A Theory of Durable Dominance

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Abstract. This paper demonstrates how increased competition entrenches dominants—a causal relationship I term durable dominance. Established theories of sustained dominance predict that a dominant’s grasp on its position weakens when resources become widely available and the number of competitors increases. This prediction stems from the assumption, from economics, that sustained dominance is only possible when competition is limited; perfectly competitive markets with large numbers of competitors, widely accessible resources, and no barriers to entry provide no basis for dominants to maintain their size. I challenge this assumption, first arguing that a dominant can benefit when the number of its competitors increases, garnering more consumer recognition and facing fewer challenges from its nearest competitors, then asserting that even when a dominant does not directly benefit, increased competition can decrease nondominants’ motility—magnitude of size changes—trammeling their growth to dominant size. Numerical simulations show reduced motility significantly lengthens the expected duration of a dominant’s reign when many new competitors enter the market. Durable dominance reverses established understandings of competitive dynamics and holds implications for an increasing number of settings as markets become global and financialized, and scalable resources for production and distribution become widely available.

Keywords: durable dominance • industrial organization • evolutionary economics

1. Introduction  
“How do dominant firms maintain their positions?” is a central question in strategy. Underlying this question is the assumption, from economics, that competition is antithetic to sustained dominance. Perfectly competitive markets with large numbers of competitors, widely accessible resources, and no barriers to entry provide no basis for dominants to maintain their size. Sustained dominance then requires explanation, which is typically provided by pointing out how the real market differs from the idealized model.

Canonical theories of sustained dominance suggest that dominant firms maintain their dominance by excluding competitors, in one of two ways. First, dominants can be sheltered by barriers to competitive entry into a market and mobility within it (Bain 1968, Porter 1980). Second, dominants can accrue inimitable advantages by amassing exclusive resources and capabilities (Stigler 1968, Barney 1991). Firms are predicted to remain dominant longer to the extent they succeed in obtaining protected market positions or hoarding exclusive resources. Another seminal theory suggests that unless dominants control all market positions and all potentially useful resources—an impossible task—they are still vulnerable to attack from new competitors. The very existence of dominant firms and the disproportionate profits they enjoy motivates new competitors to enter the market with novel resource sets, eventually upending the market structure and toppling the dominants (Schumpeter [1942] 1994).

Taken together, these theories predict that freer entry, widespread availability of resources, and an increase in the number of competitors spell the death knell for a dominant’s reign. But this prediction has not held true in recent years. New technologies and business institutions have democratized access to markets and resources over the past half-century (see, e.g., Davis 2013). An individual almost anywhere in the world with an Internet connection can now establish a company and secure scalable sources of financing, manufacturing, marketing, and distribution. Close to efficient factor markets exist for most business resources and skills. These changes and the effects of globalization have led to larger numbers of competing sellers in many markets. Yet the biggest firms have increased their market share across the majority of industries (Council of Economic Advisers 2016), and the influx of new competitors has not threatened the reign of incumbent dominants, such as Fidelity and Vanguard in mutual funds and Anheuser-Busch/AB-Inbev in beer. In fact, these dominants have prospered as the number of their competitors increased.

Rather than posit competitive exclusion, other studies (e.g., Arthur 1989, Levinthal 1991, Denrell 2004, Cabral 2016; also see Sutton 2007) explain sustained dominance as a natural result of firms’ levels of endowments and the dynamics of changes in these...
levels over time. These studies—which typically model changes in endowments as Markov processes, where the “condition of the industry in each time period bears the seeds of its condition in the following period” (Nelson and Winter 1982, p. 19)—have demonstrated that long-lasting dominance can result from increasing returns to adoption (Arthur 1989) or stochastic processes (Levinthal 1991, Denrell 2004) that do not depend on the existence of barriers to entry or on the availability of disparate sets of resources. These streams of scholarship suggest that the natural period of sustained dominance can be quite long, but they do not directly address the relationship between sustained dominance and increased competition; indeed, they often implicitly assume that an increase in the number of competitors will shorten the reign of the dominant.

This paper builds on this evolutionary approach to suggest a novel mechanism of sustained dominance. The proposed theory aptly describes the dynamics of competition between dominants and nondominants in today’s increasingly democratized industries, predicting sustained dominance in markets where conditions are close to the economic ideal of perfect competition. The main assumption necessary to develop this prediction is that the size of a firm—a state variable (Winter 1987) describing the scale of its “assets, both financial and nonfinancial” (Levinthal 1991)—is sticky. A firm can grow or shrink, but size in the near future is related to its current size.

I assert that contrary to the economic assumption that increased competition displaces dominants, competition instead entrenches dominants. Dominants—firms with asset size much larger than their competitors—differ from their competitors in three important ways: (1) by definition, they possess endowments significantly greater than their competitors; (2) their numbers are few, compared to nondominants, which are often many; (3) only dominants do not face competition from larger competitors. Any of these factors can lead to durable dominance—dominance sustained longer as a causal result of increased competition.

I develop my argument in four steps. The remainder of this section describes the theoretical puzzle of sustained dominance in democratizing markets. Section 2 addresses this puzzle by showing how dominants benefit from increased competition under certain conditions. The section starts with the observation that dominants differ from nondominants in important ways, and so an exogenous shock can affect dominants and nondominants differently, and then presents two specific types of mechanisms that can entrench dominants when new competitors enter their markets. First, theory suggests that dominants benefit through behavioral mechanisms: as the number of competitors increase, consumers may find the dominant’s offering more distinctive and exemplary, competitors may become less likely to strategically target the dominant’s position, and dominants may learn more from new competitors. Second, an analytical model demonstrates that, as competition increases, the statistical dynamics of competition reduce the motility—magnitude of size changes—of nondominant firms, trammeling their growth. The model finds a roughly linear positive relationship between number of new entrants and expected duration of dominant reign, when firm revenue is on average linearly predicted by size and firm growth is linearly predicted by deviations of revenue from its predicted value.

These predictions are tested with two computer simulations, which are described in Section 3. I first map the parameter space with a Brownian random walk model (cf. Levinthal 1991, Denrell 2004), examining how long nondominants typically take to reach dominant size—i.e., the half-life of the dominant—under various conditions. For reasonable reductions in nondominant motility from increased competition, the dominant’s half-life lengths as the number of new competitors increases. As the analytical model predicts, this relationship is roughly linear when the relationships between firm revenue and size, and growth and revenue, are linear. Increased dominant advantage (from behavioral mechanisms) and reduced nondominant motility can both lead to substantial increases in dominant half-life, with the latter mechanism dominating when the number of new entrants is large. The second computer simulation removes some simplifying assumptions of the Brownian model and explicitly models individual consumer choices. The results again show a roughly linear increase in dominant half-life as the number of new entrants increases.

Section 4 starts by discussing the assumptions of the various models in the paper and delineating the boundary conditions of the theory presented in the paper. The section and paper conclude by discussing implications of the proposed theory for strategy researchers, dominants, nondominants, entrepreneurs, and policy makers.

