Flows and Performance in Optimal Intermediation Contracts

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Abstract
Previous literature documents that mutual funds’ flows increase more than linearly based on past performance. I establish that this convex flow-performance relationship is consistent with a dynamic contracting model where investors learn about the manager’s skills. Learning links the manager’s expected performance to past return realizations, while the optimal contract links the slope of flow-performance relationship to expected returns. As a result, my model predicts that the flow-performance relationship becomes steeper after a history of good returns. I provide empirical support for this prediction. Moreover, by incorporating an explicit incentive contract for the manager, my model rationalizes common compensation practices in the money-management industry, such as convex pay-for-performance schemes and deferred compensation. My findings hold robustly across multiple contracting environments, which include full commitment contracts, renegotiation-proof contracts and market-based incentives.

JEL Classification: G11, G23, J33, D86, D83

1 Introduction
Investors react asymmetrically to mutual fund performance: They supply large money inflows to top-performing funds, but they demand smaller outflows from poor performers. Some authors interpret this convex relation between flows and performance as an implicit compensation scheme for money managers and investigate its implications for managerial incentives. In reality, mutual fund managers face explicit incentive contracts written by fund advisory firms. Moreover, rational investors should anticipate the manager’s incentives and adjust money flows accordingly. In this

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paper, I show that the convex relationship between flows and performance is consistent with an optimal dynamic contract for money managers and I provide empirical support for my theory.

In my dynamic contracting model, I consider the two layers of incentives that characterize the mutual fund industry. In the first layer, competitive investors supply capital and pay proportional fees to a fund advisory firm. In the second layer, the advisory firm hires a portfolio manager and sets the terms of the manager’s compensation contract. Since the advisory firm captures the value added of the fund through the fee revenues, it faces implicit incentives to run the fund efficiently. As for the manager, his incentives are explicitly established in the contract that the advisory firm designs.

The model produces an increasing and convex relation between fund flows and performance. The motivation relies on two assumptions. First, investors learn about the manager’s skills by observing return realizations. This assumption implies that investors expect higher returns from a manager who performed better in the past. Second, the manager is subject to moral hazard. In particular, he can mismanage his portfolio and gain private benefits at the expense of investors. The advisor designs an optimal incentive contract to prevent this possibility. In the optimal contract, the flow-performance sensitivity is positive and increases when expected returns increase. Therefore, as good returns accumulate and investors expect increasingly higher future returns, flows become increasingly more sensitive to current performance. As a result, over any period of time, cumulative fund flows respond in a positive and convex way to cumulative performance. This observation has been repeatedly documented in the empirical literature, e.g. in Chevalier and Ellison (1997), Sirri and Tufano (1998) and Del Guercio and Tkac (2002).

My model also generates a managerial compensation scheme which reflects common patterns in the money-management industry. In the model, managers are compensated for their past performance. Similarly to capital flows, the pay-performance sensitivity increases after good returns, thus resulting in a convex compensation scheme on an annual basis. This result is consistent with then widespread use of convex pay-for-performance contracts in the industry (BIS, 2003; Ma et al., 2019). The motivation relies on the nature of the optimal contract, which links compensation to performance in order to motivate the manager to maximize returns. Moreover, the optimal contract includes a deferred compensation feature. After good performance, the advisor partially postpone the delivery of the promised compensation in order to provide stronger incentives to the manager. In the money-management industry, several practices effectively postpone the payout of performance-based compensation to future periods.

I then provide empirical support for my theory by testing the main prediction of the model: The sensitivity of flows to current performance increases after a history of good performance. I measure a mutual fund’s performance by comparing its return to the average return of funds with the same style. Every month, I compute the average past performance over the previous three to nine months. At a monthly frequency, past performance has a large and statistically significant

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[1] Ma et al. (2019) documents that 30% of the mutual fund managers in their sample are subject to explicit deferred compensation contracts. Moreover, they find that managers’ bonuses depend on their average performance over multiple years in the past. This last practice effectively implements a deferred compensation scheme.
effect on the current flow-performance sensitivity. If a fund beats its benchmark by 1% over the previous six months, its flow-performance sensitivity is about 18% higher than for the average fund. Consistently with the model, this effect increases as the past evaluation horizons increases, reflecting a more accurate mapping between past performance and expected returns. Interestingly, at monthly frequency, the flow-performance sensitivity does not display any economically nor statistically significant convexity. This is consistent with my model, where a convex flow-performance relationship appears over longer horizon because the flow-performance sensitivity varies in response to past returns.

I also test a second implication of the model, which relates the flow-performance relationship to the tenure of the manager. Under the assumption that the advisor can fully commit to the optimal long-term contract, the flow-performance sensitivity should not respond to past performance if the manager has a long tenure. I verify that this prediction holds in mutual funds. The motivation relies on the fact that, in order to achieve optimality ex-ante, the optimal contract imposes constraints on the incentives and on the skills of the manager in the future. If the manager’s skills play a minor role in generating returns, past performance provides less information about future performance, thus severing the link between past performance and flow-performance sensitivity. Almazan et al. (2004) provide evidence that more experienced managers are subject to more investment constraints. My model suggests that this correlation could simply represent the physiological outcome of an optimal contract.

I derive my results in a continuous time contracting model with learning about the manager’s skills. This contract design problem poses some challenges, which I overcome using duality methods. Since investors learn about the manager’s skills from returns, the manager could acquire an ex-post information rent after shirking. Intuitively, a manager who shirks and appears unskilled has better career prospects than a manager who is actually unskilled. If a contract induces larger information rents, the manager has weaker incentives to maximize returns and the advisor obtains lower revenues. Therefore, one could formulate an optimization problem where the advisor controls the manager’s information rent as an independent state variable. Unfortunately, this problem cannot be feasibly solved. To overcome this challenge, I use duality methods and I offer a tractable and intuitive formulation of the contract design problem. In the optimal contract, the advisor commits to ex-post inefficient incentives in order to reduce the ex-ante information rent of the manager. In the dual formulation, this commitment is captured by a multiplier which, over time, distorts the terms of the contract towards a lower risk exposure for the manager. By considering the dynamics of the multiplier and of beliefs, I can provide intuitive interpretations for my results on the flow-performance relationship and on the manager’s compensation scheme.

The implications of the model hold across different contracting environments. As a benchmark, I consider the case in which the advisor fully commits to the terms of the ex-ante optimal contract. I then relax this assumption and study the case in which the advisor can renegotiate the terms of the contract ex-post. Finally, I let the manager raise capital directly from investors in the spot market. These last two scenarios are actually equivalent. Moreover, in all these cases,
the flow-performance sensitivity is positive and increasing in past performance, the manager’s pay depends on performance, and his compensation is partially deferred. These results further highlight that the main predictions of the model do not rely on specific assumptions about the contracting process, but only on the presence of moral hazard and learning.

The rest of the paper is organized as follows. In Section 2, I review the related literature. In Section 3, I present the set-up of the model. In Section 4, I characterize the optimal contract. In Section 5, I show the implications of the optimal contract for flows, performance and compensation and I provide the economic intuition behind this results. In Section 6, I empirically test the predictions of the model. Finally, in Section 7, I show that the key results of the model hold also for renegotiation-proof proofs and market-based incentives. All proofs are in Appendix B.

2 RELATED LITERATURE – TBU

3 MODEL SET-UP

3.1 PLAYERS

There are three types of players: Investors, a fund advisor (the principal/she) and a portfolio manager (the agent/he). I will use the term fund to refer to the organization that the portfolio manager and the fund advisor form. The advisor collects capital from investors on the spot market and she hires the manager in order to actively manage this capital.

Investors are risk-neutral, competitive and cannot commit to long-term contracts. They interact with the advisor through a series of spot contracts, at every time \( t \), specify the assets that they supply to the fund, \( K_t \), and the proportional fee, \( f_t \), which the advisor receives. Investors then collect the returns that the fund produces, \( R_t \). Through their interaction with the fund, investors will therefore obtain a utility of

\[
E \left[ \int_0^\infty e^{-rt} \left( K_t R_t - (f_t + r)K_t dt \right) \bigg| \mathcal{F}_0 \right]
\]

where \( r > 0 \) is the market risk-free rate and represents both the discount rate of investors and their opportunity cost of capital\(^2\). Investors are therefore willing to provide any amount of capital as long as they expect the net-of-fee return to weakly exceed the risk free rate.

Through most of the paper, I will assume that the fee is variable over time and that \( K_t \) represents the amount of assets that the fund actively manages. Alternatively, one could assume, as in Berk and Green (2004), that the fee is fixed to some given sufficiently low value, \( \bar{f} \), and that investors supply additional capital, \( \bar{K}_t \), which the fund invests in a passive benchmark. Investments

\(^2\)These preferences could reflect the assumption that investors are active in a complete asset market. The fund offers an idiosyncratic return that they are able to fully diversify. The fund’s returns are thus uncorrelated with their aggregate consumption and carry no risk premium for investors.
in the passive benchmark can be easily monitored, so they are not subject to the moral hazard friction that I will describe shortly. These two spot contracts are equivalent, as long as the passively and actively managed parts of the portfolio are contractible and the implied transfers coincide, i.e. \( f_t K_t = \tilde{f}(K_t + \bar{K}_t) \).

The advisor collects the fees that investors pay, \( f_t K_t \), and provides a compensation \( \tilde{C}_t \) to the manager. The advisor is also risk-neutral and her objective is to minimize the costs of running the fund,

\[
E \left[ \int_0^\infty e^{-rt} (\tilde{C}_t - f_t K_t) dt \right].
\]

Unlike the investors, the advisor has the power to commit to a long-term contract with the manager\(^3\). Investors observe this long-term contract and understand the consequences of the contract on the manager’s incentives. In particular, they will adjust their supply of capital and their willingness to pay fees in response to the terms of the contract. Therefore, in designing the contract, the advisor will not only account for the incentives of the manager, but also for the investors’ responses.

The manager controls the fund’s active portfolio. However, he cannot be directly monitored by advisor. In particular, she cannot tell whether the manager’s investment and trading choices were made in the best interest of investors or for his own private gain. This creates a moral hazard friction. More formally, I assume that the agent could mismanage the active portfolio of the fund, thus destroying value \( m_t K_t \) for investors. From this, he obtains a private consumption value of \( \lambda m_t K_t \), where \( 1 - \lambda \in (0, 1) \) represents the inefficiency of mismanagement \( m_t \). More precisely, if a fund manager consumes \( (C_t)_{t \geq 0} \) and shirks by mismanaging at a rate \( (m_t)_{t \geq 0} \), then his expected utility is given by

\[
V_0 = E \left[ \int_0^\infty e^{-\delta t} u(C_t + m_t \lambda K_t) dt \right]|_{\mathcal{F}_0}.
\]

where \( (K_t)_{t \geq 0} \) are the fund’s assets under management. I assume that \( \delta \geq r \) and \( u(x) = \frac{x^{1+\rho}}{1-\rho} \) with \( \rho \in (0, 1/2) \)\(^4\).

The manager possess a specific skills, which are unobservable to the advisor and the investors, as well as to the manager himself. However, he can generate informative signals about his skills through costly experimentation. For example, he can search for profitable investment opportunities or implement new trading strategies. If the manger is skilled, he will produce additional excess returns. If he is not, his experimentation efforts will be worthless. By observing realized returns, all players will be able to learn, as time goes by, whether the manager possesses superior investment skills. I assume that experimentation can be represented by a variable, \( e_t \), which

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\(^3\)I will relax the full commitment assumption in Section 7.

\(^4\)The assumption that \( \rho < 1/2 \) is needed in order to obtain a finite solution to the problem. If \( \rho > 1/2 \) then the manager’s marginal utility of consumption would decline quickly enough that the advisor would find it profitable to give infinite capital and infinite consumption to the agent and overcome the incentive problem. This assumption can be relaxed if I extend the model to allow the agent to privately save. This extension would substantially complicate the model along several dimensions. Moreover, it would not add any additional insight on the economic mechanism that determines the flow-performance relationship.
takes values in a bounded set, $e_t \in [0, 1]$. Experimentation is fully observable and contractible. To introduce costs, I assume that experimentation reduces the consumption value of the manager’s compensation. If the manager receives compensation $\tilde{C}_t$ from the advisor, but he is required to undertake experimentation $e_t$, then his final consumption is given by

$$C_t = q(e_t)\tilde{C}_t,$$

(2)

for a function $q(e)$ such that $q(\cdot) \in (0, 1)$, $q'(\cdot) < 0$ and $q(1) \geq \lambda$.

### 3.2 Returns

The advisor hires the manager in order to actively manage the fund’s portfolio and generate excess returns. However, the manager’s skills are unknown to all players, including the manager himself. I assume that the manager could be either skilled or unskilled, as indexed by his hidden type $h \in \{0, 1\}$. If the manager experiments with investment ideas and trading strategies, his skills will be reflected in the fund’s returns. Players then learn over time about the manager’s skills by observing the fund’s performance.

I assume that returns follow a stochastic process

$$dR = (r + \mu(e_t, h, m_t))dt + \nu(e_t)dW_t$$

$$\mu(e_t, h, m_t) = \alpha + e_tsh - m_t$$

$$\nu(e_t) = \sigma + be_t$$

(3)

where $r$ is the risk free rate and $\alpha \geq 0$, $\sigma \geq 0$, $s \geq 0$ and $b$ are known parameters.

Returns are affected by the uncertain skills of the manager, $h$, and by the manager’s hidden shirking, $m_t$. If a manager is skilled ($h = 1$), he obtains superior returns by experimenting, i.e. by setting $e_t > 0$. If he is unskilled ($h = 0$) his experimentation efforts will not be reflected in returns. By shirking and mismanaging at rate $m_t$, the manger reduces cash flow for investors by $m_tK_t$. However, his private benefit from mismanagement is only $\lambda m_tK_t$, for $\lambda < 1$. Shirking is, therefore, inefficient, since the manager destroys more value that what he obtains. The advisor, in order to maximize her revenues, will design a contract which prevents mismanagement. When investors observe this contract, they will not fear that returns will be destroyed and they will therefore be willing to provide more capital to the advisor and pay higher fees.

Besides affecting the distribution of returns, experimentation determines the information content of returns as a signal. Consider the signal-to-noise ratio of returns at $t$. This quantity is given by

$$\frac{\mu(e_t, 1, m_t) - \mu(e_t, 0, m_t)}{\nu(e_t)} = \frac{e_t s}{\sigma + be_t}$$

and it measures how informative returns are as a signal. Suppose that a skilled manager produces, on average, much larger returns than an unskilled manager, i.e. $\mu(e_t, 1, m_t) \gg \mu(e_t, 0, m_t)$. In this case, a good (bad) return realization will be a strong signal that the manager posses high (low)
skills. However, if the volatility of returns, $\nu(e_t)$, is very large, then a skilled manager could generate a very poor return out of bad luck and, while an unskilled manager could deliver a superior return out of good luck. Therefore, volatility reduces the information content of return signals. By increasing experimentation, $e_t$, the fund will increase the signal-to-noise ratio of returns and, hence, provide more information about the manager’s skills. While beneficial in the short run, I will show that, in the long term, future experimentation worsens the present-day moral hazard problem. In the optimal contract, the advisor will therefore trade off the benefits of current experimentation with the costs of any future experimentation that she promises.

### 3.3 Contracting Environment and Learning

The two main elements of the model are the incentive contract between the advisor and the manager, and the learning process. Returns constitute the only source of information for the players. On the one hand, returns allow players to draw inference about the manager’s skills. On the other hand, return realizations change depending on the hidden action, $m_t$, of the manager. Therefore, the terms of the contract between the manager and the advisor, as well as the spot contracts between the advisor and the investors, can solely depend on the history of returns. Let $(R_s)_{0 \leq s \leq t}$ denote the history of returns up to time $t$ and let $\mathcal{F}_t = \{(R_s)_{0 \leq s \leq t}\}$ be the filtration generated by such history.

