1, depending on the administrative costs of running the promotions, it’s possible that neither or both firms offer trade promotions to their retailers. Therefore, H1 is valid for those instances where the fixed administrative trade promotion costs are low enough to make trade promotions profitable for the more durable product but not so low that trade promotions are profitable for the less durable product. Our theory provides a sharper prediction for trade promotion depth: conditional on both firms holding trade promotions, we predict that the trade promotion for the more durable product always has a greater depth.

Empirical model

We investigate H1 and H2 by specifying and simultaneously testing a two-equation system that models an automobile manufacturer’s decision to offer a trade promotion and the depth of the resulting trade promotion. The proposed specification controls for individual specific effects and the potential selectivity bias that result from the fact that we observe measures of trade promotion depth if and only if automobile manufacturers promote. Formally, we investigate H1 and H2 together using a random effects variant of the widely applied sample selection model (Verbeek 1990; Zabel 1992; Veerbeek and Nijman, 1992). By simultaneous estimation, we have the advantage of being able to get estimates of the slope parameters of interest (e.g., H1 and H2) and test for the covariance of our endogenous variables with a single econometric specification.8

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8 We thank an anonymous reviewer for suggesting this simultaneous estimation.
We model the decision to offer a trade promotion with a random effects Probit\(^9\). Define \(z_{it}^*\) to be the latent propensity of an automobile manufacturer to promote nameplate \(i\) (Toyota Camry, Ford Escort, etc) in year \(t\). In Equation 5 (found below) we let \(z_{it}^*\) to be a function of the vector of explanatory variables, \(w_{it}\), idiosyncratic disturbance \(u_{it}\), and time-invariant, random nameplate specific effect, \(\eta_i\). Similarly, we let the depth of the trade promotion \((y_{it})\) for a nameplate \(i\) in year \(t\) be a function of the vector of explanatory variables \(x_{it}\), disturbance \(\varepsilon_{it}\), and nameplate specific effect, \(\alpha_i\) (See Equation 6 below). The vector coefficients \((\beta, \lambda)\) represent the effects of the explanatory variables. Note that we observe \(y_{it}\) and \(x_{it}\) if and only if auto manufacturers promote, i.e., \(z_{it}=1\). Following standard practice, we assume the idiosyncratic error terms \((\varepsilon_{it}, u_{it})\) follow bivariate normal distributions with mean vector zero and covariance matrix, \(\Sigma\). The conditional distribution \((\varepsilon_{it} | u_{it})\) is also stated for later use (see Equation 7). Finally, we assume nameplate specific random effects, \((\alpha_i, \eta_i)\) are bivariate normal, with zero means, standard deviations \((\sigma_\alpha, \sigma_\eta)\) and CDF \(G(\alpha_i, \eta_i)\) and uncorrelated with the nameplate effects, i.e. \(\alpha_{it}, \eta_{it} \perp \varepsilon_{it}, u_{it}\). We capture selectivity through the correlation \((\rho)\) of the disturbance components, \(\varepsilon_{it}\) and \(u_{it}\). Thus, the estimable parameters in our model are the slope and intercept parameters contained in vectors \(\beta\) and \(\lambda\), the variance parameters \(\sigma_\varepsilon\), \(\sigma_\alpha\) and \(\sigma_\eta\) and the correlation parameter, \(\rho\). Lastly, we make the standard, but somewhat restrictive assumption that the observations on different nameplates are independent even though some of the nameplates in our data are marketed by the same automobile manufacturer.

\[
(5) \quad z_{it}^* = \lambda'w_{it} + \eta_i + u_{it}, \\
z_{it} = 1, \quad (z_{it}^* > 0) \text{ and } y_{it}, x_{it} \text{ observed if and only if } z_{it} = 1,
\]

\(^9\) We could have used a random effects Tobit type I specification. However, the Probit specification is simpler to estimate in the above model and does not bias the test of H1 and H2. (See L. Lee, G. S. Maddala, and R.P. Trost, 1980). Our formulation is similar to a Tobit type II formulation.
(6) $y_{it} = \beta' x_{it} + \alpha_i + \epsilon_{it}$,

(7) $(\epsilon_{it}, u_{it}) \sim N(0, \Sigma)$ where $\Sigma = \begin{bmatrix} \sigma^2 & \rho \sigma \rho \\ \rho \sigma \rho & 1 \end{bmatrix}$ and thus,

$$u_{it} | \epsilon_{it} \sim N \left( \frac{\rho \sigma \rho}{\sigma^2}, (1 - \rho^2) \right).$$

We obtain estimates of our model parameters by maximizing the theoretical unconditional likelihood of the $i^{th}$ nameplate using LIMDEP 8.0. We derive this likelihood function by first stating the contribution of the $i^{th}$ nameplate in year $t$ to the likelihood conditional on the random effects. This conditional likelihood is made up of three parts: the conditional probability that a firm offers a trade promotion, the distribution of the trade depth and the probability the firm does not offer a trade promotion. We obtain the theoretical unconditional likelihood of the $i$th nameplate by numerically integrating out the random effects, i.e.,

$$L_i(\beta, \lambda, \Sigma) = \int \int \cdots \int \prod_{i=1}^{T} f(\epsilon_{it}, u_{it} | \alpha_i, \eta_i) dG(\alpha_i, \eta_i)$$

We are able to fit the model in LIMDEP 8.0 using the maximum simulated likelihood.

**Data Description and Measures**

Our primary data source is a proprietary data set of automobile promotions that provides a description of every dealer incentive program offered by Daimler-Chrysler, Ford, GM, and the major Japanese (Honda, Mazda, Nissan and Toyota) automobile manufacturers during 1992-1997. Each of the 69 nameplates (Toyota Camry, Ford Escort, etc) on promotion was recorded along with the duration and target (dealer, consumer etc.) of the promotion. The rest of the data came from three leading public sources of automotive information: *Automotive News Market Data Book, Consumer Reports* and *Automotive Leasing Guide*. Since not every nameplate was
sold in all six years, we were left with a total of 317 nameplate/year observations. For each of these observations, we used the following measures.

**Trade Promotion**: Our dependent measure for H1 is the indicator that a nameplate was on promotion in a given model year. We define a model year to be 16 months long. Since we are able to attach the specific promotion with a specific nameplate, there is no confusion in those months where multiple model years are available, e.g. September through December.