1.1. The Problem of Sustained Dominance

Schumpeter famously argued that monopolistic structures in capitalist economies contained the seeds of their own “creative destruction,” providing an engine for continued innovation. Dominant firms accrued large profits that could fund the development of industry-changing innovations, which the dominants were forced to pursue to survive the onslaught of entrepreneurs motivated by the possibility of usurping a dominant industry position and appropriating monopoly profits. This viewpoint of repeated, new entrant-driven industry disruption has become
a recurrent theme in management discourse, especially after the very visible success of venture capital-funded, technology-based startups in the 1990s. Seminal studies portrayed a hypercompetitive world where dominant advantage was ever more fleeting. As the number of competitors and the pace of innovation increased, dominant companies needed to continually out-innovate their competitors or fall from their positions (D’Aveni 1994). Dominant companies were characterized as ill-equipped to embrace disruptive innovations, however, because they were constrained by their existing structures and the demands of their large customer base (Christensen and Bower 1996).

Earlier strategy studies focused on industry structures between disruptions and often sought to explain how dominant companies maintained their dominance during these more stable periods. The structure-conduct-performance paradigm (Bain 1968, Mason 1939, Hunt 1972) undergirded Porter’s (1980) influential viewpoint on competitive strategy, which asserted that industry structure (especially barriers to entry into the industry and between segments of the industry) explained sustained competitive advantage. Others asserted a stronger role for idiosyncratic firm conduct and characteristics, arguing that continued dominance was the result of dominant firms’ sustained innovation (Stigler 1968, Demsetz 1973) whereby dominants amassed exclusive control of unique resource combinations (Penrose 1959, Wernerfelt 1984, Barney 1991).

There are settings where the predictions of these theories do not hold, however. Dominants sometimes maintain their dominance in spite of massive influxes of new competitors and regulatory, technology, and market changes that level entry barriers and eliminate preferential access to key resources. The number of new entrants into the mutual fund industry exploded in the 1980s and 1990s, as experienced fund managers flooded the market with new offerings (Kacperczyk 2012) and barriers to entry dropped. Yet Fidelity and Vanguard maintained—even increased—their dominance throughout this period. The number of U.S. beer breweries grew from 82 in 1980 to 5,300 in 2016.1 Manufacturing and selling beer is no longer technically difficult; you can contract for both brewing and delivery, and distributors are now accessible for smaller brands. Yet Anheuser-Busch products continue to dominate the U.S. beer market, maintaining almost 50% market share.

These are not isolated examples. Dominant firms’ market shares have been increasing (Council of Economic Advisers 2016) even as markets have become more and more open since the 1980s. Globalization and deregulation have lowered barriers to entry. New technologies and business norms have democratized access to resources previously available only to dominants, making them available for easy purchase. For example, even one-person companies now have access to scalable sources of financing, manufacturing, marketing, and distribution. Increased financialization has simplified the reallocation of resources across fields. Yet more often than not, large established firms maintain control of the lion’s share of their markets.

This recurring, recent pattern of durable dominance extends beyond industrial settings. The number of new actors making their feature film starring debut each year has been increasing since 1990, yet the leading stars of 1990 maintain their ability to garner new starring roles and draw audiences. All but two of the top ten actors on Quigley’s list of the top ten money-making stars from 1990 were still active in 2013, with a starring role in at least one Hollywood release in the past year. Business academia has seen the blooming of “a thousand flowers” (Pfeffer 1995) as the number of business authors, journals, theories, and articles increased dramatically, yet “we have become stuck in theories developed in the latter half of the twentieth century” (Schoonhoven et al. 2005, p. 327). The apparent disappearance of a cohesive corporate elite (Mizruchi 2013, Chu and Mizruchi 2015, Chu and Davis 2016) suggests that elites matter less in the United States, and social elites are no longer distinguishable by their monopolization of highbrow culture (Peterson and Kern 1996, Khan 2011). Yet over the past 40 years, economic mobility in the United States has declined (Kopczuk et al. 2010), while the wealth gap between elite and nonelite has dramatically increased (Piketty and Saez 2003).

One potential explanation for these trends is that an expanded range of competitive settings is now characterized by increasing returns to adoption (Arthur 1994), where an offering becomes more useful as more consumers choose it. Increasing returns to adoption can arise from many sources. A greater number of users for an offering may lead to positive network externalities, such as a larger pool of fellow users (e.g., potential Facebook friends) with whom to connect, and better availability of complementary products (e.g., Windows software) and services (e.g., specialized repair shops) for the focal offering (Katz and Shapiro 1985). The ubiquity of the offering may make consumers likely to discover new uses for it. Laws and government policy may be tailored to increase the benefits of the leading offering (Nelson 1994). Whether these kinds of effects have grown stronger in recent years and whether such increases in effect size (if found) can explain the heightened incidence of sustained dominance are interesting and unanswered questions, but fall outside the scope of this paper. I assume no increasing returns to adoption, and focus on the effects of increased competition on sustained dominance.

Another potential explanation comes from recent economic work that suggests dominants protect their
positions by changing the competitive rules—often by influencing government policy—after they have risen to the top. For example, Chang (2002) argues that developed countries "kick away the ladder" and protect their dominant positions against developing countries by promulgating free trade and laissez-faire industrial policies, even though the developed countries themselves attained their current positions through interventionist trade and industrial policy measures. Rajan and Zingales (2003) hold the free market in higher regard, contending that well-functioning free markets optimize welfare. But they, too, assert that dominants change the rules to their benefit once on top—those controlling the most capital distort markets in their favor by influencing government rules and regulations. These studies suggest important policy implications by considering the effects of social structure and process on markets (Granovetter 1985, 2017). But I demonstrate that even an atomistic, nonsocialized model of competition can generate durable dominance. Competition itself can provide the shock that changes the dynamics of the market.

2. Mechanisms of Durable Dominance

2.1. Well-Endowed, Distinctive Dominants

To understand how this may be, consider a market where all input resources are available at the same price per unit to all competitors—i.e., strategic factor markets are perfectly competitive (Barney 1986). Money can buy everything at market price, and no firm can appropriate control of a unique set of complementary resources. Dominants cannot maintain their dominance by monopolizing exclusive sets of resources (Barney 1991), but disruptive innovation (Christensen 1997) by challengers controlling novel sets of resources is also difficult. Given these conditions, where all resources are commensurable and tradable, each firm in the market can be characterized by a single state variable (Winter 1987), which represents the overall size of its assets—financial and non-financial resources. I define a firm as being dominant if it is the largest firm in the market, with asset size significantly larger than that of the next largest competitor.

While we cannot easily measure, commensurate, and total up real firms’ assets, we do know that real-world size distributions tend to be highly skewed. In typical established markets, one firm or a very few firms are very large, and many firms are small (Gibrat 1931, Hart and Prais 1956, Simon and Bonini 1958, Cabral and Mata 2003). This holds true of firm performance measures such as revenue and profits, and also asset measures such as firm value, number of employees, and brand recognition. These measures are not perfectly correlated, however. Google and Apple are not very large by number of employees, but have very high market capitalization, brand value, and high profits. Walmart by comparison is large by any measure.