A contract between the advisor and the manager specifies the manager’s consumption, the size of his actively manage portfolio and the experimentation that he undertakes. Moreover, for completeness, I assume that the contract also specifies the mismanagement that the advisor expects. Although the agent’s mismanagement cannot be verified and written in a contract, the advisor will form a conjecture about the manager’s hidden action at any point in time. I let the contract specify this conjecture. To keep the notation parsimonious, I omit compensation from the definition of contract. Given consumption $C_t$ and experimentation $e_t$, the compensation $\hat{C}_t$ of the manager is determined by equation (2). Whenever needed, compensation can be obtained as $\hat{C}_t = \frac{C_t}{q(e_t)}$.

**Definition 1 (Contract).** A contract $\mathcal{C}$ is a set of $\mathcal{F}_t$-adapted processes

$$((C_t)_{t \geq 0}, (K_t)_{t \geq 0}, (e_t)_{t \geq 0}, (m_t)_{t \geq 0}).$$

While the principal cannot directly control the agent’s mismanagement, she understands the implications of a contract on the agent’s incentives to mismanage. If her conjecture about the agent’s hidden action coincide with the action that the manager has incentives to take, then the contract is called incentive compatible.

**Definition 2 (Incentive Compatible Contract).** A contract $\mathcal{C} = ((C_t)_{t \geq 0}, (K_t)_{t \geq 0}, (e_t)_{t \geq 0}, (m_t)_{t \geq 0})$ is incentive compatible if

$$
(m_t)_{t \geq 0} \in \arg \max_{(\hat{m}_t)_{t \geq 0}} E \left[ \int_0^\infty e^{-\delta t} u(C_t + \hat{m}_t \lambda K_t) \, dt \bigg| \mathcal{F}_0, \mathcal{C}, (\hat{m}_t)_{t \geq 0} \right].
$$
In general, it is without loss of generality to consider only contracts that are incentive compatible. Since equilibrium strategies are common knowledge, the advisor can always change any given contract to another one in which her conjecture about the manager’s hidden action are consistent with the manager’s best response to the contract.

Given an incentive compatible contract, players will symmetrically learn about the skills of the manager by observing the fund’s returns. Suppose that all players possess a common prior, \( E[h|F_0] = p \in [0,1] \). Since the advisor and the investors correctly anticipate the hidden action of the manager, at any time \( t \) all players will have common beliefs about the manager’s skills given by

\[
\phi_t = E[h|F_t].
\]

Beliefs are an important state variable of the model. Beliefs determine investors’ expectations about fund’s returns,

\[
E[\mu(e_t, h, m_t)|F_t] = \mu(e_t, \phi_t, m_t),
\]

and, through this, they determine the investor’s willingness to supply capital.

**Proposition 1.** Given an incentive compatible \( \mathcal{C} = ((C_t)_{t \geq 0}, (K_t)_{t \geq 0}, (e_t)_{t \geq 0}, (m_t)_{t \geq 0}) \), beliefs \( \phi_t = E[h|F_t] \) evolve as

\[
d\phi_t = \eta(e_t)\phi_t(1 - \phi_t) dW_t^e \tag{4}
\]

where

\[
dW_t^e = \frac{1}{\nu(e_t)} [dR_t - (r + \mu(e_t, \phi_t, m_t)) dt]
\]

is an increment to a standard Brownian motion under the measure of returns induced by \( \mathcal{C} \) and \( \eta(e_t) = \frac{e_t s}{\nu(e_t)} \) is the signal-to-noise ratio of returns.

Equation (4) remarks the role of experimentation in the production of information. If the advisor requires higher experimentation \( e_t \), the signal-to-noise ratio increases, together with the response of beliefs to any given return shock \( dW_t^e \).

Given their beliefs \( \phi_t \) and the contract \( \mathcal{C} \), competitive and risk neutral investors provide capital to the fund through a series of spot market contracts. These contracts specify the proportional fees that investors pay to the advisor. Since investors competitive, they take the fund’s net expected returns as given and they compare them to the interest rate \( r \), which represents their outside option. Given their risk neutrality, they are not concerned about the idiosyncratic risk of the fund.

**Definition 3 (Spot Market Contract).** Given a contract \( \mathcal{C} \) and beliefs \( \phi_t \), a spot market contract, \( \mathcal{C}_s \), is a pair of \( F_t \)-measurable variables, \((f_t, K_t)\) such that, for all times \( t \), investors are willing to supply capital \( K_t \) and pay fees \( f_t \),

\[
K_t \in \arg\max_K \left( \mu(e_t, \phi_t, m_t) - f_t \right) K,
\]
and such that the advisor obtains revenues \( f_t K_t \).

From this definition, we can see that competitive investors offer a perfectly elastic supply of capital at rate \( r \). The advisor can increase the fees up to the point at which they coincide with expected excess returns. For these fees, individual investors are willing to supply any amount of capital, while the advisor maximizes the revenues she collects per unit of assets under management. I call a spot market contract optimal if it maximizes the spot revenues of the advisor.

**Lemma 1.** In any optimal spot market contract, \( f_t = \mu(e_t, \phi_t, m_t) \) for all \( t \geq 0 \).

While investors’ preferences pin down fees \( f_t \), the size of the fund is pinned down by the advisor’s demand.\(^5\) By setting fees equal to expected excess returns, the advisor is capturing the value added of the fund, \( K_t \mu(e_t, h, \tilde{m}_t) \), through her revenues. She has therefore incentives to design a compensation contract that maximizes the value of the fund.

The compensation contract of the manager will therefore be optimal for the advisor if it minimizes the costs of running the fund, after taking into account the incentives that the manager receives from the contract. Formally, an optimal contract can be defined as follows

**Definition 4 (Optimal Contract).** A contract \( C = (C_t)_{t \geq 0}, (K_t)_{t \geq 0}, (e_t)_{t \geq 0}, (m_t)_{t \geq 0} \) is optimal for initial beliefs \( \phi_0 \) and for initial promised value \( V_0 \) if

\[
C \in \arg \inf_C \mathbb{E} \left[ \int_0^\infty e^{-rt} \left( \frac{\hat{C}_t}{q(\hat{e}_t)} dt - \hat{K}_t \mu(\hat{e}_t, h, \tilde{m}_t) \right) d\mathcal{F}_0, \hat{C}, (\tilde{m}_t)_{t \geq 0} \right]
\]

s.t. \( (\tilde{m}_t)_{t \geq 0} \in \arg \max_{(m_t')_{t \geq 0}} \mathbb{E} \left[ \int_0^\infty e^{-\delta t} u(\hat{C}_t + m_t' \lambda \hat{K}_t) dt \big| \mathcal{F}_0, \hat{C}, (m_t')_{t \geq 0} \right] \]

\[
\mathbb{E} \left[ \int_0^\infty e^{-\delta t} u(\hat{C}_t + \tilde{m}_t \lambda \hat{K}_t) dt \big| \mathcal{F}_0, \hat{C}, (\tilde{m}_t)_{t \geq 0} \right] \geq V_0
\]

\[
\mathbb{E}[h|\mathcal{F}_0] = \phi_0.
\]

In the optimal contract, the advisor takes explicitly into account that the manager might face incentives to mismanage and gain private benefits. By shirking, the manager changes expected returns \( \mu(\hat{e}_t, h, \tilde{m}_t) \), as well as the distribution of returns. Since contract’s terms are written as functions of the history of returns, the expectations in the definition take into account of the probability measure of return induced by the contract \( C \) and by the agent’s mismanagement process \( (\tilde{m}_t)_{t \geq 0} \). Finally, the optimality of a contract is always defined with respect to an promised value for the the agent \( V_0 \), which can be seen as an initial outside option for the manager.

In this paper, I am interested in optimal contracts. While verifying optimality in the sense of Definition 4 appears intractable, the following proposition shows it suffices to search over a restricted class of contracts.

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\(^5\)The logic would be slightly different in the alternative scenario in which fees are fixed. In this case, the advisor would determine the amount of actively managed assets. Investors would then supplying additional capital to be invested in the passive, agency-free portfolio. Hence, investors determine the total size of the fund so that the fund’s total net return coincides with the interest rate. As already mentioned, these two scenarios imply identical outcomes for the size of the active portfolio and for the value of the fund’s revenues.
PROPOSITION 2. It is without loss of generality to restrict optimal contracts to be incentive compatible with \( m_t = 0 \) for all \( t \).

The intuition for this proposition is straightforward. First, as already discussed, we lose no generality if we restrict attention to incentive compatible contracts. Second, optimal contracts must align the manager’s incentives to the advisor’s objectives. Since mismanagement is inefficient and the advisor obtains revenues from the fund’s value added, it is intuitive that the optimal contract will be designed in order to prevent mismanagement.

From an operational point of view, Proposition 2 tells that, in searching for an optimal contract, we can simply derive the conditions under which the manager has no incentive to shirk and, within the class of contracts that satisfy this condition, we can select the optimal one. In the remainder of this Section, I will derive the conditions that an optimal contract must satisfy. I will then use these conditions in Section 4 to derive the optimal contract.

3.4 Incentive Compatibility and Information Rent

In the previous subsection, I have argued that we can restrict our attention to contracts that are incentive compatible and enforce zero mismanagement. In this subsection, I provide conditions for a contract to achieve this target. As in static principal-agent problems, these conditions require the manager to be exposed to some fund-level risk. While in static models the principal exposes the agent to risk by giving him performance-contingent pay, in a dynamic model the agent is exposed to risk by adjusting his future continuation value.

In order to design an optimal contract, the advisor needs to account for the incentives of the agent. At any time \( t \) the manager has incentives to shirk increase his instantaneous utility of consumption, \( u(C_t + m_t \lambda K_t) \). However, he could be deterred from shirking if it causes a loss of future utility. This forward-looking incentives are captured by the manager’s continuation value \( V_t \). Formally, the manager’s continuation value is given by a function of the continuation contract \( C_t \) and of beliefs,

\[
V_t = V(\mathcal{C}_t, \phi_t) = E \left[ \int_t^\infty e^{-\delta(s-t)} u(C_s) ds \bigg| \mathcal{F}_t \right],
\]

which captures the present value of the future utility that the manager expect to receive from the contract\(^6\).

Using the martingale representation approach developed in previous literature (Sannikov, 2008; Williams, 2009), it is possible to obtain a law of motion for the agent’s continuation value \( V_t \).

\(^6\) A the continuation contract a time \( t \), \( \mathcal{C}_t \), is set of \( \mathcal{F}_t \)-adapted processes \( ((C_s)_{s \geq t}, (K_s)_{s \geq t}, (e_s)_{s \geq t}, (m)_{s \geq t}) \), where \( \mathcal{F}_t = \{(R_u)_{t \leq u < s}\} \).

\(^7\) Given a continuation contract \( \mathcal{C}_t \), beliefs are a sufficient statistic for the probability measure of future returns. To see this, write

\[
V_t = (1 - \phi_t) E \left[ \int_t^\infty e^{-\delta(s-t)} u(C_s) ds \bigg| h = 0 \right] + \phi_t E \left[ \int_t^\infty e^{-\delta(s-t)} u(C_s) ds \bigg| h = 1 \right].
\]

The conditional expectations on the right hand side of this equation are functions of the continuation contract \( \mathcal{C}_t \) only.
**Lemma 2.** The agent’s continuation value evolves as

\[ dV_t = (\delta V_t - u(C_t))dt + \beta_t dW^C_t \]  \hspace{1cm} (7)

for some \( \mathcal{F}_t \)-adapted process \( (\beta_t)_{t \geq 0} \), and where \( dW^C_t \) is defined as in (5).

If \( \beta_t \) is different from zero, then the agent is facing a risky consumption path. Absent moral hazard, this would be an inefficient allocation of risk. Since the agent is risk-averse, while the advisor and investors are risk neutral, the former should be fully insured by the latter. However, with moral hazard, the advisor may find it optimal to expose the manager to some risk through some suitable contract \( C \). If \( \beta_t \) is positive, then the utility of the agent increases if he delivers a return above expectations. If the manager shirks, returns decline and the manager suffers a loss in terms of future utility.

To highlight the role on learning in the provision of incentives, suppose that the skills of the manager, \( h \), were known. In case, the advisor can prevent the agent’s mismanagement by offering contract where the manager’s exposure to returns, \( \beta_t \), offsets the marginal consumption value of mismanaging.

**Lemma 3.** If \( h \) is common knowledge, then a necessary and sufficient condition for incentive compatibility is

\[ u'(C_t)\lambda \nu(e_t)K_t \leq \beta_t. \]

This condition is intuitive. If the manager shirks, he reduces returns by \( m_t \) and, hence, \( dW^C_t \) acquires a negative drift \(-\frac{m_t}{\nu(e_t)}\). The agent therefore suffers a loss of continuation value equal to \( \beta_t \frac{m_t}{\nu(e_t)} \). However, his current utility increases to \( u(C_t + m_t \lambda K_t) \). The condition in Lemma 3, ensures that \( m_t = 0 \) is the best response of the manager to the contract, i.e.

\[ 0 = \arg\max_{m \geq 0} \left\{ u(C_t + m_t \lambda K_t) - \beta_t \frac{m_t}{\nu(e_t)} \right\}. \]

If the skills of the manager are uncertain, the advisor faces some additional challenge in designing an incentive compatible contract. For example, suppose that the manager deviates to \( m_t = m > 0 \) for a small amount of time between \( s \) and \( s + \epsilon \). With learning, the agent not only gains consumption value from the deviation. He also earns an informational advantage over the advisor and the investors. Unaware of the agent’s deviation, the advisor and the investors update beliefs according to equations (4) and (5). Since the true drift of returns is now \( \mu(e_t, h, m_t) \), their beliefs, \( \tilde{\phi}_t \), will acquire a negative drift relative to the manager’s beliefs, \( \tilde{\phi}_t \) and their different, after the deviation will be given by

\[ \tilde{\phi}_{s+\epsilon} - \phi_{s+\epsilon} \approx \eta(e_s) \phi_s (1 - \phi_s) \frac{m_s}{\nu(e_s)} \epsilon. \]

This difference in beliefs is persistent and causes persistent distortions in the provision of incentives. Following a deviation, the manager will always be more optimistic than the other play-
ers, i.e. $\tilde{\phi}_t > \phi_t$ for all $t > s + \epsilon$. The agent is not only more optimistic, but he is also aware of possessing correct beliefs. This constitutes an information rent for the manager. Intuitively, a high-skilled manager who is believed to be low-skilled is better off than a manager who is actually low-skilled. The high-skilled managers can expect to surprise the market in the future with his superior skills. The truly low-skilled manager cannot expect to surprise anyone. In order to see this, consider equation (7). If $\tilde{\phi}_t > \phi_t$, then shocks $\frac{1}{\nu(e_t)} \left[ dR_t - (r + \mu(e_t, \phi_t, 0))dt \right]$ have a positive drift given by

$$\eta(e_t)(\tilde{\phi}_t - \phi_t)dt > 0.$$ 

Therefore, the agent’s continuation value acquires a positive drift

$$\beta_t \eta(e_t)(\tilde{\phi}_t - \phi_t)dt.$$

The drift $\beta_t \eta(e_t)(\tilde{\phi}_t - \phi_t)$ captures the surprise that the agent expects other players to receive. After a deviation, the advisor will promise a continuation value $V_{t+\epsilon}$ to the manager. However, the true continuation value of the manager is larger than what the advisor believes and it include the additional surplus that accrues to the manager through the future drift $\beta_t \eta(e_t)(\tilde{\phi}_t - \phi_t)$. This additional surplus constitutes the information rent that the manager has over the principal and and investors.

