Multimodality in Macro-Financial Dynamics

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Outline

1. Introduction

2. Data

3. Methodology

4. Empirical Results

5. Simulated pseudo out-of-sample forecast evaluation

6. Euro area

7. Economics

8. Density Impulse Response Function
Motivation

- Macro-financial interactions are likely to be nonlinear
- Nonlinearities are pervasive feature in firms
  - Narrative and anecdotal evidence, especially at the time of financial crises
  - Macro-finance “structural” models, often featuring occasionally binding constraints for intermediaries, households and firms.
- Need for descriptive and robust facts that can inform and validate theories
  - Many models of nonlinearities, each with his own mechanisms
  - No consensus has emerged in the literature
February 10, 2009

“Last Friday we learned that the economy had lost three million jobs last year, and an additional 600,000 just last month...Instead of catalyzing recovery, the financial system is working against recovery. And at the same time, the recession is putting greater pressure on banks.

This is a dangerous dynamic, and we need to arrest it...Today, as Congress moves to pass an Economic Recovery Plan that will help create jobs and lay a foundation for stronger economic future, we [at the US Treasury] are outlining a new Financial Stability Plan...Our plan will help restart the flow of credit, clean up and strengthen our banks, and provide critical aid for homeowners and for small businesses...

We believe that action has to be sustained until recovery is firmly established. In the United States in the 30s, Japan in the 90s, and in other cases around the world, previous crises lasted longer and caused greater damage because governments applied the brakes too early. We cannot make that mistake.”

Treasury Secretary Geithner Introduces Financial Stability Plan
Macro-Finance Literature and Nonlinearity

- He Krishnamurthy (2013) and Brunnermeier Sannikov (2014)
  - Present continuous time theories of macro-financial amplification with occasionally binding constraints
  - All shocks are conditionally gaussian, but vol and drift depend on state variables in highly nonlinear ways

- Adrian Boyarchenko (2016) and Adrian Duarte (2018)
  - Present macro-financial theories with value-at-risk constraints
  - VaR constraints directly tie conditional means and vols together, even when constraints always bind

- Gertler Kiyotaki Prestipino (2016, 2019)
  - Present macro-finance models with multiplicity
  - In a “run” equilibrium banks stop financial intermediation
From linear to nonlinear reduced form models

- Vector Autoregressive models are very flexible and powerful:
  - Forecasting, scenario and structural analysis
  - Establish stylized fact to guide and validate economic (behavioral) modeling
  - The only limit is linearity

- Extensions to nonlinear setting, several approaches:
  - Time Varying parameters VAR and Stochastic Volatility: Cogley and Sargent (2005); Primiceri (2005); Cogley and Sbordone (2008); D'Agostino, Gambetti, and Giannone (2013); Negro and Primiceri (2015); Carriero, Clark, and Marcellino (2018)
  - Thresholded VAR: Altissimo and Violante (2001); Aikman, Lehnert, Liang, and Modugno (2016)
  - Markov switching: Hamilton (1989); Sims and Zha (2006); Chang, Choi, and Park (2017); Hubrich and Tetlow (2015)
  - Quadratic autoregressions and pruning: Aruoba, Bocola, and Schorfheide (2017)
  - For a survey see Kilian and Lutkepohl (2018).

- This paper:
  - Estimation the one-step ahead predictive density \( p(y_t | y_{t-1}) \) based on Kernel smoothing
  - Montecarlo simulation for multistep predictions
What we do

- Estimate the one step ahead predictive density using Kernel smoothing
- Discretize (and bound) the support and simulate predictive paths
- Assess forecasting performances
  - Compare in-sample and out-of-sample estimates
  - Evaluate accuracy (mean square forecast errors and predictive scores) and calibration (PIT)
- Density Impulse Responses: perturb initial conditions and track the dynamic effects on the distribution of all the possible outcomes
What we find

1. Out-of-sample forecasts are reasonably accurate and calibrated
   ⇒ The empirical model is able to capture in a parsimonious way the salient features of a nonlinear dynamic economy