**Trade Promotion Depth**: For every trade promotion for a nameplate in a given year, we calculate the per car value of the trade promotion incentive and divide it by the dealer’s invoice. If there were multiple trade promotions per year, we then calculate the average depth by weighting the specific depth by the duration of the promotion. Finally, since we assume trade depth is normally distributed (and thus varies from minus infinite to plus infinite), we rescale this weighted average proportion by taking the log odds of the weighted average.

**Durability**: This is our primary independent variable. We use predicted reliabilities from *Consumer Reports* as a measure of quality for car models between 1992-1997 (e.g., See Desai and Purohit, 1999). Predicted reliabilities are measured on a five-point scale for each nameplate and are based on the frequency of repair data for earlier vintages of the same model. Thus, it is a measure of the expected durability based on past performance, i.e., a measure available to consumers when they make the buying decision.

**Degree of Product Differentiation**: Our analytic model contains the parameter, $k$, which captures the degree to which the nameplates are differentiated. Although we did not present any specific hypotheses concerning this parameter, we control for any effect by including a Herfindal index measure to capture the extent of competition or concentration in each segment. We divided the automobile market into segments using the scheme adopted by *Automotive News*.
Market Data Book. For each of these segments, we calculated Herfindal index as: $H_{it} = \sum_{i} m_{it}^2$, where $m_{it}$ is the market share of nameplate $i$ in the year $t$. Note that $H_{it}$ is higher for markets with higher concentration (lower competitive intensity), and lower for markets with lower concentration (higher competitive intensity).

**Average Monthly Excess Inventory:** To control for the possibility that trade promotions may be a response to inventory backlog, we first created a measure for the monthly changes in inventory levels.\(^{10}\) Specifically, for Month $t$, we use $\Delta I_{it} = \frac{P_{it-1} - S_{it-1}}{S_{it}}$ where $P_{it}$ is the production in month $t$ and $S_{it}$ is the sales in the same month. This is mathematically equivalent to the difference between ending and beginning inventory in the previous month as a fraction of the current month’s sales. This reflects the industry practice of thinking about inventory in terms of number of days of sales (referred to as *days supply* by the trade) and also controls for different rates of sales. Since only positive value of $\Delta I_{it}$ lead to excess inventory levels, we use as our measure of excess inventory the sum of positive values of $\Delta I_{it}$ where the summation is taken over a model year. Thus, model years with higher values of our measure indicate the model year had (at least at times) higher (excess) inventory levels compared to model years with lower values of our measure. The production and sales data for all calendar years in this study are available in the Automotive News Market Data Book.

**Car vs. Truck:** Since trade promotion policies may depend in part on whether the vehicle is a car or a truck, we include a dummy variable to reflect the fact that the promotion was for a car.

\(^{10}\) We did not have the data on absolute levels of inventory. The change in inventory could be calculated from production and sales data that are publicly available.
**Foreign vs. Domestic:** We created an indicator to flag whether the nameplate was a foreign car. This is important because empirically Japanese nameplates have higher predicted reliability than domestic nameplates. By including the foreign dummy along with our quality measure we are able to control for other factors, such as management policy, that might serve as an alternate explanation.

**Results**

The results of our analysis are provided in Table 1. As predicted, the durability coefficient in both promotion and depth equations is statistically significant and positive; that is, the likelihood of trade promotion increases with durability (H1) and the depth of trade promotion increases with durability (H2).

The foreign car indicator is positive and significant in both the promotion and depth equations. We noted earlier that the Japanese nameplates on average have a higher durability level than the American nameplates. By including both our durability measure and the foreign car indicator, we are able to rule out the possibility that our durability results (H1 and H2) are due entirely to differences in the management practices of the Japanese and American carmakers, i.e., we find the hypothesized durability effects even after accounting for any differences in managerial practices. This provides us with more confidence concerning our assertion that higher durability *causes* manufacturers to use trade promotions and give deeper promotions.

We also note that the Herfindal index is negative and significant in the promotion likelihood equation, suggesting that trade promotions are more likely to occur when markets are less concentrated, i.e., more competitive, but that increased competition does not affect the depth of the trade promotion. We conjecture that the finding on how likely promotions occur is due to
market share issues, i.e., manufacturers may also use trade promotions to capture market share, while the depth of promotion finding only reflects the manufacturers’ concerns about the double marginalization problem.

Promotions in the automobile industry are commonly thought of as an operational mechanism to manage inventory levels. Our Table 1 results support this belief, at least in terms of the depth of the trade promotion. However, we find no impact of excess inventory levels on the likelihood that a firm offers a trade promotion. Thus it appears that the manufacturers increase the target sales levels when they have extra inventory on hand.

We further explore the impact of inventory levels on trade promotions by looking at the frequency of trade promotions over the 16 months associated with a given model year, aggregating the data across models and model years. Our analytic model assumes trade promotions are offered throughout the period in order to (partially) alleviate the double marginalization problem. The hypothesis that trade promotions are used to clear inventories should show a different pattern since excess inventories often exist at the end of the model year.

The results of our analysis are shown in Table 2. Table 2 shows three interesting patterns. First, trade promotions are offered throughout the model year. Second, there are more trade promotions in effect at the end of the model year than average, although this increase is not large. This is consistent with the idea that at least some trade promotions are intended to lower retail prices in order to deplete excess end of year inventories. Perhaps more surprising is the fact that there is a slight increase in frequency starting in month 2 and running through month 4. This latter observation is consistent with an outward shift in demand due to the new model year. This shift increases the double marginalization problem and thus increases the benefits of giving a trade promotion. In summary, although there is some indication that trade promotions are used
to lower retail prices in periods of low demand, we find strong evidence that these promotions are used throughout the model year. This, in turn, supports our belief that trade promotions are used to alleviate the double marginalization problem.

Finally, we find the highly significant negative estimate of \( \rho \) to be interesting. It suggests that when trade promotions are given for reasons that are not included in our model (and therefore assumed to be random), the resulting promotion has a smaller depth. Although we don’t have a theoretical explanation for this finding, the finding has strong face validity because it suggests that unplanned trade promotions have lower trade depth than the planned trade promotions.

**SUMMARY AND CONCLUSIONS**

Trade promotions for durable goods such as automobiles are an integral part of a manufacturer’s marketing activities. We address a set of important questions related to these promotions. Starting with observations about key aspects of the automobile market, we develop a model that allows us to parsimoniously capture these aspects. In doing so, we explicitly model the strategies of three sets of players: consumers, retailers, and manufacturers. Our results provide a number of interesting insights.