This reasoning suggests that, when players are learning, an incentive compatible contract should provide incentives $\beta_t$ to offset the manager’s marginal utility of shirking as well as the the information rent that he could earn. I formalize this argument using the stochastic maximum principle introduced in the contract design literature by Williams (2011).

**Proposition 3.** In any optimal contract

$$u'(C_t)K_t \nu(e_t) + \eta(e_t) \xi_t \leq \beta_t$$

where $\xi_t$ follows

$$d\xi_t = (\delta \xi_t - \eta(e_t) \beta_t (1 - \phi_t))dt + \omega_tdW^C_t$$

for some $\mathcal{F}_t$-adapted process $(\omega_t)_{t \geq 0}$.

The term $\eta(e_t)\xi_t$ in the incentive compatibility condition (8) accounts for the information rent that accrues to the manager after a deviation. To more clearly interpret the variable $\xi_t$, consider expression (6). In particular, consider the fact that, at any point in time, the manager’s continuation value is a function of the continuation contract $C_t$ and his beliefs $\phi_t$, i.e. $V_t = V(C_t, \phi_t)$. For a given contract, $\partial_\phi V(C_t, \phi_t)$ measures the marginal change in continuation value coming from a marginal change in beliefs. Essentially, this quantity captures the information rent that the manager earns for being marginally more optimistic. The variable $\xi_t$ is related to this marginal information rent, as the following proposition shows.
**Proposition 4.**

\[
\xi_t = \phi_t(1 - \phi_t)\partial_{\phi} V(C_t, \phi_t).
\]

moreover

\[
\xi_t = E \left[ \int_t^\infty e^{-\delta(s-t)}\eta(e_s)\beta_s \phi_s (1 - \phi_s) \, ds \bigg| T_t \right],
\]

and, in any incentive compatible contract, \( \xi_t \geq 0 \).

I can now give an intuitive interpretation of the incentive compatibility condition (8). First, from equation (7), we know that incentives \( \beta_t \) correspond to the total volatility of the continuation value \( V_t \). Second, combining (4) and (10), we see that \( \eta(e_t)\xi_t = \eta(e_t)\phi_t(1 - \phi_t)\partial_{\phi} V(C_t, \phi_t) \) represents the volatility of the continuation value that originates from the changes in beliefs. Therefore, we can interpret the quantity

\[
\beta_t - \eta(e_t)\xi_t = \beta_t - \eta(e_t)\phi_t(1 - \phi_t)\partial_{\phi} V(C_t, \phi_t)
\]

as the volatility of the agent’s continuation value that originates from changes in the continuation contract while keeping beliefs fixed.

Seen from this perspective, the incentive compatibility condition (8) is extremely intuitive. Although changes in beliefs do affect the volatility of the continuation value along equilibrium path, they cannot be exploited to prevent off-equilibrium deviations. This is because the agent’s belief distribution is not affected by his deviations. After a deviation \( m_t \), the agent’s true expected future utility declines by \( (\beta_t - \eta(e_t)\xi_t)\frac{m_t}{\nu(e_t)} \), and not by \( \beta_t\frac{m_t}{\nu(e_t)} \), as the principal incorrectly thinks. Therefore, in an incentive compatible contract, it is the quantity \( \beta_t - \eta(e_t)\xi_t \) that matters for incentive provisions, and must be such that

\[
0 = \arg \max_{m \geq 0} \left\{ u(C_t + m_t\lambda K_t) - (\beta_t - \eta(e_t)\xi_t)\frac{m_t}{\nu(e_t)} \right\}.
\]

In other words, equation (8) states that, in order to give incentives to the manager, the principal cannot rely on changes in beliefs to punish him for bad performance. She must instead adjust the continuation contract so that, keeping beliefs constant, the incentives of the manager exceed the private benefit of shirking.

The presence of learning does make the provision of incentives more costly for the manager. This is captured by the result that \( \xi_t \geq 0 \) in Proposition 4. This result, together with (8) and the previous discussion, indicates that there is a source of risk in the manager’s consumption profile which cannot be exploited for the provision of incentives. Compared to a situation where \( h \) is known, for a given consumption \( C_t \), capital \( K_t \) and experimentation \( e_t \), a manager with uncertain skills will have to bear an additional quantity \( \eta(e_t)\xi_t \) of risk. The advisor will have to compensate the risk-averse manager for this additional risk, thus increasing the cost of the contract.
3.5 Verifying Incentive Compatibility

I conclude this section by presenting a condition that can be used to verify the incentive compatibility of a contract. Proposition 3 offers a condition that prevents the agent to engage in a one-shot deviation. Although this is a necessary incentive compatibility condition, alone it does not guarantee full incentive compatibility. With learning, there is a state variable, beliefs, that the agent can control. Any hidden mismanagement will cause a persistent wedge between the principal’s and the agent’s beliefs. Therefore, following a deviation, the agent may face incentives to shirk for which Proposition 3 does account. Even if (8) holds, the agent’s best response to the contract may still involve a dynamic mismanagement strategy.

Previous literature has long recognized the challenge that hidden state variables pose on the design of optimal contract. The common approach is to solve for an optimal contract by imposing the necessary condition (8) only. This is the so called relaxed problem. Then, one would verify whether the contract so obtained satisfies a sufficient incentive compatibility condition. This is the strategy undertaken by He (2012) and Di Tella and Sannikov (2018) for private savings, by Prat and Jovanovic (2014), DeMarzo and Sannikov (2017) and He et al. (2017) for learning, and Williams (2011) for generic state variables. I take the same approach and use the following proposition to verify whether the solution to the relaxed is actually incentive compatible.

**Proposition 5.** If

\[ u'(C_t)k_t \lambda \nu(e_t) + \eta(e_t) \xi \leq \beta_t \]

and

\[ \omega_t \geq \eta(e_t)(1 - 2 \phi_t)\xi_t, \tag{12} \]

then the contract is incentive compatible.

This is a sufficient condition for incentive compatibility. In general, there are incentive compatible contracts which satisfy (8), but not (12). In solving for an optimal contract, I will adopt the first order approach and impose the necessary condition (8) as a constraint on the contract. Once a candidate optimal contract is obtained, it will be possible to verify whether it satisfies condition (12).

To interpret the sufficient condition for incentive compatibility, it is useful to refer to the proof of this proposition in Appendix B. There, I show that \( \omega_t - \eta(e_t)(1 - 2 \phi_t)\xi_t \) is the volatility of \( \partial \phi V(C_t, \phi_t) \). Proposition 5, therefore, states that, as long as a contract prevents instantaneous deviations and it reduces the marginal value of beliefs after a negative shock, then this contract is fully incentive compatible.

This result bears some intuitive appeal. If the contracts makes the marginal value of skills \( \partial \phi V(C_t, \phi_t) \) decrease after a bad shock then, following a deviation, the manager would not only suffer a decrease in his continuation value, but also in his information rent. The agent thus loses part of the option to “impress” investors in the future, lowering the value of his informational advantage. These conditions are sufficient to dissuade the agent from engaging in any misman-
Equation (12) is likely to hold in an optimal contract. Looking at Proposition (4), we see that \( \partial_{\xi} \xi_t \) depends on future incentives \( \beta_t \) and signal-to-noise ratios \( \eta(e_t) \). After a good shock, the advisor has incentives to increase both future incentives and future learning. On the one hand, a good shock increases expected returns and the advisor likely takes advantage of them by increasing capital under management \( K_t \) and, to ensure incentive compatibility, incentives \( \beta_t \). On the other hand, experimentation becomes more profitable, so the principal will likely increase experimentation \( e_t \) and, consequently, the signal-to-noise ratio \( \eta(e_t) \). We therefore have reasonable economic motivations that, in the optimal contract, \( \xi_t \) increases sufficiently after good performance.

4 Optimal Contract

Starting from the incentive compatibility condition in Proposition 3, I will adopt a first order approach to solve for the optimal contract under full commitment. Through the first order approach, I can treat the optimization problem in Definition 4 in a recursive way, while imposing incentive compatibility condition (8) at every point in time. The recursive state variable include the manager’s continuation value \( V_t \), his information rent \( \xi_t \) and beliefs \( \phi_t \). Commitment by the principal is reflected in \( V_t \) and \( \xi_t \) being state variables of the problem. With full commitment, the principal always honors her past promises in terms of continuation value and information rent. The law of motions (7) and (9) represent promise keeping constraints for the principal. In order to provide incentives, she will specify how future continuation values and information rents evolve on the basis of performance. She will therefore pick incentives \( \beta_t \) and volatility \( \omega_t \) optimally.

Formally, the optimal contract is characterized as a solution to the following optimization problem.

\[
J^*(V_0, \xi_0, \phi_0) = \inf_{(C_t \geq 0, K_t \geq 0, \beta_t, \omega_t, e_t), \forall t \geq 0} \mathbb{E} \left[ \int_0^\infty e^{-rt} \left( \frac{C_t}{q(e_t)} - K_t \mu(e_t, \phi_t, 0) \right) \, dt \right] \bigg| \mathcal{F}_0
\]

s.t. \( C_t^{-\rho} \lambda \nu(e_t) K_t + \eta(e_t) \xi_t \leq \beta_t \quad \forall t \geq 0 \)

\[
dV_t = \left( \delta V_t - \frac{C_t^{1-\rho}}{1-\rho} \right) dt + \beta_t dW_t^C
\]

\[
d\xi_t = (\delta \xi_t - \eta(e_t) \beta_t (1 - \phi_t)) dt + \omega_t dW_t^C
\]

\[
d\phi_t = \eta(e_t) \phi_t (1 - \phi_t) dW_t^C,
\]

\( J^* \) represents the cost function of the principal.

Problem (13) is clearly challenging to deal with analytically. However, it is possible to characterize some of its properties. Consider its associated Hamilton-Jacobi-Bellman equation (HJB),

\(8\)While full commitment from both the agent and the principal is certainly a strong assumption, this formulation of the problem offers a key benchmark for alternative specifications. With full commitment, the principal implements an allocation that yields the best outcome given the frictions of the model. After relaxing the full commitment assumption in Section 7, I will highlight which results hold independently of the contracting assumptions and which results depend instead on the particular contractual form that is implemented.
\( rJ^*(V, \xi, \phi) = \)

\[
\inf_{C,K,\beta,\omega,e} \left\{ \frac{C}{q(e)} - (\alpha + es\phi)K \right. \\
+ J^*_V(V, \xi, \phi) \left( \delta V - \frac{C^{1-\rho}}{1-\rho} \right) + J^*_\xi(V, \xi, \phi) \left( \delta \xi_t - \eta(e_t)\beta_t(1-\phi_t) \right) \\
+ \frac{1}{2}J^*_V(V, \xi, \phi)\beta^2 + \frac{1}{2}J^*_\xi(V, \xi, \phi)\omega^2 + \frac{1}{2}J^*_\phi(V, \xi, \phi)\eta(e)^2\phi^2(1-\phi)^2 \\
+ J^*_V\xi(V, \xi, \phi)\beta\omega + J^*_\phi(V, \xi, \phi)\beta\eta(e)(1-\phi)J^*_\phi(V, \xi, \phi)\omega\eta(e)(1-\phi) \left. \right\}
\]

(14)

First, we can verify that, in any optimal contract, the incentive compatibility condition holds with equality, i.e.

\[ K_t = \frac{\beta_t - \eta(e_t)\xi_t}{\lambda\nu(e_t)} C_t^\rho. \]  

(15)

This is because excess returns are always strictly positive and the advisor collects positive fees. So, for any choice of the other control variable, the advisor would like to collect as much capital as the incentive compatibility constraint permits.

Second, the HJB equation (14) is subject to a boundary condition \( J^*(V, 0, \phi) = J^0(V) \), where the function \( J^0(V) \) is the cost function for a principal in the case in which \( h = 0 \) is common knowledge\(^9\). This boundary conditions comes from the promise keeping constraint with respect to the information rent \( \xi \). If the principal promised zero information rent to the agent, the only way to maintain this promise is to either provide no incentives \( \beta \), and hence no capital \( K \), to the manager, and/or to stop experimentation \( e \) forever. The principal clearly prefers to stop experimentation only, since she can still obtain fees equal to the baseline excess return \( \alpha \) by providing capital to the manager.

As a remark, note that we cannot obtain analogous boundary conditions for \( J^*(V, \xi, 0) \) and \( J^*(V, \xi, 1) \). Beliefs \( \phi \) have no drift and their volatility vanish as they approach 0 and 1. These singular points, however, do not constitute a problem. We can indeed think of the HJB (14) as a state constraint problem where beliefs are constrained between zero and one. Katsoulakis (1994) and Alvarez et al. (1997) show that state constraints effectively replace the role of boundary conditions in determining the solutions to partial differential equations.

Finally, we can derive an endogenous bound for the information rent \( \xi \). Suppose that \( J^*(V, \xi, \phi) \) is convex in \( \xi \). I will formally establish this property later in Corollary 1. Note that, at time 0, the advisor has no initial commitment to any information rent. As illustrated in Definition 4, the op-

\(^9\)\( J^0(V) \) satisfies the HJB equation

\[
rJ^0(V) = \inf_{C,\beta} \left\{ C - \frac{\alpha\beta}{\sigma\lambda}C^\phi + J^0_V(V) \left( \delta V - \frac{C^{1-\rho}}{1-\rho} \right) + \frac{1}{2}J^0_{VV}(V)\beta^2 \right\}.
\]
Optimal contract is initially defined only in terms of the initial promised value of the agent, $V_0$, and initial beliefs $\phi_0$. Therefore, we can think of the advisor as first setting an initial information rent $\xi_0$, and then committing to it and solve problem (13). The advisor chooses an initial information rent that minimizes her costs, so she will pick $\xi_0$ such that

$$J^*_{\xi}(V_0, \xi_0, \phi_0) = 0.$$ 

We can further characterize the behavior of the information rent and its marginal cost $J^*_{\xi}$ in the optimal contract. For any pair of continuation value $V$ and beliefs $\phi$, define $\bar{\xi}(V, \phi)$ the marginal rent that minimizes costs for the principal, i.e. $J^*_{\xi}(V, \xi(V, \phi), \phi) = 0$. Using the envelope theorem and the convexity of the value function with respect to the information rent, I can prove the following properties of the optimal contract.

**Proposition 6.** In the optimal contract, these two properties hold:

I. The marginal cost of information rent is always non-positive

$$J_{\xi}(V_t, \xi_t, \phi_t) \leq 0 \quad t \geq 0;$$

II. For all $t \geq 0$,

$$\xi_t \leq \bar{\xi}(V_t, \phi_t).$$

Information rents, in contracting model, tend to be bounded and propositions analogous to 6 have been derived in DeMarzo and Sannikov (2017) and He et al. (2017). This is because large information rents expose the agent to risk that are irrelevant for incentive provision. Consequently, the principal avoids to promise excessive information rents.

While it is possible to derive the general properties of the optimal contract that I have just discussed, numerically analyzing the HJB equation (14) is far from easy, if not far from feasible. Proposition 6 offers the reason. In the optimal contract, the information rent is bounded by $\bar{\xi}(V, \phi)$. For values of information rent far above this bound, optimal contracts might not exist, or they might violate regularity properties required for the validity of a dynamic programming approach. In order to solve for the optimal contract, the bound $\bar{\xi}(V, \phi)$ is needed. However, this bound depends on the optimal contract itself. One could attempt to numerically solve (14) by conjecturing upper bound functions $\bar{\xi}(V, \phi)$. This attempt would be extremely computationally inefficient, if not unfeasible. I take a completely different approach.

