A Quantitative Theory of the Credit Score

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This draft: May 2022

Abstract

What is the role of credit scores in credit markets? We argue that it is a stand-in for a market assessment of a person's unobservable type (which here we take to be patience). We pose a model of persistent hidden types where observable actions shape the public assessment of a person's type via Bayesian updating. We show how dynamic reputation can incentivize repayment. Importantly, we show how an economy with credit scores implements the same equilibrium allocation. We estimate the model using both credit market data and the evolution of individuals' credit scores. We find a 13% difference in patience in almost equally sized groups in the steady state population with significant turnover and a shift towards becoming more patient with age. If tracking of individual credit actions is outlawed, poor young adults of low type benefit from subsidization by high types despite facing higher interest rates arising from lower dynamic incentives to repay.

Keywords: Credit Scores, Unsecured Consumer Credit, Bankruptcy, Persistent Hidden Information.

JEL Classification Numbers: D82, E21, G51.

*This project has been a long time in the making. An earlier version circulated under the title “Credit Scoring and the Competitive Pricing of Default Risk.” We thank our editor Chad Jones and three referees for very helpful comments on an earlier version of this paper and Isaiah Andrews as well as Tsung-Hsien Li and Jan sun for this version. We thank all the economists who have had an important input into the final product: Murat Tasci, Pablo D’Erasmo, Daphne Chen, Jake Zhao, and Kuan Liu. We thank Hongchao Zhang for kindly sharing with us his DFBOLS Fortran code. We thank Cole Drier and Michael Slonkosky for research assistance. We also thank the many seminar and conference participants who commented on earlier versions of the paper. Finally, Corbae and Ríos-Rull wish to thank the National Science Foundation for support under grants SES-0751380 and SES-0351451. The views expressed in this paper are those of the authors and do not necessarily reflect views of the Federal Reserve Bank of Philadelphia or of the Federal Reserve System.

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1 Introduction

Credit scores are a fundamental ingredient of a borrower’s access to credit. In the United States, credit bureaus and credit rating agencies serve this function for individual and business credit by creating and maintaining credit scores for individual borrowers. Similar agencies exist in many other countries. Credit scores affect borrowing terms and change with credit use and repayments. Despite the widespread use of credit scores in actual credit markets, they are conspicuously absent from standard quantitative models of consumer default which are typically more concerned with allocations than the contractual arrangements that generate them.

In this paper we provide a theory of the joint behavior of unsecured credit and credit scores, that is, both allocations and arrangements, over the lifetime that is based on hidden information about some persistent, credit-relevant individual characteristic – which we take to be patience – and where maintaining a good reputation plays a central role.

Our theory is founded on the premise that an individual’s true propensity to repay — i.e., the individual’s true type — is hidden from her creditors, and it is the presence of this persistent hidden information that makes an individual’s history of actions relevant for lenders. Our theory is dynamic: at any point in time, lenders use a person’s observable history of actions to perform a Bayesian update of her type; individuals understand this and choose actions mindful of the consequence any action has on the future beliefs of lenders. A loss of reputation, rather than stigma or exogenous exclusion from future borrowing is the only dynamic punishment from default. Specifically, an individual’s credit score falls upon default and she subsequently faces worse borrowing terms. Our theory is competitive: information available to any lender is assumed to be available to all lenders and there is free entry into the business of lending. Finally, our theory respects a key feature of the institutional arrangement under which unsecured consumer credit is extended in the United States: at some monetary cost, individuals can choose to have their debts discharged via Chapter 7 bankruptcy.

We make several contributions. First, we extend the theory of unsecured credit to accommodate persistent hidden information about individual types. Our model environment is rich enough to cover four of the five characteristics lenders use to assess creditworthiness: character (reflected in credit history), capacity (reflected in debt-to-income ratio), capital (wealth), and conditions (amount of the loan).\footnote{The fifth, collateral, is not relevant for unsecured credit. See e.g. https://www.investopedia.com/terms/f/five-c-credit.asp.} Competition drives lending contracts to be indexed by all observable borrower and loan characteristics relevant for predicting the probability of default on a loan. When there is hidden information, a new
individual characteristic becomes relevant: a borrower’s type probability vector — in the terminology of this paper, the borrower’s type score — indicating the probability that a person is of each of the different types existing in the economy. The Bayesian update of an individual’s type score conditions on all relevant observables: the individual’s current type score, her current net wealth, all the relevant information to forecast future earnings, and, of course, her current action, be that to save, borrow or default. One way to interpret the large number of conditioning variables is that the lender is using “big data.” Our framework easily encompasses “small data” cases in which lenders observe only some strict subset of actions (an instance of a “small data” world is explored in Section 6.2).

Second, after proving an equilibrium with type scores exists, we show that a market arrangement which uses credit scores as in modern societies replicates the same equilibrium allocation without any reference to type scores. Specifically, we use the type score to define a credit score — an object that yields a ranking of individuals with regard to their probability of default on a particular contract. Such an ordinal ranking is widely used by credit bureaus. We provide a sufficient condition such that the equilibrium of the arrangement that uses credit scores to index contracts has the same allocation as the equilibrium of our baseline economy with type scores. Just as agents take prices as given in standard competitive equilibrium models, in our equilibrium with credit scores individuals and lenders take credit-score-dependent prices and the distribution of future credit scores conditional on their current state and actions as given; they do not need to know what is behind such functions, just that they exist. In doing so, we provide a theory of the credit score itself and of how it evolves over time in response to fundamentals. In this context, we take to heart that the actual market arrangement is a form of data and our equivalence result allows for the use of such data for empirical purposes.

Third, we take our model to the data, estimating preference parameters from the joint behavior of credit scores over an individual’s lifetime and aggregate credit market moments. It is here that our decision to model age variation in the evolution of earnings and hidden characteristics pays off. For these estimates, we verify that the sufficient condition which guarantees equivalence between the type score economy and credit score economy holds. We find what we believe are important properties of the U.S. population with regard to (hidden) patience as revealed by the properties of the credit market: the difference in discount factors between patient and impatient people are 14% annually, slightly less than one third of people are born patient but that fraction rises with age and patience is persistent but not permanent. Random changes in type (i.e., in patience) along with transitory variation in unobserved extreme value shocks to utility and earnings, prevent fast learning about an individual’s type.\(^2\)

\(^2\)An empirical literature that finds evidence of adverse selection in credit markets includes Ausubel (1999), Agarwal et al. (2010), Hertzberg et al. (2018), and Einav et al. (2013).
Fourth, we use our estimates to explore the role of hidden information in the U.S. unsecured credit market. We start by considering a policy counterfactual in which lenders are prohibited from keeping track of the history of an individual’s asset market actions but can condition on the observable length of individuals’ credit history (effectively their age). In this case, impatient types are pooled with patient types without having to bear the costs of imitating them in order to obtain better borrowing terms. Since young adults wish to borrow against their higher expected future income, and most start their adult life impatient, the policy has the possibility of improving the welfare of those young adults. However, the policy removes the incentives to maintain a good reputation which leads to individuals facing higher interest rate offerings. We find the negative incentive effects roughly offset the potential pooling benefits except for young, poor impatient adults who are made substantially better off.

Our second counterfactual considers an economy where one’s type is perfectly observable. The findings are intuitive. Since the impatient are now known, they face a more adverse situation; their interest rates are higher and they borrow less. The contrary is true for the patient. As people age, this knowledge becomes less relevant because people accumulate precautionary balances and rarely borrow. The benefits to the patient outweigh the costs to the impatient resulting in a relatively large welfare improvement associated with full information. Furthermore, individual-level allocations in the full information economy are quite different from individual-level allocations in the base economy, showing that our “big data” economy is still far being a full information economy.

Fifth, we make a couple of methodological contributions. First, we combine both screening and dynamic signaling where these screening and signaling opportunities are constrained by noise which we introduce via extreme value shocks. The shocks ensure that beliefs held by lenders following any feasible action are determined in equilibrium (reminiscent of Selten (1975) and Myerson (1978)). The shocks also eliminate multiplicity of equilibria that can arise in signalling games from variation in off-the-equilibrium-path beliefs. Second, we extend quantitative theory models of default to include hidden information which requires us to index the pricing of credit to the market assessment of individual

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3 The microeconomic literature classifies hidden knowledge/adverse selection models as “screening” or “signaling” models (Riley (2001)). As in screening models, in our paper lenders offer loans distinguished by loan characteristics (size of the loan) and observable personal characteristics (income, previous history) that give ample scope for separation (if such separation is desirable from an individual point of view and can be sustained in equilibrium). And, as in signaling models, there are actions that an individual can take that have no effect on the payoff to any creditor but which convey valuable information to them. For instance, when an individual who has no debt (and therefore is not interacting with any creditors) takes an action that reveals information about her true type (such as increasing her savings), that action has signaling value for creditors in the event she were to borrow from them in some future period. Thus there is also ample scope for individuals to signal their type for use in some future contract via an update on their type score.

4 Even if there was no hidden information and no Bayesian updating of beliefs, a continuous support of shocks would be needed to ensure the existence of a pure strategy equilibrium; otherwise existence would require that people be allowed to play mixed strategies. Despite individuals playing pure strategies, the shocks ensure different types may choose the same action analogous to a “semi-separating” equilibrium. Further, the assumption that the shocks are drawn from a Type 1 extreme value distribution delivers the tractability as in McFadden (1974) and Rust (1987).
types. While previous quantitative theory models imposed exogenous punishment, we incorporate dynamic reputation as a means of disciplining borrowers along the lines of Diamond (1989).

The paper is organized as follows. Section 2 describes our baseline economy with hidden information. Section 3 describes the equilibrium problems faced by our agents. Section 4 describes how we map the model to data and Section 5 studies the properties of the estimated model. Section 6 compares our baseline economy to alternative economies with different information structures. Section 7 considers an extension which includes delinquency. Section 8 provides a brief conclusion. There is an accompanying online appendix, where we provide additional theoretical (Section A), computational (Section B), and data (Section C) results.

2 Environment

Time is discrete and infinite. At each point in time, there is a unit measure of individuals. An individual dies with probability $1 - \rho$ at the end of the period and individuals who die are replaced by newborns. An individual’s persistent log earnings, denoted $e_t \in E = \{e_1, e_2, ..., e_E\} \subset \mathbb{R}^{++}$, are exogenously drawn from a stationary finite state Markov process $Q^p(e_{t+1}|e_t)$. In addition, there are purely transitory (log) earnings, denoted $z_t \in Z = \{z_1, z_2, ..., z_Z\} \subset \mathbb{R}^{++}$, which is exogenously drawn from a stationary probability distribution $H(z_t)$. All earnings draws are independent across individuals.
and we denote individual total earnings as $y_t(e_t, z_t) = \exp(e_t + z_t)$. A newborn’s earnings are drawn from initial distribution $G_e$.\(^{10}\)

At time $t$ individuals can choose assets $a_{t+1} \in \mathcal{A} = \{a_1, a_2, ..., a_N\} \subset \mathbb{R}$, where $a_1 < a_2 < ... < a_N$, at discount price $q_t$ determined in a competitive market. We assume the finite set $\mathcal{A}$ includes 0 with $a_1 < 0$ and $a_N > 0$. If an agent holds debt (i.e. $a_t < 0$), she (or he) can choose whether or not to file for bankruptcy $d_t \in D = \{0, 1\}$. If she files (i.e. $d_t = 1$), then in the period of filing she cannot borrow or save (i.e. $a_{t+1} = 0$) and her earnings net of the costs of bankruptcy become $y_t(e_t, z_t)(1 - \kappa_1) - \kappa$ where $\kappa > 0$ is the bankruptcy filing fee and $\kappa_1 \in (0, 1)$ proxies for the negative consequences of bankruptcy on one’s earnings prospects.\(^{11}\)

In each period $t$, the individual values consumption $c_t$ using a utility function $u(c_t) : \mathbb{R}_+ \to \mathbb{R}$ which is is continuous, increasing, and concave. At time $t$, an individual discounts her future utility at rate $\beta_t \in \mathcal{B} = \{\beta_1, \beta_2, ..., \beta_B\}$ if she survives. Her discount factor varies stochastically over time drawn from a finite state Markov process $Q^\beta(\beta_{t+1}|\beta_t)$. The $\beta_t$ are drawn independently across individuals and are unobservable to others. We will call $\beta_t \in [0, 1)$ a household’s type.\(^{12}\)

In addition, households receive action-specific, additively separable extreme value preference shocks each period. The first set of shocks attach to the bankruptcy/no bankruptcy choice ($d_t \in \{0, 1\}$) and is therefore drawn only by borrowers: \(\nu_t = (\nu_t^{d=0}, \nu_t^{d=1}).\) \(^{(1)}\)

The second set of shocks of length $N$ attach to the set of borrowing and saving choices in the event of no default: \(\epsilon_t = (\epsilon_t^{a_1}, \epsilon_t^{a_2}, ..., \epsilon_t^{a_N}).\) \(^{(2)}\)

The vectors $\nu_t$ and $\epsilon_t$ are drawn independently across individuals. Each element $\nu_t^d$ and $\epsilon_t^{a_n}$ is drawn from type I extreme value distributions $F_\nu$ and $F_\epsilon$ with scale parameters $\alpha$ and $\lambda$, respectively.\(^{12}\)

Intermediaries can observe individuals’ persistent earnings (i.e. $e_t$) and asset market behavior (i.e. $a_t$, $d_t$, and $a_{t+1}$), but cannot observe their preferences (i.e. $\nu_t$, $\epsilon_t$, and $\beta_t$) nor the transitory component of earnings (i.e. $z_t$). Since $\nu_t$, $\epsilon_t$ and $z_t$ are i.i.d. over time and individuals, nothing can be learned about their future values from their current values. In contrast, since $\beta_t$ is drawn from a persistent

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\(^{10}\)By setting $G_e$ to the degenerate distribution that has all mass on the lowest persistent earnings level, we can parsimoniously achieve a rising earnings profile over an individual’s working life as an approximation to the full life cycle model in Livshits et al. (2007).

\(^{11}\)See Corbae and Glover (2018) for a model in which a poor credit record adversely affects an individual’s earnings.

\(^{12}\)For reasons that we explain when computing the model in Section 4.1, we permit the location parameters of the extreme value shocks to depend on the options available to households.
Markov process, the probability distribution of its future values depends on its current (unobservable) value. We denote the creditor’s probability assessment that an individual is of type $\beta_i$ at the beginning of period $t$ before any actions are taken as $s_t(\beta_i) = \Pr(\beta_t = \beta_i)$. We call $s_t = (s_t(\beta_1), ..., s_t(\beta_B))$ an individual’s type score with $\sum_{i=1}^B s_t(\beta_i) = 1$.\(^{13}\)

Given an individual’s observable characteristics $\omega_t = (e_t, a_t, s_t)$ as well as their credit market actions $(d_t, a_{t+1})$, the financial intermediary revises its assessment of an individual’s type from $s_t$ via Bayes’ rule.\(^{14}\) We denote this update as $\psi_t^{(d_t,a_{t+1})}(\omega_t) \in [0,1]^B$. As a result of this assessment, the prices faced by an individual in the credit market will also depend on her observable state and her credit market actions. Thus, we denote the price function for an individual with observable characteristics $\omega_t$ who chooses assets $a_{t+1}$ by $q_t^{a_{t+1}}(\omega_t)$. The arguments of $q_t$ influence the price because they can directly affect the likelihood of repayment on a loan (as in standard debt and default models) and indirectly by revealing information about the individual’s current type (this is encoded in the update $\psi_t^{(d_t,a_{t+1})}(\omega_t)$).\(^{15}\) Importantly, note that all the future implications of current credit market actions are encapsulated in the update $\psi_t^{(d_t,a_{t+1})}$. In particular, the only punishment to bankruptcy in future periods (aside from those that follow from the requirement that saving is not permitted in the filing period) is the possible loss of reputation stemming from the intermediary’s adverse assessment of her unobservable type.

For technical reasons, we assume $s_t \in S$, a finite subset of $[0,1]^B$. This assumption makes it possible to apply standard methods to prove existence of equilibrium. Since the posterior $\psi_t$ may not lie on one of the finite points in $S$, we assign it randomly to nearby points in $S$. We denote the probability mass function implied by our random assignment rule given in equation (35) in Online Appendix A as $Q^S(s_{t+1}|\psi_t)$.\(^{16}\) There we prove:

**Lemma 1.** There exists an assignment rule satisfying: (i) $E_{s_{t+1} \in S} [Q^S(s_{t+1}|\psi_t)] = \psi_t$ (i.e. consistency), (ii) the variance of the approximation error (i.e. of $s_{t+1}$ from $\psi_t$) is arbitrarily small, and (iii) $Q^S(s_{t+1}|\psi_t)$ is continuous in $\psi_t$.

**Definition 1.** The *timing* in any given period is as follows:

1. All individuals (survivors and newborns) begin with the vector $(\beta_t, e_t, a_t, s_t)$ and receive a transitory

\(^{13}\)Of course the framework is rich enough to add more unobservables. For instance, if the persistent component of earnings are unobservable, then $s_t = (s_t(\beta_1, e_1), s_t(\beta_1, e_2), ..., s_t(\beta_B, e_E))$.

\(^{14}\)As in the original econometric use of extreme value shocks in the discrete choice literature, $(v_t, e_t)$ provide a parsimonious way to capture how a type scorer may observe different choices $(d_t, a_{t+1})$ by two individuals in the same observable starting state $(\omega_t)$ due to, for instance, unobserved preference heterogeneity.

\(^{15}\)Note that in the absence of hidden information regarding type, the pricing function would be independent of $a_t$ (as in fact is the case in Chatterjee et al. (2007a)) since past debts have no bearing on the repayment probability of newly incurred debt.

\(^{16}\)There is nothing of substance in this randomization over contiguous elements of $S$ since $S$ is finely gridded when we estimate the model.
earnings shock $z_t$.

2. Individuals who have $a < 0$ receive the random utility vector $\nu_t$ and decide whether to file for bankruptcy ($d_t = 1$) or not ($d_t = 0$).

3. Individuals who have not filed for bankruptcy receive the random utility vector $\epsilon_t$ and choose a feasible action given prices $q_{t+1}(\omega_t)$.

4. Based on each individual’s actions ($d_t$, $a_{t+1}$) and observable characteristics $\omega_t$, intermediaries revise their assessments of an individual’s type via Bayes’ rule, updating $s_t$ to $\psi_t$.

(a) Individuals who survive draw beginning-of-next-period realizations of $\beta_{t+1}$ and $e_{t+1}$ from the exogenous transition functions $Q^\beta(\cdot|\beta_t)$ and $Q^e(\cdot|e_t)$. The beginning of next period type score $s_{t+1}$ is drawn from the probability mass function $Q^s(\cdot|\psi_t)$.

(b) Newborns begin life with $\beta_{t+1}$ drawn from initial distribution $G_\beta$, earnings class $e_{t+1}$ drawn from initial distribution $G_e$, zero assets, and a type score $s_{t+1}$ equal to $G_\beta$ for consistency.

We assume $G_\beta \in \mathcal{S}$.

3 Equilibrium

3.1 Individuals’ problem

Let a current value $x_t$ be denoted $x$ and next period’s variable $x_{t+1}$ be denoted $x'$. Denote the part of the state space observable to creditors by $\Omega = \{E \times A \times S\}$ with typical element $\omega$. An individual takes as given:

- the price function $q'(\omega) : A \times \Omega \to [0, 1]$
- the type scoring functions $\psi^{(0,0)}(\omega) : A \times \Omega \to [0, 1]^B$ and $\psi^{(1,0)}(\omega) : \Omega \to [0, 1]^B$ which perform Bayesian updating of an individual’s type based on all observables following asset choice and bankruptcy, respectively.

For ease of notation, we will denote the triplet of functions $\{q'(\omega), \psi^{(0,0)}(\omega), \psi^{(1,0)}(\omega)\}$ by $f \in F$, where $F = \{ (f_1, f_2, f_3) \mid f_1 : A \times \Omega \to [0, 1], f_2 : A \times \Omega \to [0, 1]^B$ and $f_3 : \Omega \to [0, 1]^B \}$.

Definition 2. Given $(z, \omega)$ and $f \in F$, the set of feasible actions is a finite set $\mathcal{F}(z, f)$ that contains
all actions \((d, a')\) such that consumption \(c^{(d,a')}(z, \omega|f)\) is strictly positive where:\(^{17}\)

\[
c^{(d,a')}(z, \omega|f) = \begin{cases} 
  y(e(\omega), z) + a(\omega) - q^a(\omega) \cdot a' & \text{if } (d, a') = (0, a') \\
  y(e(\omega), z)(1 - \kappa_1) - \kappa & \text{if } a(\omega) < 0 \text{ and } (d, a') = (1, 0)
\end{cases}
\]  

(3)

We make the following assumption:

**Assumption 1.** \(y(e_1, z_1) + \min\{a_1, -\kappa - \kappa_1 y(e_1, z_1)\} > 0.\)

This assumption ensures that it is always feasible for an indebted individual to file for bankruptcy and always feasible for her to pay back her debt.

We work backwards from an individual’s state at stage 3 in timing. Given the functions \(f,\) for \((d, a') \in \mathcal{F}(z, \omega|f)\) we denote the conditional value function

\[
u^{(d,a')}(\beta, z, \omega|f) = u \left( c^{(d,a')}(z, \omega|f) \right) + \beta \rho \cdot \sum_{(\beta', z', e', s')} Q^\beta(\beta'|\beta)Q^e(e'|e)H(z')Q^s(s'|\psi^{(d,a')}(\omega))W(\beta', z', \omega'|f)
\]  

(4)

where the expected value function \(W\) integrates the value function over transitory preference and is defined below.

The value function \(V^{ND}(\epsilon, \beta, z, \omega|f) : \mathbb{R}^N \times \mathcal{B} \times \mathcal{Z} \times \Omega \rightarrow \mathbb{R}\) for an individual who chooses not to file for bankruptcy at stage 2 is then given by

\[
V^{ND}(\epsilon, \beta, z, \omega|f) = \max_{(0, a') \in \mathcal{F}(z, \omega|f)} \nu^{(0,a')}(\beta, z, \omega|f) + e^{a'}
\]  

(5)

and

\[
W^{ND}(\beta, z, \omega|f) = \int V^{ND}(\epsilon, \beta, z, \omega|f) dF_\epsilon(\epsilon).
\]  

(6)

Given the sequential nature of choices in our timing, the value function at stage 2 is then given by

\[
V(\nu, \beta, z, \omega|f) = \begin{cases} 
  W^{ND}(\beta, z, \omega|f) & \text{if } a(\omega) \geq 0 \\
  \max \left\{ \nu^{(1,0)}(\beta, z, \omega|f) + \nu^{D}, W^{ND}(\beta, z, \omega|f) + \nu^{ND} \right\} & \text{if } a(\omega) < 0
\end{cases}
\]  

(7)

Note that \(W^{ND}\) shows up because the assumption is that \(\epsilon\) has not yet been drawn at stage 2. Finally,

\(^{17}\)We use \(x(\omega)\) to denote the corresponding element of \(\omega\) (e.g. \(a(\omega)\) denotes the specific \(a\) in \(\omega\)).
as promised we have
\[ W(\beta, z, \omega|f) = \int V(\nu, \beta, z, \omega|f)dF_\nu(\nu). \] (8)

Given that \( \nu \) and \( \epsilon \) are drawn from type I extreme value distributions, we have simple closed form solutions for choice probabilities (see, for instance, Rust (1987)). Conditional on not filing for bankruptcy, let \( \sigma^{(0, \alpha')}(\beta, z, \omega|f) \) be the probability that the individual in state \((\beta, z, \omega)\) chooses action \(a' \in F(z, \omega|f)\). Then, \(\sigma^{(0, \alpha')}(\beta, z, \omega|f)\) is drawn from type I extreme value distributions, we have simple closed form expressions for choice probabilities. Furthermore, these choice probability expressions imply simple expressions for \( W^{ND} \) in (6) and \( W \) in (8). Specifically,
\[ W^{ND}(\beta, z, \omega|f) = \lambda \ln \left( \sum_{(\alpha', \alpha) \in F(z, \omega|f)} \exp \left( \frac{\nu^{(0, \alpha')}(\beta, z, \omega|f)}{\lambda} \right) \right) + \lambda y_E + \bar{v}(\mathcal{A}) \] (12)

and
\[ W(\beta, z, \omega|f) = \begin{cases} W^{ND} & \text{if } a(\omega) \geq 0 \\ \alpha \ln \left( \exp \left( \frac{\nu^{(1, \alpha)}(\beta, z, \omega|f)}{\alpha} \right) + \exp \left( \frac{W^{ND}(\beta, z, \omega|f)}{\alpha} \right) \right) + \alpha y_E + \bar{v}(\mathcal{D}) & \text{if } a(\omega) < 0 \end{cases} \] (13)

In Online Appendix A.2 we prove:

**Theorem 1.** Given \( f \), there exists a unique solution \( W(\beta, z, \omega|f) \) to the individual’s decision problem in (3)-(8).
3.2 Intermediaries’ problem

Competitive intermediaries with deep pockets have access to an international credit market where they can borrow or lend at the risk-free interest rate \( r \geq 0 \).