An increased influx of new competitors changes the asset size distribution in the market, typically adding competitors at sizes below the size of the largest firm. While basic economic models and almost all theory on incumbent-new entrant competitive dynamics (see Ansari and Krop 2012 for an extensive review of studies of incumbent-entrant competition) assume the impact of entrants on incumbents is uniform, it is not obvious that this should be so. When a shock occurs below the size of the largest firm, the shock creates a dividing line between the incumbent(s) with size larger than the shock and all other incumbents (Figure 1). It seems plausible that at least some effects of the shock will differ—perhaps become reversed—for incumbents of size above and below the shock size.

The largest firms are different from their competitors in other ways, too. Dominants are distinctive. There are many small firms, and only one (or at most a few) very large firm. Dominants are also the only firms that do not face competition from above. All others, by contrast, are affected by the actions and market presence of firms larger than they are, as well as by those of firms at or below their own size.

2.2. Type I Mechanisms: Behavioral Responses to Increased Competition

These differences between the largest firms and all other competitors can directly benefit dominants in many ways when competitive entry increases. First, consumers may become more likely to choose the dominant’s offering as the number of competitors increases, because the dominant is distinctive. When

![Figure 1. A Highly Skewed Size Distribution](image-url)

*Notes. Stylized example of a highly-skewed firm size distribution. One firm has size ∼10 billion, 10 firms size ∼1 billion, 100 firms size ∼100 million, 1,000 firms size ∼10 million, and 10,000 firms size ∼1 million. An exogenous shock characterized by a size of ∼1 billion (dashed vertical line) impacted the market, the dominant’s size would be many times the scale of the shock, but all 11,110 non-dominants would have sizes at or below the scale of the shock.*
faced with increased choice, consumers often default to simple heuristics, such as choosing the largest, most well-known provider (Tversky and Kahneman 1973, Schwartz 2004). If a market has 10 competing firms, consumers may attempt to compare the offerings from each of these when making a purchase decision. But when there are 100 firms in the market, search costs will preclude consumers from carefully comparing offerings from each, and the largest, most visible firm gains an advantage over its less-distinctive competitors.2

An increase in the number of competitors can also cause consumers to equate idiosyncratic characteristics of the dominant’s offering with what constitutes a desirable offering in general. A large influx of firms into a hitherto small market can direct new consumer attention to the market. As new consumers are crystallizing their expectations for what constitutes a good provider and product offering in this market, the firm that is dominant during this foundational juncture and its products are likely to become regarded as exemplars, strengthening the dominant’s advantages vis à vis its competitors. The dominant firm’s characteristics come to be taken as normal and normative, and other firms are judged against this new standard (Zuckerman 1999).

Second, increased competitive entry can deter nondominants from challenging dominants. Managers tend to prioritize defending against attacks from similar or smaller competitors (White 1981, Porac et al. 1995, Bothner et al. 2007), so an increase in similar or smaller competitors may lead managers of nondominant firms to shift their efforts to competing against other nondominants rather than challenging the dominants. (Dominants by definition never need to compete upward.) Nondominant firm managers may also be deterred from investing in growth, as the presence of more competitors decreases the projected risk-adjusted return on potential investments (Spence 1979, Fudenberg and Tirole 1983, Sutton 1991). The reasoning behind resource partitioning (Carroll 1985) suggests that large but nondominant firms will suffer disproportionately from new entry at the low end of the size distribution in segmented markets. An influx of smaller specialty firms targeting specific consumer niches may steal customers from midsize firms spanning several niches. These midsize firms then cannot accrue enough assets to challenge the dominant for the large generalist market.

Dominants benefit from increased competition in other ways also. They may be the best positioned to innovate into promising new niches uncovered by new entrants, for example. Dominants have more resources to invest in these new niches, and their large size often means they have experience with a wide breadth of niches. Such breadth of prior experience may make them more likely to successfully adapt to take advantage of new opportunities (Cohen and Levinthal 1990, Eggers 2012). Any advantage known to be enjoyed by larger, more distinctive, or less-constrained competitors over their competition can provide the basis for an argument that dominants benefit more or suffer less than other incumbents from an influx of new competitors.

2.3. Type II Mechanisms: Decreased Nondominant Motility

But even when increased competition doesn’t directly benefit dominants (through, e.g., heightened consumer recognition, fewer competitor challenges, or wider learning opportunities), dominants can become entrenched because nondominants’ motility—the scale of typical changes in firm size over a given time—decreases when the number of competitors grows.

In our idealized market, a dominant firm cannot exclude competitors by monopolizing access to critical resources. Each and every nondominant poses a threat to grow larger than the dominant. On the other hand, challengers to the dominant cannot control novel resource sets that are unavailable to the dominant. If we assume no mergers, a challenger can only usurp dominance by growing its size over time.

To the extent revenue provides the main stream of resources to maintain or grow assets, a firm will grow in size relative to its competitors if it brings in revenues that are higher than expected given its asset size, or shrink relative to competitors if revenues are lower than expected from its asset size. This relationship, because it concerns only relative sizes and revenues, will hold whether total revenues for the industry are increasing, decreasing, or stable. Relative sizes and the size ordering of firms remain constant if revenue size across firms is linearly predicted by asset size, with no randomness.

When revenues for each firm during a given period differ from the linear relationship with asset size, these deviations determine motility. Excess revenue allows the firm to grow its assets compared to other firms; revenue deficits force the firm to shrink its relative share of assets. If the deviations during each time period are large, firms can grow or shrink faster. If they are small, firms can only grow or shrink in small increments relative to their original size.

If new competitors enter the market, the variance of each nondominant incumbent’s revenue falls—motility shrinks—all other things being equal, as the probability of a nondominant’s offering being chosen for purchase decreases. To illustrate this point, consider a Luce model (Luce 1959, 1977; Bell et al. 1975) of choice, where for each purchase of the product, a firm’s offering is selected with probability proportional to the firm’s asset size. If each purchase choice is made independently of other purchase choices (so no consumer
lock-in, herding, network effects, or other behavioral mechanisms leading to nonproportional choice), then the probability distribution of revenues for a firm takes the form of a binomial distribution. On average, the relationship between a firm’s size and revenue is linear, and the expected proportion of total revenue captured by firm $i$ is equal to the firm’s size $\kappa_i$ over the sum of all firm sizes:

$$\langle R_i \rangle = \frac{\kappa_i}{\sum_j \kappa_j} R_{\text{total}},$$

where $R_i$ is the revenue of firm $i$ and $R_{\text{total}}$ the sum of revenue across all firms. From the properties of the binomial distribution, if there are $c$ purchase choices, the variance of firm revenue $R_i$ is given by

$$\sigma_i^2 = c \frac{\kappa_i}{\sum_j \kappa_j} \left(1 - \frac{\kappa_i}{\sum_j \kappa_j}\right).$$

When $\kappa_i/\sum \kappa_j$ is less than $1/2$—i.e., when the firm controls less than half the total resources of the industry, which is true of all nondominants—the equation above is a monotonously declining function of $\kappa_i/\sum \kappa_j$. As more competitors enter, increasing the total resources of the industry $\sum \kappa_j$ and reducing the ratio $\kappa_i/\sum \kappa_j$, the variance of nondominant incumbents’ revenue decreases; nondominant motility falls.