Proposition 6 also offers the intuition to solve the model efficiently. In the optimal contract, the marginal cost $J_{\xi}(V_t, \xi_t, \phi_t)$ is bounded above zero, thus always satisfying the bound on the information rent. I will then formulate a problem where the marginal cost $J_{\xi}(V_t, \xi_t, \phi_t)$ replaces the information rent $\xi$ as a state variable. This problem is connected to the initial one through a duality relation. In the remainder of this section, I introduce the dual problem of (13), derive its properties and show that the dual problem offers an efficient way to characterize the contract.
4.1 THE DUAL PROBLEM

I now introduce the dual problem of (13). Informally speaking, the purpose of the dual problem is to replace the information rent, $\xi_t$, with its marginal values, $J_\xi(V_t, \xi_t, \phi_t)$, as a state variable.

Define the multiplier

$$Y_t = e^{t(r-\delta)} \left[ - \left( \int_0^t e^{s(\delta-r)} \mu(e_t, \phi_t, 0) \frac{\eta(e_t)}{\lambda \nu(e_t)} C^\rho_t \, ds \right) + Y_0 \right]$$

where $Y_0 \in \mathbb{R}$ and consider the following problem.

$$G^*(V_0, Y_0, \phi_0) = \inf_{(C_t \geq 0, K_t \geq 0, \beta_t, e_t), \forall t \geq 0} \mathbb{E} \left[ \int_0^\infty e^{-rt} \left( \frac{C_t}{q(e_t)} - \mu(e_t, \phi_t, 0) \frac{\beta_t}{\lambda \nu(e_t)} C^\rho_t - Y_t \eta(e_t) \beta_t \phi_t (1 - \phi_t) \right) \, dt \middle| \mathcal{F}_0 \right]$$

s.t. $dV_t = (\delta V_t - u(C_t)) \, dt + \beta_t dW^C_t$

$$dY_t = (r - \delta) Y_t \, dt - \mu(e_t, \phi_t, 0) \frac{\eta(e_t)}{\lambda \nu(e_t)} C^\rho_t \, dt$$

$$d\phi_t = \eta(e_t) \phi_t (1 - \phi_t) dW^C_t.$$ (16)

I call (16) the dual problem, as opposed to problem (13) which, from now, I call the primal problem.

The dual problem in (16) has an intuitive meaning. Consider the objective function. The term $\frac{C_t}{q(e_t)} - \mu(e_t, \phi_t, 0) \frac{\beta_t}{\lambda \nu(e_t)} C^\rho_t$ represents the flow cost of an advisor who runs a fund with no information rent problem. This could be a fund where the hidden action $m_t$ is observable but not contractible. In this case, the incentive compatibility condition is the same as in Lemma 3, since shirking does not induce any belief distortion and no information rent for the manager. The terms $-Y_t \eta(e_t) \beta_t \phi_t (1 - \phi_t)$ represent a penalty for an advisor who sets large incentives $\beta$ or fast learning through a large signal-to-noise ratio $\eta(e)$. The severity of the penalty is measured by the multiplier $Y_t$. A more negative multiplier $Y_t$ gives the advisor a larger penalty for incentives and learning. In the dual problem (16), the advisor replaces the information rent of the manager with the multiplier $Y$ as a relevant state variable.

The dual cost function $G^*$ possesses some intuitive and convenient properties

**Lemma 4.** The dual cost function $G^*$ is decreasing and concave in the multiplier $Y$. Moreover, $G^*$ is decreasing in $\phi$.

The concavity of the dual cost function $G^*$ in the multiplier $Y$ is important since it allows to map the dual problem (16) into the primal problem (13).

After this introduction of the dual problem, I can finally draw the connection with the primal problem (13).
**Proposition 7.** The primal cost function $J^*$ and the dual cost function $G^*$ are related by

\[ J^*(V, \xi, \phi) = \sup_Y \{ G^*(V, Y, \phi) + Y\xi \} \]
\[ G^*(V, Y, \phi) = \inf_\xi \{ J^*(V, \xi, \phi) - Y\xi \} \]

Moreover, a solution to the dual problem with multiplier $\Lambda$ yields an optimal contract with information rent $\xi = -G^*_Y(V, Y, \phi)$. Vice versa, the optimal contract with information rent $\xi$ can be obtained from the dual problem when the multiplier is given by $Y = J^*_\xi(V, \xi, \phi)$.

Proposition 7 is extremely powerful. It states that any optimal contract can be obtained as a solution to the dual problem if the initial multiplier is chosen appropriately. Moreover, a solution of the dual problem is an optimal contract for some information rent.

Combining Proposition 7 with Proposition 6 we can immediately obtain a characterization of the optimal contract through the dual formulation.

**Corollary 1.** The optimal contract is the solution to the dual problem (16) with $Y_0 = 0$. For all $t \geq 0$, $Y_t \leq 0$.

This Corollary is key to overcome the computational challenges that I discussed in my description of the primal problem. Looking at the law of motion of the multiplier $Y_t$ in (16) we see that, by construction, it will always be non-positive as long as $Y_0$ is non-positive. Since, for a given information rent $\xi_t$, $Y_t = J^*_\xi(V_t, \xi_t, \phi_t)$, the dual problem will automatically satisfy the endogenous bound on the information rent. Therefore, standard numerical schemes will suffice to obtain a solution for the optimal contract.

Before numerically solving the dual problem, I develop some further properties of the dual value function $G^*$ in order to simplify the numerical computations. In particular, using the homogeneity of the problem, it is possible to write the dual cost function as

\[ G^*(V, Y, \phi) = \hat{v}g^*(y, \phi) \]

where

\[ \hat{v} = ((1 - \rho)V)^{\frac{1}{1-\rho}} \]

is the consumption equivalent of the agent’s continuation value, where and

\[ y = (1 - \phi)\hat{v}^{-\rho}Y \]

is a scaled version of multiplier $Y$. We can then define the scaled control variables

\[ c_t = \frac{C_t}{\hat{v}_t^{-\rho}} \quad k_t = \frac{K_t}{\hat{v}_t^{-\rho}} \quad \hat{\beta}_t = \frac{\beta_t}{(1 - \rho)V_t} \]

and derive the law of motion of continuation value $\hat{v}$ and multiplier $y$ through Ito’s Lemma, thus
obtaining
\[
d\hat{v}_t = \left( \frac{\delta}{1 - \rho} - \frac{c_1^{1-\rho}}{1 - \rho} + \frac{1}{2}\rho\hat{\beta}^2_t \right) \hat{v}_t dt + \hat{\beta}_t \hat{v}_t dW^c_t
\]
and
\[
dy_t = -(1 - \phi_t) \mu(e_t, \phi_t, 0) \frac{\eta(e_t)}{\lambda\nu(e_t)} c^\rho_t dt + y_t \left( r - \frac{\delta}{1 - \rho} + \rho \frac{c_1^{1-\rho}}{1 - \rho} + \frac{1}{2}\rho\hat{\beta}^2_t + \rho \eta(e_t) \hat{\beta}_t \phi_t \right) dt - y_t \left[ \rho \hat{\beta}_t + \eta(e_t) \phi_t \right] dW^c_t .
\]

Finally, using the homogeneity property of the dual cost function, the HJB equation that the cost function \( g^* \) satisfies is given by
\[
rg^*(y, \phi) = \inf_{c, \beta, e} \left\{ \frac{c}{q(e)} - (\alpha + e\phi) \frac{c^\rho}{\nu(e)\lambda} - y \hat{\beta} \eta(e) \phi (1 - \phi) + g^*(y, \phi) \left( \frac{\delta}{1 - \rho} - \frac{c_1^{1-\rho}}{1 - \rho} + \frac{1}{2}\rho\hat{\beta}^2_t \right) \right. \\
+ \left. g^*_y(y, \phi) \left[ -(1 - \phi_t) \mu(e_t, \phi_t, 0) \frac{\eta(e_t)}{\lambda\nu(e_t)} c^\rho_t dt + y_t \left( r - \frac{\delta}{1 - \rho} + \rho \frac{c_1^{1-\rho}}{1 - \rho} + \frac{1}{2}\rho\hat{\beta}^2_t + \rho \eta(e_t) \hat{\beta}_t \phi_t \right) \right] \\
- g^*_y(y, \phi) y \left[ \rho \hat{\beta} + \eta(e) \phi \right] + g^*_\phi(y, \phi) \eta(e) \phi (1 - \phi) \hat{\beta} \\
+ \frac{1}{2} g^*_{yy}(y, \phi) y^2 [\rho \hat{\beta} + \eta(e) \phi]^2 + \frac{1}{2} g^*_{\phi\phi}(y, \phi) \eta(e)^2 \phi^2 (1 - \phi)^2 \\
- g^*_{y\phi}(y, \phi) y [\rho \hat{\beta} + \eta(e) \phi] [\eta(e) \phi (1 - \phi)] \right\} .
\]

In the next Section, I will numerically solve the HJB equation (19) and characterize the implications of the optimal contract for the fund flows and the manager’s compensation. In interpreting the results, I will often refer to the HJB equation (19), to the dynamics of the multiplier \( y \) given by (18) and to the properties of the dual cost function \( G^* \) which are summarized in Lemma 4 and which are inherited by the function \( g^* \).

## 5 Results and Discussion

I solve the model using finite difference. Given the homogeneity of the problem, I can solve a two-dimensional partial differential equation where the state variables are beliefs \( \phi \) and the multiplier \( y \). In order to solve for the optimal contract, it is enough to restrict the state space to \( y \leq 0 \). Since the drift of \( y \) is negative at \( y = 0 \) and its volatility is 0, I do not need to impose any restrictions on the control variables to satisfy the state constraint.
Table 1: Model Parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Value</th>
<th>Source/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest rate</td>
<td>$r$</td>
<td>2.6%</td>
<td>Average real rate from 1990</td>
</tr>
<tr>
<td>Baseline excess return</td>
<td>$\alpha$</td>
<td>0.1%</td>
<td>Minimum fee in Pastor et al. (2015)</td>
</tr>
<tr>
<td>Baseline volatility</td>
<td>$\sigma$</td>
<td>18%</td>
<td>High vol funds, Pastor et al. (2015)</td>
</tr>
<tr>
<td>Discount rate of manager</td>
<td>$\delta$</td>
<td>5%</td>
<td>Di Tella and Sannikov (2018), DeMarzo et al. (2012)</td>
</tr>
<tr>
<td>Risk-aversion / IES$^{-1}$</td>
<td>$\rho$</td>
<td>1/3</td>
<td>Di Tella and Sannikov (2018)</td>
</tr>
<tr>
<td>Returns to skills</td>
<td>$s$</td>
<td>1.5%</td>
<td>High alpha funds, Fama and French (2010)</td>
</tr>
<tr>
<td>Cost of learning</td>
<td>$\bar{q}$</td>
<td>0.15</td>
<td>Convexity of cumulative flows</td>
</tr>
<tr>
<td>Curvature of learning cost</td>
<td>$d$</td>
<td>2.1</td>
<td></td>
</tr>
<tr>
<td>Gains from mismanagement</td>
<td>$\lambda$</td>
<td>0.85</td>
<td>Flow-performance slope</td>
</tr>
</tbody>
</table>

5.1 Calibration

In order to select the parameters of the model, I rely on three main strategies. If a parameter can be directly observed, I will use empirical observations of that parameter value. If a parameter cannot be directly observed, but bears a strong connection to an outcome of my model that can be observed in the data, I will select values of the parameter that yield outcomes that match the data. Finally, I discipline unobservable preference parameters by selecting values previously adopted in the contracting literature. Table 1 summarizes the parameter choice.

I set the interest rate $r$ equal to the average real rate starting from the year 2000. In order to select a value for the baseline excess return, $\alpha$, I consider the sample of mutual funds that I select in Section 6. As in Pastor et al. (2015), I exclude all funds that charge annual fees below 0.1%, since they are unlikely to be actively managed. I therefore take this value to represent the minimum excess return that a fund is able to provide.

I then consider daily mutual fund data from CRSP starting from the year 2000. From this sample, I consider only US equity mutual funds that are not index funds or ETFs. I compute the daily excess return for every fund relative to its style benchmark. Then, for every quarter, I compute each individual fund’s average daily return and daily standard deviation. I then compute the correlation between quarterly returns and quarterly volatility, finding essentially no correlation. I therefore set the volatility-performance slope, $b/s$ to 0. This value is also consistent with the findings of Jordan and Riley (2015) who show that, after controlling for a volatility factor, funds’ performance is uncorrelated with their return volatility.

Due to the linearity of the model, the volatility of returns must be large enough in order to guarantee the existence of finite solutions for the contracting problem. Intuitively, if the volatility of returns is too low, than the advisor could easily detect mismanagement. She would therefore find it very easy to provide incentives to the manager and could exploit the constant returns to scale of the technology to gain unbounded revenues. By adding decreasing returns in actively managed assets, as in Berk and Green (2004), I could easily avoid the possibility of unbounded
returns and would therefore have additional degrees of freedom in the choice of parameters. Unfortunately, this would also complicate the problem substantially, since the homogeneity property of the dual cost function $G^*$ would fail. I therefore choose a value of the volatility $\sigma$ of 18%. This is close to, but a little larger than, the highest percentile of the standard deviation of abnormal returns in Pastor et al. (2015), which is of 16.4% on an annualized basis.

I choose the agent’s preference parameters in order to match choices made in previous contracting literature. I set the discount rate of the manager, $\delta$, equal to 5% as in DeMarzo et al. (2012) and Di Tella and Sannikov (2018), while for the inverse of the agent’s IES, I select $1/3$ as in Di Tella and Sannikov (2018).

I assume that the experimentation cost is a power function of experimentation, i.e. $q(e) = 1 - \bar{q}e^d$ for some parameters $\bar{q}$ and $d$. I set $d$ to 2.1 in order to obtain sufficient convexity of cumulative flows in cumulative performance. The parameter $\bar{q}$ cannot be separately identified from $s$, as long as $e$ remains in the interior of the $[0, 1]$ set. I therefore fix $q$ to 0.15 and use excess return data to pick the value of $s$.

I consider the distribution of four-factor gross alpha in Fama and French (2010). They find that the 90th percentile of the distribution is 1.3%. I take this number to represent the excess returns of highly skilled managers. I therefore set $s = 1.5\%$ so that a fund with $\phi = 1$ and multiplier $y = 0$ yields a return of 1.3%.

Finally, I set the gains from mismanagement, $\lambda$, to 0.85. In this way, the flow-performance sensitivity of funds with $\phi = 0.5$ and with multiplier $y \in [-0.1, 0]$ lies between 15% and 20%. These values match the estimates of the flow-performance sensitivity that I will present in Tables 3, 5 and 6.
Figure 2: Convex relation between cumulative flows and cumulative returns. The solid blue curve represents the cumulative flows as a function of cumulative returns. The dashed red line is the tangent line at 0. Curves are shifted so to represent the flows relative to a fund that has a zero cumulative return. Performance and flows are computed over one year. In 2a, fees are assumed to be fixed and flows include changes in actively managed capital, as well as changes in the holdings of a passive, agency-free index. In 2b, fees match expected returns and flows include only changes in actively managed capital.

5.2 FLOW-PERFORMANCE RELATION

The flow-performance relationship $\epsilon_K$ the percentage change in assets under management for a 1%, i.e. $\epsilon_K = \frac{dK_t}{K_t}$. Using the fact that, in the optimal contract, $K_t = \hat{v}_t k_t$, for some optimal control $k_t$, the flow-performance relationship can be measured as

$$\epsilon_K = \frac{1}{\nu(e)} \left( \frac{\sigma_k}{\bar{k}} + \beta \right)$$

where $\sigma_k$ is the volatility of $k_t$.