Any given intermediary takes prices \( q \) and scoring function \( \psi \) (i.e. \( f \)) as given. We assume that both losses and gains resulting from an individual’s death accrue to the financial intermediary.\(^{18}\)

The profit \( \pi^d(\omega|f) \) on contract of size \( a' \) with agents with observables \( \omega \) is:

\[
\pi^d(\omega|f) = \begin{cases} 
\rho \cdot \frac{p^d(\omega|f)(-a')}{1+r} - q^d(\omega) \cdot (-a') & \text{if } a' < 0 \\
q^d \cdot a' - \rho \cdot \frac{a'}{1+r} & \text{if } a' \geq 0,
\end{cases}
\]

where the probability of repayment on a financial contract of size \( a' \) made to individuals with observable characteristics \( \omega \) is denoted \( p^d(\omega|f) : (\mathbb{R}^{\omega} \cap \mathcal{A}) \times \Omega \to [0, 1] \). Given perfect competition in financial intermediation and constant returns to scale in the lending technology, if a solution to the intermediary’s problem exists, then optimization by the intermediary implies zero profits for strictly positive measures of contracts issued or

\[
q^d(\omega|f) = \begin{cases} 
\rho \cdot \frac{p^d(\omega|f)}{1+r} & \text{if } a' < 0 \\
\rho & \text{if } a' \geq 0,
\end{cases}
\]

Assessing an individual’s probability \( p^d(\omega|f) \) of repaying a debt next period given her current observable characteristics \( \omega \) given unobservable \((\beta, \epsilon, z)\), takes two steps:

1. Assess the probability that an individual in state \( \omega \) who takes action \((d, a')\) will be of unobservable type \( \beta' \) next period via Bayes rule (the type scoring function \( \psi^{(d,a')}_\beta(\omega) \)).

2. For each possible future unobservable type \( \beta' \), compute the individual’s probability of future repayment conditional on being that type and transitions over observable characteristics and then compute the weighted sum over future types to obtain \( p \).

Starting with step 1, an individual’s probability of being type \((\beta'_1, \ldots, \beta'_B)\) next period is given by the

\(^{18}\)That is, an individual who borrows honors her obligation only if she survives and does not declare bankruptcy and, symmetrically, an intermediary that accepts a deposit is released from its obligation if the depositor does not survive to collect.
the third line is the probability that a newborn arrives in state
The second line in equation (19) is the probability of a survivor transitioning to
Note that in (16), the assessment uses Bayes’ rule to assign the probability of an individual in observable
where the transition function
Turning to step 2, given observable state \( \omega \), we obtain the probability of repayment the intermediary
uses for pricing debt (i.e. for \( a' < 0 \)) via:
3.3 Evolution
Let \( \mu(\beta, z, \omega|f) \) denote the beginning-of-period measure of individuals in state (\( \beta, z, \omega \)) for a given
where the transition function
The second line in equation (19) is the probability of a survivor transitioning to (\( \beta', z', e', a', s' \)) while
An invariant distribution is a fixed point \( \overline{\mu}(|f) = T \overline{\mu}(|f) \). In Online Appendix A.3 we prove:
Lemma 2. There exists a unique invariant distribution $\bar{\mu}(\cdot|f)$ and $\{\mu_0T^n\}$ converges to $\bar{\mu}(\cdot|f)$ at a geometric rate for any initial distribution $\mu_0$.

Note that although the invariant distribution is critical for computing cross-sectional moments used to map the model to the data, none of the other equilibrium objects (i.e. the set of functions $f$, the value function $V$ or the decision rule $\sigma$) takes $\mu$ as an argument. This simplifies the model and eases the computational burden, but is not necessary. Other specifications in which knowledge of the distribution is required are possible, but we do not consider these in the baseline model.

3.4 Existence

We can now give the definition of a stationary recursive competitive equilibrium.

Definition 3. A stationary Recursive Competitive Equilibrium (RCE) is a pricing function $q^*$, a type scoring function $\psi^*$, a choice probability function $\sigma^*$, and a steady state distribution $\bar{\mu}$ such that:

(i). Optimality: $\sigma^{(d,a')}(\beta, z, \omega|f^*)$ satisfies (9) and (10) for all $(\beta, z, \omega) \in \mathcal{B} \times \mathcal{Z} \times \Omega$ and $(d, a') \in \mathcal{F}(z, \omega|f^*)$,

(ii). Zero Profits: $q^{d*}(\omega|f^*)$ satisfies (15) with equality for all $\omega \in \Omega$ with $p^{d*}(\omega|f^*)$ satisfying (17) for all $\omega \in \Omega$,

(iii). Bayesian Updating: $\psi^{(d,a')}_{\beta'}(\omega|f^*)$ satisfies (16) for all $(\beta', \omega) \in \mathcal{B} \times \Omega$, and

(iv). Stationary Distribution: $\bar{\mu}(\beta, z, \omega|f^*)$ solves (18) for $T(\beta', z', \omega'; \beta, z, \omega|f^*)$.

The key step in proving the existence of a competitive equilibrium is proving that the value function $W(\beta, z, \omega|f)$ is continuous in $f$. In Online Appendix A.4 we first prove:

Lemma 3. $W(\beta, z, \omega|f)$ is continuous in $f$, and for any $(d, a') \in \mathcal{F}(z, \omega|f)$, $\sigma^{(d,a')}(\beta, z, \omega|f)$ is continuous in $f$.

Using this result, we then prove

Theorem 2. There exists a stationary recursive competitive equilibrium (RCE).

3.5 Equivalence to an Economy with Credit Scores

In the economy described thus far, an individual’s reputation is her type score. In U.S. credit markets, an important measure of reputation is the credit score. A credit score is an index that is positively related
to the likelihood of repayment. The goal of this subsection is to show that under certain conditions, the equilibrium described in the previous subsections can be implemented via an arrangement in which lenders use a model equivalent of a credit score. In this arrangement, lenders know only how credit scores and loan sizes translate into repayment probability, not why.

We formalize the vague notion that a credit score depicts a consumer’s creditworthiness, by defining it to be the probability of repayment on a loan of some standard size \( a' = \bar{a} < 0 \). According to the timing in Definition 1 part 4(a), since \( \omega = (e, a, s) \) is known at the end of \( t - 1 \), an individual’s credit score can be calculated at the end of period \( t - 1 \) to be \( m \equiv p^a(e, a, s) \) using equation (17). Newborns in period \( t - 1 \) are assigned \( m \equiv p^a(e, 0, G\beta) \). Intuitively speaking, an equivalence between the type-scoring and credit-scoring environments will exist if there is a one-to-one mapping between \( s \) and \( m \), holding fixed all other factors that affect credit scores (i.e., \( e \) and \( a \)). Then, wherever \( s \) appears in the theoretical model, it can be replaced by \( m \).

An individual in state \( \hat{\omega} = (e, a, m) \) of the financial arrangement with credit scores takes as given a pricing function \( q^d(\hat{\omega}) \) and a credit-score transition function \( Q^m(m'| (d, a'), e', \hat{\omega}) \) which tells her the likelihood of her future credit score conditional on her actions. Intermediaries take as given the pricing function (which must satisfy the zero profit condition) and the probability of repayment function \( p^d(\hat{\omega}) \) (which must be consistent with the individual’s objective likelihood of repayment). This repayment function makes use of a one-to-one and onto mapping between \( s \) and \( m \), which we denote \( M(\hat{\omega}) \).

Specifically, \( M(\hat{\omega}) \) is given by the inverse mapping defined by \( m = p^a(e, a, M(e, a, m)) \). These functions can be defined to exactly replicate the RCE with type scores.

In Online Appendix A.5, we restate the household and financial intermediary problems using this definition of a credit score. We then provide a definition of a recursive competitive equilibrium with credit scores (RCECS) and prove:

**Theorem 3.** Suppose an RCE exists and let \( m = p^a(e, a, s) \). Suppose \( M^*(\hat{\omega}) \) given by the inverse function \( m = p^a(e, a, M^*(e, a, m)) \) exists. Then an RCECS also exists in which the choice probabilities \( \sigma^{(d, a')}(\beta, z, e, a, m) = \sigma^{(d, a')}(\beta, z, e, a, s) \).

### 4 Mapping the Model to Data

We now examine the U.S. unsecured credit market through the lens of our model. We rely on the equivalence result between the model with type scores (RCE) and the model with credit scores (RCECS).
described in Theorem 3 when there are two $\beta$ types ($\beta_H > \beta_L$). Specifically, it allows us to use the model with type scores in order to target the joint behavior of earnings, aggregate credit market moments, and credit rankings over the life cycle.\footnote{An analogy would be that real business cycle models solve the simpler Planner’s problem to compute consumption and investment allocations consistent with a decentralized competitive equilibrium. It is faster to compute the RCE because Bayesian updating gives a candidate input used to generate $p^{(0,a')}(\tilde{\omega})$, while iterating on an initial guess of $p^{(0,a')}(\tilde{\omega})$ in an RCECS does not use information on how an agent’s type affects her repayment decision.} We then verify that the sufficient condition in Theorem 3 is satisfied for the estimated parameters so that the equivalence result holds.

In the data, a credit score is an ordinal measure of creditworthiness, typically ranging from around 300 to 850, not a direct estimate of the probability of repayment $m$.\footnote{See, for example, https://www.investopedia.com/terms/c/credit_score.asp.} To close this gap, we associate with $m = p^3(\omega)$ a number in the unit interval that gives $p^3(\omega)$’s position (i.e. ranking) in the overall distribution of $p^3(\omega)$ in the model economy.

**Definition 4.** An individual’s credit ranking in state $\omega$ is given by

$$\chi^3(\omega) = \sum_{\tilde{\omega} \in J^3(\omega)} \mu(\tilde{\omega}),$$

where $J^3(\omega) = \{ \tilde{\omega} : p^3(\tilde{\omega}) \leq p^3(\omega) \}$ and $\mu(\omega) = \sum_{z, \beta} \mu(\beta, z, \omega)$.

Clearly, $\chi^3(\omega) \in [0, 1]$. We construct the data analogue of $\chi^3(\omega)$ by associating with each credit score its percentile position in the overall distribution of credit scores.

Furthermore, real-world credit scores do not mention any specific level of borrowing $\tilde{a}$. One way to interpret this fact is to think that the ranking of individuals with respect to probability of repayment holds for any level of debt. For this to be true, we need the following property:

**Definition 5.** Let $\hat{a} < \tilde{a} < 0$. Then, $p^d(\omega)$ preserves order with respect to $a'$ if $p^3(\omega) \geq p^3(\tilde{\omega})$ if and only if $p^3(\omega) \geq p^3(\hat{\omega})$.

If $p^d(\omega)$ preserves order, then $J^3(\omega) = J^3(\omega)$ and $\chi^3(\omega)$ becomes invariant to the choice of $\tilde{a}$. This order preserving property holds for a wide range of debt levels for our estimated model in Section 4.

Credit rankings, earnings and assets all grow with age on average, and we want our model to capture those features. Unfortunately, we do not have access to a panel dataset which contains all these dimensions. So we use a version of simulated method of moments to estimate our model. Specifically, we take some non-controversial information from outside the model: the earnings process, the risk free
rate of return, demographics, preferences over risk, a measurement of the costs of bankruptcy filings, and a generic value of debt ($\bar{\alpha}$) to which the credit score is normalized.\footnote{We verify that the choice of $\bar{\alpha}$ does not matter provided it is higher than the bankruptcy filing costs.}

Next we obtain a set of data moments that summarize the properties of the unsecured credit market (bankruptcy filing rates, average interest rates, dispersion of interest ratios, fraction of households in debt, debt to income ratio) and we approximate the behavior of credit scores as a function of age (specifically, affine functions of the mean and the standard deviation of credit scores and the autocorrelation of the annual change in individual scores). Establishing a credit history is, in light of our model, just code for age.

We then proceed to estimate the parameters of interest which are the values of patience for both types, the transition probabilities of types and their frequency at birth, the proportional earnings loss from bankruptcy, as well as measures of noise (the variances of the extreme value shocks) by minimizing the weighted sum of squared differences between the values of the moments in the data and their model counterparts. We have tried various alternative sets of moments with minimal effects on the findings. While earnings, credit and bankruptcy statistics have been used since Chatterjee et al. (2007a) and Livshits et al. (2007), credit scores, and their evolution by age, have not. The evolution of credit scores is crucial for understanding the building of a reputation over the early part of the life-cycle.

4.1 Computation

Computation of equilibrium requires solving for two endogenous functions: the bond price function and the type-score updating function. The bond price function is standard in unsecured debt models like Chatterjee et al. (2007a), except that the endogenous type score is an additional dimension. The type scoring function is new: individuals take as given how feasible actions change the market’s perception of their type that is updated using Bayes law and this perception has to be consistent with the actions taken by both types.

While we introduced extreme value shocks in order to keep the Bayesian posterior well behaved, it can, however, exacerbate grid sensitivity associated with approximating continuous choices.\footnote{As an example, imagine that we are approximating the interval $[0, 2]$ with a discrete grid that is log-spaced. This means that there are more grid points in $[0, 1]$ than in $[1, 2]$. Now assume that the value of the action associated with any grid point $i$ is just $v + \epsilon_i$. Then, any one of these grid points has an equal chance of being selected. But, since there are more points in $[0, 1]$, it is more likely that the choice will be from that interval. In the context of our model, this effect imparts a bias toward actions close to the origin (debt or small levels of assets).} We deal with this by making the location parameters of the extreme value shock associated with each asset choice depend positively on the measure of consumption points that are associated with that action in

15
the individual’s feasible set \( \mathcal{F}(z, \omega | f) \).\(^{24}\) We describe the adjustment that mitigates such grid sensitivity in equation (43) in Appendix B.1.

4.2 Estimation

4.2.1 Parameters Chosen Outside the Model

A model period is one year. We take the relevant life span of people to be 40 years, as the bulk of borrowing is by young people, implying a survival probability of 0.975. We choose a CRRA utility function with risk aversion parameter 1.5. We pose a risk free rate of 1%, which implies an effective interest rate of 3.59% in the presence of perfect annuity markets. We take the cost of filing for bankruptcy to be about 1.5% of median earnings taken from Albanesi and Nosal (2015).\(^{25}\) Since it is a dominant action not to invoke bankruptcy on debts less than the filing cost, we choose \( \bar{a} \) (the debt value used to compute the probability of repayment for a credit score) to be 3.5% of median earnings (i.e. well above those costs). Finally we take the earnings class to be the persistent AR1 process estimated by Floden and Lindé (2001) and assume agents are born with the lowest earnings level to replicate the upward earnings path in the life cycle.\(^{26}\) This is all summarized in Table 1.

4.2.2 Data and Targets

We start by describing the data moments that we target in our classical minimum distance estimation. The set of statistics that we deem important to target pertain to the main aggregate characteristics of the U.S. unsecured credit market: credit usage (the fraction of households in net debt (FID) and the debt-to-income ratio (DTY)), credit terms (average interest rates (AI) and their dispersion (SDI)), and the bankruptcy rate (BR). Importantly, we are also interested in matching properties of the age profile of credit rankings. In particular, we match a linear approximation (i.e. intercept and slope) of the average

\(^{24}\)For instance, in the example of footnote 23, the adjustment to the location parameter of the shocks lowers the mean of the extreme value shocks associated with closely-packed choices. The result is that with the adjustment it is equally likely that the best choice is in \([0, 1]\) or \([1, 2]\). This adjustment has implications for savings behavior explored in Briglia et al. (2021). See also subsection B.5.

\(^{25}\)Albanesi and Nosal (2015) report a filing fee of $697 in 2005 pre-BAPCA. Median household income in 2004, adjusted for 3.39% inflation between 2004 and 2005, was $45,837; the ratio of these numbers yields 1.52%.

\(^{26}\)We approximate this process by a five-state Markov chain using the Adda and Cooper (2003) method, which yields

\[
\begin{pmatrix}
0.767 & 0.207 & 0.025 & 0.001 & 0.000 \\
0.207 & 0.496 & 0.253 & 0.043 & 0.001 \\
0.025 & 0.253 & 0.466 & 0.253 & 0.025 \\
0.001 & 0.043 & 0.253 & 0.496 & 0.207 \\
0.000 & 0.001 & 0.025 & 0.207 & 0.767
\end{pmatrix}
\]

support \( \mathcal{E} = \{-0.71, -0.27, 0.00, 0.27, 0.71\} \), transition matrix \( Q^e(e'|e) = \)

\[
\begin{pmatrix}
0.25 & 0.25 & 0.446 & 0.253 & 0.025 \\
0.001 & 0.043 & 0.253 & 0.496 & 0.207 \\
0.000 & 0.001 & 0.025 & 0.207 & 0.767
\end{pmatrix}
\]

transitory component with a three-point uniform distribution on support \( \mathcal{Z} = \{-0.25, 0, 0.25\} \).
Table 1: Parameters Chosen Outside the Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Demographics and preferences</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Survival probability</td>
<td>$\rho$</td>
<td>0.975 avg. life span 40 years</td>
</tr>
<tr>
<td>Risk aversion</td>
<td>$\gamma$</td>
<td>1.5 CRRA preferences</td>
</tr>
<tr>
<td>Earnings at birth</td>
<td>$e$</td>
<td>-0.71 Footnote 26</td>
</tr>
<tr>
<td><strong>Technology</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Risk-free rate (%)</td>
<td>$r$</td>
<td>1.000</td>
</tr>
<tr>
<td>Bankruptcy filing cost</td>
<td>$\kappa$</td>
<td>0.0152 1.5% of median earnings</td>
</tr>
<tr>
<td>Debt level for computing credit score</td>
<td>$\bar{a}$</td>
<td>-0.035 2.9% of median earnings</td>
</tr>
<tr>
<td><strong>Earnings</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Persistence of log(e)</td>
<td>$\rho_e$</td>
<td>0.9136 Floden and Lindé (2001)</td>
</tr>
<tr>
<td>Variance of innovations to log(e)</td>
<td>$\nu^2_e$</td>
<td>0.0426 Floden and Lindé (2001)</td>
</tr>
<tr>
<td>Variance of log(z)</td>
<td>$\nu^2_z$</td>
<td>0.0421 Floden and Lindé (2001)</td>
</tr>
</tbody>
</table>

(I:MCR and S:MCR) and standard deviation (I:SDCR and S:SDCR) of credit rankings across the age profile and the autocorrelation of credit ranking changes (AUTO).²⁷

To obtain these data targets, we use three primary sources: the Survey of Consumer Finances (SCF), the administrative records of the U.S. Bankruptcy Courts, and the Federal Reserve Bank of New York Consumer Credit Panel/Equifax (FRBNY CCP/Equifax). The first provides information on individual level variation in debt and interest rates, the second provides information on aggregate bankruptcy filing rates, and the last contains individual level information on credit records from an anonymized panel which provides us with moments on the variation and evolution of credit scores. The credit score measure is the Equifax Risk Score (hereafter Risk Score), which is a proprietary credit score similar to other risk scores used in the industry.

We choose 2004 as our baseline year. This is because this is the latest year for which the bankruptcy filing statistics are unaffected by the Bankruptcy Abuse Prevention and Consumer Protection Act of 2005 (which changed the eligibility requirements for a discharge in ways we do not model in this paper). To align with this choice, we use the 2004 SCF for our credit market moments and the data for 2004 from the CCP for the age profile of credit rankings. For the autocorrelation of year-to-year changes in credit rankings we use CCP data from 2003, 2004 and 2005.

²⁷The age profile of AUTO showed no significant slope in the data, so we only target the intercept.
In the SCF, we focus on the subset of households with heads between the ages of 20 and 60 years excluding the top 5% of the wealth distribution, for whom we think our theory is not relevant. The fraction of indebted households is the fraction of such households with negative net worth. The average debt-to-income ratio is the ratio of total unsecured debt of indebted households to 2004 per household U.S. GDP. For the mean and standard deviation of interest rates we used the interest rates reported on unsecured debt by all households with negative net worth.

The bankruptcy rate is the ratio of the total number of nonbusiness Chapter 7 filings in 2004 reported by the U.S. Bankruptcy Courts, scaled by the total number of U.S. households in 2004.

While the previous credit market data targets have been used in numerous quantitative theory papers on bankruptcy, what is novel is our use of the age profile of Risk Score moments. For this we use a 2% sample (approximately 150,000 observations) of the FRBNY CCP/Equifax panel (described in detail in Online Appendix C). With this data, we create credit rankings, defined as the percentile ranking of an individual’s Risk Score relative to the overall sample distribution of Risk Scores, and group individuals between the ages of 21 and 60 in 5 year bins. We then compute the means and standard deviations of credit rankings within each age bin. With these age bin data values, we estimate affine age profiles for means, standard deviations. To compute autocorrelations of year-to-year changes in credit rankings, we create credit rankings for 2003 and 2005 for each individual. We place individuals in the same 5-year age bins and compute the correlation between the change in rankings between 2003 and 2004 and between 2004 and 2005 for each age bin. Figure 1 shows the data and auxiliary affine approximations to the data (in red) as well as the model generated data and approximation (in black).

4.2.3 Estimated Parameters

We parameterize the CDF of the $\nu$ shocks as:

$$ F_\nu(\nu^d; \alpha) = \exp \left\{ -\exp \left( -\frac{\nu^d - \overline{\nu}}{\alpha} \right) \right\} \quad \text{for } d \in \{0, 1\}. \quad (21) $$

Given $\alpha$, we choose the location parameter $\overline{\nu}$ to eliminate the incentive for a household to choose debt simply in order obtain favorable draws of the extreme value shock associated with the bankruptcy

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28 We do not use the slope of the age profile of the autocorrelation of year-to-year credit score changes as a target because it is zero, which makes matching the relative deviation between model and data a problem.
Figure 1: Credit Ranking Age Profile: Model vs. Data

Notes: The credit ranking data is based on author calculations using FRBNY CCP/Equifax data in Table 13 in Online Appendix C. The linear approximation to the model generated credit ranking age profiles uses the regression coefficients from Table 2.

We parameterize the CDF of the $\epsilon$ shocks as:

$$F_\epsilon(\epsilon_{a_n}; \lambda) = \exp \left(-\exp \left(-\frac{\epsilon_{a_n} - \bar{\epsilon}_{a_n}(z, \omega)}{\lambda}\right)\right) \text{ for } n \in \{1, ..., \bar{n}(z, \omega)\}.$$  \hspace{0.5cm} (22)

where $\bar{n}(z, \omega)$ is the index of the largest budget feasible $a'$ for an agent with $(z, \omega)$. For reasons described above in Section 4.1, we specify choice and state-specific means for the $\epsilon$ shocks, $\bar{\epsilon}_{a_n}(z, \omega)$ defined in equation (43) of Appendix B.1. We estimate the scale parameters $\alpha$ and $\lambda$ internally.