Decreased motility reduces the odds of a given nondominant overtaking the dominant during any fixed period of time. Let’s say that before new competitors entered, nondominant $i$ needed 10 consecutive fortuitous time periods, within each of which it achieved revenue of one standard deviation above the expected value, to grow to the current size of the dominant. An influx of new competitors decreases $i$’s motility ($\sigma_i$) by a scaling factor $\gamma$ (i.e., $\sigma_{i, \text{new}} = \gamma \sigma_{i, \text{old}}$; $0 < \gamma < 1$), $i$ now needs $10/\gamma$ lucky periods in a row to reach the dominant’s size. While the minimum number of time periods required for $i$ to reach dominant size increases by a factor of $1/\gamma$, the expected number of time periods for $i$ to equal the dominant’s size increases more, as the probability of $i$ having 10 good periods in a row is very small but the probability of $i$ having $10/\gamma$ good periods in a row is even smaller. Every time period where $i$ does not grow as fast, or even worse, shrinks, adds to the time it takes $i$ to grow to dominant size.

Processes such as these, where $i$’s size can increase or decrease at each time step, with the direction and amount of change at each step drawn from a probability distribution, are called random walks. The properties of random walks have been studied extensively. For a random walk where multiple walkers start at the same point and independently walk on a line (the asset size line in our model), the expected number of time steps ($\tau$) it takes any one of them to reach a fixed destination point (the first hitting time) is related to the number of walkers ($n$), the initial distance between the walkers and the destination ($d$), and the size of each step ($s$) by the relationship:

$$\tau \sim \frac{d^2}{s^2 \ln \nu},$$

where the tilde indicates this relationship is asymptotically true when the number of steps between the origin and the destination is larger than $\sqrt{\ln \nu}$. This condition can be intuitively understood by considering the counter-case of many walkers (say 100) starting just a few steps (say 5) away from the destination. In this scenario, at least a few of the walkers will be expected to reach the destination in the minimum possible number of steps (5).

The first hitting time equates to the expected remaining duration of dominance of the dominant in our model, and we can write an equation for the effect of new competitive entry for the simple case where all nondominant incumbents and new entrants start with the same asset size, and the size of the dominant does not change over time, as

$$\frac{\tau_n}{\tau_0} = \frac{s_0^2}{s_n^2} \ln(N + n),$$

where $N(\geq 2)$ is the number of nondominant incumbents, and $\tau_n$ is the expected duration of dominance and $s_n$ the motility of nondominants with $n$ new entrants. Inserting the equation for variance earlier yields

$$\frac{\tau_n}{\tau_0} = \frac{c(\kappa/K)(1 - \kappa/K)}{c(\kappa/(K + \kappa n))(1 - \kappa/(K + \kappa n))} \ln(N + n)$$

$$= \frac{(1 + \kappa n/K)(1 - \kappa/K)}{1 - \kappa/(K + \kappa n)} \ln(N + n),$$

where $\kappa$ is the initial asset size of each nondominant firm (incumbents and new entrants) and $K$ the sum of initial asset sizes across all incumbents. For large values of $n$, this equation approaches a linearly increasing function; the mean time $\tau_n$ until the dominant is overtaken increases linearly as the number of new entrants $n$ increases. Entry of large numbers of competitors will increase the expected length of the current dominant’s reign.

3. Simulations

These arguments suggest that dominants become entrenched when the number of new entrants increases, either because dominants benefit directly from the increase or because the effect of adding new potential claimants to the dominant’s throne is smaller than the effect of decreased size motility from more firms competing against each other for revenue and resources. To test this intuition and ascertain the magnitude of expected effects, I turn to numerical simulations, first presenting a random walk model that does not assume a particular form for the relationship between incumbent motility and competitive entry, and then a full model simulating Luce-ian consumer choices.
3.1. Mapping the Parameter Space

The first simulation is a random walk Monte Carlo, which explores the effects of varying the number of new entrants \( n \) and the motility scaling factor \( \gamma \) on the expected duration of the dominant’s reign. The model also allows comparison of the effect sizes for type I (increased dominant advantage) and type II (reduced nondominant motility) mechanisms.

Random walk models have previously yielded important insights into competitive dynamics. Arthur’s (1989) random walk model illustrated that increasing returns to adoption could lock in the dominance of an initially lucky competitor. Levinthal (1991) used a Brownian model (a random walk model where step sizes are drawn from a normal distribution) to explain the lower mortality rates of older firms, finding that older organizations will on average have accrued higher endowments of assets and so be less likely to be selected out than younger firms. Using a similar model, Denrell (2004) demonstrated that profitability differences between two firms could be long-lasting even if neither was a priori better than the other. The probability of the two firms randomly walking along similar profitability trajectories was much smaller than the probability that their trajectories diverged (a pattern known as the phenomenon of long leads in random walks), allowing the luckier firm to sustain higher profitability than the other for long periods. Cabral (2016) built a more complicated two-variable model to show that the feedback loop between firm reputation and performance can endogenously cause higher-reputation firms to invest more in reputation, leading to persistent differences in firm performance. Sutton (2007) built a random walk model to predict baselines for the duration of industry leadership using parameters identified from empirical data on Japanese industries.

My model mirrors those of Levinthal (1991), Denrell (2004), and Sutton (2007). For simplicity of communication, I present results from a single set of simulations modeling a market with 11 initial competitors—the dominant and 10 others. The nondominants each had the same initial size, 1/10th that of the dominant. Different starting conditions (e.g., numbers of nondominants and distributions of initial sizes) do not change the pattern of outcomes, as long as the dominant starts with a significant size advantage over nondominant incumbents and new entrants.

At each time step \( t \) greater than 0, I modeled changes in each nondominant’s size \( \kappa \) as a step in a one-dimensional Brownian random walk on a base-10 logarithmic scale, \( \log \kappa_{i,t} \) — the size of firm \( i \) at time \( t \) — was calculated from the size of firm \( i \) at the previous time \( t - 1 \) by

\[
\log \kappa_{i,t} = \log \kappa_{i,t-1} + \epsilon_t, \quad \text{for } t \geq 1,
\]

where \( \epsilon_t \) was drawn independently for each firm at each time step from a normal probability distribution of mean 0 and standard deviation \( \gamma \sigma \). Here, \( \gamma \) is the motility scaling factor introduced earlier, and the constant \( \sigma \) denotes the base motility (the motility when \( \gamma = 1 \)). The results shown here use \( \sigma = \log 1.1 \); in the base case where motility is not reduced, a positive one-standard deviation draw of \( \epsilon \) leads to an increase in size by a factor of 1.1, a negative one-standard deviation draw leads to a decrease in size also by a factor of 1.1. The pattern of findings presented next hold so long as \( \sigma \) is significantly less than the difference in logged sizes between the dominant and the largest nondominant.