As Figure 1a shows, the flow-performance relation $\epsilon_K$ is positive, meaning that assets under management increase after good performance. This is consistent with several empirical results, including the ones in Section 6 of this paper, that show a positive flow-performance slope.

The prediction that sets my model apart from previous literature, like Berk and Green (2004), is that the flow-performance slope increases after good performance. We can see this in Figure 1a, where the flow-performance relation $\epsilon_K$ increases when when the posterior $\phi$ and the multiplier $y$ increase. Since these two state variables have positive volatility, this implies that the flow-performance slope increases after good performance. This result is new to the literature and no other theoretical or empirical paper has established the history dependence of the flow-performance relation. I will test this prediction in mutual fund data in Section 6.

Since the flow-performance slope $\epsilon_K$ increases in past performance, cumulative flows are a convex function of cumulative performance. As an illustration, Figure 2 plots the one-year flow into a fund with $y = 0$ and $\phi = 0.75$ as a function of the one-year cumulative return. In order
to make the example more empirically relevant, in Figure 2a I assume that the fund charges a
fixed fee and collects additional capital to be invested in a costless, agency-free index. We can
clearly observe that the net flows into the fund are a convex function of the cumulative returns.
Unlike previous theories (Berk and Green, 2004) the convexity that we observe on a yearly basis
in Figure 2a does not derive from any contemporaneous convex relation between fund flows and
performance. On the contrary, the convexity arises since, as good performance accumulates over
time, additional positive returns have a stronger impact on additional flows.

My model distinguishes itself from the implicit incentive model of Berk and Green (2004),
as it does not rely on specific functional form assumption or fee-setting restrictions. Under the
decreasing-return form assumptions made by Berk and Green (2004), flows of actively managed
capital are a linear function of returns. Convexity arises as the manager, being unable to change
fees, needs to adjust the amount of capital he invests in a passive index the In 2b, I allow the
fund to change fees in order to match its expected returns so that these flows reflect the change
in the actively managed capital. As predicted by my model, also actively manged capital flows
are convex in cumulative performance and the result does not rely on assumptions on decreasing
returns to scale or restriction fee changes.

By establishing that the flow-performance slope is increasing in past performance, my model
provides a new economic mechanism to justify the convex relation between cumulative flows and
cumulative performance. In my model, as returns accrues over time, they will change the relation
between future returns and future flows. This mechanism is completely absent in other models
of flow-performance convexity, which model only implicit incentives for the fund and ignore the
moral hazard problem and the optimal contracts that are designed in response. For example, in
Berk and Green (2004) past returns have no effect on the flow-performance relationship. In their
paper, the flow-performance slope and the its convexity decline deterministically over time.

**Economic Mechanism** The flow-performance slope increases after good returns in response
to the two main forces of the model: learning and moral hazard. On the one hand, learning about
the manager’s skills naturally link past performance to future expected performance. If players
observe good (bad) returns in the past they will expect good (bad) returns also in the future. On
the other hand, the optimal contract designed in response to managerial moral hazard stipulates
that the flow-performance sensitivity increases when expected returns increase. The intuition for
this mechanism requires some further elaboration.

Capital and incentives are tightly linked by the incentive compatibility condition. In order to
actively manage more capital, the agent must face steeper incentives. Therefore, providing capital
to the manager is costly. However, capital also brings revenues to the advisor. When expected
returns increase, this trade-off moves in favor of more assets under management, since the advisor
can collect higher fees per unit of capital. Moreover, after a good return, the multiplier \( y \) increases
(it becomes less negative) and, hence, the advisor faces a smaller penalty to provide incentives
\( \beta \). Consequently, after a history of positive performance, the advisor wants to increase capital
Incentives must increase too. This explains why $\hat{\beta}$ increases with beliefs $\phi$ and with multiplier $y$, thus driving the variation in the flow-performance sensitivity $\varepsilon_K$.

We can gain some further intuition for the reason why incentives $\hat{\beta}$ determine the volatility of flows in the first place. Since assets are tied to the risk that the manager bears, his risk tolerance determines how costly it is to provide capital to the manager. If the manager is wealthier, i.e. has a larger promised value $\hat{v}$, then he will be more tolerant to risk. If the promised value of the agent is very volatile, i.e. $\hat{\beta}$ is very large, then his future risk tolerance will also be very volatile. This means that future flows will also be very volatile, as they will have to reflect the realizations of the agent’s risk tolerance.

Therefore, while learning links expected returns to past performance in a straightforward way, the optimal contract links the flow-performance sensitivity to expected returns by balancing the costs and the benefits of flows in an intertemporal perspective. Current flows require higher risk for the manager. However, higher risk for the manager result in ex-post volatile capital flows.

5.3 Pay for Performance and Back-Loading

Similarly to capital, I define the pay-performance sensitivity as the percentage change in compensation for a 1% increase in returns, i.e. $\epsilon_C = \frac{dC_t/C_t}{dR_t}$. Since in the optimal contract compensation takes the form $\tilde{C}_t = \hat{v}_t \tilde{c}_t$ for some optimal control $\tilde{c}_t$, then pay-performance sensitivity takes the form

$$\epsilon_{\tilde{C}} = \frac{1}{\nu(\epsilon)} \left( \frac{\sigma_{\tilde{c}}}{\tilde{c}} + \hat{\beta} \right)$$

where $\sigma_{\tilde{c}}$ is the volatility of $\tilde{c}$.

From Figure 3a, we can observe that the contract implies a bonus for good performance, since the value of the pay-performance sensitivity, $\epsilon_{\tilde{C}}$, is always positive. Moreover, since the pay-performance sensitivity $\epsilon_{\tilde{C}}$ is increasing in both beliefs $\phi$ and the multiplier $y$, compensation will
appear convex in cumulative performance. This is consistent with the widespread use of convex pay schemes in the money management industry. For example, Ma et al. (2019) documents that mutual fund managers’ compensation is often composed by a base salary plus a bonus for good performance. The mechanism driving the convexity of cumulative compensation with respect to cumulative performance is identical to the one behind the convexity of cumulative capital flows.

In addition to the pay-performance sensitivity, we can obtain additional insights about the back-loading or front-loading of compensation by looking at the volatility of current compensation relative to future promises, \( C/\hat{v} \). The volatility of this ratio captures how consumption changes relative to promised value for a given return shock. If the volatility of \( C/\hat{v} \) is positive, then consumption increases relative to promised value after good performance. I interpret this outcome as meaning that the principal front-loads compensation after good performance. If, instead, the volatility of \( C/\hat{v} \), then the principal is back-loading compensation since, after a good shock, she will increase future promises more than current compensation.

The optimal contract implies that consumption is back-loaded after good performance, as shown in Figure 3b. This result is consistent with common practices in the mutual fund industry which effectively postpone the delivery of compensation to the agent. For example, Ma et al. (2019) show that more than 30% of mutual fund managers are subject to deferred compensation schemes. At the same time, they illustrate that the vast majority of the pay-for-performance bonuses rely on the average return over multiple years (on average, three years) to determine the bonus. This latest feature effectively implies that, following good performance, the manager can expect an increase in compensation for the next few years, thus effectively back-loading his compensation.

**ECONOMIC MECHANISM** The manager is paid for his performance as a consequence of the optimal incentive contract with the advisor. In order to provide incentives to the manger, the advisor needs to deliver risky compensation to the manager. After good performance, the advisor rewards the agent with higher compensation while, after bad performance, the agent is punished with lower compensation. However, the riskiness of the compensation must be limited, in an optimal contract, since the agent is risk-averse. With the exception of highly productive managers, who are subject to very steep incentives, managers face a compensation path that is smoother than returns. This is consistent with the low point estimates of the pay-performance sensitivity in Ibert et al. (2018).

Moreover, the pay-performance sensitivity increases after good performance, similarly to the flow-performance sensitivity. After good performance, the advisor expects higher returns from the manager. Therefore she desires to allocate more capital to the manager. However, in order to offset the rents that the manager acquires form a larger amount of assets under management, the principal needs to expose the agent to more risk. Eventually, this higher risk exposure for the manager translates into a more risky compensation scheme.

In order to understand the reason why the advisor back-loads the manager’s compensation,
consider the trade-off that changes in compensation involve. By increasing the compensation-to-promised value ratio, the principal decreases the growth rate of the manager’s promised value, which has two effects. On the one hand, a larger promised value is costly for the principal who has to deliver larger future compensation. On the other hand, the principal can extract some benefits from a larger promised value. If the manager has a large promised utility, he can tolerate higher absolute levels of risk. Therefore, the principal can exploit this increased risk tolerance to expose the manager to more risk. This enables the principal to increase assets under management and fee revenues.

After a good return, the trade-off tilts in favor of a larger continuation value and, thus, of a lower compensation-to-promised value ratio. As for the costs of the promised value, the principal can deliver any given promised utility at a lower cost since, after a good shock, the manager is expected to be more productive. As for the benefits, since the principal now expects the manager to generate higher returns, she has stronger incentives to increase the agent’s assets under management. Hence, the principal desires to increase the absolute risk tolerance of the manager through a higher promised value. Consequently, after good performance, the principal reduces the ratio of compensation to promised utility, thus generating a back-loaded consumption path.

5.4 Long-Term Implications and the Dynamics of Multiplier $y$

In order to later interpret the outcomes of the model in terms of flow, performance and compensation, I need to first discuss the dynamics of the state variables. While beliefs $\phi$ follows a martingale with positive volatility, the multiplier, $y$, has negative drift and positive volatility, as shown in Figure 4. The volatility of the marginal cost $y$ is negative by construction. Therefore, after a good return, the multiplier $y$ will unambiguously increase.

A full commitment contract bears implications for flows, incentives and performance in the
long run. While beliefs $\phi$ are martingales, the multiplier $y$ drifts down over time. The negative drift of $y$ is a robust outcome that I find across all the parameterizations that I have explored. In order to understand the consequences of a negative drift on flows, incentives and performance, recall that we can interpret $-y$ as a penalty on incentives and learning. If $y$ becomes more negative, the principal faces a large penalty for incentives $\beta$ and expected performance through its effect on learning $\eta(e)$. Consequently, by giving a negative drift to $y$, the principal is committing herself to reduce incentives and expected performance over time.

By reducing incentives $\beta$ over time, the principal is reducing the flow-performance sensitivity and the pay-performance relation. Figures 1a and 3a show that, for increasingly more negative values of the multiplier $y$, the relation of flows and pay to performance becomes increasingly smaller. Moreover, for very negative values of the multiplier $y$, the flow-performance sensitivity is barely affected by past performance, thus suggesting that the history-dependence of the flow-performance sensitivity is, on average, substantially reduced for managers with long tenure. I test this prediction in Section 6.3.

In the optimal contract with full commitment, the principal also commits to lower the productivity of the agent over time. In Figure 5, we can see that, for very negative values of multiplier $y$, the principal requires lower experimentation $e$ from the agent, which results in lower expected returns. Almazan et al. (2004) show that older fund manager are subject to more investment constraints, which, effectively, may limit the informativeness of their returns. As long as age and tenure correlate, my model provides one additional framework to interpret these empirical findings. Consistently with an optimal contract, more experienced managers may simply be required to undertake less sophisticated investments and to be subject to lower risk.

**Economic Mechanism.** In order to understand why the principal wants to reduce the productivity and the incentives of the agent over time, it is instructive to consider the incentive con-
straint and the manager’s information rent of the agent once again. The manager’s information rent is costly since it is a risk for the manager that the advisor cannot exploit to provide incentives. Moreover, from equation 11, the manager’s information rent corresponds to a present value of future incentives and learning. Therefore, if a contract implies large learning rates and steep incentives in the future, it will also imply a large information rent today. Consequently, in designing the optimal contract, the principal is willing to ex-ante commit to decrease learning and incentives over time. By taking ex-post inefficient actions, she can reduce the ex-ante information rent of the agent, and thus relax the current incentive compatibility condition (8).

Full commitment by the principal is crucial for this result. After any period of time, the principal would be better off if she could re-negotiate the contract, leaving the agent with an unchanged continuation value, but increasing his information rent to the point at which the multiplier $γ$ is equal to zero. In Section 7, I study contracts where there is no commitment by a principal to an information rent. As expected, the long-term decline in performance and incentives cannot be observed. However, all the other predictions of the model in terms about the flow-performance relation, the pay for performance and the consumption back-loading continue to robustly hold regardless on the commitment of the principal.

In Appendix A I provide a simple two period model to further highlight the mechanism behind the inefficient ex-post learning. This two period model is substantially different from the dynamic model of the paper. It therefore brings support to my claim that this result is a general outcome of contracting models under uncertainty.

6 Empirical Tests

As shown in the previous section, the model implies a novel explanation for the convex flow-performance relationship. In my model, the slope of the relationship depends on the history of past returns. Everything else being equal, if a fund experienced good performance in the past, the slope of its flow-performance relationship will be steeper.

I test the model’s prediction in mutual fund data. I focus on US mutual funds investing in domestic equity, which provide a sample of money managers with very liquid and reasonably ample investment opportunities. In principle, the general economic mechanism of my model could apply to other money managers like hedge funds, private equity and bond funds. However, previous studies point out that these intermediaries tend to undertake more illiquid investments. These frictions on the assets side of the intermediary would mechanically introduce concavity in the flow-performance relationship. Since my model, for tractability reasons, abstracts from illiquidity and size constraints, the sample of US equity mutual fund constitutes the natural testing ground of my theory.
6.1 Data and Variables of Interest

My sample of mutual funds is from Center for Research in Security Prices (CRSP) database. Although CRSP provides data starting from 1962, I restrict the sample to monthly observations starting from January 1984. As Fama and French (2010) and Elton et al. (2001) observe, before 1983 there is a considerable selection bias in funds that report monthly returns.

I include only actively managed funds that invest in U.S. stocks, thus excluding index funds, ETFs, bond and commodity funds. In order to avoid the incubation bias identified by Evans (2010), I consider only funds that have at least two years of age. I also exclude funds whose assets under management are below 15 million of 2011 dollars. I finally restrict my attention to funds that are open to new investors.

CRSP reports net monthly returns and annual expense ratio for every share class. I compute the gross returns of every share class by adding 1/12 of the expense ratio to the net monthly return. I then compute fund-level gross returns and expense ratios by taking a weighted average of these quantities across every fund’s share classes. As Pastor et al. (2015) observe, actively managed funds are unlikely to charge less than a 0.1% annual fee. Thus, following their procedure, I exclude observations whose fees are below the 0.1% threshold, since these observations might represent index funds or data entry mistakes.

My model yields predictions about the relation between flows of capital into a fund, its idiosyncratic returns and its past performance. I measure the net flow of capital into fund $i$ at time $t + 1$ as

$$ F_{it+1} = \frac{K_{it+1} - K_{it}}{K_{it}} - R_{it+1}^N $$

(20)

where $K_{it}$ are the assets under management in fund $i$ at time $t$ and $R_{it+1}^N$ is the net return that fund $i$ delivers to investors from time $t$ to time $t + 1$.

In order to measure the idiosyncratic return of a fund, I consider its gross return relative to the average gross return of funds with the same style, i.e., I consider the excess return

$$ \tilde{R}_{it} = R_{it} - \bar{R}_{s(i)t} $$

(21)

where $R_{it}$ is the gross return of fund $i$ from $t$ to $t + 1$, $s(i)$ is the style of fund $i$ (as described by the CRSP objective classification) and $\bar{R}_{s(t)}$ is the average return of all funds with the same style $s$.