Perhaps the most general insight is that trade promotions can partially mitigate the channel coordination problem. Moreover, counter to McGuire and Staelin’s (1986) finding that it is not always best for the manufacturer to get total channel coordination in a very competitive market, this form of (partial) channel coordination provides benefits over the total feasible parameter space, i.e., all levels of product differentiation. Interestingly the benefits of channel
coordination problem are greater when the product durability is higher. Therefore, if there are administrative costs independent of durability associated with running trade promotions, we find the firm with higher durability is more likely to offer trade promotions. Moreover, regardless of administrative costs this firm will offer trade promotions at a greater depth.

When trade promotions are used, our model shows that the increase in current sales comes from converting some potential used car buyers into new car buyers. As a result, the remaining pool of potential used car buyers is made up of people who have a relatively lower valuation for the used cars. This puts a downward pressure on the future used car prices of the manufacturer running the trade promotion. Since supporting high used car prices can be a very important concern for automakers, our model highlights an important controllable variable for the automakers to achieve that objective.

We also find that retailers will be negatively affected by trade promotions when both manufacturers run trade promotions even though participation in the program is voluntary. In designing a trade promotion, each manufacturer has to provide a certain minimum profit to his retailer so that the retailer accepts the trade deal. The necessary minimum profit is reduced when the other manufacturer also runs a trade promotion. Thus, a manufacturer’s trade promotion allows the other manufacturer to extract a greater surplus from his retailer. We also find that the incremental benefits of promotion to a manufacturer may be higher when the other manufacturer also runs a promotion. This provides an interesting contrast to the results about promotions arising from prisoners’ dilemma.

We test two of our theoretical results and one of our basic assumptions and find that the US automobile market trade promotion data are consistent with our premise that a) promotions are given throughout the model year, b) they are more likely to be used by manufacturers of high
quality cars, and c) when these high quality manufacturers give trade promotions, they give deeper promotions.

In coming to these conclusions, we note the limitations in our model. By using a two period model we are able to capture, to some degree, the impact of first period sales on the second-period outcomes. We say to some degree since our formulation assumes all high-valuation consumers buy new cars in the second period and thus, second period new car sales are fixed. Consequently, we do not allow first period sales to impact second period new car sales. To do this, we would need to allow some high-valuation consumers to keep their first period cars if new car prices are too high in the second period, i.e., the cost of trading up is too high. We have separately analyzed such a model and find that, as in the model presented here within, the higher durability of the car makes the first-period demand for the product stronger (by shifting the demand curve out). However the second-period demand is weaker since the lower used car prices make it less economical for the high-valuation consumers to replace their old cars with new cars in the second period. Therefore, first-period trade promotions are followed by a reduction in new car sales in period 2. Interestingly, when we modify our model to allow for this to occur, we still find that our main results hold.11

In our model, we do not have trade promotions in the second period. Allowing trade promotions in the second period and also determining the effects of durability would require us to model a third period and analyze third-period used car prices. We believe adding a third period to our model increases the complexity without any significant benefits. Moreover, the primary reason that we have a second period is to capture the future effects of first period trade promotions.

11 These results are available from the authors.
In this study, we have restricted our attention to trade promotions. A more general model would allow the manufacturers to offer either trade or consumer promotions (i.e., promotions given directly to the consumer) or both. We did not pursue this more general model since the underlying motivation for consumer promotions must be different from trade promotions. We say this because retailers are free to adjust their retail price upward to match the size of the consumer promotion, but must lower their retail price if they are to take advantage of the trade promotion. Consequently, our explanation for trade promotions cannot be directly applied to customer promotions. Instead, one needs to postulate another mechanism for why they occur. We leave this postulation to future research.

We acknowledge that there may be other possible explanations for manufacturers to run trade promotions. For example, trade promotions may help manufacturers to motivate dealers or reflect a reduction of the wholesale price during periods of weak demand. Still we believe our model and the empirical evidence provides a deeper understanding of why we observe manufacturers using trade promotions differentially within the auto industry and the impact of trade promotions on dealer profits and new and used car prices.
References


Figure 1

Consumer Distribution and Market Segments

Firm $a$  

\[ x \rightarrow \]  

High-Valuation Segment

Firm $a$  

\[ x \rightarrow \]  

Low Valuation Segment
Figure 2
Assumed Consumer Strategies By Market Segment

Firm $a$                      Firm $b$

<table>
<thead>
<tr>
<th>BB$_{aa}$</th>
<th>BB$<em>{ab}$ or BB$</em>{ba}$</th>
<th>BB$_{bb}$</th>
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</thead>
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<td>$x_1$</td>
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High-Valuation Segment

Firm $a$                      Firm $b$

<table>
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<tr>
<th>BK$_a$</th>
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<th>II</th>
<th>IU$_b$</th>
<th>BK$_b$</th>
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Low Valuation Segment
### Table 1

Estimates of Empirical Models for Hypotheses H1 and H2

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<th>Models:</th>
<th>Estimate</th>
<th>Standard Deviation</th>
<th>t-stat</th>
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<td><strong>Promotion Likelihood: (H1)</strong></td>
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<td></td>
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<tr>
<td>Constant</td>
<td>-1.7532*</td>
<td>0.3276</td>
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<tr>
<td>Durability</td>
<td>0.3826*</td>
<td>0.1036</td>
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<td>Herfindahl Index</td>
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<td>1.2432</td>
<td>-2.096</td>
</tr>
<tr>
<td>Foreign/Domestic</td>
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<td>0.2315</td>
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<tr>
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<td>$\sigma_\eta$</td>
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<td>0.1252</td>
<td>10.767</td>
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**Sample Size (N=317)**

Panels (69 Nameplates)

*: significant at 1% significance level

**: significant at 5% significance level
Table 2
Aggregate Frequency Distribution of Trade Promotions Over the Model Year (16 month period)

<table>
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<th>Month</th>
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<th>Cumulative Percent</th>
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<td>8.96</td>
<td>748</td>
<td>91.78</td>
</tr>
<tr>
<td>16</td>
<td>67</td>
<td>8.22</td>
<td>815</td>
<td>100</td>
</tr>
</tbody>
</table>
Appendix