To estimate the parameters, we proceed with a mixture of simulated method of moments for the aggregate statistics and indirect inference for the credit score auxiliary affine model of means, standard deviations, and autocorrelation across the age profile. Our system is overidentified (we have 8 parameters $\theta = (\beta_H, \beta_L, Q^B(L'|H), Q^D(H'|L), G_{\beta_H}, k_1, \alpha, \lambda)$ and 10 moments $S = (BR, AI, SDI, FID, DTY, I:MCR, S:MCR, I:SDCR, S:SDCR, AUTO)$) so not all moments will be exactly replicated. Specifically, the consistent estimated parameter values in Table 3 solve

$$\hat{\theta} = \arg \min_\theta g'(\theta) \hat{W} g(\theta)$$

where $g(\theta) = (\hat{S} - S(\theta))$ is the difference between data $\hat{S}$ and model $S(\theta)$ moments and $\hat{W}$ is a weighting

29 We set $\bar{\nu} = -\alpha \cdot (\gamma_E + \ln(|D|))$ where $\gamma_E$ is the Euler-Mascheroni constant so that $E[\max(\nu^D, \nu^ND)] = 0$ described in Appendix B.1. This correction implies that for an indebted household $a(\omega) < 0$ for whom $\nu(1,0) = W^{ND}$ in (7), the ex-ante value $W$ in (8) is equal to $\nu(1,0) = W^{ND}$. In other words, the presence of the default/no-default shocks do not add any extra utility.
Table 2: Estimation Targets

<table>
<thead>
<tr>
<th>Moment (%)</th>
<th>Data (Ma)</th>
<th>Model (M)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Aggregate credit market moments</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bankruptcy rate</td>
<td>BR 1.00</td>
<td>1.02</td>
</tr>
<tr>
<td>Average interest rate</td>
<td>AI 11.9</td>
<td>11.5</td>
</tr>
<tr>
<td>Interest rate dispersion</td>
<td>SDI 7.00</td>
<td>7.08</td>
</tr>
<tr>
<td>Fraction of HH in debt</td>
<td>FID 7.92</td>
<td>9.16</td>
</tr>
<tr>
<td>Debt to income ratio</td>
<td>DTY 0.40</td>
<td>0.26</td>
</tr>
<tr>
<td><strong>Credit ranking age profile moments</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept, mean credit ranking</td>
<td>I:MCR 0.278</td>
<td>0.320</td>
</tr>
<tr>
<td>Slope, mean credit ranking</td>
<td>S:MCR 0.038</td>
<td>0.038</td>
</tr>
<tr>
<td>Intercept, std. dev. credit ranking</td>
<td>I:SDCR 0.215</td>
<td>0.218</td>
</tr>
<tr>
<td>Slope, std. dev. credit ranking</td>
<td>S:SDCR 0.011</td>
<td>0.011</td>
</tr>
<tr>
<td>Average autocorrelation of credit ranking changes</td>
<td>AUTO -0.220</td>
<td>-0.215</td>
</tr>
</tbody>
</table>

| **Sum of squared errors**          |           |           |
| Aggregate credit market moments    |           | 0.144     |
| Credit ranking age profile moments |           | 0.023     |
| Total                              |           | 0.167     |

Notes: The credit ranking data is based on author calculations using FRBNY CCP/Equifax data. MSEs are computed in percentage deviation terms to control for relative magnitudes of moments. That is, for data moment \( x \) with model analog \( \hat{x} \), the contribution of \( x \) to the MSE is \((\hat{x}/x - 1)^2\). Each moment receives equal weight.

The estimated values for patience are \( \beta \in \{0.809, 0.930\} \), so that low types have a 13% lower discount factor than high types. This differential allows reputation acquisition to play a role in equilibrium: types are far enough apart to want to behave differently, but close enough that imitation is not too costly.

30 For more details on computation and estimation see Online Appendix B.1.
31 Our annual estimates of discount factors translate into quarterly values \( \beta_H = 0.982 \) and \( \beta_L = 0.948 \).
Table 3: Parameters Chosen Within the Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evolution of types</td>
<td></td>
</tr>
<tr>
<td>High discount factor $\beta_H$</td>
<td>0.930</td>
</tr>
<tr>
<td>Low discount factor $\beta_L$</td>
<td>0.809</td>
</tr>
<tr>
<td>High to low $\beta$ transition $Q(\beta')</td>
<td>H)$</td>
</tr>
<tr>
<td>Low to high $\beta$ transition $Q(\beta'</td>
<td>L)$</td>
</tr>
<tr>
<td>Fraction high $\beta$ at birth $G_{\beta_H}$</td>
<td>0.318</td>
</tr>
<tr>
<td>Proportional default cost $\kappa_1$</td>
<td>0.067</td>
</tr>
</tbody>
</table>

| Extreme value parameters              |        |
| Scale in $F_v(\nu; \alpha)$          | $\alpha$ | 0.029 |
| Scale in $F_\epsilon(\epsilon; \lambda)$ | $\lambda$ | 0.002 |

when individuals start with low earnings and zero assets, i.e., when they are young. This is explained in more detail in Section 5.3.

The estimated transition matrix $Q^\beta$ implies an ergodic distribution featuring 48% high types, but the life-cycle demographic structure implies a slightly lower stationary fraction equal to 47%.\(^{32}\) There is demographic improvement in the average assessment of an individual's type (and hence creditworthiness) over one's lifetime: agents' initial type scores are consistent with the estimated initial fraction of high types, $G_{\beta_H} = 32\%$, with the average assessment updating via $\bar{s}' = \bar{s} \cdot Q^\beta(H'|H) + (1 - \bar{s}) \cdot Q^\beta(H'|L)$ which converges to 48%. This is independent of alternative credit arrangements we consider. The fact that $G_{\beta_H}$ is estimated to be below the stationary fraction of high types is a robust consequence of the rising average credit score over the age profile.

The estimated earnings loss from filing for bankruptcy is 6.7 percent of the persistent component of earnings. This relatively large estimated cost indicates that there are costs to bankruptcy that are not captured by the loss of reputation in unsecured credit markets only. For example, reputation loss can impact one's job finding prospects, secured borrowing costs like mortgages, and even insurance premia. For reasons of parsimony, these other channels are captured by our estimate of $\kappa_1$.

\(^{32}\)The fraction of type $H$ in the stationary distribution (call it $\mu_H$) solves $\mu_H = \rho \cdot (1 - Q^\beta(L'|H))\mu_H + Q^\beta(H'|L)(1 - \mu_H) + (1 - \rho) \cdot G_{\beta_H}$ or $\mu_H = \frac{\rho Q^\beta(H'|H)(1 - \rho) G_{\beta_H}}{1 - \rho (1 - Q^\beta(L'|H)) + \rho Q^\beta(H'|L)}$. For our estimated parameter values in Table 3, $\mu_H = 0.47$. In a cohort, the fraction of type $H$ asymptotes to 0.48 (the value of $\mu_H$ corresponding to $\rho = 1$).
Our estimates of the parameters $\alpha$ and $\lambda$ of the extreme value distributions imply that there is more noise in the bankruptcy decision than in asset choices, but not so much that fundamentals are overridden (i.e. fundamental heterogeneity in unobservable type and earnings are the key drivers of default and asset choice). One measure of the size of the extreme value shocks is how noisy consumption decisions are; at an individual level, the variance of consumption decisions is zero without the extreme value shocks. In our model, the average coefficient of variation of consumption across all agents is only modestly higher at 2.03%. This is especially true for the $\epsilon$ shock process associated with the asset choice decision where the share of total borrowing and saving actions by “modal agents” (i.e. those for whom an action in the set under consideration is the mode) is 85.6% and 99.9%, respectively. On the other hand, the default/no-default action is associated with more variability in the shock process $\nu$ where the share of modal defaulters is only 5.25%. This can be explained by the fact that our parsimonious $\nu$ shock process is capturing other unobservable factors behind the default decision not included in our model (for example, Chatterjee et al. (2007b) include other shocks to capture medical expenses and lawsuits which survey respondents cited as reasons for filing for bankruptcy).

### 4.2.4 Sensitivity Analysis

In lieu of standard errors, we provide a sensitivity measure of the relationship between parameter estimates and the moments of the data they depend on using the local methods in Andrews et al. (2017). Table 4 presents a version of their sensitivity measure $\Lambda$ in Definition 1 applied to classical minimum distance estimation given by

$$\Lambda = -(G'WG)^{-1}G'W$$

(23)

where $G = \mathbb{E}[\nabla_{\theta} g(\theta)]$ is the $10 \times 8$ Jacobian and we take $W$ to be an identity matrix. In particular, $\Lambda$ measures how sensitive the parameter estimates in Table 3 are to local perturbations of the data moments. Further, there is a tight connection between $\Lambda$ and standard errors in GMM/SMM given a variance-covariance matrix $\Omega$. Specifically, given (23), the limiting distribution of the estimates can be written

$$\sqrt{T}(\hat{\theta} - \theta_0) \xrightarrow{d} \mathcal{N}[0, \Lambda\Omega\Lambda']$$

(24)

where $\Omega = \mathbb{E}[g(\theta)g(\theta)']$ is the variance-covariance matrix of the data moments, $\theta_0$ is the true parameter value, and $T$ is sample size. For a given $\Omega$, (24) makes clear that small values of $\Lambda$ are associated with

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33 At each point in the state space, we compute the standard deviation and mean of consumption implied by the decision rule $\sigma$. We then take the ratio of these numbers at each point and average over the stationary distribution.

34 See Online Appendix B.5.3 for a description of these calculations (in particular equation (52)).
Table 4: Sensitivity Analysis: Implied Percentage Change in Parameter given 1% Change in Empirical Moment

<table>
<thead>
<tr>
<th></th>
<th>BR</th>
<th>AI</th>
<th>SDI</th>
<th>FID</th>
<th>DTY</th>
<th>I:MCR</th>
<th>S:MCR</th>
<th>I:SDCR</th>
<th>S:SDCR</th>
<th>AUTO</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_H$</td>
<td>0.00</td>
<td>-0.08</td>
<td>-0.06</td>
<td>-0.08</td>
<td>0.09</td>
<td>-0.12</td>
<td>-0.12</td>
<td>-0.11</td>
<td>-0.04</td>
<td>0.02</td>
</tr>
<tr>
<td>$\beta_L$</td>
<td>0.34</td>
<td>-0.11</td>
<td>0.02</td>
<td>-0.04</td>
<td>-0.21</td>
<td>0.51</td>
<td>0.44</td>
<td>0.47</td>
<td>0.13</td>
<td>-0.02</td>
</tr>
<tr>
<td>$Q^{\beta}(L'</td>
<td>H)$</td>
<td>-0.47</td>
<td>0.93</td>
<td>0.36</td>
<td>2.33</td>
<td>-2.67</td>
<td>23.42</td>
<td>14.19</td>
<td>19.71</td>
<td>4.29</td>
</tr>
<tr>
<td>$Q^{\beta}(H'</td>
<td>L)$</td>
<td>-5.83</td>
<td>6.06</td>
<td>0.54</td>
<td>3.12</td>
<td>2.44</td>
<td>5.57</td>
<td>-0.03</td>
<td>2.60</td>
<td>0.00</td>
</tr>
<tr>
<td>$G_{\beta_H}$</td>
<td>-2.49</td>
<td>2.45</td>
<td>0.06</td>
<td>1.24</td>
<td>1.39</td>
<td>-10.66</td>
<td>-0.69</td>
<td>7.17</td>
<td>-0.29</td>
<td>0.39</td>
</tr>
<tr>
<td>$\kappa_1$</td>
<td>0.17</td>
<td>-0.19</td>
<td>-0.03</td>
<td>-0.06</td>
<td>0.01</td>
<td>-0.64</td>
<td>-0.43</td>
<td>-0.88</td>
<td>-0.13</td>
<td>0.01</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-0.35</td>
<td>-0.22</td>
<td>-0.03</td>
<td>0.48</td>
<td>0.03</td>
<td>-0.59</td>
<td>-0.43</td>
<td>-0.77</td>
<td>-0.13</td>
<td>0.02</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>7.22</td>
<td>-6.67</td>
<td>-0.22</td>
<td>-7.07</td>
<td>2.02</td>
<td>-3.32</td>
<td>-3.46</td>
<td>-5.06</td>
<td>-1.03</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Notes: Each entry corresponds to the implied percentage change in the estimated parameter in the row associated with a 1% change in the indicated empirical moment in the column. Abbreviations for moments are available in Table 2. All numbers are reported in percentage points, i.e. the 0.00 (which when expanded is actually 0.0011%) in the top left cell implies that if the bankruptcy rate were 1% higher, our estimate of $\beta_H$ would increase by 0.0011%, from 0.930 to 0.931.

Looking across the rows of Table 4, there are several parameters ($\beta_H$, $\beta_L$, $\kappa_1$, and $\alpha$) which appear to be relatively insensitive to a 1% change in our moments. Depending on $\Omega$, these would correspond to precisely estimated parameters. In contrast, there are parameters like $Q^{\beta}(L'|H)$, $Q^{\beta}(H'|L)$, and $G_{\beta_H}$ which appear to be sensitive to variation in moments like the intercept of the age-profile of mean credit rankings (I:MCR). This suggests that if we had parameterized the age-profile of credit rankings differently (say with a quadratic instead of affine function), the estimate of these parameters might be affected. The fact that these parameters are all jointly sensitive to the intercept of the mean credit ranking profile is related to the fact that the affine function (intercept and slope) depends on the transition of high types across age bins which depends explicitly on all those parameters (see footnote 32). There also appears to be sensitivity in the estimate of the variance of extreme value shocks associated with asset

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35Section B.3 of Appendix B provides a detailed explanation of our implementation of Andrews et al. (2017). There we also provide the Jacobian matrix $G$ to make clear its relation to $\Lambda$ and discuss how small changes in model parameters lead to local changes in model moments.
choices with respect to several of the moments but this may be related to our very small estimate of $\lambda$ in Table 3.

4.3 Model Fit

We next assess how the model does against certain non-targeted properties in the data.

4.3.1 Credit Ranking of Bankruptcy Filers

Figure 2 shows the non-targeted bankruptcy rate by credit ranking quintiles in the data and in the model. As in the data, the model generates high filing rates among individuals with low credit rankings and low filing rates among individuals with high credit rankings. The model replicates the decreasing pattern in the data.

4.3.2 Credit Rankings Around Bankruptcy Filer

A key property of real world Risk Scores is that they fall upon bankruptcy and mean revert. We illustrate this property for credit rankings in both the data and the model in Figure 3. Specifically, we conduct an event study of the average change in credit rankings around a bankruptcy filing for various

Notes: The credit ranking data is based on author calculations using FRBNY CCP/Equifax data.
Figure 3: Event Study: Credit Rankings around Bankruptcy Filings by Age

Notes: The credit ranking data is based on author calculations using FRBNY CCP/Equifax data. Model results are obtained by simulating a panel of 10,000 individuals for 1,000 periods and dropping the first 100 periods. Bankruptcies are then isolated, and each data point reported represents the mean of credit rankings across all bankruptcies for the given lead or lag from the date of the bankruptcy (normalized to 0). The results are binned by 5-year age groups consistent with our earlier results.

age bins. While the model underpredicts the fall in credit rankings for younger cohorts and overpredicts the long run recovery in credit rankings, it does remarkably well in matching the rank at the time of filing and the patterns we see in the data despite not being targeted in our estimation.

5 Model Mechanics

5.1 Choice Mechanics

The workings of our model depend on differences in patience among types as well as their earnings. The fact that patience matters in the simple sense that agents of different types take different actions
is illustrated by means of likelihood ratios in Figure 4.\(^{36}\) The left panel shows the bankruptcy likelihood ratio across different persistent earnings levels as a function of debt. While for all earnings levels, the low type is more likely than the high type to invoke bankruptcy, the likelihood ratio is increasing in earnings which indicates the importance of differences in type. While both types have the same current gain from default, type \(\beta_H\) cares more about the future consequences of a drop in type scores which incentivizes her to choose a relatively lower bankruptcy probability. Further, for any given earnings level, the difference in bankruptcy probabilities are declining as debt increases. This is because as debt increases the current gain from bankruptcy rises sufficiently that it offsets the future consequences of a drop in one’s type score so that even type \(\beta_H\) bankruptcy probabilities become more similar to \(\beta_L\) type.

The right panel of Figure 4 shows likelihood ratios associated with asset choices. Type \(\beta_L\) borrows much more frequently than type \(\beta_H\), regardless of earnings. Regarding savings, type \(\beta_L\) saves more frequently than type \(\beta_H\) (the likelihood ratio is greater than 0.5) for small levels of savings but saves less frequently for higher levels of savings. Furthermore, the level of savings beyond which type \(\beta_L\) saves less frequently than type \(\beta_H\) is increasing in earnings. These properties are consistent with type \(\beta_L\) tending to choose lower values of \(a'\) (i.e., higher values of current consumption) than type \(\beta_H\) in the same circumstances.

As is a feature of many default models, the probability of bankruptcy is increasing in debt and

\[ \frac{\sigma(d, a')(\beta_L, z, \omega)}{\sigma(d, a')(\beta_L, z, \omega) + \sigma(d, a')(\beta_H, z, \omega)} \]

which lies in \([0, 1]\). For an action which is uninformative about an agent’s type, this ratio is 0.5.
decreasing in earnings for those with sufficiently large debt. The latter implies that, all else equal, realizations of an individual’s earnings have important implications for credit rankings as evident in equation (17). This highlights the composite nature of credit scores: it depends on earnings, asset positions, and type scores.

**Figure 5: Average Moments by Age and Type in Baseline Model**

![Graphs showing average moments by age and type in baseline model](image)

**Notes:** In each panel, each line corresponds to the average moment indicated at the specified age in the baseline model. For the type-specific measures, the average is computed conditional on type. The share of total debt by age is the share of economy-wide debt across all ages for the indicated type accounted for by agents of that type at that age. For example, high-$\beta$ 22 year-olds account for 10.6% of the debt held by high $\beta$ agents.

While age is not a state variable in the decision problems of individuals and lenders, demographics play a role through how we model the arrival of newborns and the Markov process for hidden type. Specifically, since all newborns begin with the lowest earnings class, our Markov process for earnings implies that earnings are expected to rise through an individual’s life as shown in the top left panel of Figure 5. The earnings profile induces an increasing wealth profile in the top center panel. Given the

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$^{37}$Figure 16 in Appendix B.7.1 documents these facts. For very small debt, however, the lowest earners (who have the highest marginal utility of consumption) are least likely to default in order to avoid bearing the costs ($\kappa_0$ and $\kappa_1 \times \exp(e)$) of bankruptcy.
rising earnings profile, the young do the lion’s share of borrowing as evident in the top right panel.\textsuperscript{38} Using our estimates from Section 4 of $G_\beta$ and $Q^\beta$ where the fraction of type $H$ newborns is lower than the long run fraction of type $H$, there is a rise in the average type score with age given by $\bar{s}' = \bar{s} \cdot Q^\beta(H|H) + (1 - \bar{s}) \cdot Q^\beta(H|L)$ evident in the bottom left panel of Figure 5. The age profile for types scores induces a similar ordering for credit rankings in the bottom left panel. Finally, the bottom right panel shows that the “within-group” variance of consumption is higher for type $L$ than type $H$, consistent with more precautionary saving by type $H$ as well as greater credit access evidenced in the top right panel via higher credit rankings in the center bottom panel of our baseline model.\textsuperscript{39} A notable feature of the bottom right panel of Figure 5 is that the cross-sectional variance of consumption is growing across the early part of the life cycle consistent with the empirical evidence in Figure 14 of Heathcote et al. (2010).

5.2 Scoring Mechanics

Figure 6 plots the change in the public assessment of an individual’s type resulting from Bayesian updating as a function of her current type score and her current actions (i.e. $\psi^{(d,a')}(e, a, s)$ in (16)). Because our estimates exhibit non-zero off-diagonal elements of $Q^\beta$, an individual’s type can switch from one period to the next even if their action reveals themselves to be one type or another. Thus, the domain of the type scoring function in Figure 6 lies between ($s = 0 + Q^\beta(H'|L) = 0.205$ and $s = 1 - Q^\beta(L'|H) = 0.774$). The left plot shows the different updates for bankruptcy filers and non-filers for $a = -0.02$ integrated over earnings and all $a'$ choices in the case of non-filers. It also plots the posterior of an agent’s type even if their actions are not observed, which we call “no inference.” The mean reversion in type score accounts for why a person’s score falls upon repayment if the current score is sufficiently high or rises upon default if it is sufficiently low. Still, it remains true that repaying leads to a higher type score than filing for bankruptcy. Since the $\nu$ shock is noisy, the choice to not file does not reveal much (the reason why no bankruptcy is imperceptibly higher than no inference in the left panel of Figure 6).

\textsuperscript{38}Given that economy-wide debt across all ages by type $H$ is lower than that for type $L$ (i.e. the denominator of the share), type $H$ have a higher share than type $L$ when each are poor, which reverses as type $H$ accumulate more savings through time.

\textsuperscript{39}Krueger and Perri (2006) term "across-group" variation owing to observable differences like education and "within-group" variation the residual which includes idiosyncratic income. Here we are grouping people on observables like age and also unobservables like type.
Figure 6: **Type Score Responses**

The center and right plots of Figure 6 show the Bayesian updates that result from different actions taken by an individual either already in debt (center at $a = -0.02$) or with zero assets (right) for the median earner ($e = z = 0$). For an individual already in debt, staying in debt (i.e. $a' = -0.02$) signals the individual is likely to be type $\beta_L$ as we saw in Figure 4 leading to a drop in their posterior score. As the individual chooses higher $a'$ their posterior rises. It is not until sufficiently high savings choices (e.g. $a' = 0.05$) that there is enough separation to raise an individual’s posterior higher than what would be associated with mean reversion only (i.e. no inference). The right panel documents that starting from a higher asset position ($a = 0$), all the assessments shift down; that is, the smaller net change in asset position makes the inference less likely to be a high type.

Figure 7 illustrates some important points. First, the figure verifies a form of the sufficient condition
(one-to-one mapping between type scores and credit scores conditional on persistent earnings and assets) in Theorem 3 that establishes the equivalence between the fundamental type score equilibrium (RCE) and the credit score equilibrium (RCECS). Second, it shows the important effect of earnings on credit rankings; higher earnings are correlated with higher rankings. Third, higher beginning of period assets are not necessarily correlated with higher rankings (e.g. for \( e = 0.71 \), borrowing \( \bar{a} \) from a higher initial asset holding lowers one’s credit ranking). Finally, while higher type scores are associated with higher credit rankings for most earnings levels, it is not true when earnings are very low. In this case, type \( \beta_H \) actually file for bankruptcy slightly more frequently than type \( \beta_L \) generating the negative relation between type score and credit ranking. At low earnings, it may actually be optimal to borrow rather than default (see Figure 1 in Chatterjee et al. (2007a)).

In our current case where we have different types, the gain from borrowing is stronger for type \( \beta_L \) than for \( \beta_H \) since they care less about the relative drop in their future type score.

**Figure 8: Outcomes by Credit Ranking Quintiles**

![Graphs showing debt to income ratio, fraction in debt, and debt, conditional on borrowing across credit ranking quintiles.](image)

**Notes:** Average moments are computed as the average conditional on credit ranking quintile. The debt to income ratio is the average of individual debt to individual income across the population. The size of debt conditional on borrowing averages across all choices made by each agent in a given state.

In Figure 8 we plot the cross-section of debt choices across credit ranking quintiles (behind their aggregate counterparts in Table 2). The figure illustrates that borrowers with low credit rankings are more likely to be in debt and have high debt-to-income ratios. It also illustrates that conditional on actually borrowing, those with high credit rankings tend to borrow more (since they can do so at lower interest rates). These relations about debt, income, and credit status are consistent with the empirical findings in Diaz-Gimenez et al. (2011) [Table 17, p. 19].

### 5.3 Selection and Reputation Effects

We first consider how one’s current asset choice affects the price they face today because of what it reveals about the individual’s unobservable type. If an individual of observable type \( \omega = (e, a, s) \) were
to borrow $a'$ she would be facing a price that depends on the default probabilities of her type tomorrow $\omega' = (e', a', s')$. Since $a'$ is given at the beginning of the next period and does not affect the probability distribution of $e'$, what matters is how $a'$ affects the update $s'$. To isolate the contribution of the asset choice on the update, we compare the baseline equilibrium price which depends on the update through $Q^e(s' | \psi^{(d,a')})$ with a price schedule that results from excluding $a'$ from the Bayesian updating formula. This price schedule is now given by

$$\tilde{q}^d(a, s, e) = \frac{\rho \tilde{p}^d(a, s, e)}{(1 + r)}$$

where

$$\tilde{p}^d(a, s, e) = \sum_{\beta', e', z', s'} H(z') Q^e(e' | e) Q^e(s' | \beta') \tilde{s}'(\beta') \left[ 1 - \sigma^{(1.0)}(\beta', e', z', a', s') \right]$$

(25)

where $\tilde{s}' = s \cdot Q^\beta(H|H) + (1 - s) \cdot Q^\beta(H|L)$ updates the prior $s$ using only $Q^\beta$ and ignores the information in the borrowing choice $a'$.

Figure 9 shows the percentage increase in $\tilde{q}$ relative to the baseline equilibrium price $q$. Recall from Figure 4 that borrowing reveals oneself to very likely be type $\beta_L$ and therefore when lenders cannot take this information into account, ignoring what can be learned from selection effects induces the price $\tilde{q}$ to exceed $q$. The figure shows that if an individual starts with a low prior $s$ (here we take the 10th percentile $s = 0.21$), the price effect is smaller than if the individual starts with a high prior $s$ (here we take the 90th percentile $s = 0.76$). Further, the positive price effects for the two priors are amplified the more borrowing is undertaken. The latter effect arises since for higher debt levels the pool of borrowers will tend to contain less creditworthy type $\beta_L$ - that is, more adverse selection.
In addition to the contemporaneous effects discussed above, asset market choices can also have long lasting effects. This requires two conditions: (i) prices depend on an individual’s current reputation (i.e. type score) for a given current action and (ii) her choice today affects her future reputation (and hence future prices). Having established the second condition in Figure 6, we next establish the first in Figure 10.