I examined the effects of changing two factors. The first was the number, \( n \), of new entrants introduced to the system. Each new entrant entered at time 0 with size equal to that of the incumbent nondominants. The second was the motility scaling factor \( \gamma \). The number of new competitors varied from 0 to 100 in increments of 1 (\( n = 0 \) modeled the base scenario with no new entrants) and the motility scaling factor from 0.01 to 1 in increments of 0.01 (\( \gamma = 1 \) modeled the base scenario with no decrease in motility). At each value of \( n \) and \( \gamma \), I simulated 1,000 runs of 1,000 steps each, for a total of 10,000,000 runs across all conditions.

In the base scenario with no new competitors (\( n = 0 \)) and no decrease in step size (\( \gamma = 1 \)), dominant reigns lasted an average of 216.5 time steps. This mean value does not include three runs, however, where no nondominant grew to be larger than the dominant by step 1,000. This is a typical result for random walks, where the single-walker hitting time distribution has a heavy right tail; there is always a non-zero probability that the dominant will remain dominant after any arbitrarily long, finite time. Since more and more runs suffered distorted mean first hitting times from missing data points in the right tail as conditions diverged from those of the base scenario, I present results using the median first hitting time instead.

The median first hitting time represents the typical length of dominants’ reigns—i.e., the dominant half-life where for a given scenario the dominant is overtaken in half the runs and still dominant in the other half. Unlike the mean, the median is unaffected by extreme values from the long right tail of the hitting time distribution, so the median will be lower than the mean. The median length of dominant reign was 179.5 time steps (at least one nondominant had grown larger than the dominant in 498 of the 1,000 runs by time step 179 and in 502 runs by time step 180) for the base scenario.

If the effects of adding new challengers (an increase in \( n \)), each of which can overtake the dominant, were to be balanced perfectly by the effects of shrinking motility (decrease in \( \gamma \)), then the median length of dominant reigns would remain constant as \( n \) increases. We can calculate the relationship between \( \gamma \) and \( n \) that needs
to hold for this balance to occur using a survival analysis. For a given time step \( t \), let \( S(y) \) denote the survival probability of the dominant (i.e., the probability that no nondominant grows to its size during the period) in the face of competition from \( j \) competitors each with the same motility \( \gamma \sigma \) and initial size. For a single Brownian random walk, we know the survival probability at time \( t \):

\[
S = \text{erf}\left(\frac{x_c - x_0}{2\sqrt{Dt}}\right),
\]

where \( \text{erf} \) denotes the error function, \( x_c \) and \( x_0 \) the destination and starting points of the walker, respectively, and \( D \) is the diffusion coefficient. For one-dimensional Brownian motion, the diffusion coefficient and the standard deviation of a single step are related by the following (Einstein 1905, p. 559):

\[
\sigma = \sqrt{2D}.
\]

Using this relationship, and substituting the scaled motility \( \gamma \sigma \) for \( \sigma \), the initial size of the dominant \( x_c \) and the initial size of the nondominant \( x_0 \) yields

\[
S(y) = \text{erf}\left(\frac{K_{\text{dom}} - K_{\text{nondom}}}{\gamma \sigma \sqrt{2t}}\right).
\]

Since the probability of each nondominant catching the dominant is independent, we can write \( S(y) = [S(y)]^j \). To obtain the median curve, along which the dominant half-life does not change, we set \( S_{10^\text{th}}(y) = 1/2 \) (we start with \( N = 10 \) nondominant incumbents and add \( n \) competitors and set the survival probability of the dominant at time \( t \) to 50%) and find that \( y \) depends only on \( n \) and not on \( t \):

\[
y = \frac{\text{erf}^{-1}\left(\frac{1/2}{\text{erf}^{-1}\left(\frac{1/2}{1/100+n}\right)}\right)}{\text{erf}^{-1}\left(\frac{1/2}{1/10+1}\right)}.
\]

where \( \text{erf}^{-1} \) denotes the inverse error function.

The simulation results bear out this prediction. The solid gray lines in Figure 2 plot contour lines for the probability of a dominant remaining dominant until time step 180. At each observed value of \( n \) and \( y \) along the 10%-tile line, 100 of the 1,000 simulation runs resulted in the dominant still maintaining its position until step 180, while 900 of the simulations resulted in at least one nondominant growing larger than the dominant by step 180. The shaded area at the bottom of the figure contains values of \( n \) and \( y \) for which dominants remained dominant throughout this period in all runs; no nondominant grew as large as the dominant. The dashed dark line is the predicted median contour line from the equation above. The prediction closely matches the observed 50%-tile values from the simulation.

If motility decreases so that \( y \) falls below the median contour line, dominants are entrenched by competitive entry. For values of the number of new competitors \( n \) and motility scaling factor \( \gamma \) along the median contour line, there is a 50% chance that the dominant will be overtaken by step 180. At values of \( y \) below this line, the dominant becomes more likely to remain dominant until the same time step.

The median contour line decreases slowly compared to a linear function of \( n \) and falls at a slower pace as \( n \) increases, leaving a large range of \( \gamma \) where new competitive entry increases dominant half-lives. If competition between incumbents pushes \( \gamma \) into this range, dominants are entrenched; motility decreases below the level that balances the effects of increased number of competitors.

The analytical model in Section 2.3 predicted declining \( \gamma \) as \( n \) increased:

\[
y = \frac{\sigma_n}{\sigma_0} = \sqrt{\frac{c(\kappa/K)(1 - \kappa/K)}{c(\kappa/(K + \kappa n))(1 - \kappa/(K + \kappa n))}},
\]

Notes. The solid gray lines are observed contour lines for the probability of a dominant remaining dominant until time step 180. For example, along the 70% contour line, 70% of dominants across runs maintained dominance. For values of \( n \) and \( y \) within the shaded area, dominants across all runs sustained dominance throughout this period. The dashed dark line (largely overlapping the unlabelled 50% contour line) is the predicted median contour line, along which the predicted effects of increased competition for the dominant and decreased motility for nondominants are balanced—dominant half-lives remain constant along this line. The solid dark line shows predicted values of the motility scaling factor \( \gamma \) from the analytical model. Results displayed are from runs where the dominant had initial asset size 1,000,000 and nondominants and new entrants initial asset size 100,000.
where again $\kappa$ is the initial asset size of each non-dominant firm (incumbents and new entrants) and $K$ the sum of initial asset sizes across all incumbents. The solid dark line in Figure 2 plots this relationship. This line is always lower than the median contour line and falls farther away as $n$ increases, crossing several lower contour lines. When the number of competitors increases, the probability of the dominant being overtaken during any given time period drops.

### 3.2. New Entry Entrenches Dominants

How large are the effects of new competitive entry on the expected length of a dominant’s reign? Are they comparable to the effects of type I behavioral mechanisms? How do these two types of effects interact with each other?