Finally, I capture a fund’s past performance by considering the average excess return of the fund over the previous $k$ months,

$$ \tilde{\mu}_{jt-1} = \frac{1}{k} \sum_{j=1}^{k} \tilde{R}_{it-j} $$

(22)

and I will consider $k = 3, 6, 9$.

Berk and van Binsbergen (2015) and Pastor et al. (2015) document that the CRSP database contain data entry mistakes, some of them representing large outliers in the distribution of flows.
and returns. To avoid bias in my statistical analysis, I remove the tails of the distributions of capital flows and gross returns, keeping only observations within the 1st and 99th percentile.

6.2 Prediction 1: History-Dependence of the Flow-Performance Relationship

The first prediction of the model can be summarized as follows.

**Prediction 1.** The slope of the flow-performance relationship is increasing in past performance.

Econometrically, I test this prediction by running a linear regression of fund flows on past returns, as

\[ F_{it+1} = a_1 \tilde{R}_{it} + a_2 \tilde{\mu}_k^{it-1} \tilde{R}_{it} + c'X_{it} + \epsilon^F_i + \epsilon^{S}_{s(i)t} + u_t \]  

I test the model’s novel prediction by verifying that, in the data, \( a_2 > 0 \). A positive coefficient on the term that capture the interaction between returns and past performance, \( \tilde{\mu}_k^{it-1} \tilde{R}_{it} \) implies that the flow performance slope is an increasing function of past performance. While the model predicts that also \( a_1 \) is positive,

Controls \( X_{it} \) include the square of the monthly return, \( \tilde{R}_{it}^2 \), in order to show that, at monthly frequency, no convexity can be detected. The additional controls in \( X_t \) include lagged returns, lagged flows, the logarithm of assets under management, the logarithm of the manager’s tenure and fund’s fees. I also control for fund fixed effects, \( \epsilon^F_i \) and style-month fixed effects, \( \epsilon^S_{s(i)t} \).

In Table 2, I report the summary statistics of the data used in my empirical analysis.

6.2.1 Results

Table 3 provides estimates of model (23) where the past performance \( \tilde{\mu}_k^{it-1} \) is computed over the previous six months, i.e. \( k = 6 \). Tables 5 and 6 in Appendix C report results for \( k = 3 \) and \( k = 9 \).
A predicted by the model, monthly flows positively respond to performance, as indicated by the positive coefficients on $\tilde{R}_{it}$. Moreover, the flow-performance sensitivity is increasing in the history of performance, as reflected by the positive coefficients on the $\hat{\mu}_{it-1}^{k} \tilde{R}_{it}$.

Coefficients remain positive and statistically significant across all specifications, while their numerical values depend on whether I control for fund fixed effects and on the horizon used to measure past performance. In particular, the inclusion of fund fixed effects reduces the numerical values for the coefficients on $\tilde{R}_{it}$ and $\hat{\mu}_{it-1}^{k} \tilde{R}_{it}$. This should be expected. In the cross-section, some funds perform consistently better and other and attract consistently larger flows than other. By adding fund fixed effects, I remove this source of cross-sectional correlation between flows and performance. However, coefficients remain positive and significant, in line with my model that predicts that, even in the time series of a single fund, we should observe a positive flow-performance relationship and a positive relation between the history of returns and the flow-performance slope.

In Table 3, a 1% excess return is associated with a 0.22% increase in fund flows when a fund matches its style benchmark over the previous months. This effect on flows is reduced to 0.17% when I control fund fixed effects. As for the history dependence, a 1% excess return over the past six months is going to increase the effect of a 1% monthly return on flows by 0.04%, while, after including fixed effect, this incremental flow-return sensitivity is down to 0.03%. This increment is not negligible in relative terms. If a fund which beats competitors by 1% over a six month period, then its flow-performance sensitivity is 18% steeper than the average.

Furthermore, the flow performance slope is more positively dependent with average past performance over longer horizons, as we can notice from the fact that the coefficients on $\hat{\mu}_{it-1}^{k} \tilde{R}_{it}$ increase in $k$. This is again a reasonable outcome. Good performance over longer horizon is more informative over the fund’s expected returns. In terms of the model, a 1% average excess return over nine months result in a larger increase in beliefs than a 1% average excess return over three months. Consequently, the slope of the flow-performance relationship depends more strongly on past performance when it is evaluated over longer periods.

Finally, it is interesting to note that no convexity can be detected in monthly data. The coefficients on $\tilde{R}_{it}^{2}$ are small and not statistically significant. This suggests that, at monthly frequency, there is no convexity in the flow-performance relationship and that, over longer horizon, convexity appears as, over time, the flow-performance slope varies in response to previous returns.

### 6.3 Prediction 2: Flow and Performance in Long-Term Contracts

The second prediction of the model can be summarized as follows.

**Prediction 2.** For managers with longer tenure, the flow-performance relationship is flatter and its slope depends less on past performance.
Table 3: Effect of past performance on flow-performance sensitivity. Past performance is calculated as the average return of the fund over its style benchmark in the past six month, i.e. $k = 6$ in equation (22). The dependent variable is the net flow of capital, as defined in equation (20). $\hat{R}$ is the excess gross return of the fund relative over the average gross return of funds with the same style, as in equation (21). The effect of past performance on the flow-performance sensitivity is measured by the coefficient on $\tilde{\mu}_{it} \cdot \hat{R}_{it}$. Columns (3) and (4) include fund fixed effects. Observations are weighted by the logarithm of real fund size, $\log K_{it}^{2011}$. Standard errors are in parenthesis and they are clustered at the fund and month level.

<table>
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<td>$\hat{R}_{it}$</td>
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<td></td>
<td>(0.014)</td>
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<td>$\log K_{it}$</td>
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<tr>
<td>$\text{ExpRatio}_t$</td>
<td>0.130</td>
</tr>
<tr>
<td></td>
<td>(0.897)</td>
</tr>
<tr>
<td>$\log \text{FundAge}_t$</td>
<td>−0.007***</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
</tr>
<tr>
<td>Style-time FE</td>
<td>Yes</td>
</tr>
<tr>
<td>Fund FE</td>
<td>No</td>
</tr>
<tr>
<td>Observations</td>
<td>150,456</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.159</td>
</tr>
</tbody>
</table>

Notes: * $p \leq .10$; ** $p \leq .05$; *** $p \leq .01$
Table 4: Effect of long-term contracts on the flow-performance sensitivity. Past performance is calculated as the average return of the fund over its style benchmark in the past six months, i.e. $k = 6$ in equation (22). The dependent variable is the net flow of capital, as defined in equation (20). $\bar{R}$ is the excess gross return of the fund relative to the average gross return of funds with the same style, as in equation (21). The effect of long-term contracts on the flow-performance sensitivity is measured by the coefficient on $\text{LongTenure}_{it} \cdot \tilde{R}_{it}$, while the effect of long-term contracts on the history-dependence of the flow-performance sensitivity is measured by the coefficient on $\text{LongTenure}_{it} \cdot \tilde{\mu}_{it-1} \cdot \tilde{R}_{it}$. Columns (3) and (4) include fund fixed effects. Observations are weighted by the logarithm of real fund size, $\log K_{it}^{2011}$. Standard errors are in parenthesis and they are clustered at the fund and month level.

<table>
<thead>
<tr>
<th>$F_{it+1}$ (Net Flow)</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
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<tbody>
<tr>
<td>$\tilde{R}_{it}$</td>
<td>0.233***</td>
<td>0.227***</td>
<td>0.192***</td>
<td>0.187***</td>
</tr>
<tr>
<td>(0.016)</td>
<td>(0.016)</td>
<td>(0.017)</td>
<td>(0.017)</td>
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<tr>
<td>$\tilde{\mu}<em>{it-1} \cdot \tilde{R}</em>{it}$</td>
<td>5.044***</td>
<td>4.529***</td>
<td>3.700***</td>
<td>3.517***</td>
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<tr>
<td>(1.080)</td>
<td>(1.062)</td>
<td>(1.054)</td>
<td>(1.046)</td>
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</tr>
<tr>
<td>$\text{LongTenure}<em>{it} \cdot \tilde{R}</em>{it}$</td>
<td>-0.110***</td>
<td>-0.108***</td>
<td>-0.087***</td>
<td>-0.080***</td>
</tr>
<tr>
<td>(0.023)</td>
<td>(0.022)</td>
<td>(0.023)</td>
<td>(0.023)</td>
<td></td>
</tr>
<tr>
<td>$\text{LongTenure}<em>{it} \cdot \tilde{\mu}</em>{it-1} \cdot \tilde{R}_{it}$</td>
<td>-5.989**</td>
<td>-3.171</td>
<td>-3.221**</td>
<td>-3.213**</td>
</tr>
<tr>
<td>(2.427)</td>
<td>(2.079)</td>
<td>(1.560)</td>
<td>(1.561)</td>
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<tr>
<td>$\tilde{R}_{it}^2$</td>
<td>0.070</td>
<td>0.019</td>
<td>0.498</td>
<td>0.526</td>
</tr>
<tr>
<td>(0.330)</td>
<td>(0.330)</td>
<td>(0.382)</td>
<td>(0.363)</td>
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<tr>
<td>$\tilde{R}_{it-1}$</td>
<td>0.168***</td>
<td>0.160***</td>
<td>0.129***</td>
<td>0.123***</td>
</tr>
<tr>
<td>(0.014)</td>
<td>(0.014)</td>
<td>(0.013)</td>
<td>(0.013)</td>
<td></td>
</tr>
<tr>
<td>$\tilde{R}_{it-2}$</td>
<td>0.194***</td>
<td>0.183***</td>
<td>0.151***</td>
<td>0.142***</td>
</tr>
<tr>
<td>(0.016)</td>
<td>(0.016)</td>
<td>(0.013)</td>
<td>(0.013)</td>
<td></td>
</tr>
<tr>
<td>$\text{LongTenure}<em>{it} \cdot \tilde{R}</em>{it-1}$</td>
<td>-0.051**</td>
<td>-0.052**</td>
<td>-0.032</td>
<td>-0.026</td>
</tr>
<tr>
<td>(0.023)</td>
<td>(0.021)</td>
<td>(0.021)</td>
<td>(0.020)</td>
<td></td>
</tr>
<tr>
<td>$\text{LongTenure}<em>{it} \cdot \tilde{R}</em>{it-2}$</td>
<td>-0.061**</td>
<td>-0.057**</td>
<td>-0.035*</td>
<td>-0.028</td>
</tr>
<tr>
<td>(0.025)</td>
<td>(0.023)</td>
<td>(0.021)</td>
<td>(0.020)</td>
<td></td>
</tr>
<tr>
<td>$F_{it}$</td>
<td>0.013***</td>
<td>0.010***</td>
<td>0.010***</td>
<td>0.003</td>
</tr>
<tr>
<td>(0.005)</td>
<td>(0.005)</td>
<td>(0.003)</td>
<td>(0.003)</td>
<td></td>
</tr>
<tr>
<td>$F_{it-1}$</td>
<td>0.012***</td>
<td>0.009***</td>
<td>0.007***</td>
<td>0.002***</td>
</tr>
<tr>
<td>(0.004)</td>
<td>(0.004)</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td></td>
</tr>
<tr>
<td>$\log \text{MgrTenure}_{it}$</td>
<td>0.001***</td>
<td>0.002***</td>
<td>0.002***</td>
<td>0.000</td>
</tr>
<tr>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td></td>
</tr>
<tr>
<td>$\log K_{it}$</td>
<td>0.001***</td>
<td>-0.004***</td>
<td>-0.004***</td>
<td>0.000</td>
</tr>
<tr>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td></td>
</tr>
<tr>
<td>$\text{ExpRatio}_{it}$</td>
<td>0.145</td>
<td>4.347*</td>
<td>(2.409)</td>
<td>(2.409)</td>
</tr>
<tr>
<td>(0.896)</td>
<td>(0.896)</td>
<td>(2.409)</td>
<td>(2.409)</td>
<td></td>
</tr>
<tr>
<td>$\log \text{FundAge}_{it}$</td>
<td>-0.007***</td>
<td>-0.016***</td>
<td>-0.016***</td>
<td>0.000</td>
</tr>
<tr>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.002)</td>
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<td></td>
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<tr>
<td>Style-time FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
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<tr>
<td>Fund FE</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Observations</td>
<td>129,392</td>
<td>129,392</td>
<td>129,392</td>
<td>129,392</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.163</td>
<td>0.194</td>
<td>0.332</td>
<td>0.342</td>
</tr>
</tbody>
</table>

Notes: *$p ≤ .10$; **$p ≤ .05$; ***$p ≤ .01$
This prediction is different from those from models where age matters only because the volatility of beliefs declines over time deterministically, like Berk and Green (2004). In my model, capital responds to performance mainly as a consequence of the incentive problem in the fund. In Berk and Green (2004), capital responds to performance only because beliefs are updated by investors. If investors cease to update their beliefs on the fund, then they will also cease to adjust the amount of capital they provide. In sum, Berk and Green (2004) only predict that the sensitivity of flow to performance will decline over time. Through my model, I also predict that the history dependence of the sensitivity declines over time.

I test this prediction by running a linear regression of fund flows on past returns with interaction dummies for managers with long tenure, as

\[
F_{it+1} = a_1 \tilde{R}_{it} + a_2 \tilde{\mu}^{k-1}_{it} \tilde{R}_{it} + \text{LongTenure}_{it} \cdot (b_1 \tilde{R}_{it} + b_2 \tilde{\mu}^{k-1}_{it} \tilde{R}_{it}) \\
+ c' X_t + \epsilon_t^{F} + \epsilon_t^{S(i)t} + \nu_t. 
\]

(24)

The dummy variable LongTenure<sub>it</sub> takes value 1 if the manager of fund <i>i</i> in month <i>t</i> has a tenure above the 90th percentile of the distribution of tenure length, which, in the sample, is 13 years. In order to make the comparison across tenure lengths meaningful, I consider only fund-month observations for which CRSP contains information on the manager start date. This, for example, excludes funds managed by teams. The controls are the same as above, except that I also include interaction terms between LongTenure<sub>it</sub> and lagged returns. Controls <i>X_t</i> are the same as in model (23) with the addition of interactions between lagged returns and the LongTenure dummy variable.

I test the model prediction on long term contracts by verifying that \(b_1 < 0\) and \(b_2 < 0\). A negative \(b_1\) coefficient indicates that the flow-performance slope decreases with the manager’s tenure, while a negative \(b_2\) coefficients indicate that the flow-performance slope is less sensitive to past performance.

### 6.3.1 Results

In Table 4, I provide estimates of model (24) when past performance is measured over six months, i.e. \(k = 3\), while in Tables 7 and 8 in Appendix C I repeat the study with \(k = 3\) and \(k = 9\).

While the coefficients on \(\tilde{R}_{it}\) and \(\tilde{\mu}^{k-1}_{it} \tilde{R}_{it}\) maintain the same signs as before for managers in short contractual relationships, the results change when managers with long tenure are considered.

Consistently with the model, longer tenure decreases the sensitivity of flows to returns, as indicated by the negative coefficient on LongTenure<sub>it</sub> \(\cdot \tilde{R}_{it}\), but it nevertheless leaves a positive and statistically significant relation between returns and flows, as, with reference to equation (24), a Wald test can always comfortably reject the hypothesis that \(a_1 + b_1 = 0\) in every econometric specification. Moreover, the history-dependence of the flow-performance slope is weaker for managers with long tenure, as the model predicts. The coefficient on LongTenure<sub>it</sub> \(\cdot \tilde{\mu}^{k-1}_{it} \tilde{R}_{it}\) is significantly
negative and it quantitatively offset the coefficient on $\mu_{it-1}$. In the data, managers with long tenure do not display any significant history dependence in their flow-performance slope, as a Wald test of the null hypothesis that $a_2 + b_2 = 0$ fails to reject it.