Specifically, Figure 10 plots the percentage change in debt prices that an individual with current type score $s$ can obtain relative to a person with the highest type score $s = 0.77$ for two different debt choices. The fact that both lines are downward sloping establishes that type scores matter; for a given debt choice, a higher $s$ fetches a higher price. The reason is simple; since $\beta_H$ types repay with higher probability than the $\beta_L$ types, there is information about the probability of repayment in the current type score (which, in turn, reflects the history of the individual’s past actions). The fact that the line in Figure 10 is less steep for high amounts of borrowing is consistent with the fact in Figure 4 that type $\beta_L$ are almost perfectly separated from type $\beta_H$ at those high levels so the prior $s$ does not matter as much for assessing the probability of future repayment (i.e. pricing).

Finally, we examine signalling costs. In models with hidden types, the “bad” types have an incentive to imitate the “good” types in order to pool with them and obtain better terms of trade, while the “good” types have an incentive to separate themselves from the “bad” types to get even better terms. Here we assess the costliness for an impatient type $L$ to imitate the actions of a patient type $H$.

Changing one’s action has three effects: (i) a change in today’s consumption; (ii) a change in
Table 5: Signaling Costs and Benefits

<table>
<thead>
<tr>
<th>% Average Gain in:</th>
<th>Consumption ($\hat{C}$)</th>
<th>Wealth ($\hat{A}$)</th>
<th>Credit Ranking ($\hat{\chi}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td>-3.65</td>
<td>3.80</td>
<td>1.31</td>
</tr>
<tr>
<td>Newborns</td>
<td>-0.77</td>
<td>0.81</td>
<td>0.37</td>
</tr>
</tbody>
</table>

Notes: The first column measure is defined as

$$\hat{C} = \frac{\sum z, \omega \mu(\beta_L, z, \omega) \sigma^{(d, d', z)}(\beta_H, z, \omega) - \sigma^{(d, d', z)}(\beta_L, z, \omega)}{\sum z, \omega \mu(\beta_L, z, \omega) \sigma^{(d, d', z)}(\beta_L, z, \omega) \sigma^{(d, d', z)}(\beta_L, z, \omega)}$$

while the second and third columns substitute $a'(z, \omega)$ and $\chi(0,3)(\omega)$ for $c^{(d, d', z)}(z, \omega)$.

tomorrow’s net wealth; and (iii) a change in tomorrow’s reputation. To explore these three effects for a type L to imitate a type H, we conduct the following experiment. To imitate a type H individual, we assume the type L follows the choice probability function $\sigma(\beta_H, z, \omega)$ instead of $\sigma(\beta_L, z, \omega)$. One measure of the consumption cost from a type L individual mimicking a type H is the average difference in consumption between H types and L types implied by the differences in their choice probabilities relative to the average consumption of a type L individual. Similar measures can be computed for next period net wealth and credit ranking.

Table 5 provides these calculations for our calibrated parameters (i.e. where $(\beta_H - \beta_L)/\beta_H = 13\%$). The table illustrates an important point. Type L newborns have a much lower consumption loss to mimicking a type H individual than their older counterparts. This is because the imitation costs are increasing in earnings and assets, both of which rise on average through one’s life as evident in Figure 5. Since type H choose to save more, this imposes a bigger consumption loss to type L from mimicking later in life. Alternatively, it is easier to mimic when young, as the dispersion in assets and scores are lower in youth. The fact that it is less costly to mimic when young implies there is more pooling among the young and the fact that it is more costly to mimic when old implies there will be more separation among the old. The consequence is that while there is a bigger jump in credit ranking of $\beta_L$ type when mimicking in old age, it is more costly to do so.

6 Impact of Alternative Information Structures

How important is the information structure for allocations? What would happen if society outlawed tracking of individual credit histories and with it the incentives to build a good reputation? Would the credit market shrink dramatically as the usefulness of maintaining a good reputation disappears? These are natural questions that we can provide quantitative answers to in the context of our model by
comparing the outcomes of economies that differ from our baseline only in their information structures.

Before we get into the details of our answers to these questions it is important to keep in mind certain features of hidden information that are present in our model. First, because of imperfect separation, low types are being subsidized by high types. Second, there are additional incentives to repay one’s debt and to save more so as to imitate a high type. Third, there can be important interactions of hidden information across the age profile. Specific to our model, all newborns are low earners and face an (expected) upward sloping age-earnings profile. Thus, newborns and young have a life-cycle reason to borrow and so are more impacted by hidden information. Finally, all individuals face idiosyncratic earnings shocks against which direct insurance is unavailable. Since borrowing to smooth consumption is costly in all the economies that we explore, all individuals have a precautionary savings reason to accumulate assets.

Differences between the economies imply not only that individuals behave differently on account of the incentives that they face, but also that the equilibrium prices reflect these changes. Accordingly we have to look at both aspects simultaneously.

6.1 Description of alternative economies

We now consider two stark alternative information structures in which reputation plays a limited role: one where past actions cannot be used to price discriminate but demographic drift can be used to infer type and another where type is public information.40

Our first alternative economy poses hidden information as in the baseline model (hereafter termed BASE) but creditors are prohibited from using a person’s past to price loans. We assume that the length of one’s credit history (proxied here by age) is both publicly observable and legally used to price debt. We make this assumption to isolate the role of reputation from the role of the demographic drift in the credit market. In this economy some information about the individual’s type is learned from her current asset choice but this information cannot be carried across periods. We refer to this economy as the no-tracking (NT) economy since an individual’s asset market actions cannot tracked over time. To be concrete, in this alternative economy individuals start their adult life with a type score equal to the fraction of high types among newborns and it evolves thereafter according to the demographic drift improving over time.41 Note that this implies a one-to-one mapping between an individual’s age and the prior that she is a high type (her type score in this alternative economy). Consistent with no-tracking, lenders are also not allowed to use information about an individual’s beginning-of-period asset holdings

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40 For the formal specification of these alternative economies, see Online Appendix B.6.
41 As before, the evolution is simply given by \( \bar{s}' = \bar{s} \cdot Q^B(H|H) + (1 - \bar{s}) \cdot Q^B(H|L) \) with initial condition \( \bar{s} = 0.32 \).
when pricing loans since this also contains information about her past actions. However, lenders are able to use the current action and the cross-sectional distribution of agents in the NT economy when forming a posterior about the likelihood of repayment necessary to price loans. In this NT economy there is hidden information and indeed, there is cross-subsidization, but there are no dynamic incentives for a $\beta_L$ type to imitate a $\beta_H$ type or a $\beta_H$ type to act differently from a $\beta_L$ type, as actions cannot be used to impute type as in the BASE economy.

Our second alternative features full information (hereafter termed FI) where the type is directly observed by lenders that use it to price discriminate. Except for our extreme value shocks, this alternative economy is similar in many ways to Chatterjee et al. (2007a). There is no need to infer a person’s type, and the price for a loan of size $a'$ depends directly on $\beta$ and all other relevant observables. Importantly, prices do not depend on $s$ nor $a$ because they are not directly payoff relevant. Comparing the FI economy to the BASE reveals the full impact of hidden information: in the FI there is no cross-subsidization nor are there any incentives to imitate or separate.

Some of the main distinguishing features of the three economies can be seen already in Figure 11. The right panel shows the CDF of the type scores for each type indicating the degree to which information about type is revealed to creditors. In the BASE economy, the CDF of type $L$ individuals rises steeply and fast, indicating that most type $L$ individuals have low scores. In contrast, the CDF of type $H$ individuals rises more gradually, indicating that type $H$ individuals have more dispersed type scores. In the NT economy, type scores (priors) are trapped between 0.32 (score at birth) and 0.48 (score at age
infinity). Although people of all types share the same age-specific type score at each age, there are more type $L$ individuals at each age than type $H$ and, consequently, the CDF of low types rises somewhat faster. Most importantly, the CDFs are closest to each other for the NT economy, indicating that less is being learned about an individual’s type as she ages, compared to the BASE and FI economies.

The left panel of Figure 11 plots the mean and standard deviation of type scores for each age across the alternative economies. Importantly, the mean type score at each age is correctly assessed in all economies to be equal to the fraction of high types. In the NT economy the standard deviation of type scores at all ages is zero, as nothing is learned.\textsuperscript{42} For the FI economy, the dispersion in type scores at any age is the dispersion of types themselves in the economy. In the BASE economy it is increasing at a faster rate than in the FI economy, consistent with learning.

6.2 No-Tracking Economy Results

The key feature of NT is that the only information that can be used by lenders in the future (i.e. earnings class and the type score implied by their age or length of credit history) is exogenous so incentives to maintain one’s reputation in asset markets are absent. This can cause equilibrium price menus to drop, an example of which we provide in Appendix B.6.1. Figure 14 which is also evident by the fact that average interest rates rise in Table 6. In response to the rise in the menu of interest rates, it is evident in Table 6 that the fraction of the population in debt falls. However, owing to the absence of reputation effects, those who choose to borrow are willing to borrow more despite higher interest rates. The result as shown in Table 6 is an equilibrium rise in the debt-to-income ratio and interest rates. Higher debt in turn leads to an increase in the bankruptcy rate.

Table 6 also decomposes the aggregate statistics attributable to agent type. It shows that type $\beta_H$ borrowing and default is more sensitive to the change in incentives compared to type $\beta_L$. This arises because, in contrast to the base economy where borrowing and default signaled the person was more likely to be a bad type with reputational costs lasting into the future, that cost now, which high types care about more than the low types, is absent.

Regarding welfare, we focus on newborns.\textsuperscript{43} Rising interest rates associated with the effect of

\textsuperscript{42}Strictly speaking, because the evolution of $s$ implied by the Markov type transition function for the NT economy typically does not yield scores which fall on the grid points in $S$, there is some negligible dispersion in type scores.

\textsuperscript{43}One consequence of focusing on newborns is that we do not need to compute a transition. At the moment of the policy switch, the average asset holdings of older cohorts are potentially different from those of the same age group in the steady state of the NT economy. Hence, even in a small open economy, all except the newborns face a transition of prices as the cross-sectional distribution used to infer future default probabilities evolves to the invariant distribution.

Besides this, there are multiple ways to think of how the switch from the BASE to NT would be implemented for people already alive. One possibility is to immediately outlaw the use of personal asset market history beyond the length of one’s credit history (i.e. age). Alternatively, one could treat older individuals just like newborns, using information on their asset
eliminating incentives that rely on credit histories generally make newborns worse off. Only the type $\beta_L$ newborns with lowest transitory earnings shock $z$ benefit from the cross-subsidization that comes with no tracking. Since their gain is large (0.089) relative to the small losses in all other cases, the mean overall gain for newborns is positive (0.020). 44

Table 6: Comparison of Baseline, No Tracking, and Full Information Economies

<table>
<thead>
<tr>
<th>economy</th>
<th>No Tracking (NT)</th>
<th>Full Information (FI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>discount factor</td>
<td>high</td>
<td>low</td>
</tr>
<tr>
<td>type</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Panel A: % difference from BASE

- bankruptcy rate: 1.40 0.95 1.12 -0.98 0.36 -0.13
- average interest rate: 2.02 1.02 1.44 -7.15 0.83 -2.52
- interest rate dispersion: 10.7 0.75 7.57 -5.12 0.71 -2.15
- fraction in debt: -0.21 -0.13 -0.15 0.23 -0.10 0.00
- debt-to-income ratio: 0.45 0.34 0.39 -0.30 0.12 -0.04

Panel B: wealth equivalent welfare measure, newborns (% of mean wealth)

- low $z$: -0.001 0.089 0.060 0.187 0.089 0.121
- median $z$: -0.000 -0.000 -0.000 0.089 0.044 0.058
- high $z$: -0.000 -0.000 -0.000 0.135 0.089 0.104
- mean: -0.001 0.030 0.020 0.137 0.074 0.094

Notes: Each entry in Panel A is the difference, in percentage points of the BASE moment, of the moment in the indicated alternative economy (FI or NT) relative to the BASE economy. Panel B reports the amount of additional wealth an agent would have to be given in the baseline economy in order to be indifferent between being born into the indicated alternative economy in the indicated state and being born in the baseline economy. The units for Panel B are percentages of mean wealth. Table 12 in Appendix B.6.3 explores how variations in $\alpha$ and $\lambda$ affect the “all” columns in this table.

### 6.3 Full Information Results

Under full information (FI), types are observed, and cross-subsidization ends, as do incentives for a type $\beta_L$ to imitate a type $\beta_H$. An important consequence of this is that equilibrium debt price menus change fundamentally for each type, an example of which we provide in Appendix B.6.2 Figure 15. Specifically, as one might expect, type $\beta_L$ in the FI economy face lower loan prices (higher interest rates) for each given age and type $H$ face higher prices (lower interest rates) than the BASE economy

44See Online Appendix B.4 for the derivation of our wealth equivalent welfare measure.
where there is some cross-subsidization. Price differences also change with age. Interestingly, as agents accumulate assets through time, the act of borrowing is assessed to be even more likely to come from a low type so there is little difference between prices in FI and BASE for type $\beta_L$ but large differences for type $\beta_H$. These differences in interest rates faced by the two types are clearly illustrated in Table 6.

In response to the changes in the menu of interest rates, Table 6 documents that the fraction of type $H$ (type $L$) who borrow rises (falls) as one would expect. While there are also no reputation effects in the FI case, the changes in debt-to-income ratio come about for different reasons for the two types. Debt-to-income for type $L$ increases despite the rise in interest rates because they were holding so little debt in the BASE economy in order to raise their reputation by mimicking high types (i.e. the rise is not that they are holding more debt in FI but they were holding so little debt in BASE). The lower debt-to-income ratio for type $H$ arises from the large increase in $q_{BASE}^{a'}$ which makes it cheaper to achieve a desired inflow for consumption (i.e. since $q_{BASE}^{a'}$ rises, one can achieve the desired inflow $q_{BASE}^{a'}a'$ with a smaller $a'$). Lower (higher) debt-to-income for type $H$ ($L$) explains the fall (rise) in bankruptcy across type in Table 6. One important takeaway from the differences across type is that they tend to cancel out in the aggregate, leading to slight differences in the all statistics except for the impact on equilibrium interest rates.

As Table 6 documents, all newborns are better off in the FI economy. While it is clear that the newborn high types would rather live with full information where they do not subsidize the low types, even low types prefer (albeit less so) full information since they transit to type $H$ with a relatively high probability $Q^{\beta}(H'|L) = 0.205$. The aggregate welfare gains from eliminating cross-subsidization are quite high ($0.094$) in the FI economy relative to the gain ($0.020$) in the NT economy. Thus our “big data” BASE economy yields welfare properties for newborns which are very close to the “small data” no-tracking economy. This is in contrast to the relatively large welfare gains that can come from eliminating hidden information.

7 Delinquency

So far the only possible actions for an indebted agent were to either payback its debt completely and choose another asset position facing prices that are based on its observables or to file for bankruptcy at a cost and have its debts discharged. In the real world, indebted agents can also go delinquent. Here we modify our model along the lines of Athreya et al. (2019) to include a delinquency option and quantitatively assess its implications for credit market outcomes. Notably, the following basic results from the BASE model hold in the extended model: (i) type $\beta_L$ are more likely to go bankrupt; (ii) each type is more likely to file for bankruptcy at higher levels of debt; and (iii) bankruptcy on average leads
to a downward revision of one’s type score.

Specifically, default can arise either through delinquency, whereby an agent can choose not to pay its debts yet it does not file for bankruptcy to discharge its debt, or bankruptcy. In our modified model, the unobservable income loss from default can depend on one’s type by a factor of proportionality \( \tau(\beta) \), where \( \tau(\beta_H) \geq \tau(\beta_L) = 1 \) so that a default can be weakly more costly for high types than low.\(^{45}\)

Under the former choice, the household’s income net of any costs associated with delinquency (which we take to be \( y(e,z)(1-\kappa_2 \cdot \tau(\beta)) \) with \( \kappa_2 < \kappa_1 \) so there are lower costs than in bankruptcy) is used for consumption and its obligation next period is the face value of its current debt plus a penalty specified in the contract that we take to be a factor \( \eta > 0 \) of the debt. Upon becoming delinquent, a household can pay back its debts, file for bankruptcy, or become delinquent again.

Lenders with delinquent debt are required (by law) to remove (charge-off) such debts from their books which they do by selling delinquent debt to third-party collectors. For simplicity, we assume all delinquent debt is pooled and sold to third party collection agencies at an equilibrium price \( \bar{q}_{d} \) to be described below. Buyers of delinquent debt operate at a per unit cost \( \gamma \) and are competitive.

### 7.1 The household problem

We modify the problem in Section 3.1 by expanding the set \( D = \{0, 1, 2\} \) where \( d = 2 \) signifies delinquency. Delinquency adds a new option in the choice set allowing the household to avoid repaying its debt without incurring a bankruptcy fee but saddling it with more debt next period. In this case equation (3) becomes:

\[
\begin{align*}
c^{(d,a')} (z, \omega | f) = \begin{cases} 
y(e(\omega), z) + a(\omega) - a^2 (\omega) \cdot a' & \text{if } (d, a') = (0, a'), 
y(e(\omega), z)(1-\kappa_1 \cdot \tau(\beta))) - \kappa & \text{if } a(\omega) < 0 \text{ and } (d, a') = (1, 0), 
y(e(\omega), z)(1-\kappa_2 \cdot \tau(\beta))) & \text{if } a(\omega) < 0 \text{ and } (d, a') = (2, a(\omega)(1 + \eta) \geq a_1).
\end{cases}
\end{align*}
\]

The addition is the last line of (26). For any \( a(\omega) \in [a_1, 0) \) such that \( a(\omega)(1 + \eta) < a_1 \) (i.e. a delinquency would take the agent past the lowest grid point) we assume the agent cannot go delinquent and must choose either bankruptcy or repayment (both of which are feasible by Assumption 1). These assumptions imply that delinquency can only happen a finite number of times in a row.\(^{46}\)

---

\(^{45}\) Corbae and Glover (2018) provides an adverse selection labor matching model with pre-employment credit screening which generates a larger income loss for type \( \beta_H \) than type \( \beta_L \).

\(^{46}\) When \( a(\omega)(1 + \eta) \) is not on the grid \( \mathcal{A} \), similar to what we did with type scores, we distribute \( a(\omega)(1 + \eta) \in [a_j, a_{j+1}] \) with probability \( w \) to \( a_j \) and probability \( 1-w \) to \( a_{j+1} \) where \( w = (a_{j+1} - a(\omega)(1 + \eta))/(a_{j+1} - a_j) \).
Recall that earlier a household first chose whether to file for bankruptcy or not, and if not, how much to save. We now pose that the household chooses whether to default or not and the mode of default. If the household chooses to default, it also chooses whether to file for bankruptcy or to become delinquent; if it does not, it chooses how much to save, receiving a vector of shocks $\epsilon$ attached to each $a'$ choice exactly as in the baseline model according to (22). To allow for correlation between the shocks associated with the default actions we posit a nested logit structure for the shocks no default / bankruptcy / delinquency shocks. That is, rather than the independent draws from (21) as in the baseline, the vector $\nu$ is now drawn from

$$F(\nu) = \exp \left\{ -\exp \left( -\frac{\nu^d = 0 - \bar{v}}{\alpha} \right) - \left[ \exp \left( -\frac{\nu^d = 1 - \bar{v}}{\phi \alpha} \right) + \exp \left( -\frac{\nu^d = 2 - \bar{v}}{\phi \alpha} \right) \right] \right\}^{\phi}$$

(27)

where the new parameter $\phi$ specifies the correlation between the shocks associated with bankruptcy ($d = 1$) and delinquency ($d = 2$). Note that the adjustment to insure that there is no bonus on average associated with the extra options available to debtors changes in this setting: now we set $\bar{v} = -\alpha y - \alpha \ln(1 + 2^\phi)$. The value functions conditional on each one of the choices in the feasible set $F(z, \omega)$ follow trivially.

### 7.2 Pricing

All that remains is to determine how lenders price debt in the presence of two different types of default. Regulation requires that banks charge off loans that are severely past due.\(^{47}\) Hence, unlike Athreya et al. (2019) where delinquent debt can be held on a lender’s balance sheet as long as the individual is delinquent, we assume all delinquent debt is pooled after a period and sold at price $q_0 [q_2]$ per unit. Competition ensures that debt collectors obtain zero profits net of the transaction (collection) costs to the lending process.

Turning first to the new pricing equation of loans by the financial intermediary, the probability of repayment on a new loan of size $a'$ is altered from that given in equation (17) to

$$p^d(\omega) = \sum_{\beta', z', e', s'} H(z') \cdot Q^a(e'|e) \cdot Q^s(s'|s) \psi(\beta'|\omega) \cdot s'$$

$$\cdot \left[ 1 - \sigma^{(1,0)}(\beta', z', e', a', s') - \sigma^{(2,(1+\eta)a')}(\beta', z', e', a', s') \right].$$

\(^{47}\)From https://en.wikipedia.org/wiki/Charge-off, In the United States, federal regulations require creditors to charge off installment loans after 120 days of delinquency, while revolving credit accounts must be charged-off after 180 days.
The probability of delinquency on that new loan is

\[ \delta^d(\omega) = \sum_{\beta', z', e', s'} H(z') \cdot Q^d(e'|e) \cdot Q^s(s' | \beta') | \psi^{(0, d')}_\beta(\omega) \cdot s'(| \beta') \]  

(29)

Consequently, the competitive price of a new loan offered by lenders is altered from that given in equation (15) to

\[ q^d(\omega) = \frac{p^d(\omega)}{(1 + r)} \left[ p^d(\omega) + \delta^d(\omega) \cdot q_\delta \cdot (1 + \eta) \right], \]  

(30)

where the second term on the right hand side is the lender’s recovery from selling off the delinquent debt to a collector.

Turning next to the value of debt held by a collection agency, the probability of repayment on delinquent debt \( a \) of a household in state \( \omega \) held by a collector is

\[ p_\delta^{(1 + \eta)a}(\omega) = \sum_{\beta', z', e', s'} H(z') \cdot Q^d(e'|e) \cdot Q^s(s' | \beta') | \psi^{(2, (1 + \eta)a)}_\beta(\omega) \cdot s'(| \beta') \]  

(31)

\[ \cdot \left[ 1 - \sigma^{(1, 0)}(\beta', z', e', (1 + \eta)a', s') - \sigma^{(2, (1 + \eta)a)}(\beta', z', e', (1 + \eta)a, s') \right] \]

noting the key differences between (28) and (31) are the type updates \( \psi^{(d, a')}_\beta \) and the future debt obligations \( a' \). Equation (31) makes clear that punishment associated with delinquency arises from being saddled with penalties augmenting what is owed and delinquency’s impact on type score.

We assume a collector does not need to discharge its own debt holdings if a person becomes delinquent again, but it pays collection costs \( \gamma \) each period. Denoting by \( q_\delta^{(1 + \eta)a}(\omega) \) the value per unit of delinquent debt \( a \) of a person in state \( \omega \) held by a collector, we have

\[ q_\delta^{(1 + \eta)a}(\omega) = \frac{p}{(1 + r)(1 + \gamma)} \left[ p_\delta^{(1 + \eta)a}(\omega) + \right. \]  

(32)

\[ \sum_{\beta', z', e', s'} H(z') \cdot Q^d(e'|e) \cdot Q^s(s' | \beta') | \psi^{(2, (1 + \eta)a)}_\beta(\omega) \cdot s'(| \beta') \]  

\[ \cdot \sigma^{(2, (1 + \eta)a)}(\beta', z', e', (1 + \eta)a, s') \cdot q_\delta^{(1 + \eta)a}(e', (1 + \eta)a, s') \cdot (1 + \eta) \right] \]

The zero profit condition for debt collectors is then

\[ \bar{q}_\delta = \frac{\sum_{\beta, z, \omega} q_\delta^{(1 + \eta)a}(\omega) \cdot a \cdot \sigma^{(2, (1 + \eta)a)}(\beta, z, \omega) \cdot \mu(\beta, z, \omega)}{\sum_{\beta, z, \omega} a \cdot \sigma^{(2, (1 + \eta)a)}(\beta, z, \omega) \cdot \mu(\beta, z, \omega)}. \]  

(33)
Table 7: Delinquency Related Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recovery rate</td>
<td>( \bar{q}_\delta ) 0.22</td>
</tr>
<tr>
<td>Penalty rate in delinquency</td>
<td>( \eta ) 0.30</td>
</tr>
<tr>
<td>Extreme value correlation parameter</td>
<td>( \phi ) 0.10</td>
</tr>
<tr>
<td>Variable cost in delinquency</td>
<td>( \kappa_2 ) 0.03</td>
</tr>
<tr>
<td>Collection cost</td>
<td>( \gamma ) 2.07</td>
</tr>
<tr>
<td>Factor of proportionality</td>
<td>( \tau(\beta_H) ) 1.25</td>
</tr>
</tbody>
</table>

Substituting (32) into (33) we can solve for \( \gamma \) residually given an observed price \( \bar{q}_\delta \). A requirement is that \( \gamma \geq 0 \).