Figure 3 presents dominant half-lives along the predicted $\gamma-n$ curve from the equation above. Triangles plot results from a set of simulation runs where the dominant received an initial size increase of 20% and crosses a set where the dominant received a 10% initial boost. This 20% or 10% boost in initial dominant size crudely models an increased advantage for the dominant from a type I mechanism (say increased consumer attention), with the assumption that the size of the boost is independent of the number of new entries (the dominant benefits when consumers start to rely on heuristics as the number of firms crosses a threshold, but increasing the number of firms further doesn’t shift consumer behavior more). The circles plot results where the dominant received no added advantage. Solid lines are linear fits to each distribution.

Initial boosts of 10% or 20% to a dominant’s endowments substantially lengthened the dominant’s expected reign. But the effects of endogenous changes to competitive dynamics from increased numbers of competitors were larger than these initial endowment effects at higher values of $n$. Boosting the initial dominant size by 10% lengthened dominant half-life by 8.5%, and boosting by 20% lengthened half-life by 16%. Adding 10 new entrants increased dominant half-life by 17.7%, 17.6%, and 17.8%, respectively, for dominants with no initial size boost and size boosts of 10% and 20%. Adding 30 new entrants lengthened half-life by 53%, 52.8%, and 53.3%, respectively. At each level of boost, adding more new entrants had a positive, linear ($R^2 > 0.995$ for all three linear fits) effect on dominant half-life. Increased competition entrenched dominants, lengthening their expected duration of dominance.

### 3.3. An Explicit Model of Consumer Choice

This pattern of results still holds if we relax the simplifying assumptions of the random walk model. Figure 4 displays results from a more involved and computationally intensive simulation, where each consumer choice was modeled explicitly. Instead of randomly drawing the change in size of a firm from a normal distribution, each consumer choice was simulated one by one. The probability a given consumer choice selected a given firm was set proportional to the firm’s size. Firms that were selected more often than expected given their size grew; those that were selected less than expected shrank:

$$
\kappa_{i,t} = \kappa_{i,t-1} + \mu (c_{i,t-1} - \langle c_{i,t-1} \rangle),
$$

where $c_{i,t-1}$ is the number of times firm $i$ was chosen in the previous time step, $\langle c_{i,t-1} \rangle$ is the a priori expected number of times the firm was selected, and $\mu$ is a scaling constant. The results in Figure 4 are from simulations with $\mu = 32.44$. This value was selected so the

![Figure 3. Dominant Half-Life as a Function of Number of New Entrants from Simple Brownian Model](image)

**Notes.** Circles, crosses, and triangles are observed dominant half-lives (time steps until dominants in half the runs in the condition were displaced) from simulations where the dominant was given a 0%, 10%, or 20% initial size boost, respectively. The lines are linear fits to these observations.

![Figure 4. Dominant Half-Life as a Function of Number of New Entrants from Consumer Choice Model](image)

**Notes.** Circles are observed dominant half-lives (time steps until dominants in half the runs in the condition were displaced) from simulation. Results displayed are from runs where the dominant had initial asset size 1,000,000 and nondominants and new entrants initial asset size 100,000.
standard deviation of size changes for nondominants when there were no new entrants was 10%, consistent with the choice of \( \sigma = \log 1.1 \) for the previous model. Using different values for \( \mu \) does not substantially change the dynamics of the simulation, but does shorten or lengthen time scales.

I modeled the same starting conditions (1 dominant, 10 nondominant incumbents) as before, varying the number of new entrants from 0 to 100. Two million consumer choices were modelled at each of the 1,000 steps in each of the 1,000 runs for each condition. The results shown have dominants starting with 10 times the size of nondominants and new entrants. The pattern of results is robust to varying the number of nondominant incumbents, the number of new entrants, and the starting sizes, as long as the dominant is initially many standard deviations of size changes larger than the nondominants and new entrants.

Unlike the previous simulation, the motility scaling factor \( \gamma \) is not modeled explicitly. The dynamics of consumer choice will determine the motility of each firm, and this motility can scale differently for each firm as their sizes vary. The current simulation allows the size of the dominant to vary also, allowing for the possibility that the dominant is overtaken largely because it shrinks, and not because other firms grow to its size.

Explicitly modelling consumer choice yields the same roughly linear relationship between increased competitive entry and dominant longevity. Dominants remain dominant longer as new entry increases.

4. Discussion
4.1. Conditions of Durable Dominance
This study proposes a shift in how we think about the relationship between competition and sustained dominance. When the stickiness of firm size is taken into account, in almost-perfect markets with widely available resources, increased competition acts to entrench dominants—a causal relationship I term durable dominance.

Durable dominance provides an explanation for sustained dominance that applies where established theories do not. Whereas established theory posits exclusionary structures—e.g., barriers to entry (Porter 1980) and preferential control of important resources (Barney 1991)—durable dominance explains sustained dominance under more open conditions: competitive entry is frequent and competitive advantage comes only from overall asset size rather than depending on the type of assets controlled. Accordingly, durable dominance may predominate in more democratized settings—where factor markets are efficient and exogenous barriers to entry are low—such as today’s financialized world where markets exist for scalable procurement of almost all types of resources. If money can buy all needed resources, then aggregate levels of cash (and assets that can be traded for cash) become the key to determining success.

The theory and analysis in this paper demonstrate that durable dominance is feasible—perhaps even inevitable—under the right conditions. The obvious first prerequisite for durable dominance is that dominants exist and differ meaningfully from nondominants. These differences can lead to increased competition affecting dominants and nondominants in opposite ways.

I presented two types of mechanisms that can lead dominants to benefit from competition. First, where behavioral processes exist that favor larger or more distinctive firms, increased competitive entry can benefit dominants by granting them disproportionate consumer recognition, decreased competitor attention, and greater learning opportunities.

Second, even when they do not benefit from such behavioral processes, dominants can become entrenched by increased competition because new competitive entry hampers nondominants’ ability to grow. Analytical and simulation models show that nondominant motility can decrease more than enough to offset the increased threat to the dominant from facing a larger number of competitors.

This second type of mechanism does not present a theory of dominant thriving, but rather a theory of nondominant frustration and entrepreneurial futility. While dominants are predicted to remain dominant longer—stay number one in the industry—their market share and revenue may fall. The models assume that dominant firm strategies are not affected by these changes, but in the real world this assumption may not hold. If a dominant’s management team starts to make big bets on risky initiatives, perhaps because they value market share over market position, the dominant can quickly fall from its position.

The models contain other simplifying assumptions. Growth was assumed to come only from excess revenue, and other sources of growth—such as mergers—were bracketed out. Efficient factor markets were assumed, but the dynamics of competition for production resources were not explicitly modeled. The possibility of firms exiting the market was not incorporated in the models. The simple Brownian model related changes in firm size linearly to the existing size of the firm. The full consumer choice model held the number of consumer choices for each time period constant and assumed that the probability a firm’s offering was chosen was linearly proportionate to the firm’s size and that each choice was made independently and resulted in the same amount of revenue.