7 Alternative Contractual Environments

The design and implementation of the optimal contract required the full commitment of the principal to the initial promises. The ex-ante optimal contract implies actions that are, ex-post, inefficient. For example, the principal would like to renegotiate the contract at any time to reset $\Lambda = 0$. Similarly, a contract with full commitment is not robust to competition between principals or to the assumption that the manager could leave the principal to open his own fund. In this section, I explore this two alternative scenarios. In the first one, I consider a renegotiation-proof (or state contingent) contract. In the second one, I explore the incentives that a market of atomistic investors can provide to a manager without the full commitment of any principal.

I will first present these two contractual environment formally, I will then report and compare their outcomes.

7.1 Renegotiation-Proof Contract

In order to understand the economic mechanism behind renegotiation-proof contract, imagine that multiple principals are competing for the same agent. Imagine that, one of them, offer the contract of Section 4 to the agent. After some time has passed, the principal will be committed to undertaking ex-post inefficient actions, captured by a strictly negative multiplier $\Lambda_t < 0$. At that point, the principal is willing to transfer wealth $J(V_t, \hat{\xi}(V_t, \Lambda_t, \phi_t), \phi_t)$ to another principal and employ the agent the agent starting with the same continuation value $V_t$ and beliefs $\phi_t$, but with a re-set information rent $\hat{\xi}(V_t, 0, \phi_t)$. At these terms, the first principal would be indifferent between keeping the agent and transferring him and the agent is indifferent since he obtains the same continuation value with the old and the new principal. However, the new principal makes a strictly positive gain. She obtains a wealth transfer of $J(V_t, \hat{\xi}(V_t, \Lambda_t, \phi_t), \phi_t)$ from the first principal, but it will cost her $J(V_t, \hat{\xi}(V_t, 0, \phi_t), \phi_t)$ to employ the agent.

Therefore, if the agent can transfer across funds and a principal has no power to commit to retain the agent, the the contract of Section 4 would not be credible. The terms of the contract would be continuously re-negotiated, making it impossible to implement an ex-ante optimal contract that requires ex-post inefficient choices.

Define the following set

$$I(V, \phi, t) = \left\{ \mathcal{C}_t : \mathcal{C}_t \text{is IC, } \mathbb{E}\left[ \int_t^\infty e^{-R(s-t)}u(C_s + m_s\lambda K_s) ds|\mathcal{F}_t, \mathcal{C}_t \right] = V, \mathbb{E}[h|\mathcal{F}_t] = \phi \right\},$$

which represents the set of all contracts that are incentive compatible (as in Definition 2) and that provide the agent with continuation value $V$ starting from beliefs $\phi$. 36
To introduce some notation for the definition of renegotiation-proof contract, let $J(C)$ denote the costs for the principal that offers contract $C$ and let $O(C,t)$ denote the time $t$ continuation contract implied by $C$.

**Definition 5 (Weakly Renegotiation-Proof Contract).** $C \in \mathcal{J}(V_0, \phi_0, 0)$ is weakly renegotiation-proof if, for all $t \geq 0$ and $t' \geq 0$ such that $V_t = V_{t'}$ and $\phi_t = \phi_{t'}$, $J(O(C,t)) = J(O(C,t'))$.

In a renegotiation-proof contract, the principal must be unable to renegotiate the terms of the contract in order to leave the agent indifferent and reduce costs for herself. In this environment, the principal still fully commits to a promised value for the agent, but she is unable to commit to the manager’s information rent.

Even when the principal cannot commit to an information rent for the manager, it will be always optimal for her to enforce zero mismanagement. In other words, Proposition 2 remain valid, since it does not rely on any particular assumption about the principal’s commitment. Moreover, (3) also remains valid, as it characterize incentive compatibility in any generic principal-agent setting.

For any given contract, the manager has an information rent given by equation (11). However, unlike in the optimal contract with full commitment, the principal does not take into account the effects of a contract on the information rent. Rather, the principal will the the process for the information rent as given and design a contract that is optimal for this information rent process. This capture the idea that the manager could leave the principal to join another fund, which might promise the agent a contract with an information rent the initial principal has no power to control.

We can refine the search for a renegotiation-proof contract by looking at the set of Markovian contracts, under the assumption that the optimal renegotiation-proof contract is unique for a given promised value and posterior.

**Definition 6.** A contract $C \in \mathcal{J}(V_0, \phi_0, 0)$ is Markovian if, for all $t \geq 0$ and $t' \geq 0$ such that $V_t = V_{t'}$ and $\phi_t = \phi_{t'}$, $O(C,t) = O(C,t')$.

**Lemma 5.** Any Markovian contract is weakly renegotiation-proof. If the optimal renegotiation-proof contract is unique, then it is Markovian. For any optimal renegotiation-proof contract there exist a payoff-equivalent Markovian contract.

In this paper I will not explore the issue of multiple optimal contract. I will simply focus on the optimal Markovian contract which, given the previous lemma, is payoff-equivalent to any optimal renegotiation-proof contract. Consequently, given expression (11) for the agent’s information rent and the Markovian structure of the contract, we conclude that also the agent’s information rent is Markovian in the agent’s continuation value and beliefs. Using the homogeneity of the problem, I can characterize an optimal renegotiation-proof contract as follows.

**Proposition 8.** As before, let $\hat{v}_t = ((1 - \rho) V_t)^{1/\phi_t}$. In an optimal renegotiation-proof contract, $C_t = \hat{v}_t c_R(\phi_t)$, $\beta_t = (1 - \rho) V_t \hat{\beta}_R(\phi_t)$ and $E_t = e_R(\phi_t)$, where $c_R(\phi)$, $\hat{\beta}_R(\phi)$ and $e_R(\phi)$ are the optimal controls.
in the HJB equation

\[ r J_R(\phi) = \min_{c, \hat{\beta}, e} \left\{ \frac{c}{1 - q e^d} - \mu(e, \phi, 0) \hat{\beta} - \eta(e) g_R(\phi) \right\} + J_R(\phi) \left( \frac{R}{1 - \rho} - \frac{c^{1 - \rho}}{1 - \rho} + \frac{1}{2} \rho \hat{\beta}^2 \right) \]

\[ + \beta \eta(e) \phi (1 - \phi) J_R'(\phi) + \frac{1}{2} \eta(e(\phi))^2 \phi^2 (1 - \phi)^2 J_R''(\phi) \right\}. \]

\( y_R \) solves the differential equation

\[ y_R(\phi) c_R(\phi)^{1 - \rho} - \beta_R(\phi) \eta(e_R(\phi)) \phi (1 - \phi) - (1 - \rho) \beta_R(\phi) \eta(e_R(\phi)) \phi (1 - \phi) y_R'(\phi) \]

\[ = \frac{1}{2} \eta(e_R(\phi))^2 \phi^2 (1 - \phi)^2 y_R''(\phi) \]

The cost function for the principal is \( \hat{v}_t J_R(\phi_t) \) and the agent’s information rent is \( \xi_t = (1 - \rho) V_t y_R(\phi_t) \)

### 7.2 Market-based Incentives

I now consider the case of an agent that directly collects money from investors. No explicit contracts are written, however, the manager can maintain a stake in the fund he runs in order to ensure incentive compatibility. To maintain comparison with the problem I have studied so far, I assume that the agent’s consumption is observable by the market, together with the amount of asset under management and the manager’s investment share. Consequently, these must be measurable processes with respect to the information of the market, since the market could “punish” observed deviations by leaving the manager in autarky.

In this framework, the manager possess some wealth that can allocate between a risk-free asset and the fund he runs. He raise capital from investors, who supply capital perfectly elastically at the risk free rate \( r \). Therefore, the manager can collect fees on the investor’s capital for the difference between the expected return of his fund and the risk free rate.

Absent a principal designing an explicit incentive contract, the manager must resort to other implicit incentives schemes to mitigate his moral hazard. In particular, the manager needs to hold a stake in the fund. If the manager had none of his wealth invested in the fund, then he would have no motivation not to shirk and obtain private benefits. Expecting this, investors would not be willing to provide capital. If, instead, the manager had some of his own wealth invested in the fund, then investors can, for some extend, trust the manager with their own money.

As in the previous cases, Proposition (2) continues to hold and the manager will choose a strategy that will credibly enforce no mismanagement. The proof of Proposition (2) holds by simply reinterpreting the principal’s cost function as the wealth of the manager. However, the incentive compatibility condition needs to be modified to account for the way in which incentives are provided.

The incentive compatibility condition, in this case, remains essentially unchanged: Changes in the continuation value of the agent should exceed the marginal utility of shirking, keeping the
agent’s beliefs fixed. However, since there is no principal committing to a promised value for the agent, the implementation of this incentive compatibility condition has to rely on the dynamics of the agent’s wealth and beliefs.

In equilibrium, the manager’s continuation value will depend on his wealth $A$ and the common beliefs about his skills $\phi$ and it will be given by the function $V(A, \phi)$.

The agent’s wealth evolves as

$$dA_t = \left( rA_t - \frac{C_t}{1 - qe^t} + \mu(\phi_t, e_t, 0)K_t \right) dt + \Theta_t\nu(e_t)dW^e_t$$

where $\Theta$ is the amount of wealth that the agent invests into the fund. He collects fees on the capital he raised from investors, thus accounting for a quantity $\mu(\phi_t, e_t, 0)(K_t - \Theta_t)$ of his instantaneous cash flow. The remaining part, $\mu(\phi_t, e_t, 0)\Theta_t$ is the expected excess return from his equity in the fund. In an incentive compatible contract, the fee and the expected return coincide, so that the inside equity $\Theta_t$ does not affect expected cash-flows, but only the volatility of cash-flows.

If the agent mismanages, he will be effectively punished through two channels. The first is a loss of wealth from his exposure to the fund, $\Theta_t$. The second is a decline in market’s beliefs about his skills and a subsequent persistent decline in fees. However, this second channel does not correspond to a decline in equilibrium beliefs. The market beliefs will affect only the fees the manager collects, not the expected return that the manager realizes on his investment. More precisely, after a deviation the wealth of the manager evolves as

$$dA_t = \left( rA_t - \frac{C_t}{1 - qe^t} + \mu(\phi^M_t, e_t, 0)K_t + (\mu(\phi^A_t, e_t, 0) - \mu(\phi^M_t, e_t, 0))\Theta_t \right) dt + \Theta_t\nu(e_t)dW^e_t$$

where the positive term $\mu(\phi^A_t, e_t, 0) - \mu(\phi^M_t, e_t, 0)$ captures the more optimistic expectations of the manager. Moreover, the manager will also expect the market’s beliefs to increase over time.

As in the principal-agent contract, therefore, the incentive compatibility condition will have to take this information rent of the manager into account. Using the Markovian characterization of the manager’s continuation value I can therefore conclude that

**Proposition 9.** If the market-based allocation is incentive compatible, then

$$V_W(A_t, \phi_t)\Theta_t\nu(e_t) + V_\phi(A_t, \phi_t)\eta(e_t)\phi_t(1 - \phi_t)\eta(e_t) \geq u'(C_t)K_t\lambda\nu(e_t) + \eta(e_t)\xi_t$$

where $\xi_t$ evolves as

$$d\xi_t = [R\xi_t - \eta(e_t)\phi_t(1 - \phi_t)(V_W(A_t, \phi_t)\Theta_t\nu(e_t) + V_\phi(A_t, \phi_t)\eta(e_t)\phi_t(1 - \phi_t)\eta(e_t))]dt + \delta_t dW_t$$

Since there is no intermediary with full commitment, but rather, the manager and investors interact in a spot market, the implicit market-based contract must also be renegotiation-proof the
manager’s information rent will be a function of his wealth and the beliefs

$$\xi_t = \xi(A_t, \phi_t)$$

Given the functional form of the utility function, the manager’s continuation value and information rent are homogeneous in wealth, i.e. $$V(A, \phi) = \frac{A^{1-\rho}}{1-\rho} v(\phi)$$ and $$\xi(A, \phi) = \frac{A^{1-\rho}}{1-\rho} y_M(\phi)$$. Therefore, the IC constraint can be written as

$$(1 - \rho)v(\phi_t)\hat{\Theta}_t v(e_t) + v'(\phi_t)\eta(e_t)\phi_t(1 - \phi_t)\eta(e_t) \geq (1 - \rho)c_t^{-\rho}k_t\lambda v(e_t) + \eta(e_t)y_M(\phi_t)$$

where I use $$\hat{\Theta}_t = \frac{\Theta_t}{A_t}$$, $$c_t = \frac{C_t}{A_t}$$ and $$k_t = \frac{K_t}{A_t}$$.

This contracting environment can thus be characterized as follows.

**Proposition 10.** In an optimal market-based contract, $$C_t = A_t c_M(\phi_t)$$, $$\Theta_t = A_t \hat{\Theta}_M(\phi_t)$$ and $$e_t = e_M(\phi_t)$$, where $$c_M(\phi), \hat{\Theta}_M(\phi)$$ and $$e_M(\phi)$$ are the optimal controls in the HJB equation

$$R v_M(\phi) = \max_{e, \hat{\Theta}, \eta(\phi), c} \left\{ c^{1-\rho} + v_M(\phi)(1 - \rho) \left[ r - \frac{c}{1 - \rho e^d} + \mu(e, \phi, 0)k - \frac{1}{2} \rho \hat{\Theta}^2 \nu(e) \right] + (1 - \rho)\nu(e)\hat{\Theta}(1 - \phi)v_M'(\phi) + \frac{1}{2} \eta(e)^2(1 - \phi)^2v_M''(\phi) \right\}$$

s.t. $$(1 - \rho)v_M(\phi_t)\hat{\Theta}_t v(e_t) + v'_M(\phi_t)\eta(e_t)\phi_t(1 - \phi_t) = (1 - \rho)c_t^{-\rho}k_t\lambda v(e_t) + \eta(e_t)y_M(\phi_t)$$

$$y_M$$ solves the differential equation

$$\left[ R - (1 - \rho) \left( r - c_M(\phi) + \mu(e_M(\phi), \phi, 0)k_M(\phi) - \frac{1}{2} \rho \hat{\Theta}(\phi)^2 \nu(e_M(\phi))^2 \right) \right] y_M(\phi) =$$

$$\left((1 - \rho)\eta(e_M(\phi))\phi(1 - \phi)\hat{\Theta}(\phi)\nu(e_M(\phi))v_M(\phi) + (1 - \rho)\eta(e_M(\phi))^2(1 - \phi)^2v_M'(\phi) \right)$$

$$+ \left((1 - \rho)\eta(e_M(\phi))\phi(1 - \phi)\hat{\Theta}(\phi)\nu(e_M(\phi))y_M(\phi) \right)$$

$$+ \frac{1}{2} \eta(e_M(\phi))^2(1 - \phi)^2y_M''(\phi)$$

The value function for the manager is $$\frac{A^{1-\rho}}{1-\rho} v(\phi_t)$$ and his information rent is $$\xi_t = \frac{A^{1-\rho}}{1-\rho} y_M(\phi_t)$$

**7.3 Results**

While I introduced the two problems separately to highlight the substantially different contracting environment that surrounds them, they are, as the next proposition shows, outcome-equivalent. Therefore, as long as the principal and the manager have the same commitment power and the same actions are verifiable in the two scenarios, there is no change in social welfare nor in observable outcomes between a contract designed by the principal and a contract that arises from market incentives.