Consumer Choice and Demand Generation

We start with the second period. In period 2 consumers who bought a new car first period can choose to a) sell their current car and either buy another new car or remain inactive, or b) keep the car and use it for the second period. It is easy to show that the strategy of trading in the old car and then being inactive is dominated. Thus, we only need to determine if the consumer prefers to trade up to a new car or keep the old one. If the consumer keeps the current car, her utility is $\theta_i \gamma_j - r_{2uij} - d_j$ where $d_j$ is the distance between the consumer and brand j where $r_{2uij}$ is the imputed cost of the car in the second period and is endogenously determined. If she replaces the current car with a new car, then her utility is $\theta_i - p_{2k} - d_k$ where k=A,B is the brand of the new car and j is the identity of the car sold. The consumer’s choice depends on whether $\theta_i \gamma_j - r_{2uij} - d_j$ is greater (less) than $\theta_i - p_{2k} - d_k$ i.e., the intrinsic valuation of the used car is greater or less than the cost of trading up to the new car. To simplify the analysis and yet capture both types of behavior, we assume that $\theta_{H}$ and $\theta_{L}$ are such that

$$\theta_{H} (1 - \gamma_j) > (r_{2uij} + d_j) - (p_{2k} + d_k) \quad \text{and} \quad \theta_{L} (1 - \gamma_j) < (r_{2uij} + d_j) - (p_{2k} + d_k), \quad \text{i.e., the high valuation consumers trade up and the low valuation consumers holds on to their previously purchased new car.}$$

The high-valuation consumers in period 2 buy the brand that gives them the higher

---

12 Although the consumer pays the price $p_{ij}$ at the beginning of period 1 to buy a new car, she can get $r_{2uij}$ by selling her used car at the beginning of period 2. Therefore, when a consumer buys a new car, we subtract $r_{ij}$ from her first period gross utility and $r_{2uij}$ from her second period gross utility only if she holds on to the car in period 2. It should be clear that the other approach of subtracting $p_{ij} = r_{ij} + r_{2uij}$ in the first period and then adding $r_{2uij}$ in the second period if the consumer sells the car is exactly the same as our approach.
utility. The location of the high-valuation consumer who is indifferent between the two brands in period 2 is determined by equating the two second period net utilities and solving for $x$. We denote this location by $x_2$, where $x_2 = \frac{(k - p_{2a} + p_{2b})}{2k}$. Therefore, since consumers are assumed to be uniformly distributed over the unit lines, $q_{2a} = n_H x_2$ and $q_{2b} = n_H (1 - x_2)$. Let $x_1$ be the location of the high-valuation consumer who is indifferent between the two brands in period 1 (we determine this value later). Since $x_1$ need not equal $x_2$, there may be some brand switching. However, this switching can only go in one direction depending on whether $x_1$ or $x_2$ is greater. If $x_1 > x_2$, then those consumers who lie between $x_1$-$x_2$ switch from brand $a$ to brand $b$. If $x_1 < x_2$, then the converse is true (See Figure 2).

Consumers who bought cars in period 1 don’t gain anything by selling their used cars and replacing those used cars with other used cars. Moreover, low valuation consumers who bought a car in period 1 do not trade up by assumption. Therefore, the only potential buyers for used cars in period 2 are low-valuation consumers who did not buy any car in period 1. However, we don’t force all of these potential buyers to buy used cars. Therefore, we need to make sure that a) the market of used cars is cleared and b) there still are some consumers who do not buy a used car. This means that, among the potential buyers of used cars, there are three sub-segments: the segment that buys the brand $a$ used cars, the segment that remains inactive and the segment that buys the brand $b$ used cars. A consumer who did not buy a car in period 1 gets utility $\theta_L \gamma_a - r_{2ua} - kx$ from buying a brand $a$ used car and utility $\theta_L \gamma_b - r_{2ub} - k(1-x)$ from buying a brand $b$ used car. The option of not buying any car gives consumer zero utility. Comparing the

13 These requirements are important because we want to a downward sloping industry demand curve. We need to assume $n_L > n_H$ and that $\theta_L$ is not too large compared to $k$ to ensure these requirements.
utilities of buying a brand a used car and staying inactive, it’s clear that this difference is
decreasing in x. On the other hand, the difference between utilities of buying a brand b used car
and staying inactive is increasing in x. Therefore, the consumers who buy brand a used cars will
be to the left of inactive consumers and consumers who buy brand b used cars will be to the right
of inactive consumers (See Figure 2). The cutoff points described in Figure 2 are given
by: \[ x_4 = \frac{-r_{2ua} + \gamma_a \theta_L}{k} \] and \[ x_5 = \frac{k + r_{2ub} - \gamma_b \theta_L}{k} \]. We later use these values to derive used car
prices.

We now derive the demand for new cars in period 1. We start our analysis by noting
there are no used cars in the first period. Thus, consumer choice is limited to either buying a
new brand a car, a new brand b car or remaining inactive. The net utility of buying a brand j
car is \[ \theta_i - r_{1j} - k \theta_j \]. Buying a new car in period 1 does not commit the consumer to any specific
strategy in period 2—she can keep the car, replace it with another car of either brand or sell the
car and not replace with any car in period 2. Therefore, consumers will buy a new car as long as
\[ \theta_i - r_{1j} - k \theta_j \] is positive. We assume all consumers in the high valuation segment value new
cars enough that they will always buy a new car. Since \[ \theta_i - r_{1a} - kx \] is decreasing in x and
\[ \theta_i - r_{1b} - k(1 - x) \] is increasing in x, consumers toward the left of product space will buy brand
a and those toward the right will buy brand b. Moreover, since all high valuation consumers will
buy a new car, the break point between buying brand a and brand b, \[ x_1 \], is that point where the
two net utilities are equal: \[ x_1 = \frac{k - r_{1a} + r_{1b}}{2k} \]. We assume that only a fraction of the low
valuation consumers will find positive utility from buying a new car. We determine the group of
low valuation consumers who will buy brand a by identifying the value of x for which the net
utility in period one from buying brand a is zero. This value of x, denoted by \[ x_3 \], is given by
\[ x_3 = \frac{-r_{1a} + \theta_L}{k} \]. Similarly, the value of x for which the net utility from buying brand b is zero,

\[ 14 \] The choice of not buying a car in the first period and then buying a new car in the second period can be ruled out. More details are available from the authors.
is denoted by \( x_6 \) and is given by \( x_6 = \frac{k + r_{lb} - \theta_L}{k} \). In the low-valuation segment, consumers in the range \( x \in [0, x_3) \) buy brand \( a \), consumers in the range \( x \in (x_6, 1] \) buy brand \( b \), and consumers in the range \( x \in [x_3, x_6] \) stay inactive in period 1. Then, the demand for brand \( a \) in period 1 is given by \( q_{1a} = n_H x_1 + n_L x_3 \) while the demand for brand \( b \) is

\[
q_{1b} = n_H (1 - x_1) + n_L (1 - x_6) .
\]

After simplification, we get:

\[
q_{1a} = \chi' - \left(\frac{2 - n_H}{2k}\right) r_{1a} + \left(\frac{n_H}{2k}\right) r_{1b},
\]

and

\[
q_{1b} = \chi' - \left(\frac{2 - n_H}{2k}\right) r_{1b} + \left(\frac{n_H}{2k}\right) r_{1a}. \tag{A1}
\]

where \( \chi' = \frac{kn_H + 2\theta_L - 2n_H\theta_L}{2k} \).