### 7.3 Parameterization

To illustrate the workings of our model with both bankruptcy and delinquency, here we simply supplement the estimated parameter values from the BASE model with parameter values \( \bar{q}_\delta, \eta, \kappa_2, \gamma, \tau(\beta_H) \) chosen to approximate certain moments like credit card recovery rates, delinquency rates and penalties, and certain restrictions implied by the model on the data.

We set the recovery rate \( \bar{q}_\delta \) to 0.22 as in Chatterjee and Gordon (2012). We set the penalty rate in delinquency \( \eta \) to 30% consistent with industry averages.\(^4^8\) The extreme value parameter \( \phi \) and variable cost in delinquency \( \kappa_2 \) are set to be roughly consistent with the bankruptcy and delinquency rate (measured as being delinquent for four quarters in a row consistent with our annual model period). The collection cost \( \gamma \) is set to satisfy (33) given (32). Finally, the proportional income loss from default \( \tau(\beta_H) \) is 25% higher for type \( \beta_H \) than \( \beta_L \). The parameters are given in Table 7.

### 7.4 How does a delinquency option change equilibrium outcomes?

In Table 8, we provide the moments from our delinquency extension of the BASE model (adding the 4 quarter delinquency rate that was absent from Table 2). While there are some differences, perhaps the most noteworthy result is that the addition of the delinquency option yields model moments not very different from their data counterparts despite not re-estimating the model.

\(^{48}\) See for example, https://www.thebalance.com/credit-card-default-and-penalty-rates-explained-960643: text=The%20average%20credit%20card%20penalty,in%20interest%20after%20one%20year.
Table 8: **Target Moments, Delinquency v Baseline**

<table>
<thead>
<tr>
<th>Moment (%)</th>
<th>Data</th>
<th>Baseline</th>
<th>Delinquency</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Aggregate credit market moments</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bankruptcy rate</td>
<td>1.00</td>
<td>1.02</td>
<td>0.74</td>
</tr>
<tr>
<td>Average interest rate</td>
<td>11.9</td>
<td>11.5</td>
<td>16.6</td>
</tr>
<tr>
<td>Interest rate dispersion</td>
<td>7.00</td>
<td>7.08</td>
<td>2.76</td>
</tr>
<tr>
<td>Fraction of HH in debt</td>
<td>7.92</td>
<td>9.16</td>
<td>11.4</td>
</tr>
<tr>
<td>Debt to income ratio</td>
<td>0.40</td>
<td>0.26</td>
<td>0.36</td>
</tr>
<tr>
<td>Delinquency rate</td>
<td>1.54</td>
<td>N.A.</td>
<td>1.11</td>
</tr>
<tr>
<td><strong>Credit ranking age profile moments</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept, mean credit ranking</td>
<td>0.278</td>
<td>0.325</td>
<td>0.394</td>
</tr>
<tr>
<td>Slope, mean credit ranking</td>
<td>0.038</td>
<td>0.037</td>
<td>0.022</td>
</tr>
<tr>
<td>Intercept, std. dev. credit ranking</td>
<td>0.215</td>
<td>0.219</td>
<td>0.267</td>
</tr>
<tr>
<td>Slope, std. dev. credit ranking</td>
<td>0.011</td>
<td>0.010</td>
<td>0.003</td>
</tr>
<tr>
<td>Average autocorrelation of change in credit ranking</td>
<td>-0.220</td>
<td>-0.204</td>
<td>-0.238</td>
</tr>
</tbody>
</table>

**Notes:** Given our model period is a year, we classify a person as delinquent if they have been delinquent for 4 consecutive quarters.
Notes: The individual state for the top panels of this figure is $s = 0.48$ and $e = z = 0$. The left panel presents the probability of either type of default for each type while the right panel presents the likelihood ratio for each type of default. The bottom left panel plots the share of agents from a simulated panel who file for bankruptcy in year 0 who are delinquent in year $t$. This share is zero in the year of the bankruptcy (declaring bankruptcy precludes delinquency) and the year after (bankruptcy in year 0 implies $a = 0$ in year 1, so delinquency is infeasible).
As in the BASE model, type $\beta_L$ default (i.e. choose either delinquency or bankruptcy) more than type $\beta_H$ as evident in the top right panel of Figure 12 since their likelihood ratio for default exceeds 0.5 (similar to the earlier results in Figure 4). The novel aspects stem from the fact that, as evident in the budget sets of equation (26), delinquency provides a low current resource cost way to default at the expense of incurring more future debt and lowering one’s future reputation. Since type $\beta_L$ care less about the future and more about current consumption than type $\beta_H$ and have lower costs of default, they are more likely to choose delinquency and bankruptcy. This difference is clearly evident in the top row of Figure 12. Since type $\beta_L$ is more likely to go delinquent and bankrupt, the bottom left panel of Figure 12 shows that such default decisions lead to a fall in their type scores similar to the earlier results in Figure 6.\textsuperscript{49} It also shows that a bankruptcy leads to a bigger downward revision of type score than a delinquency.\textsuperscript{50} As one might expect, the bottom right panel of Figure 12 shows that bankruptcies often follow delinquencies; a little more than 20% of the individuals who choose bankruptcy are already delinquent (for a model period of one year).\textsuperscript{51}

The extended model provides testable predictions. For example, the top left panel shows that both types are more likely to choose delinquency for low debt levels and more likely to choose bankruptcy for high debt levels (for a given earnings and type score). This generates a pattern where for both types as debt grows, they substitute out of delinquency into bankruptcy as a form of default. This is intuitive as the future debt cost of delinquency is more severe with higher debt. If one integrates across all individuals who default, this provides a prediction that those who go bankrupt have higher debt levels than first-time delinquents. Our model generates a 17% higher level of debt held by bankrupts than first time delinquents while the data generates a 37% higher level.\textsuperscript{52}

8 Conclusion and Directions for Future Research

In this paper, we present a hidden information model of unsecured consumer credit with risk of default. People are subject to unobserved persistent and transitory shocks, and the history of people’s asset market actions helps forecast future defaults. The setup is possibly the simplest environment to quantitatively study the role of credit scores in regulating consumer credit. We showed how this can be

\textsuperscript{49}The bottom left panel also shows that repayment raises one’s score relative to a model where there is no inference about one’s type from their actions.

\textsuperscript{50}This ordering is sensitive to the choice of $\tau(\beta_H)$. A bankruptcy can have a less severe impact on type score than delinquency for $\tau(\beta_H)$ values closer to 1.

\textsuperscript{51}There is a slightly smaller spike in delinquencies preceding bankruptcy for the older cohort as they less likely to have higher levels of debt.

\textsuperscript{52}The average credit card debt of delinquents is $5564 and of bankrupts is $7625 and the ratio is 1.37 (estimates are for individuals with positive credit card debt at the time of delinquency or bankruptcy in 2004 (authors’ calculations using FRBNY CCP/Equifax data)).
done using shocks drawn from an extreme value distribution and recursive updating of beliefs.

Our quantitative model with persistent hidden heterogeneity in types is capable of not only accounting for aggregate credit market moments but also the joint behavior of the age profile of credit rankings. Furthermore, the model generates an age profile of credit ranking patterns like those in U.S. data. In this sense, our model provides a quantitative theory of the credit score.

Two implications of our theory are worth highlighting. First, we found that restricting lenders’ access to an individual’s history of asset market actions (no tracking) leads to an overall welfare gain for young adults. Since the young tend to borrow against their future income, the insurance afforded poor young adults of low type who are cross-subsidized by others in better standing outweighs the costs of higher interest rates associated with negative incentive effects from not having to maintain a good reputation. Our “big data” baseline model suggests that the intratemporal insurance for a subset of the population in a “small data” economy can outweigh the incentive effects worsening intertemporal insurance.

Second, even though our model allowed lenders unrestricted access to the history of all actions relevant for inferring an individual’s type, the equilibrium allocations at an individual level remain far removed from those of a full information economy. This stems from the fact that individuals select actions that only partially reveal their type, while in the full information economy they get that revelation for free. Despite big differences at the micro level, the macro (aggregate) differences can be small.

Where next? First, type does not have to correspond to an individual’s hidden time preference. Alternatively, it could correspond to hidden ability differences that exogenously affect earnings. Hidden time preference can affect a hidden human capital decision (i.e. moral hazard) to endogenously affect earnings or a variety of other personal traits. Second, reputation in the unsecured credit market can spill over to other markets, reinforcing reputation effects. A person’s reputation (or type score) in the unsecured credit market may have implications for other markets (e.g. insurance, labor, housing) and other interactions (marriage) that are worth exploring. Finally, considering the interaction of financial literacy and imperfect competition in the unsecured consumer credit market are important directions for future research.

References

(????): .


Online Appendices

A Model Appendix

A.1 Construction of $Q^s(s'|\psi)$ and Proof of Lemma 1

Let $G \equiv \{0, 1/K, 2/K, \ldots \}$ be a uniform discrete approximation of $[0, 1]$. Let $D = 1/K$ denote the distance between adjacent (grid) points of $G$. Let $S = \{(s_1, s_2, \ldots, s_B)|s_i \in G$ and $\sum_{i=1}^B s_i = 1\}$ be the associated probability simplex.

**Lemma A.1.** Let $s_i \in G$ for $i = 1, 2, \ldots, B - 1$. If $\sum_{i=1}^{B-1} s_i < 1$, then $1 - \sum_{i=1}^{B-1} s_i \in G$.

**Proof.** $\sum_{i=1}^{B-1} s_i < 1 \Rightarrow \sum_{i=1}^{B-1} (\ell_i/K) < 1 \Rightarrow \sum_{i=1}^{B-1} \ell_i < K$ where the $\ell_i$’s are integers between 0 and $K$. Since a sum of integers is an integer and a difference of two integers is also an integer, $K - \sum_{i=1}^{B-1} \ell_i$ is a positive integer and it is less than $K$. Therefore, by the definition of $G$, $1 - \sum_{i=1}^{B-1} \ell_i/K \in G$. □

**Definition A.1.** All elements of the matrix $Q^\beta$ are strictly positive.

**Lemma A.2.** Let $\psi = (\psi_1, \psi_2, \ldots, \psi_B)$ be any vector of type scores resulting from the Bayesian update. Then, $\psi_i \geq Q > 0$.

**Proof.** Let $Q$ be the smallest element of $Q^\beta$. By Assumption 1, $Q > 0$.

$$\psi_i = \sum_j Q^\beta(i|j) \times \text{posterior probability of } j | \text{actions}$$

$$\geq \sum_j Q \times \text{posterior probability of } j | \text{actions}$$

$$= Q.$$ 

The first equality follows from the definition of $\psi_i$, the inequality follows from Assumption 1 and the last line follows from the fact that the sum of posterior probabilities is 1. □

We now identify the elements of $S$ that approximate any given type-score vector $\psi$ resulting from the Bayesian update. Let $s_{i,L} = \max_{s \in G} s \leq \psi_i$ and $s_{i,H} = s_{i,L} + D$. Consider the collection of $2^{B-1}$ vectors:

$$S_\psi = \left\{(s_{1,l(1)}, s_{2,l(2)}, \ldots, 1 - \sum_{i=1}^{B-1} s_{i,l(i)}) \right\} \text{ where for each } i, l(i) \in \{L, H\}$$

**Lemma A.3.** If $D < Q/(B - 1)$ then $S_\psi \subset S$.

**Proof.** By construction, $s_{i,L} \in G$. Next, observe that $s_{i,L}$ cannot be 1 since that would imply that $\psi_i = 1$ and, therefore, $\psi_{j \neq i} = 0$ in contradiction to Lemma A.2. Therefore, $s_{i,H} = s_{i,L} + D$ must belong in $G$.
for all \( i \). To show that \( (s_1,l(1), s_2,l(2), \ldots, 1 - \sum_{i=1}^{B-1} s_i,l(i)) \) belongs in \( S \) it is sufficient to show, by virtue of Lemma A.1, that \( \sum_{i=1}^{B-1} s_i,l(i) < 1 \).

\[
\sum_{i=1}^{B-1} s_i,l(i) \leq \sum_{i=1}^{B-1} s_i,H
\leq \sum_{i=1}^{B-1} (\psi_i + D)
= (1 - \psi_B) + (B - 1)D
\leq 1 - Q + (B - 1)D
< 1
\]

The first inequality follows because \( s_i,l(i) \leq s_i,H \). The second inequality follows because \( s_i,L = s_i,H + D \) and \( \psi_i \geq s_i,L \). The third equality follows because \( \sum_{i=1}^{B-1} \psi_i = 1 \). The fourth inequality follows from Lemma A.2 and the final inequality follows from the hypothesis of the lemma.

By virtue of Lemma A.3 we can take \( S_\psi \) to be the collection of approximating vectors. Note that for each member of this set, the first \( B - 1 \) components are within \( \psi_i \pm D \) so, in this sense, the vectors are close to \( \psi \).

We now determine the probability assigned to each of these vectors. To this end, let

\[ p(s_i,L) = \frac{s_i,H - \psi_i}{D} \quad \text{and} \quad p(s_i,H) = \frac{\psi_i - s_i,L}{D} \quad \text{for } i = 1, 2, 3, \ldots B - 1. \quad (34) \]

Since \( s_i,L \leq \psi_i < s_i,H \) and \( s_i,H - s_i,L = D \) both \( p(s_i,L) \) and \( p(s_i,H) \) are nonnegative and their sum is 1. We set

\[
\Pr \left[ \left( s_1,l(1), s_2,l(2), \ldots, 1 - \sum_{i=1}^{B-1} s_i,l(i) \right) \right] = \prod_{i=1}^{B-1} p(s_i,l(i)), \quad l(i) \in \{L, H\}, i = 1, 2, \ldots B - 1.
\]

Then our assignment rule \( Q^S(s'|\psi) : S \rightarrow [0, 1] \) is given by:

\[
Q^S(s'|\psi) = \begin{cases} 
\prod_{i=1}^{B-1} p(s'_i,l(i)) & \text{if } s' \in S_\psi \\
0 & \text{otherwise.}
\end{cases} \quad (35)
\]

For this assignment rule, we can prove:

**Lemma 1.** (i) \( \sum_{s' \in S} s'_i Q^S(s'|\psi) = \psi_i, \forall i \) (consistency), (ii) \( \sum_{s' \in S} (s'_i - \psi_i)^2 Q^S(s'|\psi) \leq 2(B - 1)D^2, \forall i \) (variance of the approximation error can be made arbitrarily small), and (iii) \( Q^S(s'|\psi) \) is continuous in \( \psi \) (continuity).

**Proof.** (i) First, note that

\[
\sum_{s' \in S} s'_i Q^S(s'|\psi) = \sum_{s' \in S_\psi} s'_i Q^S(s'|\psi)
\]

since (35) assigns positive probability only to vectors that are in \( S_\psi \).
Let \( i \in \{1, 2, \ldots, B-1\} \). Now, group the collection of vectors in \( S_v \) into two: In the first group are all vectors for which \( s'_i = s_{i,L} \) and in the second group are all vectors for which \( s'_i = s_{i,H} \). Denote these groups as \( S_v^L \) and \( S_v^H \). Then,

\[
\sum_{s' \in S_v} s'_i Q^s(s'|\psi) = \sum_{s' \in S_v^L} s'_i Q^s(s'|\psi) + \sum_{s' \in S_v^H} s'_i Q^s(s'|\psi)
\]

\[
= s_{i,L} \sum_{s' \in S_v^L} Q^s(s'|\psi) + s_{i,H} \sum_{s' \in S_v^H} Q^s(s'|\psi)
\]

\[
= s_{i,L} p(s_{i,L}) + s_{i,H} p(s_{i,H})
\]

\[
= \psi_i.
\]

The third equality follows from the fact that the first and second summation terms in the second line are the probabilities of selecting a vector that belongs to group \( L \) and group \( H \), respectively. Since the assignment of \( s_{i,L} \) or \( s_{i,H} \) for \( s'_i \) is done independently of the assignments to the other \( B-2 \) components, the probability of selecting a vector in group \( L \) is \( p(s_{i,L}) \) and in group \( H \) is \( p(s_{i,H}) \). The last equality follows from (34).

Next, let \( i = B \). Then,

\[
\sum_{s' \in S_v} s'_B Q^s(s'|\psi) = \sum_{s' \in S_v} [1 - s'_1 - s'_2 - \ldots - s'_{B-1}] Q^s(s'|\psi)
\]

\[
= \sum_{s' \in S_v} Q^s(s'|\psi) - \sum_{i=1}^{B-1} \sum_{s' \in S_v} s'_i Q^s(s'|\psi)
\]

\[
= 1 - \sum_{i=1}^{B-1} \psi_i
\]

\[
= \psi_B.
\]

(ii) Let \( i \in \{1, 2, \ldots, B-1\} \).

\[
\sum_{s' \in S_v} (s'_i - \psi_i)^2 Q^s(s'|\psi) = \sum_{s' \in S_v^L} (s'_i - \psi_i)^2 Q^s(s'|\psi) + \sum_{s' \in S_v^H} (s'_i - \psi_i)^2 Q^s(s'|\psi)
\]

\[
= \sum_{s' \in S_v^L} (s_{i,L} - \psi_i)^2 Q^s(s'|\psi) + \sum_{s' \in S_v^H} (s_{i,H} - \psi_i)^2 Q^s(s'|\psi)
\]

\[
\leq D^2 \sum_{s' \in S_v^L} Q^s(s'|\psi) + D^2 \sum_{s' \in S_v^H} Q^s(s'|\psi)
\]

\[
= D^2 (p(s_{i,L}) + p(s_{i,H}))
\]

\[
= D^2.
\]

Let \( i = B \). Then,

\[
\sum_{s' \in S_v} (s'_B - \psi_B)^2 Q^s(s'|\psi) = \sum_{s' \in S_v} \left( 1 - \sum_{i=1}^{B-1} s'_i - 1 + \sum_{i=1}^{B-1} \psi_i \right)^2 Q^s(s'|\psi)
\]
can be made arbitrarily small. \( \nu \) and \( p \) number of (uniformly-placed) grid points.

Note that by reducing the distance \( \varepsilon \) between adjacent points of \( G \), or, equivalently, increasing the number of (uniformly-placed) grid points \( K \) approximating the unit interval, the dispersion of \( s' \) around \( \psi \) can be made arbitrarily small.

A.2 Proof of Theorem 1 (Existence of the Value Function)

**Theorem 1.** Given \( f \), there exists a unique solution \( W(\beta, z, \omega|f) \) to the individual’s decision problem in (3)-(8).

**Proof.** The proof relies on the Contraction Mapping Theorem. However, since the extreme value shocks \( \nu \) and \( \varepsilon \) can take any value on the real line, it is mathematically more convenient to seek a solution to (3), (4), (12), and (13) since the extreme value shocks do not appear in these. Define the operator \((T_f)(W) : \mathbb{R}^{B+Z+|\Omega|} \to \mathbb{R}^{B+Z+|\Omega|}\) as the map that takes a vector \( W \) in \( \mathbb{R}^{B+Z+|\Omega|} \) and returns a vector \((T_f)(W)\) via (4), (12), and (13) using (3). We may easily verify that \( T_f \) satisfies Blackwell’s sufficiency condition for a contraction map (with modulus \( \beta \rho \)). Since \( \mathbb{R}^{B+Z+|\Omega|} \) is a complete metric space (with, say, the uniform metric \( \rho(W, W') = \max_{1 \leq s \leq B+Z+|\Omega|} \| W_s - W'_s \| \)), by Theorem 3.2 of Stokey and Lucas Jr. (1989), there exists a unique \( W(\beta, z, \omega|f) \) satisfying \((T_f)(W) = W\). \( \square \)
A.3 Proof of Lemma 2 (Existence of the Invariant Distribution)

Lemma 2. There exists a unique invariant distribution $\bar{\mu}(\cdot \mid f)$ and $\{\mu_0T^n\}$ converges to $\bar{\mu}(\cdot \mid f)$ at a geometric rate for any initial distribution $\mu_0$.

Proof. We will use Theorem 11.4 in Stokey and Lucas Jr. (1989) to establish this result. To connect to that theorem, let $i$ be a typical element of the finite state space $B \times Z \times \Omega$. Let the transition matrix $\Pi$ in their theorem correspond to $T$ in (19) and let $\pi_{ij}$ denote the probability of transitioning to $j$ from $i$. Further, let $\epsilon_j = \min_i \pi_{ij}$ and $\epsilon = \sum_j \epsilon_j$. Then it is sufficient to establish that $\epsilon > 0$. To this end, consider the state $\hat{j} = (\hat{\beta}, \hat{z}, \hat{e}, 0, G_e)$ with the property that $g(\hat{\beta})H(\hat{z})G_e(\hat{e}) > 0$ (this should be $G_h(\hat{\beta})H(\hat{z})G_e(\hat{e}) > 0$ for all $i$). Hence $\epsilon_j \geq (1 - \rho)G_h(\hat{\beta})H(\hat{z})G_e(\hat{e}) > 0$. Since $\epsilon_j \geq 0$ for all other $j$, it follows that $\epsilon > 0$.

A.4 Proof of Lemma 3 (Continuity of the Value Function) and Theorem 2 (Existence of Equilibrium)

The fact that there are zero profits in equilibrium implies $q^{(0,a')}(\omega \mid f) = \frac{\rho}{1+\rho}$ for $a' \geq 0$ (i.e. the price on savings is a function only of parameters). In what follows we take $F^* \subset F$ to contain only those $f_1$ for which $f_1(a', \omega) = \frac{\rho}{1+\rho}$ for $a' \geq 0$.

Lemma 3. $W(\beta, z, \omega \mid f)$ is continuous in $f$ and for any $(d, a') \in F(z, \omega \mid f)$, $\sigma^{(d,a')}(\beta, z, \omega \mid f)$ is continuous in $f$.

Proof. To prove continuity of $W(\beta, z, \omega \mid f)$ in $f$ we first show that the operator $T_f$ defined in Theorem 1 is continuous in $f$ (meaning that for any given $W$, small changes in $f$ lead to small changes in $T_f(W)$). Inspection of (6) and (8) shows that this will be true if the conditional value functions $v^{(d,a')}(\beta, z, e, a, s \mid f)$ in (4) are continuous in $f$. Let $\hat{f} \in F^*$ and let $(\hat{d}, \hat{a})' \in F(z, \omega \mid \hat{f})$. Let $f^n \in F^*$ be a sequence converging to $\hat{f}$. By Assumption 1, $(0,0)$ and $(1,0)$ are feasible choices regardless of the value of any inherited debt (i.e. $a < 0$), so all debt choices ($a' < 0$) and the default choice belong in $F(z, \omega \mid f^n)$. Furthermore, if an asset choice (i.e. $a' \geq 0$) is feasible for $\hat{f}$, that asset choice remains feasible for $f^n$ since the price of any asset is the same in $\hat{f}$ and $f^n$ (namely, $\rho/(1+r)$). Therefore, $(\hat{d}, \hat{a})' \in F(z, \omega \mid f^n)$ and so $v^{(d,a')}(\beta, z, e, a, s \mid f^n)$ is well-defined for all $n$. Observe that $f^n$ affects $v^{(d,a')}(\beta, z, e, a, s \mid f^n)$ in (4) via how $q^n$ affects the feasible set given in (3) and how $\psi^n$ affects $Q^s(s' \mid \psi^n)$ in (4). Since $\lim_{n \to \infty} c^{(d,a')}(z, \omega \mid f^n) = c^{(d,a')}(z, \omega \hat{f})$, the continuity of $u$ gives $\lim_{n \to \infty} u(c^{(d,a')}(z, \omega \mid f^n)) = u(c^{(d,a')}(z, \omega \hat{f}))$. From Lemma 1, $\lim_{n \to \infty} Q^s(s' \mid \psi^n) = Q^s(s' \mid \psi \hat{f})$. It follows that $v^{(d,a')}(\beta, z, e, a, s \mid f)$ is continuous in $f$ and hence $\lim_{n \to \infty} T_f^n = T_{\hat{f}}$. Since $F$ is a Banach space and $T_f$ is a contraction map, we may apply Theorem 4.3.6 in Hutson and Pym (1980) to conclude that $W$ is continuous in $f$. The continuity of $\sigma^{(d,a')}(\beta, z, \omega \mid f)$ in $f$ follows directly by continuity of $\sigma$ in $W$.