Some of these assumptions are more substantively important than others. The absence of large-scale mergers between near-dominants is an important boundary condition, and a rapid increase in the number
of consumers can increase nondominants’ motility enough to threaten dominants’ reign: Type II mechanisms may only matter in more stable, established industries, where mergers are rare and the number of consumers is not growing rapidly. (Note, though, that dominant firms may be more likely to be in a position to benefit by acquiring new and promising upstarts, and new consumers may be more likely to choose the most exemplary companies.)

Other assumptions are less troublesome. A model of factor market competition, say for employees, looks very similar to the consumer choice model. As long as the net number of firms and their total asset size remain as modeled—i.e., exit is balanced by entry—the models still relate dominant entrenchment to levels of competition. Growth in real industries appears to scale linearly with some function of size (Sutton 2007). Non-independent consumer choices may tend to create advantages for the dominant due to herding or platform effects (Arthur 1994, Katz and Shapiro 1985). If some purchases are very large, only large firms may be able to fulfill them.

The mechanisms described in this paper are not the only ones that can create durable dominance. Consider a variation of the Brownian model, for example. The original model posited size changes for each firm correlated to the current size of the firm. This rule buffered a larger firm from those much smaller, as firms that were much smaller needed to take several successive positive steps to grow to the larger firm’s size. If a pro-small business policy change removed the relationship between current size and step size, and instead imposed a large, uniform step size, the motility of smaller firms would increase and the larger firm would face more risk from the much smaller firms. The larger firm could be overtaken in a single time step if the new step size was large enough.

But this type of policy change can also entrench the dominant. A concrete example may be useful to help understand why this is so. Consider what would happen if everyone in the world could gain or lose exactly $1,000 a day. A penniless, but extremely lucky graduate student could gain almost $20 million dollars over 50 years. An unlucky multimillionaire could lose her whole fortune and more over this same time period. The new rule increases the likelihood of the multimillionaire becoming poorer than the graduate student. The rule constrains billionaires also, removing their ability to compound their wealth in rich-get-richer fashion. But this constraint entrenches Bill Gates atop the wealth distribution. Even if he lost $1,000 every day for 50 years, he would lose less than $20 million of his $86 billion fortune.10 Warren Buffett, the second richest man in the world, with $75.6 billion,11 could only gain $20 million. Gates’s position is secure. Note that the graduate student could catch up to Gates’s wealth if the daily step size was the same order of magnitude as Gates’s wealth. This scenario, though, is equivalent to devaluing U.S. currency so that $100 million is a reasonable daily wage.

The existence of many different types of mechanisms entrenching the dominant when competition increases suggests that durable dominance may be multiply-determined, and thus likely to be observed in diverse settings. Indeed, patterns of dominance in several competitive markets intimate durable dominance, indicating promising settings for empirical investigation. Dominants in mutual funds, beer, and cola seem to do better when their markets are more open and competition increases, for example. Similar patterns also occur in non-industrial markets, such as markets for Hollywood starring roles, academic paper citations, and wealth in society. Even nonsocial phenomena, such as the duration of an allele atop the allele frequency distribution in a population subject to mutations, may be amenable to analysis applying predictions from this study.

4.2. Implications for Theory and Practice

If empirically proven, durable dominance holds important implications for future studies of dominance and competition. The theory’s predictions are counter to straightforward economic assumptions: increased competition, rather than dislodging dominants, entrenches them. The greater the number of competitors, the longer the expected term of dominance. Like other random variation models (Denrell et al. 2015), the stochastic models of this paper provide a baseline for the expected effect of increased competition on dominants. We may need to update our intuitions of what is normal in close-to-perfect market competitive contexts, where significant size differences exist between a dominant and its competition. Durable dominance could be the rule, not the exception.

More generally, this study highlights the need for better theory about dominants. We cannot assume that the most endowed few atop the distribution behave like the less-endowed majority. Indeed, it might be safer to assume the opposite. Dominants need to be analyzed separately. Taking the average effect of an independent variable over the population will not yield meaningful predictions for dominants.

The theory of durable dominance suggests policy implications for dominants and nondominants alike. Dominants may benefit from a strategy of opening their markets to new entrants and fostering smaller competitors. These upstarts will weaken potential challenges to the dominants’ dominance from large but nondominant incumbents. Whether guided by altruism or cunning, this strategy seems precisely the one that Google, e.g., executes when it provides funding, employees, technology, and platform access to startups. An elite business school worried about losing
its elite status because of challenges from nearly elite competitors could attempt to entrench its position by investing resources in democratizing the creation and distribution of massive open online courses (MOOCs). Giving away access to world-class production and distribution to even small colleges would flood the marketplace with new entrants. Other schools immediately below elite status would be forced to invest resources in competing against these now-strengthened upstarts, rather than taking measures to challenge elite schools. The hypothetical protagonist’s elite position would become further entrenched. Dominants may also protect their positions by sharing resources with competitors, e.g., by opening a network platform (e.g., Google ecosystem, MOOC network) to competitors. Sharing the same resources (such as the network platform) is better for maintaining dominance than allowing competing—and possibly disruptive—sets of resources to be developed.

Nondominants and policy makers face more difficult choices. Schumpeter ([1942] 1994) asserted that monopoly profits motivate innovation and disruption. When nondominants and dominants possess different types of resources, nondominants can disrupt the industry and dethrone the dominant (Christensen 1997). But if access to markets and resources is democratized, companies may come to compete primarily on sheer scale of financial and nonfinancial (but financially commensurable) assets. Disruptive innovation becomes unlikely in such a scenario. Would-be entrepreneurs will find it more difficult to overturn the status quo, and monopoly can no longer be defended as motivating innovation.

The theory presented here predicts that a dominant’s hold on its position strengthens as the market becomes more open and democratized. This implies that current taken-for-granted policy measures for weakening monopoly—lowering barriers to entry and democratizing access to key resources—may backfire, hurting near-dominants but benefiting the largest firms, unless the measures involve significant and rapid reductions in dominants’ assets. Even splitting a dominant into pieces may exacerbate the problem if the new pieces are significantly larger than other competitors.

How then can dominants be reined back? The answer, counter-intuitively, may be to make the market less perfect. While market distortions can entrench dominants (e.g., Rajan and Zingales 2003), this study shows that the lack of market frictions and social constraints (Granovetter 1985) can do the same. Allowing nondominants to control idiosyncratic resources and enjoy protected niches, providing mechanisms to socially connect nondominants, and giving preferential treatment to larger nondominants rather than smaller may provide remedies against durable dominance.

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Appendix
Characteristics of \( \tau_n \)
From the analytical model, we have

\[
\tau_n = \tau_0 \frac{(1 + \kappa n/K)(1 - \kappa/K)}{1 - \kappa/(K + \kappa n)} \ln N \frac{\ln (N + n)}{\ln (N + n)}.
\]

By observation, this is asymptotically a linearly increasing function of \( n \), since for large values of \( n \)

\[
\frac{\kappa}{K + \kappa n} \rightarrow 0,
\]

and \( n \) grows much faster than \( \ln (N + n) \).