Next, we derive used car prices \((r_{2ua}, r_{2ub})\) by equating the supply of used cars from consumers who replace their cars in period 2 and the total demand generated by used car consumers in period 2. Thus, \( n_H x_1 = n_L (x_4 - x_3) \) and \( n_H (1 - x_1) = n_L (x_6 - x_3) \). \(^{15}\) Solving for \( r_{2ua} \) and \( r_{2ub} \) yield:

\[
\begin{align*}
\text{(A1)} & \quad r_{2ua} = \alpha_{11} + \left(\frac{2 - n_H}{2n_H}\right) r_{1a} - \left(\frac{n_H}{2 - 2n_H}\right) r_{1b}, \\
\text{(A2)} & \quad r_{2ub} = \alpha_{12} - \left(\frac{n_H}{2 - 2n_H}\right) r_{1a} + \left(\frac{2 - n_H}{2 - 2n_H}\right) r_{1b} \text{ where }
\end{align*}
\]

\[
\alpha_{11} = \frac{kn_H + 2(1 - \gamma_a)(1 - n_H)\theta_L}{-2(1 - n_H)}, \quad \alpha_{12} = \frac{kn_H + 2(1 - \gamma_b)(1 - n_H)\theta_L}{-2(1 - n_H)}.
\]

**Analysis**

**Neither Firm Promotes (NN):** We assume that the two retailers move simultaneously to choose their retail prices for a given set of wholesale prices by the manufacturers. The manufacturers act as Stackelberg leaders with respect to the dealers but move simultaneously against each other in choosing their wholesale prices. The second-period analysis with this sequence of moves
provides the following equilibrium solution:

\[ w_{2a}^{NN^*} = w_{2b}^{NN^*} = 3k \quad \text{and} \quad p_{2a}^{NN^*} = p_{2b}^{NN^*} = 4k. \]

Thus, prices increase as the products become more differentiated, i.e., k increases.

In period 1, retailers and manufacturers maximize their two period profits. Note that period 1 new car prices will reflect the cost of owning the car for both periods: \( p_{1a} = r_{1a} + r_{2a} \) and \( p_{1b} = r_{1b} + r_{2b} \). Since used car prices are endogenous, dealer a chooses \( r_{1a} \) to maximize

\[ \pi_{1a} = (p_{1a} - w_{1a})q_{1a} + \pi_{2a}^{NN^*} \]

and dealer b chooses \( r_{1b} \) to maximize

\[ \pi_{1b} = (p_{1b} - w_{1b})q_{1b} + \pi_{2b}^{NN^*} \]

where \( \pi_{2j}^* \) (j = a, b) is dealer j’s second period profit. The first-period equilibrium outcome is as follows:

\[ r_{1a}^{NN^*} = \frac{z_1(w_{1a} - \alpha_{11}) + z_2(w_{1b} - \alpha_{12})}{(8 - 5n_H)(8 - 7n_H + n_H^2)} + \frac{\chi_1 k(4 - 3n_H)}{(1 - n_H)(8 - 5n_H)} \]

\[ r_{1b}^{NN^*} = \frac{z_2(w_{1a} - \alpha_{11}) + z_1(w_{1b} - \alpha_{12})}{(8 - 5n_H)(8 - 7n_H + n_H^2)} + \frac{\chi_2 k(4 - 3n_H)}{(1 - n_H)(8 - 5n_H)} \]

where: \( z_1 = (2 - n_H)^2(4 - 3n_H); \) \( z_2 = (2 - n_H)n_H(3 - 2n_H) \).

\[ w_{1a}^{NN^*} = \frac{(1 - n_H)(h_1\alpha_{11} + h_2\alpha_{12}) + 4k(32 - 22n_H + n_H^2)(8 - 7n_H + n_H^2)\chi_1}{(1 - n_H)(32 - 22n_H + n_H^2)(32 - 26n_H + 3n_H^2)} \]

\[ w_{1b}^{NN^*} = \frac{(1 - n_H)(h_2\alpha_{11} + h_1\alpha_{12}) + 4k(32 - 22n_H + n_H^2)(8 - 7n_H + n_H^2)\chi_1}{(1 - n_H)(32 - 22n_H + n_H^2)(32 - 26n_H + 3n_H^2)} \]

where \( h_1 = 512 + n_H(-768 + (348n_H - 44n_H^2 + n_H^3)) \) and

\( h_2 = (-2 + n_H)n_H(16 - 12n_H + n_H^2) \).

**Both Firms Promote (PP):** The second-period analysis is the same as in the above case. In the first period, both manufacturers simultaneously choose optimal quantity objectives (targets) as well as wholesale prices. To ensure voluntary dealer participation, each manufacturer

\[ 15 \quad \text{The used car supply-demand equations clarify why need to assume} \quad n_L > n_H \quad \text{to ensure that the supply of used cars is cleared.} \]
competes by offering its exclusive dealer a trade deal that leaves the dealer just indifferent between accepting and not accepting the promotion. This requires us to analyze PN and NP games and determine retailers’ profits in PN and NP games. The equilibrium prices and quantities in the PP game are as follows:

\[
\begin{align*}
q_{1a}^{PP} &= \frac{(16 - 20 n_H + 4 n_H^2) \alpha_{11} + 2(-1 + n_H) n_H \alpha_{12} + 4 k(8 - 3 n_H) \chi_1}{k(64 - 32 n_H + 3 n_H^2)}, \\
q_{1b}^{PP} &= \frac{(16 - 20 n_H + 4 n_H^2) \alpha_{12} + 2(-1 + n_H) n_H \alpha_{11} + 4 k(8 - 3 n_H) \chi_1}{k(64 - 32 n_H + 3 n_H^2)}, \\
w_{1a}^{PP} &= \frac{4(4 - n_H)(2(-4 + n_H) \alpha_{11} + n_H \alpha_{12}) + 8 k(4 - n_H) \chi_1 + k(8 - n_H)^2 (8 + 3 n_H) (k n_H - \pi_a^{NP*})}{z_3 z_5}, \\
w_{1b}^{PP} &= \frac{4(4 - n_H)(2(-4 + n_H) \alpha_{12} + n_H \alpha_{11}) + 8 k(4 - n_H) \chi_1 + k(8 - n_H)^2 (8 + 3 n_H) (k n_H - \pi_b^{NP*})}{z_3 z_5},
\end{align*}
\]

where \( z_3 = -8 + 3 n_H \), \( z_4 = (1 - n_H)(2(-4 + n_H) \alpha_{11} + n_H \alpha_{12}) + 2 k(-8 + 3 n_H) \chi_1 \), 
\( z_5 = 4(8 - n_H) \) and \( z_6 = (1 - n_H)(2(-4 + n_H) \alpha_{12} + n_H \alpha_{11}) + 2 k(-8 + 3 n_H) \chi_1 \).