Theorem 2. There exists a stationary recursive competitive equilibrium.

Proof. The proof of existence uses Brouwer’s Fixed Point Theorem (Theorem 17.3 in Stokey and Lucas Jr. (1989)). To connect to that theorem, we reinterpret the function $f$ as a point in a unit (hyper)cube in high-dimensional Euclidean space. To this end, let $G = \{(d, a') : (d, a') \in \mathcal{Y}, \beta \in B, z \in \Omega\}$.
\( \mathcal{Z}, \omega \in \Omega \subset \mathcal{Y} \times \mathcal{B} \times \mathcal{Z} \times \Omega \) where \( \mathcal{Y} = \{(d, a') : (d, a') \in \{0\} \times \mathcal{A} \) or \( (d, a') = (1, 0) \}. Let \( M \) and \( K \) be the cardinalities of \( \mathcal{G} \) and \( \mathcal{Y} \setminus \{(1, 0)\}. \) Then, \( f \in F^* \) can be thought of as a vector composed by stacking \( q \in [0, 1]^K \) and \( \psi \in [0, 1]^{B \cdot M}. \) Then \( f \in [0, 1]^{K+B \cdot M} \) and \( F^* \subset [0, 1]^{K+B \cdot M}. \) Next, use (15) (with equality) to construct the vector \( q_{new}^{d, a'}(\omega|f) \) and use (16) to construct the vector \( \psi_{new}^{d, a'}(\omega|f). \) Then, let \( J \) be the mapping

\[
   f_{new} \equiv \left( q_{new}^{d, a'}, \psi_{new}^{(0, a'), (1, 0)} \right) = J(f) : F^* \rightarrow F^*.
\]

Since \( \sigma^{d, a'}(\beta, z, \omega|f) \) is a continuous function of \( f \) (Lemma 3), \( J \) is a continuous self-map as (15) and (16) are continuous functions of \( \sigma^{d, a'}(\beta, z, \omega|f). \) And since \( F^* \) is a nonempty, closed, bounded and convex subset of a finite-dimensional normed vector space, by the Brouwer’s FPT there exists \( f^* \in F^* \) such that \( f^* = J(f^*). \)

**A.5 Equivalence**

Suppose an RCE exists. Let \( \mathcal{P}(e, a) = \bigcup_{s \in [0, 1]} \{ m : m = p^s(e, a, s) \} \) and \( \hat{\Omega} = \{(e, a, m) : m \in \mathcal{P}(e, a), \forall (e, a) \in \mathcal{E} \times \mathcal{A} \} \) with typical element \( \hat{\omega} \in \hat{\Omega}. \) An individual in state \( (\beta, z, \hat{\omega}) \) chooses whether to default \( d \) and conditional on not defaulting chooses asset \( a' \) taking as given

- a price function \( q^{d'}(\hat{\omega}) : \mathcal{A} \times \hat{\Omega} \rightarrow [0, 1], \)
- credit-score transition functions \( Q^m(m'|0, a'), e', \hat{\omega}) \) : \( \mathcal{P}(e', a') \times \mathcal{A} \times \mathcal{E} \times \hat{\Omega} \rightarrow [0, 1] \) and \( Q^m(m'|1, 0), e', \hat{\omega}) : \mathcal{P}(e', a') \times \mathcal{E} \times \hat{\Omega} \rightarrow [0, 1]. \)

As in (3), this implies that an individual of type \( \beta \) in state \( (z, \hat{\omega}) \) chooses \( (d, a') \in \mathcal{F}(z, \hat{\omega}) \) inducing consumption

\[
   c_{d, a'}^{(d, a')}(z, \hat{\omega}) = \begin{cases} 
   y(e(\hat{\omega}), z) + a(\hat{\omega}) - q^{d'}(\hat{\omega}) \cdot a' & \text{if } (d, a') = (0, a') \\
   y(e(\hat{\omega}), z)(1 - \kappa_1) - \kappa & \text{if } a < 0 \text{ and } (d, a') = (1, 0)
   \end{cases}
\]

(36)

For all \( (d, a') \in \mathcal{F}(z, \hat{\omega}), \) the value functions given by equations (5), (7), (12), and (13) and choice probabilities given by equations (9), (10), and (11) associated with the individual’s problem are unchanged in form after substituting \( \hat{\omega} \) for \( \omega \) except for equation (4) now given by:

\[
   v_{d, a'}^{(d, a')}(\beta, z, \hat{\omega}) = (1 - \beta \rho) v_c^{(d, a')}(z, \hat{\omega}) + \beta \rho \cdot \sum_{\beta', z', e', m'} Q^\beta(\beta'|\beta) Q^e(e'|e) H(z') Q^m(m'|d, a'), e', \hat{\omega}) W(\beta', z', \hat{\omega}).
\]

(37)

Intermediaries issue a positive measure of contracts taking the price function \( q^{d'}(\hat{\omega}) \) and probability of repayment function \( p^{d'}(\hat{\omega}) \) as given to maximize profits:

\[
   \pi_{d'}^{\hat{\omega}} = \begin{cases} 
   \rho \cdot \frac{p^{d'}(\hat{\omega}) \cdot (a') - q^{d'}(\hat{\omega}) \cdot (-a')}{1 + r} & \text{if } a' < 0 \\
   q^{d'} \cdot a' - \rho \cdot \frac{a'}{1 + r} & \text{if } a' \geq 0
   \end{cases}
\]

(38)
If the intermediary issues a strictly positive measure of credit contracts, then zero profits require:

\[ q^d(\hat{\omega}) = \begin{cases} \frac{\rho \cdot \rho^d(\hat{\omega})}{1 + r} & \text{if } a' < 0, \\ \frac{\rho}{1 + r} & \text{if } a' \geq 0 \end{cases} \]  

which is the analogue of (15).

Consistency requires that the probability of repayment satisfy the analog of (17), namely,

\[ p^d(\hat{\omega}) = \sum_{\beta', z', e', m'} H(z') \cdot Q^e(e'|e) \cdot Q^m(m'(d, a'), e', \hat{\omega}) \cdot M_{\beta}(\hat{\omega}') \cdot \left(1 - \sigma^{(1,0)}(\beta', z', \hat{\omega}')\right). \]  

Here, \( M_{\beta}(\hat{\omega}) : \hat{\Omega} \to S \) is a function mapping \( m \) to the probability an individual is of a given type \( \beta \). We will denote \( M(\hat{\omega}) = (M_{\beta_1}(\hat{\omega}), ..., M_{\beta_\beta}(\hat{\omega})). \)

The transition function in equation (19) which tracks the probability that an individual in state \((\beta, z, \hat{\omega})\) transitions to state \((\beta', z', \hat{\omega}')\) is now given by:

\[ T(\beta', z', \hat{\omega}'; \beta, z, \hat{\omega}) = \rho \cdot Q^d(\beta'|\beta) \cdot H(z') \cdot Q^e(e'|e) \cdot \sigma^{(d,a')}(\beta, z, \hat{\omega}) \cdot Q^m(m'(d, a'), e', \hat{\omega}) \]

\[ + (1 - \rho) \cdot G_{\beta}(\beta') \cdot H(z') \cdot G_{e}(e') \cdot 1_{a'=0} \cdot 1_{z'=0}. \]

We can now give the definition of a stationary recursive competitive equilibrium with credit scores.

**Definition 6. Stationary Recursive Competitive Equilibrium with Credit Scores** A stationary Recursive Competitive Equilibrium with Credit Scores (RCECS) is a pricing function \( q^{d*}(\hat{\omega}) \), a credit-scoring function \( Q^{m*}(m'|d, a', e', \hat{\omega}) \), and a choice probability function \( \sigma^{(d,a')}^{(1,0)}(\beta, z, \hat{\omega}) \), a probability of repayment function \( p^{d*}(\hat{\omega}) \), and a steady state distribution \( \mu^{*}(\hat{\omega}) \) such that:

(i). **Optimality**: Given \( q^{d*}(\hat{\omega}) \) and \( Q^{m*}(m'|d, a', e', \hat{\omega}) \), \( \sigma^{(d,a')}^{(1,0)}(\beta, z, \hat{\omega}) \) satisfies (10) and (11) for all \((\beta, z, \hat{\omega}) \in \beta \times Z \times \hat{\Omega} \) and \((d, a') \in \mathcal{F}(z, \hat{\omega}) \).

(ii). **Zero Profits**: Given \( Q^{m*}(m'|d, a', e', \hat{\omega}) \), \( M^{*}(\hat{\omega}) \), and \( \sigma^{(1,0)}^{(1,0)}(\beta, z, \hat{\omega}) \), \( p^{d*}(\hat{\omega}) \) satisfies (40) for all \( \hat{\omega} \in \hat{\Omega} \) and given \( p^{d*}(\hat{\omega}) \), \( q^{d*}(\hat{\omega}) \) satisfies (39) with equality for all \( \hat{\omega} \in \hat{\Omega} \).

(iii). **Stationary Distribution**: Given \( Q^{m*}(m'|d, a', e', \hat{\omega}) \) and \( \sigma^{(1,0)}^{(1,0)}(\beta, z, \hat{\omega}) \), \( \mu^{*}(\beta, z, \hat{\omega}) \) is a fixed point of \( \mu^{*}(\beta', z', \hat{\omega}'): \beta \mapsto T^{*}(\beta', z', \hat{\omega}'|\beta, z, \hat{\omega}) \cdot \mu(\beta, z, \hat{\omega}) \) for \( T^{*} \) in (41).

Note the difference between Definition 3 of an RCE and Definition 6 of an RCECS: An RCE requires the updating function to be consistent with Bayes Law (in (iii) used in of Definition 3), while Definition 6 simply postulates the existence of \( Q^{m*} \) and \( M^{*} \) and requires that these be consistent with zero profits.

**Theorem 3. Equivalence**: Suppose an RCE exists and let \( m = p^{d*}(e, a, s) \) using (17). Suppose \( M^{*}(\hat{\omega}) \) given by the inverse function \( m = p^{d*}(e, a, M^{*}(e, a, m)) \) exists. Let \( Q^{m*}(m' = m)(d, a', e', \hat{\omega}) = Q^d(s' (\beta')) = M_{\beta}^{*}(e', a', \hat{\omega}) \psi^{(d,a')}^{(d,a')} (e, a, s) \) using (16) \( \forall \hat{m} \in \mathcal{P}(e', a') \). Then an RCECS also exists in which the choice probabilities \( \sigma^{(d,a')}^{(d,a')} (\beta, z, e, a, m) = \sigma^{(d,a')}^{(d,a')} (\beta, z, e, a, s) \).
Proof. Provided \( q^{d'}(\hat{\omega}) = q^{d'}(\omega) \), then \( \mathcal{F}(z, \omega) = \mathcal{F}(z, \hat{\omega}) \) in (3) and (36). Since \( Q^{m'}(m' = \hat{m} | (d, a'), e', \hat{\omega}) = Q^{s}(s'(\beta') = M'_{\beta}(e', a', \hat{\omega}) | q^{(d, \hat{a}')}(e, a, s)) \), then \( v^{(d, \hat{a}'}(\beta, z, \omega) \) in (4) is identical to \( v^{(d, a')}(\beta, z, \hat{\omega}) \) in (37) as well as all other value functions. Hence \( \sigma(d, a') = \sigma^{(d, a')}(\beta, z, \omega) \) satisfying (i). If the choice probabilities are the same, then repayment probabilities in (17) and (40) are the same since \( s'(\beta') = M'_{\beta}(\hat{\omega}') \) and \( Q^{m'} = Q^{s} \), thereby satisfying the requirement on \( p_{d} \) in (ii). If the repayment probabilities in (17) and (40) are the same, then prices in (15) and (39) are the same, thus satisfying the requirement on \( q^{d'} \) in (ii) and our initial conjecture \( q^{d'}(\hat{\omega}) = q^{d'}(\omega) \). Since \( \sigma^{(d, a')}(\beta, z, \omega) = \sigma^{(d, a')}(\beta, z, \hat{\omega}) \) and \( Q^{m'} = Q^{s} \), then (19) is the same as (41) so that (iii) holds. 

\( \square \)

B Computational Appendix

B.1 Computational Algorithm for the Baseline Model

In this subsection, we describe the algorithm used to compute the RCE stated in Definition 3. The model is calibrated by using the procedure below to solve the model for a given set of parameters, and then updating parameters to minimize the distance between the model moments and the data moments. This outer minimization is performed using the Nelder-Mead simplex method over hundreds of (randomly-chosen) initial conditions.

1. Specify all grids and parameters. Relevant details:
   (a) asset grid is log-spaced in both directions from 0 with 50 points between \([-0.15, -0.00001]\) and 130 points between \([0, 15]\)
   (b) type score grid is linearly-spaced with 40 points between \(\min\{G_{\beta_{H}}, Q^{\beta}(\beta_{H} | \beta_{L})\}\) and \(Q^{\beta}(\beta_{H} | \beta_{L})\).
   (c) equilibrium convergence is on \( p \) and \( \psi \) functions with gradual updating; since \( \psi \) is more sensitive, we use a relaxation parameter of \( \theta \in (0, 1) \) on \( p \) and \( \eta \theta \) on \( \psi \) for \( \eta \in (0, 1) \).
   (d) persistent earnings has 5 grid points; transitory has 3.
   (e) all newborns have no assets, lowest \( e \), and \( s = G_{\beta_{H}} \). They are distributed across \( \beta \) and \( z \) according to \( G_{\beta_{H}} \) and \( H(z) \), respectively.
   (f) given a scale parameter on the default / no default shock \( \nu \) of \( \alpha \), we set the mean of the \( \nu \) shocks to be
      \[ \bar{\nu} = -\alpha(\gamma_{E} + \ln 2) \Rightarrow E[\max\{\nu_{D}, \nu_{ND}\}] = 0 \]
   (g) compute consumption associated with all non-borrowing actions (since \( r \) is exogenous, these don’t change iteration to iteration).
      i. Savings: for each \( \omega = (a, s, e) \) and \( z \), compute the consumption associated with each feasible action \( a' \geq 0 \) such that
         \[ c^{(0,a')}(z, \omega) = y(e(\omega), z) + a(\omega) - \frac{\rho}{1 + r} a' > 0 \]
         Let \( \tilde{\nu}(z, \omega) \) denote the index of the largest budget feasible \( a' \) for an agent with \( (z, \omega) \).
      ii. Default: define the consumption for a defaulter to be
         \[ c^{(1,0)}(z, \omega) = y(e(\omega), z)(1 - \kappa_{1}) - \kappa \]
         where \( \kappa \) is a fixed bankruptcy filing cost and \( \kappa_{1} \) is a cost that scales with earnings.
2. **Main equilibrium loop.** Every iteration \( j \) starts with a value of: (i) \( f_j = (q^s_j(\omega), \psi_j^{(0, s')}(\omega), \psi_j^{(1, 0)}(\omega)) \); and (ii) the (ex-ante) value function \( W_j(\beta, z, \omega) \).\(^{53,54}\)

(a) Compute consumption associated with all borrowing actions given the current prices: for all \( a' < 0 \) on \( \mathcal{A} \), compute

\[
c^{(0, a')}(z, \omega|f_j) = y(e(\omega), z) + a(\omega) - q^a_j(\omega)a'
\]

Note that our Assumption 1 implies that all debt choices are always feasible, which is critical for keeping our Bayesian updates well-defined.

(b) Compute mean of extreme value shock associated with each \( a' \in \mathcal{F}(z, \omega|f_j) \):

i. For \( n = 1, \ldots, \overline{n}(z, \omega) \), compute

\[
c^{(0, a)_n}(z, \omega|f_j) = y(e(\omega), z) + a(\omega) - q^{a_n}_j(\omega)\hat{a}_n,
\]

where

\[
\hat{a}_1 = a_1,
\]

\[
\hat{a}_n = a_{n-1} + \frac{a_n - a_{n-1}}{2} \text{ for } n = 2, \ldots, N.
\]

and \( q^{a_n}(\omega) \) is given by the linear interpolation of the \( q \) function

\[
q_j^{a_1}(\omega) = q_j^{a_1}(\omega) \text{ for } a' = a_1,
\]

\[
q_j^{a_n}(\omega) = \frac{q_j^{a_{n-1}}(\omega) + q_j^{a_n}(\omega)}{2} \text{ for } n = 2, \ldots, N.
\]

ii. define the measure of consumption associated with choice \( a' = a_n \) as

\[
\theta^{a_n}(z, \omega|f_j) = \begin{cases} 
c^{(0, a)_n}(z, \omega|f_j) - c^{(0, a_{n+1})}(z, \omega|f_j) & \text{for } n = 1, \ldots, \overline{n}(z, \omega) - 1, \\
c^{(0, a)_n}(z, \omega|f_j) - 0 & \text{for } n = \overline{n}(z, \omega).
\end{cases}
\]

iii. the mean of \( \epsilon^{a_n} \) for \( n = 1, \ldots, \overline{n}(z, \omega) \) is taken to be

\[
\overline{c}^{a_n}(z, \omega|f_j) = -\lambda y_E + \lambda \ln \theta^{a_n}(z, \omega|f_j)
\]

where \( \lambda \) is the common scale parameter for all shocks.

(c) Iterate to convergence on the value function. Starting with \( W_{j, k=1}(\beta, z, \omega) = W_j(\beta, z, \omega) \)

i. Compute the conditional value function in (4):

\[
\nu_k^{(d, s')}(\beta, z, \omega|f_j) = \begin{cases} 
u_k^{(d, s')}(\beta, z, \omega|f_j) + & \\
\beta \rho \cdot \sum_{(\beta', z', \epsilon, s')} Q^d(\beta'|\beta)Q^s(\epsilon|e)H(z')Q^s(s'|\psi_j^{(d, s')}(\omega))W_{j, k}(\beta', z', \omega') & \text{for } \nu_k^{(d, s')}(\beta, z, \omega|f_j) \text{ is defined}.
\end{cases}
\]

---

\(^{53}\)While we index these functions by \( f = (q^s(\omega), \psi^{(0, s')}(\omega), \psi^{(1, 0)}(\omega)) \), the algorithm actually iterates on \( p^d(\omega) \) which directly yields \( q^d(\omega) \) via (15).

\(^{54}\)Since the full information version of the model solves very quickly, for the initial \( j = 0 \) values, the value functions and loan price schedules provide a good initial guess. For type scores, a consistent initial guess is \( \psi^{(d, s')}(e, a, s) = sQ^d(\beta_H|\beta_H) + (1 - s)Q^d(\beta_L|\beta_L) \).
ii. As in (12), let

\[ W^\text{ND}_k(\beta, z, \omega|f_j) = \mathbb{E} \left[ \max_{n=1, \ldots, \mu(z, \omega)} \nu_k^{(0, \delta_n)}(\beta, z, \omega|f_j) + e^{\lambda \nu} \right] \]

\[ = \lambda \gamma_E + \lambda \ln \left( \sum_{n=1}^{\mu(z, \omega)} \exp \left( \frac{\nu_k^{(0, \delta_n)}(\beta, z, \omega|f_j) + \bar{\nu}^\delta(\omega)}{\lambda} \right) \right) \]

\[ = \lambda \ln \left( \sum_{n=1}^{\mu(z, \omega)} \eta^{\delta_n}(\omega|f_j) \exp \left( \frac{\nu_k^{(0, \delta_n)}(\beta, z, \omega|f_j)}{\lambda} \right) \right). \] (44)

Note that this step applies the definition in (43) from step (2b iii).

iii. As in (13), update

\[ W_{j,k+1}(\beta, z, \omega) = \begin{cases} 
W^\text{ND}_k(\beta, z, \omega|f_j) & \text{if } a(\omega) \geq 0 \\
\mathbb{E} \left[ \nu_k^{(1,0)}(\beta, z, \omega|f_j) + \nu^D, W^\text{ND}_k(\beta, z, \omega|f_j) + \nu^\text{ND} \right] & \text{if } a(\omega) < 0 .
\end{cases} \]

For the \( a(\omega) < 0 \) case, using \( \bar{\nu} \) from step (1f) we simply have

\[ W_{j,k+1}(\beta, z, \omega) = \alpha \gamma_E + \alpha \ln \left( \exp \left( \frac{W^\text{ND}_k(\beta, z, \omega|f_j) + \bar{\nu}}{\alpha} \right) + \exp \left( \frac{\nu_k^{(1,0)}(\beta, z, \omega|f_j) + \bar{\nu}}{\alpha} \right) \right) \]

\[ = -\alpha \ln 2 + \alpha \ln \left( \exp \left( \frac{W^\text{ND}_k(\beta, z, \omega|f_j)}{\alpha} \right) + \exp \left( \frac{\nu_k^{(1,0)}(\beta, z, \omega|f_j)}{\alpha} \right) \right) . \]

iv. If

\[ \sup |W_{j,k+1}(\beta, z, \omega) - W_{j,k}(\beta, z, \omega)| \]

is less than desired tolerance go to step (2d) otherwise go to step (2c) starting with \( W_{j,k+1}(\beta, z, \omega) \).

(d) Compute decision densities:

i. As in (9) in the text, the probability of choosing \( a' \in F(z, \omega|f_j) \) conditional on not defaulting is

\[ \sigma^{(0, \delta_n)}(\beta, z, \omega|f_j) = \frac{\eta^{\delta_n}(\omega|f_j) \exp \left( \frac{\nu_k^{(0, \delta_n)}(\beta, z, \omega|f_j)}{\lambda} \right)}{\sum_{n=1}^{\mu(z, \omega|f_j)} \eta^{\delta_n}(\omega|f_j) \exp \left( \frac{\nu_k^{(0, \delta_n)}(\beta, z, \omega|f_j)}{\lambda} \right)} \] (45)

and 0 if \( a' \notin F(z, \omega|f_j) \).

ii. As in (10) in the text, the probability of default for \( a(\omega) < 0 \) is

\[ \sigma^{(1,0)}(\beta, z, \omega|f_j) = \frac{\exp \left( \frac{\nu_k^{(1,0)}(\beta, z, \omega|f_j)}{\alpha} \right)}{\exp \left( \frac{\nu_k^{(1,0)}(\beta, z, \omega|f_j)}{\alpha} \right) + \exp \left( \frac{W^\text{ND}_k(\beta, z, \omega|f_j)}{\alpha} \right)} \]

and 0 if \( a(\omega) \geq 0 \).
iii. combining these, we obtain the unconditional probability

\[ \sigma^{(0,\alpha)}(\beta, z, \omega | f_j) = (1 - \sigma^{(1,0)}(\beta, z, \omega | f_j)) \sigma^{(0,\alpha)}(\beta, z, \omega | f_j). \]

(e) Given the decision probabilities \(\sigma^{(1,0)}(\beta, z, \omega | f_j)\) and \(\sigma^{(d,\alpha')}(\beta, z, \omega | f_j)\), compute the new set of equilibrium functions, \(f_{j+1} = (q_{j+1}'(\omega), \psi_{j+1}^{(0,\alpha')}(\omega), \psi_{j+1}^{(1,0)}(\omega))\):

i. Compute \(\psi_{j+1}^{(0,\alpha')}(\omega)\) and \(\psi_{j+1}^{(1,0)}(\omega)\) according to (16).

ii. Compute \(q_{j+1}'(\omega)\) according to (15) using \(p_{j+1}'(\omega)\) in (17).

(f) Assess equilibrium function convergence in terms of the sup norm metric

\[ \max \left\{ \sup |\psi_{j+1}^{(d,\alpha')}(\omega) - \psi_j^{(d,\alpha')}(\omega)|, \sup |q_{j+1}'(\omega) - q_j'(\omega)|, \sup |W_{j+1}(\beta, z, \omega) - W_j(\beta, z, \omega)| \right\} \]

If less than tolerance, proceed to step 3; otherwise, start step 2 with \(f_{j+1}\) and \(W_{j+1}(\beta, z, \omega)\).

3. Compute the stationary distribution.

(a) Given \(f_j\) from step (2), compute \(\mu_{k+1}(\beta, z, \omega | f_j)\) using the transition operator \(T\) in (19) applied to \(\mu_k(\beta, z, \omega | f_j)\).