This asymptotic relationship can be verified by examining the first and second derivatives of \( \tau_n \):

\[
\frac{\partial \tau}{\partial n} = \frac{(K - \kappa)(K + \kappa n)}{K^2(N + n)(K + \kappa n - \kappa)^2 \ln^2(N + n)} \times [\kappa(N + n)]
\]

\[
\frac{\partial^2 \tau}{\partial n^2} = \frac{K^2(N + n)^2(K + \kappa n - \kappa)^3 \ln^2(N + n)}{K^2(N + n)^2(K + \kappa n - \kappa)^2 \ln(N + n)} \times 2(K + \kappa n)^2 + 2\kappa^2(N + n)^2 \ln^2(N + n)
\]

\[
\times \left[(K + \kappa n - \kappa)(K + \kappa n)^3 \ln(N + n)ight]
\]

\[
\times [\kappa^2(n^2 + 2Nn - 3n - 4N) + K\kappa(2N + 1) - K^2].
\]

The first derivative is positive for large \( n \) by inspection since the fraction in the first line of the equation is always positive, and the term being subtracted from the second line is positive and has a higher degree in \( n \) than the term being subtracted (\( n^2 \ln n \) versus \( n^2 \)). The second derivative goes to zero for large \( n \) since the denominator has higher degree in \( n \) than the numerator (\( n^3 \ln^2 n \) versus \( n^4 \ln n \)). The first derivative converges to a positive constant as \( n \) becomes large; \( \tau_n \) is asymptotically positive linear in \( n \).

For the simulation runs described in the text, even when \( n \) was small, \( \tau_n \) was always larger than \( \tau_0 \). We can rewrite the equation for \( \tau_n \) to see why this was so in this case, and to shed light on whether we should always expect even small numbers of entrants to increase the expected length of dominant reign. Substituting \( g = \kappa n/K \) and \( p = \kappa/K \) and simplifying

\[
\tau_n = \tau_0 (1 + g^2(1 - p) \frac{\ln N}{1 + g - p}) \frac{\ln (N + n)}{\ln (N + n)}.
\]

Setting \( N = 10 \) and solving for \( \tau_n > \tau_0 \) yields

\[
p > \frac{g^2 \ln 10 + 2g \ln 10 - \ln(g + 10)}{g^2 \ln 10 + 2g \ln 10 - \ln(g + 10) + \ln 10}.
\]

Figure A.1 plots the right-hand side of this inequality. The dominant’s expected reign is decreased if \( p \) and \( g \) fall in the shaded area. Across all possible values of \( g \), the minimum value of \( p \) that results in decreased dominant reign is slightly
below 0.5. But $p$ cannot be this high, by definition, since $p$ is the ratio of a single nondominant’s asset size over the total assets of all incumbents (including the dominant).

Even when we start with only two nondominant incumbents, resulting in the lowest possible $\tau_n/\tau_0$ for any given $p$ and $g$, the lowest possible value of $p$ that can lead to shortened dominant reign from adding new entrants is slightly above 0.2 and increases as $g$ increases. If a single new entrant enters with the same size as a nondominant incumbent, so $g = p$, then the minimum $p$ that decreases dominant reign is 0.256. If two new entrants enter of the same size as a nondominant incumbent ($g = 2p$), then the required minimum $p$ to decrease dominant reign is 0.6, which is again by definition impossible. Relaxing the restriction that new entrants enter with the same size as nondominant incumbents alters the equation for $\tau_n$ slightly, but the conditions for new entrants decreasing dominant reign still follow the basic shape of the inequality curve shown in Figure A.1.

In almost all cases where a dominant of much larger size than other incumbents and new entrants exists, then, the dominant’s expected reign lengthens when new entrants enter. Exceptions to this rule only occur when there are very few nondominant incumbents and when the summed size of new entrants is very small compared to the summed size of incumbents.

**Derivation of $\gamma$-$n$ Median Contour Line**

We define $\lambda$ to simplify the representation of $S_n(\gamma)$:

$$S_n(\gamma) = \text{erf} \left( \frac{\frac{\lambda - \gamma}{\sigma \sqrt{2T}}}{\sqrt{n}} \right) = \frac{\lambda}{\sqrt{n}}.$$

The median contour curve starts at the base scenario, where $n = 0$, $\gamma = 1$, and the survival probability is $1/2$. Inserting these values into the equation for $S_{10}$ allows us to solve for $\lambda$:

$$S_{10}(1) = [S_1(1)]^{10} = \left[ \text{erf}(\lambda) \right]^{10} = \frac{1}{2},$$

$$\lambda = \text{erf}^{-1} \left( \frac{1}{2} \right)^{1/10}.$$

We can now solve for the full median contour curve giving $\gamma$ as a function of $n$:

$$S_{10+n}(\gamma) = [S_1(\gamma)]^{10+n} = \left[ \text{erf} \left( \frac{\lambda}{\sqrt{n}} \right) \right]^{10+n} = \frac{1}{2},$$

$$\gamma = \frac{\lambda}{\text{erf}^{-1} \left( \frac{1}{2} \right)^{1/10}}.$$

**Endnotes**


2. The dominant could benefit even more, depending on the characteristics of the new entrants. If, e.g., new offerings were inferior in all respects to the dominant’s offering, but superior in some respects to nondominant offerings, consumers would become more likely to choose the dominant’s offering—a phenomenon dubbed the asymmetric dominance or decay effect (Huber et al. 1982).

3. The effect of new competition on the dominant’s motility depends on the asset size of the dominant compared to its incumbent competitors and on the size of the total assets of new competitors. If the dominant initially controls more than half the total resources of all firms in the market and the summed size of new competitors’ assets is relatively small, the dominant’s motility will increase. If the dominant does not initially control more than half the total resources of the industry or the influx of new competitor resources is very large, the dominant’s motility will decrease.

4. This result has been derived multiple times in the mathematics and physics literatures. Larralde et al. (1992) provide asymptotic expressions for the relationship between maximum distance travelled and time for one, two, and three dimensions, for example. The relationship stated in the main text follows from their results for one-dimensional random walks.

5. Even at smaller values of $n$, $\tau_n$ ($n > 0$) is longer than $\tau_0$ if the sum of new entrants’ sizes is not insignificant compared to the sum of incumbent sizes. See the appendix for characteristics of $\tau_n$ as a function of $n$.

6. Dominants did not grow or shrink in this initial model, allowing a simpler interpretation of the model results and a direct correspondence to the analytical model of the previous section. The full consumer choice model introduced later in this section removes this restriction.

7. I use the log scale to echo the order of magnitude differences found in firm size in the real world, but the arguments hold even if dominants are only linearly larger than nondominants. One need only replace log $x$ with $x$ in the analysis that follows.

8. See the appendix for derivation.

9. This is a slight simplification. The actual condition was whether more than 100 dominants remained dominant at step 179 and 100 or fewer dominants remained dominant at step 180.


11. Ibid.

**References**


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