**Proofs of Propositions:**

**Proof of Proposition 1:**

We show the result for Firm \( a \). The proof for Firm \( b \) is similar and is omitted.

Let \( \Delta_a = w_{1a}^{PP*} - w_{1a}^{NP*} \). Then

\[
\frac{\partial \Delta_a}{\partial \gamma_a} = \left[ \frac{\partial w_{1a}^{NP*}}{\partial \alpha_{11}} - \frac{\partial w_{1a}^{PP*}}{\partial \alpha_{11}} - \frac{\partial w_{1a}^{PP*}}{\partial \pi_a^{NP*}} \frac{\partial \pi_a^{NP*}}{\partial \alpha_{11}} \right] \frac{\partial \alpha_{11}}{\partial \gamma_a}.
\]

Since \( \frac{\partial \alpha_{11}}{\partial \gamma_a} = \theta > 0 \), we need to show that \( z_7 = \frac{\partial w_{1a}^{NN*}}{\partial \alpha_{11}} - \frac{\partial w_{1a}^{PP*}}{\partial \alpha_{11}} - \frac{\partial w_{1a}^{PP*}}{\partial \pi_a^{NP*}} \frac{\partial \pi_a^{NP*}}{\partial \alpha_{11}} > 0 \).

After taking the partial derivatives, and simplifying,
\[ z_{\gamma_a=\gamma_b} = -\frac{1}{5} + \frac{2}{8-n_H} + \frac{2}{9(4-n_H)} - \frac{10}{24-9n_H} + \frac{8-5n_H}{32+(-22+n_H)n_H} - \frac{8-5n_H}{96-78n_H + 9n_H^2} \]

\[ \frac{2048(7-n_H)}{75(128 + n_H(-64 + 5n_H))^2} + \frac{8096 - 560n_H}{225(128 + n_H(-64 + 5n_H))}. \]

Since we have assumed that \( n_H + n_L = 1 \) and \( n_L > n_H, n_H \in (0, 0.5) \). It can be verified that for any value of \( n_H \in (0, 0.5) \), the above expression is positive.

\[ \frac{\partial z_7}{\partial \gamma_a} = \frac{(8-n_H)(1-n_H)n_H^6(8-3n_H)\theta_L(n_H + 2(1 + \gamma_b)(1-n_H)\theta_L)^2}{(4-n_H)(128 + n_H(-64 + 5n_H))^2(k(8-3n_H)n_H + 2(-1 + n_H)(-8 + 2\gamma_b(-4+n_H) + (3 + \gamma)n_H)\theta_L)^2} > 0. \]

Therefore, \( z_7 > 0 \) for any \( \gamma_a > \gamma_b \).

**Proof of Proposition 2(a):**

\[
(\Pi_a^{PP^*} - \Pi_a^{NN^*}) - (\Pi_b^{PP^*} - \Pi_b^{NN^*}) = \frac{1}{2k}(\alpha_{11} - \alpha_{12})((1-n_H)(\alpha_{11} + \alpha_{12}) + 4k\chi_1)f(n_H) \]

where

\[
f(n_H) = \frac{4(4-n_H)}{(8-n_H)(8-3n_H)} - \frac{4(4-n_H)(16-2n_H + n_H^2)}{(32-22n_H + n_H^2)(32-26n_H + 3n_H^2)} - \frac{(64 - 24n_H + n_H^2)(64 - 40n_H + 5n_H^2)}{(4-n_H)(128 - 64n_H + 5n_H^2)^2} > 0
\]

When \( \gamma_a > \gamma_b, \alpha_{11} > \alpha_{12} \). With \( n_H \in (0, 0.5) \) and \( k > 0, (\Pi_a^{PP^*} - \Pi_a^{NN^*}) - (\Pi_b^{PP^*} - \Pi_b^{NN^*}) \) is positive.

**Proof of Proposition 2(b):**

Let \( A_{ab} = (w_{1a}^{NN^*} - w_{1a}^{PP^*}) - (w_{1b}^{NN^*} - w_{1b}^{PP^*}) = \frac{kz_4^2(8-n_H)(32-22n_H + n_H^2)\delta_1 - \delta_2(\alpha_{11} - \alpha_{12})}{4(8-n_H)(32-22n_H + n_H^2)z_4z_6} \]

where \( z_4 \) and \( z_6 \) are defined above and, \( \delta_1 = kn_H(z_6 - z_4) - 2z_6\pi_a^{PP^*} + 2z_4\pi_b^{PP^*} \) and

\[
\delta_2 = 4n_H(8-4n_H - n_H^2)z_4z_6. \]

It can be shown that \((z_6 - z_4)(\alpha_{11} - \alpha_{12})(8-9n_H + n_H^2) > 0 \)

and when \( \gamma_a > \gamma_b, \pi_a^{PP^*} - \pi_b^{PP^*} = \frac{(\alpha_{11} - \alpha_{12})((1-n_H)(\alpha_{11} + \alpha_{12}) + 4k\chi_1)}{2kg(n_H)} > 0 \).
where \( g(n_H) = \frac{(4 - n_H)(128 - 64n_H + n_H^2)^2}{(64 - 24n_H + n_H^2)(64 - 40n_H + 5n_H^2)} > 0 \). If we substitute, \( z_4, z_6 \)

and \( \pi^{NP*}_a = \pi^{NP*}_b + g(n_H) \) into \( \delta_i \) and simplify we get,

\[
\Delta_{ab} = \frac{(\alpha_{i1} - \alpha_{i2})\eta}{4(8 - n_H)(32 - 22n_H + n_H^2)z_4z_6}, \text{ where}
\]

\[
(A6) \quad \eta = \left(-\delta_2 + kz^2(8 - n_H)(32 - 22n_H + n_H^2)(1 - n_H)(k(8 - n_H)(kn_H - 2\pi^{NP*}_b) - \frac{z_6((\alpha_{i1} + \alpha_{i2}) + 4k\chi_i)}{g(n_H)}) \right).
\]