(b) Assess convergence based on the sup norm metric \(\sup |\mu_{k+1}(\beta, z, \omega | f_j) - \mu_k(\beta, z, \omega | f_j)|\). If less than tolerance, stop; otherwise, iterate on \(\mu_{k+1}(\beta, z, \omega | f_j)\) using \(T\).

### B.1.1 Grids

Table 9 presents the key grids used in the computational analysis. Note in particular that the asset and type score grids are quite dense in order to insure convergence, while in contrast the earnings and type grids are coarse in order to ease the computational burden and simplify the analysis.

**Table 9: Grids used in computational analysis**

<table>
<thead>
<tr>
<th>Variable</th>
<th>(x)</th>
<th>(N(x))</th>
<th>Range / Values</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor</td>
<td>(\beta)</td>
<td>2</td>
<td>{0.930, 0.809}</td>
<td>2-point support makes Bayesian functions scalar-valued.</td>
</tr>
<tr>
<td>Earnings (per)</td>
<td>(e)</td>
<td>5</td>
<td>{−0.71, −0.27, 0, 0.27, 0.71}</td>
<td>See Table 1</td>
</tr>
<tr>
<td>Earnings (trans)</td>
<td>(z)</td>
<td>3</td>
<td>{−0.18, 0.00, 0.18}</td>
<td>See Table 1</td>
</tr>
<tr>
<td>Assets</td>
<td>(a)</td>
<td>180</td>
<td>[−0.15, 15.00]</td>
<td>50 points in neg. region, 130 in pos. Density close to 0 is critical.</td>
</tr>
<tr>
<td>Type score</td>
<td>(s)</td>
<td>40</td>
<td>[0.205, 0.774]</td>
<td>bounded below by low (\beta) to high (\beta) transition, above by high to low.</td>
</tr>
</tbody>
</table>
B.2 Model moment definitions

Bankruptcy rate The bankruptcy rate is computed as the total fraction of the population who files for bankruptcy within a given period. The probability of a given state is given by $\mu(\cdot)$, and the probability of bankruptcy given a state is $\sigma^{(1,0)}(\cdot)$, and so the aggregate bankruptcy rate is

$$\text{aggregate bankruptcy rate} = \sum_{\beta, z, \omega} \sigma^{(1,0)}(\beta, z, \omega) \cdot \mu(\beta, z, \omega).$$

(46)

By type, we have $\sum_{\omega} \sigma^{(1,0)}(\beta, z, \omega) \cdot \mu(\beta, z, \omega) / \sum_{\omega} \mu(\beta, z, \omega)$. (Analogous type conditions hold for all other moments as well; we omit them here for brevity.)

Fraction in debt This is the fraction of the population choosing $a' < 0$ in a given period, so

$$\text{fraction in debt} = \sum_{\beta, z, \omega, a' < 0} \mu(\beta, z, \omega) \sigma^{(0,a')}(\beta, z, \omega).$$

(47)

Debt to income The debt to income ratio is the ratio of average debt to average income:

$$\text{debt to income} = \frac{\sum_{\beta, z, \omega, a' < 0} a' \sigma^{(0,a')}(\beta, z, \omega) \mu(\beta, z, \omega)}{\sum_{\beta, z, \omega, a' < 0} y(e(\omega), z) \mu(\beta, z, \omega)}. $$

(48)

Average interest rate The average interest paid (or received) by the agents in the economy is the weighted average of the interest rates paid, $1/q - 1$, over the stationary distribution and decision probabilities

$$\text{average interest rate} = \frac{1}{\sigma} \frac{\sum_{\beta, z, \omega} \mu(\beta, z, \omega) \sum_{a'} \frac{\sigma^{(0,a')}(\beta, z, \omega)}{\sum_{\beta, z, \omega} \sigma^{(0,a')}(\beta, z, \omega)} \left( \frac{1}{q^{a'}(\omega)} - 1 \right) }{\sum_{\beta, z, \omega} \mu(\beta, z, \omega)}.$$

(49)

where $\sum_{\omega} \mu(\beta, z, \omega)$. The standard deviation is the square root of the second moment of this object computed in an analogous fashion.

B.3 Sensitivity Analysis Details

B.3.1 Implementation of Andrews et al. (2017)

We begin by computing the $10 \times 8$ Jacobian matrix $\hat{G}$ of the 10-vector of model moments with respect to the 8-vector of internally estimated model parameters. We approximate this matrix by taking numerical derivatives. Using a parameter step size of $\delta \hat{p} \cdot \hat{\theta}_p$ for $p = 1, \ldots, 8$ (i.e. proportional scaling, where $\delta_p$ is the proportional increase and $\hat{\theta}_p$ is the estimated parameter), we solve the model for the baseline calibration $\hat{\theta} = \{\hat{\theta}_p\}_{p=1}^8$, obtaining moment vector $\hat{m} = \{\hat{m}_n\}_{n=1}^{10}$, and for a sequence of 8 perturbations in with the $p$-th parameter is increased by the step size. We set $\delta_p = \delta = 0.1\%$ for all $p$. The entry of the estimated Jacobian matrix $\hat{G}$ corresponding to moment $n$ and parameter $p$, is then $\hat{g}_{np} = (\hat{m}_n - \hat{m}_n) / \delta \hat{\theta}_p$. The transpose of this matrix, $\hat{G}'$ is presented in Table 10.

Given our estimate of $\hat{G}'$, we compute an estimate of Andrews et al. (2017) sensitivity matrix $\hat{\Lambda}$ using equation (23) with the identity weighting matrix $W = I_0$. What is presented in Table 4 is not $\hat{\Lambda}$ directly, but a more easily interpretable transformation which we now describe.
Our goal is to answer the question: “by what percent would the estimated parameter $\hat{\theta}_p$ change if target moment $m_n$ changed by $\delta_n$ percent?” We assume a change in moment $m_n$ of $\delta_n \hat{m}_n$; for ease of exposition, we choose $\delta_n = 1\%$ for all $n$. Then, the bias in the estimated parameter $\hat{\theta}_p$ associated with the perturbation to moment $m_n$ is $b_{pn} = \hat{\lambda}_{pn} \delta_n \hat{m}_n$, where $\hat{\lambda}_{pn}$ is the corresponding entry of the $\hat{\Lambda}$ matrix. We then report the implied percentage change relative to the estimated value of the parameter, $\hat{\ell}_{pn} = \frac{\hat{\theta}_p - b_{pn}}{\hat{\theta}_p} - 1$. Each cell of Table 4 is the relevant $\hat{\ell}_{pn}$ entry.

B.3.2 Jacobian Matrix

The estimated sensitivity matrix $\hat{\Lambda}$ and the transformation presented in Table 4 are useful for thinking about which moments of the data drive our parameter estimates, as we describe in the main text. While the estimated Jacobian matrix $\hat{G}$ is an essential input into that analysis (per equation (23)), it also contains useful information on its own for understanding how model parameters drive model moments. To this end, we present the (transpose of the) estimated Jacobian in Table 10.

The Jacobian makes certain key properties of the model clear. We start by thinking about implications for the aggregate credit market moments (five leftmost columns of the table). If either the high or low $\beta$ type becomes more patient, there is less borrowing (FID and DTY drop) which leads to less bankruptcy (BR drops) and lower interest rates (AI drops). If the transition from type $H$ to $L$ ($L$ to $H$) rises, leading to more (less) $L$ types on average, there is more (less) borrowing, higher (lower) interest rates, and higher (lower) bankruptcy rates. A higher initial fraction of high types yields less borrowing and fewer bankruptcies. Finally, a higher variable filing cost (i.e. a higher punishment to filing for bankruptcy) yields much lower interest rates, inducing more borrowing and ultimately leading to more bankruptcy in equilibrium.

Increasing either $\alpha$ or $\lambda$ increases the noisiness of the associated decision: bankruptcy vs. no bankruptcy and the choice of $a'$, respectively. Increasing either parameter clouds lenders’ ability to infer types based on actions. Consider bankruptcy ($\alpha$) first. Since bankruptcy is generally not optimal for most borrowers, raising $\alpha$ increases the rate at which borrowers file all else equal. In equilibrium, though, this leads to a surge in interest rates across all loans and a sharp decline in both the share of borrowers and the overall amount of borrowing, ultimately lowering the bankruptcy rate. Next, consider borrowing and saving ($\lambda$). Raising $\lambda$ increases the likelihood with which households will deviate from their “optimal choices” or, put differently, be “off their Euler equations.” The first order effect of this is that households are much more willing to take on small debts which carry non-trivial interest rates due to default risk, driving up the fraction in debt and the average interest rate. At the same time, though, since DTY falls, the total volume of debt decreases, lowering the bankruptcy rate in equilibrium.

Finally, consider the credit ranking life cycle moments (five rightmost columns of the table). In general, these moments are less sensitive to our target parameters than the aggregate credit market moments because: (i) a sizable portion of the life cycle of credit rankings is driven by the life cycle of earnings, which is exogenous; and (ii) the endogenous trend upward in types is not changed much in a small neighborhood of our initial parameters. This second point, in particular, explains the relatively small magnitudes in the first five rows and last five columns of Table 10. Turning to the final three rows, then, we can highlight several intuitive patterns. Increasing the variable filing cost makes bankruptcy less attractive for the young, increasing their credit ranking at birth ($I$:$MCR$ increases) but lowering the upward trend in credit ranking ($S$:$MCR$ decreases). Since default becomes a clearer signal of type, then, we see a similar pattern for the cross-sectional variation of credit ranking ($I$:$SDCR$ and $S$:$SDCR$). Raising either $\alpha$ or $\lambda$ has the opposite effect: by driving up the incentive to file for bankruptcy or borrow when
Table 10: Jacobian Analysis:
(Numerical) Derivative of Target Moments with Respect to Estimated Parameters, $\hat{G}'$

<table>
<thead>
<tr>
<th></th>
<th>BR</th>
<th>AI</th>
<th>SDI</th>
<th>FID</th>
<th>DTY</th>
<th>I:MCR</th>
<th>S:MCR</th>
<th>I:SDCR</th>
<th>S:SDCR</th>
<th>AUTO</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_H$</td>
<td>-3.03</td>
<td>-19.6</td>
<td>151</td>
<td>-20.8</td>
<td>-0.65</td>
<td>0.07</td>
<td>-0.33</td>
<td>0.05</td>
<td>1.02</td>
<td></td>
</tr>
<tr>
<td>$\beta_L$</td>
<td>-8.80</td>
<td>-30.9</td>
<td>59.7</td>
<td>-64.1</td>
<td>-2.05</td>
<td>0.05</td>
<td>-0.32</td>
<td>0.05</td>
<td>2.09</td>
<td></td>
</tr>
<tr>
<td>$Q^B(H'</td>
<td>L)$</td>
<td>0.81</td>
<td>4.29</td>
<td>-6.47</td>
<td>5.79</td>
<td>0.18</td>
<td>-0.02</td>
<td>0.07</td>
<td>-0.02</td>
<td>-0.71</td>
</tr>
<tr>
<td>$Q^B(H'</td>
<td>L)$</td>
<td>-1.11</td>
<td>-6.24</td>
<td>-1.53</td>
<td>-7.59</td>
<td>-0.24</td>
<td>-0.10</td>
<td>-0.07</td>
<td>0.02</td>
<td>-0.22</td>
</tr>
<tr>
<td>$G_{\beta_H}$</td>
<td>-0.05</td>
<td>0.11</td>
<td>-2.49</td>
<td>-0.24</td>
<td>-0.01</td>
<td>0.06</td>
<td>-0.1</td>
<td>-0.02</td>
<td>0.00</td>
<td>-0.28</td>
</tr>
<tr>
<td>$\kappa_1$</td>
<td>7.90</td>
<td>-209</td>
<td>-195</td>
<td>215</td>
<td>10.1</td>
<td>5.74</td>
<td>-1.15</td>
<td>7.83</td>
<td>-1.35</td>
<td>-58.6</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-24.2</td>
<td>536</td>
<td>-104</td>
<td>-628</td>
<td>-20.6</td>
<td>-2.66</td>
<td>0.54</td>
<td>-3.31</td>
<td>0.54</td>
<td>19.0</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>-15.5</td>
<td>846</td>
<td>-1802</td>
<td>8.38</td>
<td>-19.5</td>
<td>-5.07</td>
<td>1.09</td>
<td>-6.66</td>
<td>1.36</td>
<td>-25.5</td>
</tr>
</tbody>
</table>

Notes: Each entry is the numerical derivative of the target moment (column) with respect to a 0.1% change in the corresponding parameter (row).

young, it lowers I:MCR and increases S:MCR, and by lowering the informational content of decisions it lowers I:SDCR.

B.4 Welfare Metric

Table 6 reports welfare using a wealth equivalent measure whose construction we describe in this section. Given that utility flows in our model are derived not only directly from consumption flows but also from the extreme value shocks attached to consumption choices, we are unable to use a standard Lucas consumption equivalent measure. As an alternative, we construct a measure that answers the question: “how much additional wealth must an agent in state $(\beta, z, \omega)$ be given in the BASE economy in order to be indifferent between being born into the BASE economy or into a given alternative (ALT) economy?” Denoting the value functions in the BASE and ALT economies by $W_{\text{BASE}}(\beta, z, \omega)$ and $W_{\text{ALT}}(\beta, z, \omega)$, respectively, we formally solve for a set of numbers $\phi_{\text{ALT}}(\beta, z, \omega)$ that satisfies

$$W_{\text{BASE}}(\beta, z, a(\omega) + \phi_{\text{ALT}}(\beta, z, \omega), s(\omega), e(\omega)) = W_{\text{ALT}}(\beta, z, a(\omega), s(\omega), e(\omega))$$

(50)

Note that if the ALT economy is the FI economy, then $s$ is not a state variable. However, $\phi_{\text{ALT}}$ still depends on $s$ since this shifts the value in the BASE economy. The numbers reported in Table 6 are the ratios of the numbers computed via this expression to mean wealth in the BASE economy, $\bar{a} = \sum a \cdot \sum_{\beta, z, s, e} \mu(\beta, z, a, s, e)$.

This measure has several attractive properties. First, by varying wealth in the BASE economy only, we avoid any issues related to the fact that in the NT economy there are no agents with the type score of a newborn and debt, since type scores increase monotonically with age and all newborns are born with zero wealth. Second, the units are directly interpretable as physical quantities of wealth rather than

\footnote{In particular, the indirect utility function for an agent in state $(\beta, z, \omega)$ may not be written as an appropriately discounted infinite sum of period utility flows.}
than utils. Relatedly, the change in wealth is purely discretionary and not imposed to be translated into consumption immediately. Third, it involves only a trivial calculation given the value functions from each equilibrium; the set of numbers $\phi$ may be solved via simple bisection.

### B.5 The Role of Extreme Value Preference Shocks and Modal Choice Metrics

One of the key modifications in our model relative to standard consumer bankruptcy models in macroeconomics is the inclusion of the additive, action-specific preference shocks. The mean of these shocks is adjusted to insure that the utility bonus scales with the measure of feasible consumption rather than the density of the grid used for computation (see Briglia et al. (2021) for details). In contrast, we calibrate the scale parameters $\alpha$ and $\lambda$ which govern the variance of the default and $a'$ shocks, respectively. How does behavior in the model change with respect to these parameters? In this section we address this question in two ways. First, we use an analytical approach to show how these parameters directly affect choice probabilities, taking the value of specific actions (i.e. all equilibrium functions) and earnings processes as given. Then, in the context of the full information version of the model, we compute actual decision rules under different parameter combinations and describe the differences graphically.

#### B.5.1 Deriving the impact of EV parameters

To ease notation in this section, let an agent’s entire state be denoted by $x = (\beta, e, z, a, s)$, and the set of feasible actions for that agent be denoted by $\mathcal{F}(x)$. The goal of this section is to show how the choice probability function $\alpha$ varies with the extreme value scale parameters $\alpha$ and $\lambda$. We first cover the repayment ($d = 0$) actions, and then bankruptcy ($d = 1$). To ease computations in this section rather than compute derivatives with respect to $\lambda$ or $\alpha$ directly, we will compute them with respect to $1/\lambda$ or $1/\alpha$. Throughout this section, we focus on the first order effects of changes in these parameters by ignoring all derivatives with respect to action-specific value terms, i.e. $\frac{\partial \alpha(x)}{\partial 1/\lambda}$.

**Saving and borrowing actions** Equation (9) describes the probability of choosing a feasible action $(0, a')$ conditional on not filing for bankruptcy. Considering first $\lambda$,

$$
\frac{\partial \alpha(0, a')(x)}{\partial (1/\lambda)} = \exp \left( \frac{\nu(0, a')(x)}{\lambda} \right) \sum_{(0,3) \in \mathcal{F}(x)} \exp \left( \frac{\nu(0,3)(x)}{\lambda} \right) - 
\exp \left( \frac{\nu(0, a')(x)}{\lambda} \right) \sum_{(0,3) \in \mathcal{F}(x)} \exp \left( \frac{\nu(0,3)(x)}{\lambda} \right) \left[ \sum_{(0,3) \in \mathcal{F}(x)} \exp \left( \frac{\nu(0,3)(x)}{\lambda} \right) \right]
$$

$$
= \exp \left( \frac{\nu(0, a')(x)}{\lambda} \right) \sum_{(0,3) \in \mathcal{F}(x)} \exp \left( \frac{\nu(0,3)(x)}{\lambda} \right) \left[ \nu(0, a')(x) - \nu(0,3)(x) \right]
$$

$$
= \alpha(0, a')(x) \sum_{(0,3) \in \mathcal{F}(x)} \alpha(0,3)(x) \left[ \nu(0, a')(x) - \nu(0,3)(x) \right].
$$

---

56 Dvorkin et al. (2021) have employed extreme value shocks to smooth out decision rules in models of sovereign default.

57 We choose the full information model purely for convenience, since the reduction in the state space reduces computation time and the number of states we need to account for in showing results. The same insights hold for other model variants.

58 This keeps the analysis clean by avoiding repeated applications of the quotient rule for derivatives to the extent possible.
We can sign this derivative according to
\[
\frac{\partial \hat{\sigma}^{(0,a')}(x)}{\partial (1/\lambda)} > 0 \iff \sum_{(0,\bar{a})\in \mathcal{F}(x)} \hat{\sigma}^{(0,\bar{a})}(x) \left[ v^{(0,a')}(x) - v^{(0,\bar{a})}(x) \right] > 0
\]
\[
\iff v^{(0,a')}(x) > \sum_{(0,\bar{a})\in \mathcal{F}(x)} \hat{\sigma}^{(0,\bar{a})}(x) v^{(0,\bar{a})}(x),
\]
where the second line uses the fact that \(\sum_{(0,\bar{a})\in \mathcal{F}(x)} \hat{\sigma}^{(0,\bar{a})}(x) = 1\) by construction. Therefore, the probability of choosing \((0, a')\) conditional on not filing for bankruptcy increases in \(1/\lambda\) (decreases in \(\lambda\)) if and only if the conditional value of choosing \((0, a')\), \(v^{(0,a')}(x)\), exceeds the expected value of choosing from the set of alternative actions \((0, \bar{a})\) at the current decision rule.

The inclusive value of repaying, \(W_{\text{ND}}(x)\), takes the familiar log-sum form of (6). Since it will be useful in computing how \(\sigma^{(1,0)}(x)\) varies with \(\lambda\), we compute:
\[
\frac{\partial W_{\text{ND}}(x)}{\partial (1/\lambda)} = \lambda \left[ \sum_{(0,a')\in \mathcal{F}(x)} \frac{v^{(0,a')}(x)}{\lambda} \sigma^{(0,a')} \right] - \lambda \ln \left( \sum_{(0,a')\in \mathcal{F}(x)} \frac{v^{(0,a')}(x)}{\lambda} \right)
\]
\[
= \lambda \left[ \sum_{(0,a')\in \mathcal{F}(x)} \sigma^{(0,a')} v^{(0,a')}(x) - W_{\text{ND}}(x) \right]
\]
which is positive if and only if the average action-value weighted by decision probabilities exceeds the value of filing for bankruptcy.

**Bankruptcy** Equation (10) defines the probability of filing for bankruptcy as a function of the conditional value of filing for bankruptcy and the inclusive value of repaying. We obtain
\[
\frac{\partial \sigma^{(1,0)}(x)}{\partial (1/\lambda)} = -\sigma^{(1,0)}(x) \left( 1 - \sigma^{(1,0)}(x) \right) \frac{\partial W_{\text{ND}}(x)}{\partial (1/\lambda)} \frac{\partial W_{\text{ND}}(x)}{\partial (1/\lambda)}
\]
\[
= \sigma^{(1,0)}(x) \left( 1 - \sigma^{(1,0)}(x) \right) \left( v^{(1,0)}(x) - W_{\text{ND}}(x) \right)
\]
The first expression above implies that \(\frac{\partial \sigma^{(1,0)}(x)}{\partial (1/\lambda)}\) takes the opposite sign of \(\frac{\partial W_{\text{ND}}(x)}{\partial (1/\lambda)}\). If raising \(\lambda\) raises \(W_{\text{ND}}(x)\), it makes repaying more attractive and therefore lowers the probability of filing. The second expressions shows that as \(\alpha\) decreases, the probability of filing increases if and only if the conditional value of filing exceeds the inclusive value of repaying.

**B.5.2 Changing EV shocks in the baseline model**

Figure 13 demonstrates the impact of changing \(\alpha\) and \(\lambda\) on decisions in our baseline model. Each figure contains three lines, corresponding to: (i) the baseline parameterization of Table 3 (black solid); (ii) a parameterization with low variance \(\alpha\) on the bankruptcy decision (blue dotted) in which \(\lambda\) is held fixed; and (iii) a parameterization with low variance \(\lambda\) on the \(a'\) decision (red dashed) in which \(\alpha\) is held fixed. All figures are presented for an agents with \((\beta, s, e, z) = (\beta_H, G_{\beta_H}, 0, 0)\). In each parameterization, the equilibrium pricing function, and therefore the conditional action values, are held fixed, and so the changes in response shown here can be thought of as partial equilibrium in order to highlight the direct
Figure 13: Impact of extreme value preference shocks

![Graphs showing the impact of extreme value preference shocks on decisions.]

Notes: "Benchmark" refers to the parameterization of the extreme value shock process from Table 3. Low $\alpha$ ($\lambda$) is half the baseline value: $\alpha' = \alpha/2$ ($\lambda' = \lambda/2$). All panels fix the state of an agent at $(\beta, s, e, z) = (\beta_H, 0.32, 0, 0)$. In the top right panel, a modal choice of -1 corresponds to bankruptcy.

Consider first the bankruptcy filing decision. The top left panel shows how this decision varies over a range of levels of debt. By lowering $\alpha$, the slope of increase in filing probability as the level of indebtedness increases is much sharper than in the baseline parameterization. This is because there is less chance for a high value shock to be realized for an action with lower fundamental value, so the decision rule becomes more centered at the mode for each level of $\alpha$. By lowering $\lambda$, the expected value of repaying increases, and so the bankruptcy filing probability shifts down.

The remaining three figures show how $a'$ decisions are affected by changes in the extreme value parameters. The top right panel depicts the modal decision across each case (with bankruptcy depicted as choosing $a' = -1$ for simplicity). Conditional on repaying, there is little change in the modal decision, but lowering either $\alpha$ or $\lambda$ makes bankruptcy the modal decision only for larger levels of debt. The bottom left and bottom right panels show the mean and standard deviation of the savings decision rule, conditional on repaying, respectively. Changing $\alpha$ has virtually no effect conditional on repaying. Mean decisions are nearly linear in wealth for positive $a$ given the low risk aversion, but there is convexity in the decision rule when in debt since default risk changes the return on borrowing relative to saving. Lowering $\lambda$ lowers both the mean and standard deviation of savings choices, with the latter effect being more pronounced.

Finally, we note that these changes in decision rules are similar (holding price and type score functions fixed) in the full information and no tracking economies as well. This is discussed below in Section B.6.
B.5.3 Modal choice metrics

In this section, we describe a series of metrics which quantify the dispersion in decision implied by the extreme value shocks in our framework. These results are summarized in Table 11, but we first describe the construction of the metrics.