**Proposition 11.** The renegotiation-proof contract and the market-based contract are equivalent
While in Appendix B, I provide a verification argument based on the recursive representation of the problems given in Propositions 8 and 10, one can immediately observe that the renegotiation-proof contract is formally equivalent to the problem of a manager in the market that wants to find the minimal wealth that could support an given lifetime utility.

I then study the flow-performance relation and the pay for performance that arise in these contractual environments. As the Figures in this section show, the results are qualitatively identical to the full-commitment model with only one exception. Since the principal or the manager are unable to commit to ex-post inefficient actions, the flow-performance relation and the pay-performance relation are stable over time. In the full commitment model the principal could commit to reduce future information rents through a multiplier with a negative drift. As a result, over time the principal would change the incentives of the agent and its learning, even for the same value of beliefs . This channel of non-stationarity is shut down when the principal lacks commitment.

As before, the flow-performance relation is positive and increasing in beliefs (Figure 6a). A good return is associated not only with a positive flow of capital into the fund, but also to an increase in the flow-performance slope, since beliefs increase after good performance. The increase flow-performance sensitivity is necessary in order to ensure incentive compatibility, as the amount of capital available to the manager increases as a function of beliefs (Figure 6b).

The relation between compensation and performance also matches the full commitment contract. Compensation increases with returns and the pay for performance sensitivity increases with past performance (Figure 7a), which would then result in a convex relation between cumulative pay and cumulative performance. Moreover, compensation is again back-loaded after positive returns (Figure 7b). Interestingly, the agent does not need a principal to defer his compensation and increase his value-at-stake in the fund. This is consistent with anecdotal evidence that hedge fund managers tend to invest most of their performance fees in the fund itself (Agarwal et al., 2009).

Although the outcomes of the two models are identical, the implementation is technically different. In the contract designed by the principal, the principal will adjust the explicit contractual terms of the manager to ensure incentive compatibility (Figure 8a). In the market-based allocation, the manager will adjust his implicit incentives by holding a larger fraction of the fund (Figure 8b). In the optimal mechanisms, these two implementations will provide the manager with identical incentives and, consequently, result in an identical flow-performance relationship and pay for performance.

8 Conclusions

I presented a continuous time contracting model between a portfolio manager, a fund advisor and a population of the investors. In this setting, moral hazard interacts with learning about managerial skills in order to deliver the main predictions of the model. I provide a novel rational

10I numerically solve for the models using the same parameters of Section 5.
explanation for the convexity of the flow-performance relationship in mutual fund, which complements the economic rationale for the convex pay-performance relationship often used in the money management industry. In my model, convexity arises since the optimal contract exposes the manager to increasing levels of risk as his expected performance increases. Through learning, expected performance is positively related to past performance. The key prediction of my model is, therefore, that the slope of the flow-performance relationship increases after good performance. My empirical analysis provides evidence for this prediction.
(a) Pay for performance

(b) Consumption back-loading

Figure 7

(a) Explicit Incentives \( \hat{\beta}_R \)

(b) Market-based Incentives \( \hat{\Theta}_M \)

Figure 8: Incentives
A Static Model

In this Appendix, I consider a two-period contracting model in order to provide an analytically tractable example of how, when moral hazard interacts with uncertainty, a principal optimally design a contract that features reduced experimentation in the future.

The assumptions on the agent’s preferences are different from the main model in order to improve tractability. However, rather than being a shortcoming of this example, this further remark the generality of my result.

Before considering the two-period model, consider a one-period model to establish a benchmark. Suppose that returns are normally distributed and given by

\[ R \sim N(\alpha + \eta h, \sigma^2) - a \]

where \( a \) is the agent’s hidden action, which gives him a private benefit \( \lambda a \), and \( h \in \{0, 1\} \) are the agent’s unknown skills. The principal is risk neutral, while the agent has CARA utility with absolute risk aversion \( \gamma \). Since returns are normal, the agent’s utility can be expressed in the mean-variance form. I focus on linear contracts that enforce \( a = 0 \). In these contracts, the agent’s final consumption is given by

\[ V = \Delta + C \]

where

\[ \Delta = \beta (R - (\alpha + \eta_0)) \]

captures the risk exposure of the agent, while \( C \) represents his expected consumption.

The principal controls experimentation through \( \eta \). I assume that \( \eta \in \{0, 1\} \) and that it involves no costs. In order to ensure incentive compatibility, we must impose \( \beta \geq \lambda \). Then, the optimal contract solves

\[
\begin{align*}
\max & \mathbb{E}[R] - C \\
\text{s.t.} & \beta \geq \lambda \\
& C - \frac{\gamma}{2} \beta^2 \geq U_0
\end{align*}
\]

It is immediate to verify the following proposition

**Lemma 6.** In a one-period contract, \( \beta = \lambda \) and \( \eta_1 = \mathbb{I}\{\phi_0 \geq 0\} \).

Now consider a two-period model. Output in period \( t \) is normally distributed and given by

\[ R_t \sim N(\alpha + \eta_t h, \sigma^2) - a_t \]

where \( a_t \) is the hidden action of the agent, which gives him a private benefit \( \lambda a_t \), with \( \lambda \in (0, 1) \). To reduce unnecessary complications, I assume that \( \eta_0 = 0 \) and that \( \phi_0 > 0 \). The principal can choose \( \eta_1 \in \{0, 1\} \). Since \( a_t > 0 \) is inefficient, I focus on contracts that implement \( a_t = 0 \).
Both the principal and the agent do not discount the future. The principal is risk neutral while the agent has CARA preferences over the final payout. The final compensation of the agent is given by

\[ V = \Delta_0 + \Delta_1 + C \]

where \( \Delta_t \) is the performance-based compensation at time \( t \), while \( C \) is a promised expected compensation that is pinned down by a participation constraint.

I focus on linear contracts where \( \Delta_t \) takes the form \( \Delta_t = \beta_t(R_t - (\alpha + \eta_t \phi t)) \) and where \( \beta_t \) is chosen in order to make the contract incentive compatible. Since the agent is risk averse, in an optimal contract \( \beta_t \) will be chosen to be as small as possible, as long as it ensures incentive compatibility.

Finally, once the principal and the agent sign a two-period contract, they have full commitment to it. In particular, the principal can commit to a future choice of \( \eta_1 \).

It’s immediate to notice that \( \beta_1 = \lambda \), since this would be a standard static problem. However, the choice of \( \beta_0 \) depends on future learning. In particular, in order to ensure IC, we must have

\[ \beta_0 \geq \lambda (1 - \mathbb{E}[\eta_1 \phi_a(R_0, 0)]) \]

where \( \phi(R_0, a) \) is the principal’s posterior as a function of the time 0 output \( R_0 \) and the time 0 action \( a \).

By Bayes’s Law, the posterior is given by

\[ \phi_1 = (1 - \bar{v})\phi_0 + \bar{v}(R_0 - \alpha) \]

so that the constraint on \( \beta_0 \) becomes

\[ \beta_0 \geq \lambda + \lambda \bar{v}\mathbb{E}[\eta_1] \]

This is the counterpart of (8) in the paper and \( \lambda \bar{v}\mathbb{E}[\eta_1] \) represents the agent’s information rent in this two-period model.

Similarly to the one-period model, the principal solves

\[
\begin{align*}
\max & \mathbb{E}[R_0 + R_1] - C \\
\text{s.t.} & \beta_0 \geq \lambda + \lambda \bar{v}\mathbb{E}[\eta_1] \\
& C - \frac{\gamma}{2}(\beta_0^2 + \lambda^2) \geq U_0
\end{align*}
\]

Since the IC and participation constraints must bind, the problem reduces to finding a \( R_0 \)-measurable learning strategy \( \eta_1 \) that maximizes

\[ \mathbb{E}[\eta_1 (\phi_1 - \gamma \lambda^2 \bar{v})] - \frac{\gamma}{2} \lambda^2 \bar{v}^2 (\mathbb{E}[\eta_1])^2. \]
Unlike the one-period model, now it is not always optimal to set \( \eta_1 = 1 \) when expected returns are positive.

**Proposition 12.** There exists a \( \phi \) such that \( \eta_1 = 0 \) if \( \phi_1 < \phi \), while \( \eta_1 = 1 \) if \( \phi_1 > \phi \). Moreover, \( \phi > 0 \).

As in the dynamic model of the paper, the principal commits to a reduced amount of future learning in order to improve current incentives. The mechanism in this static model is very clear: future experimentation \( \eta_1 \) requires stronger incentives \( \beta_0 \). But since exposing the agent to risk is costly, the principal has an incentive to reduce future experimentation below the instantaneously-optimal level in order to improve dynamic incentives.

**B Proofs – TBU**
Table 5: Effect of past performance on flow-performance sensitivity. Past performance is calculated as the average return of the fund over its style benchmark in the past three months, i.e. $k = 3$ in equation (22). The dependent variable is the net flow of capital, as defined in equation (20). $\tilde{R}$ is the excess gross return of the fund relative to the average gross return of funds with the same style, as in equation (21). The effect of past performance on the flow-performance sensitivity is measured by the coefficient on $\tilde{\mu}_{it}^6 - 1 \cdot \tilde{R}_{it}$. Columns (3) and (4) include fund fixed effects. Observations are weighted by the logarithm of real fund size, $\log K_{it}$. Standard errors are in parenthesis and they are clustered at the fund and month level.

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Notes: *$p \leq .10$; **$p \leq .05$; ***$p \leq .01$
Table 6: Effect of past performance on flow-performance sensitivity. Past performance is calculated as the average return of the fund over its style benchmark in the past nine month, i.e. $k = 9$ in equation (22). The dependent variable is the net flow of capital, as defined in equation (20). $\tilde{\bar{R}}$ is the excess gross return of the fund relative over the average gross return of funds with the same style, as in equation (21). The effect of past performance on the flow-performance sensitivity is measured by the coefficient on $\tilde{\bar{R}}_{it}$, Columns (3) and (4) include fund fixed effects. Observations are weighted by the logarithm of real fund size, log $K_{it}$. Standard errors are in parenthesis and they are clustered at the fund and month level.

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Notes: $^* p \leq .10; ^{**} p \leq .05; ^{***} p \leq .01$
Table 7: Effect of long-term contracts on the flow-performance sensitivity. Past performance is calculated as the average return of the fund over its style benchmark in the past three months, i.e. $k = 3$ in equation (22). The dependent variable is the net flow of capital, as defined in equation (20). $\tilde{R}$ is the excess gross return of the fund relative to the average gross return of funds with the same style, as in equation (21). The effect of long term contracts on the flow-performance sensitivity is measured by the coefficient on LongTenure$_{it} \cdot \tilde{R}_{it}$, while the effect of long term contracts on the history-dependence of the flow-performance sensitivity is measured by the coefficient on LongTenure$_{it} \cdot \tilde{\mu}_{it-1} \cdot \tilde{R}_{it}$. Columns (3) and (4) include fund fixed effects. Observations are weighted by the logarithm of real fund size, $\log K_{it}$. Standard errors are in parenthesis and they are clustered at the fund and month level.

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<td>(0.880)</td>
<td>(2.347)</td>
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<td>$\log FundAge_{it}$</td>
<td></td>
<td>-0.007***</td>
<td>-0.018***</td>
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<td>(0.000)</td>
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<td>Style-time FE</td>
<td>Yes</td>
<td>Yes</td>
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<td>Yes</td>
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<td>$R^2$</td>
<td>0.160</td>
<td>0.192</td>
<td>0.335</td>
<td>0.346</td>
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Notes: *$p \leq .10$; **$p \leq .05$; ***$p \leq .01$
Table 8: Effect of long-term contracts on the flow-performance sensitivity. Past performance is calculated as the average return of the fund over its style benchmark in the past nine month, i.e. $k = 9$ in equation (22). The dependent variable is the net flow of capital, as defined in equation (20). $\tilde{R}$ is the excess gross return of the fund relative over the average gross return of funds with the same style, as in equation (21). The effect of long term contracts on the flow-performance sensitivity is measured by the coefficient on $\text{LongTenure}_{it} \cdot \tilde{R}_{it}$, while the effect of long term contracts on the history-dependence of the flow-performance sensitivity is measured by the coefficient on $\text{LongTenure}_{it} \cdot \tilde{\mu}_{it-1} \cdot \tilde{R}_{it}$. Columns (3) and (4) include fund fixed effects. Observations are weighted by the logarithm of real fund size, $\log K_{it}$. Standard errors are in parenthesis and they are clustered at the fund and month level.

<table>
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<tr>
<th></th>
<th>$F_{it+1}$ (Net Flow)</th>
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<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
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<tr>
<td>$\tilde{R}_{it}$</td>
<td>0.230***</td>
<td>0.225***</td>
<td>0.189***</td>
<td>0.185***</td>
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<td>(0.017)</td>
<td>(0.017)</td>
<td>(0.018)</td>
<td>(0.017)</td>
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<tr>
<td>$\tilde{\mu}<em>{it-1} \cdot \tilde{R}</em>{it}$</td>
<td>5.902***</td>
<td>5.581***</td>
<td>4.859***</td>
<td>4.638***</td>
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<td></td>
<td>(1.603)</td>
<td>(1.529)</td>
<td>(1.471)</td>
<td>(1.462)</td>
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<tr>
<td>$\text{LongTenure}<em>{it} \cdot \tilde{R}</em>{it}$</td>
<td>-0.111***</td>
<td>-0.107***</td>
<td>-0.084***</td>
<td>-0.078***</td>
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<td>(0.024)</td>
<td>(0.023)</td>
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<tr>
<td>$\text{LongTenure}<em>{it} \cdot \tilde{\mu}</em>{it-1} \cdot \tilde{R}_{it}$</td>
<td>-6.960**</td>
<td>-4.431***</td>
<td>-4.116***</td>
<td>-4.279**</td>
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<td>(3.404)</td>
<td>(2.875)</td>
<td>(1.997)</td>
<td>(2.061)</td>
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<tr>
<td>$\tilde{R}_{it}^2$</td>
<td>0.170</td>
<td>0.131</td>
<td>0.466</td>
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<td>(0.296)</td>
<td>(0.297)</td>
<td>(0.328)</td>
<td>(0.313)</td>
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<tr>
<td>$\tilde{R}_{it-1}$</td>
<td>0.165***</td>
<td>0.157***</td>
<td>0.126***</td>
<td>0.120***</td>
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<td>(0.015)</td>
<td>(0.014)</td>
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<td>(0.013)</td>
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<tr>
<td>$\tilde{R}_{it-2}$</td>
<td>0.191***</td>
<td>0.180***</td>
<td>0.145***</td>
<td>0.138***</td>
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<td>(0.017)</td>
<td>(0.016)</td>
<td>(0.014)</td>
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<tr>
<td>$\text{LongTenure}<em>{it} \cdot \tilde{R}</em>{it-1}$</td>
<td>-0.054**</td>
<td>-0.054***</td>
<td>-0.032</td>
<td>-0.026</td>
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<td>(0.021)</td>
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<td>(0.019)</td>
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<td>-0.058**</td>
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<td>$F_{it}$</td>
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<td>0.010***</td>
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<tr>
<td>$F_{it-1}$</td>
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<td>0.008***</td>
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<td>$F_{it-2}$</td>
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<td>0.007***</td>
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<td></td>
<td>(0.002)</td>
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<td></td>
<td>0.002***</td>
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<td></td>
<td>(0.000)</td>
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<tr>
<td>$\log K_{it}$</td>
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<td></td>
<td>-0.004***</td>
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<td>-0.014***</td>
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<td>0.330</td>
<td>0.339</td>
</tr>
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</table>

Notes: $^* p \leq .10$; $^{**} p \leq .05$; $^{***} p \leq .01$
REFERENCES