Since \( z_4 < 0, z_6 < 0 \) and \( n_H \in (0, 0.5) \) the denominator of \( \Delta_{ab} \) is always positive. We need to show that the numerator is also positive. Since \( (\alpha_{i1} - \alpha_{i2}) > 0 \) when \( \gamma_a > \gamma_b \), we need to show that \( \eta > 0 \). At \( \gamma_a = \gamma_b \) if we substitute values, \( z_4, z_6, \alpha_{i1}, \alpha_{i2} \) into \( \eta \), we obtain:

\[
\eta = \frac{(8 - 3n_H)^2}{2}(kn_H + 2(1 + \gamma_B)(1 - n_H)\theta_L)^2
\]

\[
\left(\frac{(8 - n_H)(32 - 22n_H + n_H^2)(8 - 3n_H)}{g(n_H)} - 2n_H(8 + 4n_H + n_H^2) - 2k(8 - n_H)(1 - n_H)(kn_H - 2\pi^{NP*}_b) \right) > 1.
\]

In (A6) only \( \alpha_{i1} \) is a function of \( \gamma_a \). Therefore, \( \frac{\partial \eta}{\partial \gamma_a} = \left(\frac{\partial \eta}{\partial \alpha_{i1}} + \frac{\partial \eta}{\partial \pi^{NP*}_b} \frac{\partial \pi^{NP*}_b}{\partial \alpha_{i1}} \right) \frac{\partial \alpha_{i1}}{\partial \gamma_a} \). It can be shown that \( \frac{\partial \eta}{\partial \pi^{NP*}_b} < 0, \frac{\partial \pi^{NP*}_b}{\partial \alpha_{i1}} < 0, \frac{\partial \eta}{\partial \alpha_{i1}} > 0 \) and \( \frac{\partial \alpha_{i1}}{\partial \gamma_a} = \theta_L > 0 \). Therefore, \( \frac{\partial \eta}{\partial \gamma_a} > 0 \).

Since \( \eta > 0 \) at \( \gamma_a = \gamma_b \), and \( \frac{\partial \eta}{\partial \gamma_a} > 0 \), \( \eta > 0 \) for any \( \gamma_a > \gamma_b \).

**Proof of Proposition 3:**

(a) **Percentage Decline in Retail Prices:**

We need to show that \( \frac{p^{NN*}_{ia} - p^{PP*}_{ia}}{p^{NN*}_{ia}} > \frac{p^{NN*}_{ib} - p^{PP*}_{ib}}{p^{NN*}_{ib}} \) which equivalent to showing that
\[
\frac{p_{1a}^{pps}}{p_{1a}^{NN*}} < \frac{p_{1b}^{pps}}{p_{1b}^{NN*}}. \quad \text{It can be shown that at } \gamma_a = \gamma_b, \quad \frac{p_{1a}^{pps}}{p_{1a}^{NN*}} = \frac{p_{1b}^{pps}}{p_{1b}^{NN*}}. \quad \text{Therefore, we show that } \frac{p_{1a}^{pps}}{p_{1a}^{NN*}}
\]
is decreasing in \( \gamma_a \). After some simplification, \[
\frac{\partial p_{1a}^{pps}}{\partial \gamma_a} = \frac{2(4 - n_H)^2 \theta_L}{64 - 32 n_H + 3 n_H^2} \quad \text{and}
\]
\[
\frac{\partial p_{1a}^{NN*}}{\partial \gamma_a} = \frac{(192 - 288 n_H + 126 n_H^2 - 13 n_H^3)(512 - 768 n_H + 348 n_H^2 - 44 n_H^3 + n_H^4) \theta_L}{2(8 - 5 n_H)(32 - 22 n_H + n_H^2)(8 - 7 n_H + n_H^2)(32 - 26 n_H + 3 n_H^2)}. \quad \text{It can be}
\]
verified that for any \( n_H \in (0, \frac{1}{2}) \) and \( \theta_L > 0, \quad \frac{\partial p_{1a}^{pps}}{\partial \gamma_a} - \frac{\partial p_{1a}^{NN*}}{\partial \gamma_a} < 0 \) and therefore \( \frac{p_{1a}^{pps}}{p_{1a}^{NN*}} \) is decreasing in \( \gamma_a \).

(b) Percentage Increase in Quantities:

Using the proof strategy in (a) above, we show that \( \frac{q_{1a}^{pps}}{q_{1a}^{NN*}} \) is increasing in \( \gamma_a \). After some simplification, \[
\frac{\partial q_{1a}^{pps}}{\partial \gamma_a} = \frac{4(4 - n_H)(1 - n_H) \theta_L}{(64 - 32 n_H + 3 n_H^2) k} \quad \text{and}
\]
\[
\frac{\partial q_{1a}^{NN*}}{\partial \gamma_a} = \frac{(1 - n_H)(32 - 40 n_H + 14 n_H^2 - n_H^3)(512 - 768 n_H + 348 n_H^2 - 44 n_H^3 + n_H^4) \theta_L}{2(64 - 96 n_H + 43 n_H^2 - 5 n_H^3)(1024 - 1536 n_H + 700 n_H^2 - 92 n_H^3 + 3 n_H^4) k}. \quad \text{It can be}
\]
verified that for any \( n_H \in (0, \frac{1}{2}) \), \( k > 0 \) and \( \theta_L > 0, \quad \frac{\partial q_{1a}^{pps}}{\partial \gamma_a} - \frac{\partial q_{1a}^{NN*}}{\partial \gamma_a} > 0 \) and therefore \( \frac{q_{1a}^{pps}}{q_{1a}^{NN*}} \) is increasing in \( \gamma_a \).