Let \( x = (\beta, z, \omega) \) be the state variable of an agent, let \( \sigma^{(d,a')}(x) \) denote her decision rule, and let \( \mu(x) \) be the stationary distribution over individual states in the baseline economy. We want to get a sense of dispersion around the highest value (or modal) choice, which may be defined as

\[
y^*(x) \equiv \arg\max_{(d,a') \in F(x)} \sigma^{(d,a')}(x).
\]

Let \( Y \subseteq \{(1,0), ((0,a'))|a' \in A\} \) denote a set of possible actions. The share of agents for whom an action in set \( Y \) is modal is

\[
m(Y) = \sum_x \mu(x) 1 \left[y^*(x) \in Y\right]
\]

where \( 1[S] \) is an indicator function which which takes on the value 1 if \( S \) is true. The total mass of agents choosing an action in the set \( Y \) includes those for whom the action is not modal, and so we can compute the share of the actions in this set accounted for by “modal agents,” those for whom an action in this set is the mode, via

\[
\frac{\sum_{x,(d,a') \in Y} \mu(x) \sigma^{(d,a')}(x) 1 \left[y^*(x) \in Y\right]}{\sum_{x,(d,a') \in Y} \mu(x) \sigma^{(d,a')}(x)}.
\]

Lastly, for agents whose modal action is not default, we can compute the share of decisions within \( k \) grid points of the modal action. For a given individual (whose modal action is not default), let \( i^*(x) \) denote the grid index of the modal choice \( y^*(x) \). Then let a \( k \)-band of actions around the mode be defined by

\[
Y_k(x) = \{i^*(x) - k, \ldots, i^*(x), \ldots, i^*(x) + k\}.
\]

Finally, define the total weight on decisions in the \( k \)-band of the mode for agent \( x \) via

\[
\zeta_k(x) = \frac{\sum_{(0,a') \in Y_k(x)} \sigma^{(0,a')}(x)}{1 - \sigma^{(1,0)}(x)},
\]

where the denominator renormalizes the weights on actions to exclude default. We can aggregate this metric over any group of actions \( Y \)

\[
\overline{\zeta}(Y_k) = \frac{\sum_{x|y^*(x) \in Y_k} \zeta_k(x) \mu(x)}{m(Y)}.
\]

B.6 Details of Alternative Economies

B.6.1 No tracking (NT)

The key departure in the formal specification of this economy from the baseline comes from the separation of the type score updating which follows individuals and the static assessment of types that is relevant for pricing.
Table 11: Modal Choice Metrics

<table>
<thead>
<tr>
<th>Action type</th>
<th>share for whom action type is modal (%)</th>
<th>share of total</th>
<th>share of decisions w/in k grid pts. of mode (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>modal (%)</td>
<td>modal agents (%)</td>
<td>k = 0</td>
</tr>
<tr>
<td>Default</td>
<td>1.84</td>
<td>87.14</td>
<td>-</td>
</tr>
<tr>
<td>Non-Default</td>
<td>98.16</td>
<td>99.13</td>
<td>18.96</td>
</tr>
<tr>
<td>Borrowing</td>
<td>7.28</td>
<td>66.20</td>
<td>30.32</td>
</tr>
<tr>
<td>Saving</td>
<td>90.88</td>
<td>99.36</td>
<td>18.05</td>
</tr>
</tbody>
</table>

Notes: For the right 3 columns, the share is computed over the total population of agents for whom the indicated action type is the modal action.

Evolution of type score Here, an individual’s type score updates only based on the exogenous transition probabilities, and so there is no dynamic incentive to acquire reputation. As a result, $s'$ evolves from $s$ according to

$$
\psi_{NT, \beta}'(s) = \sum_{\beta} Q^\beta(\beta'|\beta)s(\beta).
$$

In the two-type case we employ in our quantitative model, we have

$$
\psi_{NT}(s) = sQ^\beta(\beta_H|\beta_H) + (1 - s)Q^\beta(\beta_H|\beta_L).
$$

Static assessment of type In this version of the model, lenders perform intraperiod updating of type assessments based on the $a'$ chosen by the borrower. That is, the lenders compute

$$
\psi_{NT, \beta}'(a', s, e) = \Pr(\beta'|a', s, e) = \sum_{\beta} Q^\beta(\beta'|\beta)\Pr(\beta|a', s, e).
$$

All of the action is in the last term of the expression above, and so we analyze it here:

$$
\Pr(\beta|a', s, e) = \frac{\Pr(\beta, a', s, e)}{\Pr(a', s, e)} = \frac{\sum_{z,a} \Pr(\beta, a', s, e, z, a)\sigma(z,a)|\beta, e, z, a, s)}{\sum_{\beta, z,a} \sigma(z,a)|\beta, e, z, a, s)}.
$$

where the first line uses Bayes' Rule, the second sums over unobserved idiosyncratic states, and the third once more applies Bayes' Rule via

$$
Pr(a', \beta, e, z, a, s) = Pr(a'|\beta, e, z, a, s)Pr(\beta, e, z, a, s) = \sigma^{(0,a')}(\beta, e, z, a, s)\mu(\beta, e, z, a, s).
$$
Therefore, we obtain

\[
\psi_{NT, \beta}(a', s, e) = \sum_{\beta} Q^\beta(\beta'|\beta) \frac{\sum_{z, a} \sigma^{(0,a')}(\beta, e, z, a, s) \mu(\beta, e, z, a, s)}{\sum_{\beta, z, a} \sigma^{(0,a')}(\beta, e, z, a, s) \mu(\beta, e, z, a, s)}.
\]

(55)

**Repayment probability function**  What the lender must compute is the probability that \( a' \) is repaid tomorrow given \( s, e \) observed today. For each choice of \( a' \), the lender revises the **borrower’s assessed type today** via (55). At the same time, though, due to the implicit “anonymity” assumption in this economy, they recognize that the **borrower’s type score tomorrow** (which is relevant for tomorrow’s default decision) will be determined via (54). Therefore, the \( p(\cdot) \) function in this economy is

\[
p(a', s, e) = \frac{\Pr(\text{repay } a'|s, e)}{\Pr(s, e)} = \frac{\Pr(\text{repay } a', s, e)}{\Pr(s, e)}
= \frac{\sum_{\beta', e', z', s'} \Pr(\text{repay } a'|\beta', e', z', a', s, e) \Pr(\beta', e', z', s'|a', s, e)}{\sum_{\beta, z, a} \Pr(\beta, e, z, a, s)}
= \frac{\sum_{\beta, \beta', e', z'} \left[ 1 - \sigma^{(1,0)}(\beta', e', z', a', \psi_{NT}(s)) \right] \Pr(\beta', e', z'|a', s, e)}{\sum_{\beta, z, a} \mu(\beta, e, z, a, s)}
= \frac{\sum_{\beta, \beta', e', z', a'} \left[ 1 - \sigma^{(1,0)}(\beta', e', z', a', \psi_{NT}(s)) \right] Q^e(\psi(\cdot)|e) H(z') Q^\beta(\beta'|\beta) \Pr(\beta|a', s, e)}{\sum_{\beta, z, a} \mu(\beta, e, z, a, s)}
= \psi_{NT}^2(a', s, e) \sum_{\beta', e'} \left[ 1 - \sigma^{(1,0)}(\beta, e', z', a', \psi_{NT}(s)) \right] Q^e(\psi(\cdot)|e) H(z')
+ \left( 1 - \psi_{NT}^2(a', s, e) \right) \sum_{\beta', e'} \left[ 1 - \sigma^{(1,0)}(\beta, e', z', a', \psi_{NT}(s)) \right] Q^e(\psi(\cdot)|e) H(z'),
\]

(56)

where the last line once more applies the two-type implementation from our quantitative model.

**Price schedule**  Figure 14 shows the percentage differences between the price menus faced by some agents in the NT economy relative to the BASE economy in comparable states (specifically the lowest persistent earning state \( e = -0.71 \) since they are the likely to borrow). For newborns (i.e. 20 year olds in our mapping to the data) the comparison is easy, as all newborns begin life in the same observable state in both economies (and it is common knowledge they do), i.e., they all have zero assets, are in the low earnings class, and are high types with probability 0.32. The price difference comes only from the different probabilities of repayment across the two economies owing to differences in dynamic incentives. Prices are comparable up to a loan size of 0.01 and for larger loans the prices are lower in NT, reflecting higher default probabilities at each loan size. This is due to the lower incentives to repay in the NT economy. These incentive effects, though mitigated, are present even at older ages.
Figure 14: Loan Price Comparison Between BASE and NT Economies

Notes: Let $s_j$ denote the average type score for an agent of age $j$, and let $a_{NT}^{j}$ be the average wealth of an agent of age $j$ in the NT economy. Each line in each panel represents $100 \cdot \left( \frac{q^{a'}_{NT}(j, e)}{q^{a'}_{BASE}(e, a_{NT}^{j}, s_j)} - 1 \right)$. The black lines in the left and right panels are the same by construction. The “alternative” NT line in the right panel replaces $\sigma$ in equation (55) with $\sigma_{BASE}$, the decision rules from the BASE economy. All price schedules are for the lowest $e = -0.71$.

A decomposition exercise In order to highlight the role of dynamic reputational incentives, we construct an alternative price schedule for the NT economy by replacing the decision rule $\sigma(\cdot)$ in the definition of the static inference function $\psi_{NT}(\cdot)$ defined in equation (55) with the decision rule from the baseline economy, $\sigma_{BASE}(\cdot)$. Having obtained this alternative $\tilde{\psi}_{NT}(\cdot)$, we then compute repayment probabilities (and therefore prices) according to (56) with $\psi_{NT}(\cdot)$ replaced by $\tilde{\psi}_{NT}(\cdot)$.

The alternative price schedule is depicted – relative to the analogous price schedule for the baseline economy – for the youngest cohort in the red dashed line in the right panel of Figure 14. For convenience, we also present the standard NT price schedule (solid black line) in this figure. The alternative price schedule is virtually indistinguishable from the baseline price schedule, while the NT prices differ significantly from the baseline. What drives this? In the alternative, the static inference of type reflects the dynamic reputational incentives of the baseline model by construction. The fact that the alternative and baseline so closely resemble each other while the NT and baseline differ markedly highlights the role of dynamic reputational incentives.

B.6.2 Full information (FI)

Since there is no incentive to infer one’s type, there is no type score in this model. Therefore, an agent’s full state is $(\beta, e, z, a)$, and the set of equilibrium functions does not include $\psi$. For comparability, and since it is purely i.i.d. and contains no information for inference, we maintain the assumption that $z$ is unobservable. Therefore, the lender can observe $\omega_{FI} = (\beta, e, a)$ for each individual.

The household problem and equilibrium stationary distribution are exactly the same as in the main text, with the state variable $s$ removed. The only substantial change is in the pricing and repayment probability equations. The repayment probability function in this case is $P_{FI}^{(0, a')}(\omega_{FI}) = \Pr(\text{repay } a'|\omega_{FI})$.

---

59 We choose only the youngest cohort to avoid having to integrate over $a$ given the common assumption across all model economies that all individuals start with no wealth and the different arguments to the pricing functions in the NT and BASE economies.
Figure 15: **Loan Price Comparison Between Full Information and Baseline Economies**

Notes: Let $s_j$ denote the average type score for an agent of age $j$, and let $a_j^{FI}$ be the average wealth of an agent of age $j$ in the FI economy. Each line in the figure represents $100 \cdot \left( \frac{q_{a'}^{FI}(\beta, e)}{q_{a'}^{BASE}(e, a_j^{FI}, s_j)} - 1 \right)$. All price schedules are for the lowest $e = -0.71$.

Since $\omega_{FI}$ directly includes $\beta$ and $z$ is i.i.d., there is no further inference to be done. Therefore, $a$ has no impact on pricing, and we obtain

$$
p_{a'}^{FI}(\beta, e) = \sum_{\beta', e', z'} \left[ 1 - \sigma_{FI}^{(1,0)}(\beta', e', z', a') \right] Q^\beta(\beta' | \beta) Q^e(e' | e) H(z').
$$

(57)

The loan pricing function, $q_{a'}^{FI}(\beta, e)$, simply adjusts the expression for the interest rate as in the baseline model.

**Price schedule** In Figure 15, we compare the prices that individuals of a given age face in the FI economy with their counterpart in the BASE economy who has the average type score for that age and the FI economy’s average asset holdings for that age. Since we use the average type score for a given age, the price comparison does take into account the learning that naturally occurs in the Base economy (i.e. type $H$ ($L$) have a higher (lower) type score than the average type score for their given age).

As one might expect, Figure 15 shows that for each age, type $L$ in the FI economy face lower loan prices (higher interest rates) and type $H$ face higher prices (lower interest rates) than the BASE economy where there is some cross-subsidization. The figure also shows that these price differences change with age. Recall that current asset holdings do not affect debt prices in the FI model but do in the BASE model. As individuals age and accumulate assets, this has an impact on $q_{BASE}^{(0,a')}$. Any type in the BASE economy who borrows by age 30 having accumulated precautionary assets is very likely assessed to be type $\beta_L$. Thus there is not much difference between the economies for a 30 year old type $\beta_L$ which explains the imperceptible price difference, but since type $\beta_H$ is pooled with type $\beta_L$ by their borrowing,
hence facing much lower $q^{(0,a')}_{\text{BASE}}$, the price difference is magnified.

### B.6.3 Extreme value shocks in the alternative economies

In this section, we explore our alternative economies under different extreme value parameterizations in order to measure how these parameters affect our counterfactuals. We present two robustness exercises relative to our results in Table 6 of Section 6. Specifically, the “benchmark” column in Table 12 simply provides our model moments and welfare measures in the FI and NT economies relative to the BASE economy using our estimated parameters (i.e. replicates Table 6). We compare these same measures under two alternative parameterizations: one in which we raise $\alpha$ by 10% in both the base and alternative economies and one in which we similarly raise $\lambda$ by 10%. The results of this analysis are presented in the remaining columns of Table 12. It should be noted that these 10% increases in $\alpha$ and $\lambda$ can have large effects on outcomes in the BASE economy (to which we are comparing the alternative outcomes) since the estimated values were chosen to match the data in the benchmark. Furthermore, 10% parametric decreases in $\alpha$ and $\lambda$ can yield even bigger deviations between model and data moments, since this correspond to significant increases in the informational content of actions, especially for $\alpha$.

Starting with Panel A for the NT economy in Table 12, we note that the differences across columns are not large relative to the BASE values. This continues to be the case for the FI economy. Panel B shows that the positive welfare mean gain in the NT economy is eliminated as we raise $\alpha$ and $\lambda$ indicating that the negative effects on dynamic incentives outweigh the static insurance benefits at already high levels of pooling. The welfare numbers in Panel B for the FI economy indicate that there can be larger gains in the presence of higher $\alpha$ and $\lambda$ since inference is harder in the BASE economy, so that FI revelation of type benefits welfare.

### B.7 Other Results

#### B.7.1 Default probabilities by earnings level

Figure 16 illustrates that, as is a feature of many default models, the probability of default is increasing in debt. It is also evident that default probabilities are decreasing in earnings for those with sufficiently large debt, another standard feature of default models. For very small debt, however, the lowest earners (who have the highest marginal utility of consumption) are least likely to default in order to avoid bearing the costs ($\kappa$ and $\kappa_1 \times \exp(e)$) of default.

#### B.7.2 Credit Access Following Default

For all $a' < 0$, define the two price schedules

\[
q_D^{a'}(e, a, s) \equiv q^{a'}\left(e, 0, \psi^{(0,0)}(e, a, s)\right), \\
q_N^{a'}(e, a, s) \equiv q^{a'}\left(e, 0, \psi^{(1,0)}(e, a, s)\right),
\]

where the former corresponds to default ($D$) and the latter corresponds to no default ($N$). In order to compute an “average” effect of defaulting, we can weight the price differences for each action by the
Table 12: Alternative Economies and Extreme Value Parameters

<table>
<thead>
<tr>
<th>model parameterization</th>
<th>No Tracking (NT)</th>
<th>Full Information (FI)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>benchmark</td>
<td>high $\alpha$</td>
</tr>
<tr>
<td>$\alpha$ value</td>
<td>0.0290</td>
<td><strong>0.0319</strong></td>
</tr>
<tr>
<td>$\lambda$ value</td>
<td>0.0015</td>
<td>0.0015</td>
</tr>
</tbody>
</table>

**Panel A: % difference from BASE model with same parameters**

<table>
<thead>
<tr>
<th></th>
<th>BASE</th>
<th>NT</th>
<th>FI</th>
</tr>
</thead>
<tbody>
<tr>
<td>bankruptcy rate</td>
<td>1.12</td>
<td>1.02</td>
<td>1.07</td>
</tr>
<tr>
<td>average int. rate</td>
<td>1.44</td>
<td>1.49</td>
<td>1.45</td>
</tr>
<tr>
<td>int. rate dispersion</td>
<td>7.57</td>
<td>5.28</td>
<td>5.80</td>
</tr>
<tr>
<td>fraction in debt</td>
<td>-0.15</td>
<td>-0.23</td>
<td>-0.17</td>
</tr>
<tr>
<td>debt to income ratio</td>
<td>0.39</td>
<td>0.35</td>
<td>0.35</td>
</tr>
</tbody>
</table>

**Panel B: wealth equivalent welfare measure, newborns**

<table>
<thead>
<tr>
<th></th>
<th>low $z$</th>
<th>median $z$</th>
<th>high $z$</th>
<th>mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>BASE</td>
<td>0.060</td>
<td>-0.003</td>
<td>-0.001</td>
<td>0.121</td>
</tr>
<tr>
<td>NT</td>
<td>-0.000</td>
<td>-0.001</td>
<td>0.000</td>
<td>0.058</td>
</tr>
<tr>
<td>FI</td>
<td>-0.000</td>
<td>-0.001</td>
<td>0.000</td>
<td>0.104</td>
</tr>
<tr>
<td>high</td>
<td>0.200</td>
<td>-0.002</td>
<td>0.000</td>
<td>0.094</td>
</tr>
<tr>
<td>low</td>
<td>0.547</td>
<td>0.188</td>
<td>0.163</td>
<td>0.299</td>
</tr>
<tr>
<td>median</td>
<td>0.139</td>
<td>0.064</td>
<td>0.103</td>
<td>0.102</td>
</tr>
<tr>
<td>high</td>
<td>0.139</td>
<td>0.064</td>
<td>0.103</td>
<td>0.102</td>
</tr>
</tbody>
</table>

Notes: Each entry in Panel A is the difference, in percentage points of the BASE moment, of the moment in the indicated alternative economy (FI or NT) relative to the BASE economy for the same parameterization. The high $\alpha$ and high $\lambda$ parameterizations raise the value of these parameters by 10% in each case. Panel B reports the amount of additional wealth an agent would have to be given in the baseline economy in order to be indifferent between being born into the indicated alternative economy in the indicated state and being born in the baseline economy. The units for Panel B are percentages of mean wealth. The “base” columns for each economy match the “all” columns for each economy from Table 6.

stationary distribution of agents who have the option to default. Specifically, define

$$\bar{\mu}(e, a, s) = \frac{\sum_{\beta, z} \mu(\beta, e, z, a, s)}{\sum_{\beta, z, \tilde{a} < 0} \mu(\beta, e, z, \tilde{a}, s)}$$

for all $a < 0$.

Then, we can compute the aggregate metrics for each debt choice $a' < 0$

$$\Delta^q(a') = \sum_{e, a < 0, s} \bar{\mu}(e, a, s) \left[ \frac{q_N^e(e, a, s)}{q_D^e(e, a, s)} - 1 \right].$$

C Data Appendix

This appendix describes the construction of the data underlying the life cycle credit ranking moments reported in Table 2 and Figures 1, 2, and 3.
We begin with a 2 percent random sample of the FRBNY CCP/Equifax anonymized panel containing an individual’s birth year and an individual’s credit score in each quarter of 2003, 2004, and 2005. The credit score measure is the Equifax Risk Score (hereafter Risk Score), which is a proprietary credit score similar to other risk scores used in the industry. We consider only living individuals who were between the ages of 21 and 60 years in 2004 and had a Risk Score value in each quarter of the three years. This yields our base sample.

For this sample, we compute the percentile ranking of an individual’s Risk Score in each quarter relative to the overall distribution of credit scores in that quarter. We call this the individual’s credit ranking — it is a number that gives the fraction of people who had Risk Scores not exceeding the individual’s score in that quarter.

We then placed individuals in 5-year age bins according to their age in 2004.

Table 13 reports the mean and standard deviations of the credit rankings in each bin, averaged over the four quarters of 2004. These moments were used in the regressions that determine the coefficients in the first 4 rows of the middle panel of Table 2. To obtain the autocorrelation statistics, we computed, for each quarter of 2004, the change in an individual’s credit ranking from the same quarter in 2003 and the change to the same quarter in 2005. For each age bin, we then computed the correlation between these pair of individual changes for each quarter of 2004. The correlations reported in the final column of Table 13 are the averages of these correlations over the four quarters of 2004. The final row of the middle panel of Table 2 reports the average over age bins of the correlations in the final column Table 13.

Turning next to the construction of the data for the default event study in Figure 3, we first isolated individuals 26 years or older who filed for Chapter 7 bankruptcy sometime in 2004 in our base sample. This yielded our base sample of bankrupts. For each individual in this sample, we recorded birth year and Risk Score in the filing quarter and in the 16 quarters preceding and following the filing quarter. We converted each Risk Score into a credit ranking by computing the percentile of each Risk Score in
Table 13: **Age Profile of Credit Rankings**

<table>
<thead>
<tr>
<th>Age Bins</th>
<th>Mean, Score Pctl</th>
<th>SD, Score Pctl</th>
<th>Corr($\Delta$Pctl$^{04}$, $\Delta$Pctl$^{05}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>21-25 years</td>
<td>0.32</td>
<td>0.20</td>
<td>-0.22</td>
</tr>
<tr>
<td>26-30 years</td>
<td>0.35</td>
<td>0.23</td>
<td>-0.18</td>
</tr>
<tr>
<td>31-35 years</td>
<td>0.40</td>
<td>0.26</td>
<td>-0.20</td>
</tr>
<tr>
<td>36-40 years</td>
<td>0.44</td>
<td>0.28</td>
<td>-0.21</td>
</tr>
<tr>
<td>41-45 years</td>
<td>0.47</td>
<td>0.28</td>
<td>-0.21</td>
</tr>
<tr>
<td>46-50 years</td>
<td>0.50</td>
<td>0.28</td>
<td>-0.21</td>
</tr>
<tr>
<td>51-55 years</td>
<td>0.54</td>
<td>0.28</td>
<td>-0.19</td>
</tr>
<tr>
<td>56-60 years</td>
<td>0.58</td>
<td>0.28</td>
<td>-0.20</td>
</tr>
</tbody>
</table>

**Notes:** The credit ranking data is based on author calculations using FRBNY CCP/Equifax data. All entries are averages over the four quarters of 2004.

The overall distribution of Risk Scores. We then placed each individual in the appropriate 5-year age bin based on her age in 2004. We computed the average credit ranking (percentile) in each age bin for each of the 33 quarterly observations. This gave us the average credit ranking by age bin of people in the base sample of bankrupts reported in Table 14.
### Table 14: Default Event Study Data

<table>
<thead>
<tr>
<th>Years</th>
<th>26-30 years</th>
<th>31-35 years</th>
<th>36-40 years</th>
<th>41-45 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>0.25</td>
<td>0.28</td>
<td>0.27</td>
<td>0.29</td>
</tr>
<tr>
<td>-3</td>
<td>0.22</td>
<td>0.26</td>
<td>0.26</td>
<td>0.27</td>
</tr>
<tr>
<td>-2</td>
<td>0.19</td>
<td>0.23</td>
<td>0.22</td>
<td>0.24</td>
</tr>
<tr>
<td>-1</td>
<td>0.14</td>
<td>0.18</td>
<td>0.17</td>
<td>0.20</td>
</tr>
<tr>
<td>0</td>
<td>0.11</td>
<td>0.13</td>
<td>0.12</td>
<td>0.13</td>
</tr>
<tr>
<td>1</td>
<td>0.17</td>
<td>0.19</td>
<td>0.20</td>
<td>0.21</td>
</tr>
<tr>
<td>2</td>
<td>0.19</td>
<td>0.21</td>
<td>0.22</td>
<td>0.24</td>
</tr>
<tr>
<td>3</td>
<td>0.20</td>
<td>0.22</td>
<td>0.23</td>
<td>0.25</td>
</tr>
<tr>
<td>4</td>
<td>0.20</td>
<td>0.23</td>
<td>0.22</td>
<td>0.25</td>
</tr>
</tbody>
</table>

**Notes:** The credit ranking data is based on author calculations using FRBNY CCP/Equifax data. The data presented in this table corresponds to the black lines in Figure 3. Since calculations are performed quarterly, the indicated year is the start of the year; that is, year -4 is the observation preceding the bankruptcy by 16 quarters.