(c) Decline in Used Car Prices:

Using the same proof strategy as in (a) above, we show that \( \frac{r_{2a}^{pps}}{r_{2a}^{NN*}} \) is decreasing in \( \gamma_a \). After simplification,
\[
\frac{\partial r_{2a}^{NN*}}{\partial \gamma_a} = \frac{(114688 - 348160 n_H + 423552 n_H^2 - 261920 n_H^3 + 86176 n_H^4 - 14484 n_H^5 + 1120 n_H^6 - 29 n_H^7) \theta_L}{2(64 - 96 n_H + 43 n_H^2 - 5 n_H^3)(1024 - 1536 n_H + 700 n_H^2 - 92 n_H^3 + 3 n_H^4)}. \]
and \( \frac{\partial r_{2ua}^{pp^*}}{\partial \gamma_a} = \frac{(48 - 28n_H + 3n_H^2)\theta_L}{(64 - 32n_H + 3n_H^2)} \). It can be verified that for any \( n_H \in (0, \frac{1}{2}) \) and \( \theta_L > 0 \),

\[
\frac{\partial r_{2ua}^{pp^*}}{\partial \gamma_a} - \frac{\partial r_{2ua}^{NN^*}}{\partial \gamma_a} < 0 \quad \text{and therefore} \quad \frac{r_{2ua}^{pp^*}}{r_{2ua}^{NN^*}} \quad \text{is decreasing in} \quad \gamma_a. 
\]

Proof of Proposition 4

We will show the difference in profits for retailer a when both manufacturers offer trade promotions and when both manufacturers stay away from trade promotions, \( \pi_{a}^{NN^*} - \pi_{a}^{pp^*} \), is positive. The proof for retailer b is similar and therefore omitted.

\[
\pi_{a}^{NN^*} = \frac{kn_H}{2} + \frac{v_1((1 - n_H)(h_1\alpha_{11} - h_2\alpha_{12}) + 2k(h_1 - h_2)\chi_1)^2}{k(h_1 + h_2)^2}
\]

\[
\pi_{a}^{pp^*} = \frac{kn_H}{2} + \frac{(u_1\alpha_{11} - u_2\alpha_{12})(1 - n_H) + 2k\chi(u_1 - u_3))^2}{2ku_3}
\]

where \( v_1 = \frac{(2 - n_H)(4 - 3n_H)(16 - 12n_H + n_H^2)}{(1 - n_H)(8 - 5n_H)(32 - 26n_H + 3n_H^2)}, u_1 = (8 - 5n_H)(8 - 3n_H), u_2 = 2(4 - n_H)n_H \)

and \( u_3 = (1 - n_H)(4 - n_H)(128 - 64n_H + 5n_H^2) \).

If we substitute the values for \( \alpha_{11}, \alpha_{12} \), set \( \gamma_a = \gamma_b + \sigma \) and simplify, we obtain the following:

\[
\pi_{a}^{NN^*} - \pi_{a}^{pp^*} = \frac{1}{4}\left((h_1 - h_2)^2(kn_H + 2(1 + \gamma_B)(1 - n_H) - (2u_1v_1 - (u_1 - u_2)^2)) + \alpha X_1 + X_2 \sigma^2, \right)
\]

where \( X_1 = ((h_1 - h_2)(1 - n_H)\theta_L(kn_H + 2\theta_L(1 + \gamma_B)(1 - n_H)))(2h_1u_1v_1 - (h_1 - h_2)(u_1 - u_2)u_1) \)

and \( X_2 = \theta_L^2(1 - n_H)^2(2h_1^2u_3v_1 - (h_1 - h_2)^2u_1^2) \).

Since \( h_1, h_2, u_1, u_2, v_1 \) are functions of only \( n_H \in (0, 0.5) \), we can easily verify the following:

\[
((2u_1v_1 - (u_1 - u_2)^2)) > 0, \ (2h_1^2u_3v_1 - (h_1 - h_2)^2u_1^2) > 0 \quad \text{and} \quad ((2h_1u_1v_1 - (h_1 - h_2)(u_1 - u_2)u_1) > 0.
\]

Since \( \theta_L > 0, k > 0, n_H \in (0, 0.5) \), and \( \gamma_B \in (0, 1) \), we have \( \pi_{A}^{NN^*} > \pi_{A}^{pp^*} \).
Proof of Proposition 5:

Manufacturer Profits:

\[
\Pi^{PP*}_a = 2kn_H - \pi^{NP*}_a + \frac{2(4 - n_H)z_4^2}{k(8 - 3n_H)^2 (8 - n_H)^2 (1 - n_H)}
\]

\[
\Pi^{NN*}_a = \frac{3kn_H}{2} + \frac{h_z((1 - n_H)(h_z\alpha_{11} - h_z\alpha_{12}) + 4h_zk\chi_1)^2}{2h_zk(n_H)(1 - n_H)(32 - 22n_H + n_H^2)(32 - 26n_H + 3n_H^2)^2}
\]

\[
\Pi^{NP*}_a = \frac{3kn_H}{2} + \frac{\left((z_3z_3\alpha_{11} + (32 - 8n_H)\alpha_{12})(1 - n_H) + 8k\chi_1(64 - 40n_H + 5n_H^2)\right)^2}{16k(1 - n_H)(4 - n_H)(128 - 64n_H + 5n_H^2)^2}
\]

\[
\Pi^{PN*}_a = 2kn_H - \pi^{NN*}_a + \frac{((16 - 4n_H)\alpha_{11} - n_H\alpha_{12})(1 - n_H) + 2k\chi_1(16 - 5n_H)^2(32 - 16n_H + n_H^2)}{k(1 - n_H)(4 - n_H)(128 - 64n_H + 5n_H^2)^2}
\]

where

\[
\pi^{NP*}_a = \frac{kn_H}{2} + \frac{\left((z_3z_3\alpha_{11} + (32 - 8n_H)\alpha_{12})(1 - n_H) + 8k\chi_1(64 - 40n_H + 5n_H^2)\right)^2}{16k(1 - n_H)(4 - n_H)(128 - 64n_H + 5n_H^2)^2}
\]

\[
\pi^{NN*}_a = \frac{kn_H}{2} + \frac{(2 - n_H)(4 - 3n_H)(16 - 12n_H + n_H^2)^2((1 - n_H)(h_z\alpha_{11} - h_z\alpha_{12}) + 4h_zk\chi_1)^2}{4k(1 - n_H)(8 - 5n_H)^2 (32 - 26n_H + 3n_H^2)^2 h_3^2}
\]

We show existence with the help of an example. Consider the following set of parameter values: \(n_H = 0.48\), \(\gamma_a = 0.9\), \(\gamma_b = 0.5\), \(\theta_L = 2\), \(k = 0.5\). With these values,

\[
\Pi^{PP*}_a - \Pi^{NP*}_a = 0.554 > \Pi^{PN*}_a - \Pi^{NN*}_a = 0.551
\]