2. Evidence of multiple modes
   ■ Bi-modal distributions when financial conditions are tight

3. Density impulse responses are nonlinear
   ■ Policies affecting financial conditions can make the economy coordinate on good (or bad) equilibria
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Measuring economic and financial conditions

- Chicago Fed National Activity Index (CFNAI)
  - Common factor extracted from 85 indicators of economic activity
    1. Production and income
    2. Employment, unemployment, and hours
    3. Personal consumption and housing
    4. Sales, orders, and inventories
  - Principal component (Stock and Watson, 1999)

- National Financial Conditions Index (NFCI)
  - Common factor extracted from 105 financial indicators
    1. Money markets
    2. Debt markets
    3. Equity markets
    4. Traditional and shadow banking
  - Quasi Maximum Likelihood (Doz, Giannone, and Reichlin, 2012)

- Quarterly averages since 1973Q1
Measuring economic and financial conditions
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The estimator

- **Data:** \( y_t = (y_{1,t}, ..., y_{n,t})', x_t = (y'_{t-1}, ..., y'_{t-p})' \)

- **Estimator:**
  \[
  \hat{p}(y|x) = \frac{1}{T-p} \sum_{t=p+1}^{T} K^y_{\omega_y} (y - y_t) K^x_{\omega_x} (x - x_t)
  \]

- **Kernels:** multivariate (independent) normal
  \[
  K^y_{\omega_y} (y - y_t) = \prod_{i=1}^{n} \frac{1}{\omega_{y_i}} \phi\left( \frac{y_i - y_{i,t}}{\omega_{y_i}} \right) \\
  K^x_{\omega_x} (x - x_t) = \prod_{j=1}^{np} \frac{1}{\omega_{x_j}} \phi\left( \frac{x_j - x_{j,t}}{\omega_{x_j}} \right)
  \]

- **Bandwidths:** \( \omega_{y_i} = c_{0,i} \sigma_{y_i}, \omega_{y_i} = c_{1,j} \sigma_{x_j} \)
  - Set \( c_{0,i} = c_{1,j} = c \) for all \( i,j \)
  - Estimate \( c \) it by maximizing out-of-sample predictive accuracy

- **Discuss alternatives:**
  - Splines: Gallant, Rossi, and Tauchen (1993)
  - Quantile or distributional regression: Koenker, Leorato, and Peracchi (2013), Adrian, Boyarchenko, and Giannone (2019)
Density Impulse Responses

- Comparison of a baseline forecast with a counterfactual forecast
- Perturb initial conditions and track the dynamic effects

Gallant, Rossi, and Tauchen (1993)
Koop, Pesaran, and Potter (1996)
Potter (2000)
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US: Bandwidth selection
U.S.: Marginal quantiles, 1 quarter ahead

**Economic conditions 1 quarter ahead**

-4 -2 0 2 4

**Financial conditions 1 quarter ahead**

-4 -2 0 2 4
U.S.: Marginal quantiles, 1 quarter ahead

Economic conditions 1 quarter ahead

Financial conditions 1 quarter ahead
U.S.: Marginal quantiles, 1 quarter ahead

Economic conditions 1 quarter ahead

Financial conditions 1 quarter ahead
U.S.: Marginal quantiles, 2 quarters ahead

Economic conditions 2 quarters ahead

Financial conditions 2 quarters ahead
U.S.: Marginal quantiles, 2 quarters ahead

Economic conditions 2 quarters ahead

Financial conditions 2 quarters ahead
U.S.: Marginal quantiles, 2 quarters ahead

Economic conditions 2 quarters ahead

Financial conditions 2 quarters ahead
U.S.: Marginal quantiles, 4 quarters ahead

Economic conditions 4 quarters ahead

Financial conditions 4 quarters ahead

In-sample
U.S.: Marginal quantiles, 4 quarters ahead

### Economic conditions 4 quarters ahead

![Economic conditions 4 quarters ahead](image)

### Financial conditions 4 quarters ahead

![Financial conditions 4 quarters ahead](image)
U.S.: Marginal quantiles, 4 quarters ahead

**Economic conditions 4 quarters ahead**


**Financial conditions 4 quarters ahead**

U.S.: Marginal quantiles, 8 quarters ahead

Economic conditions 8 quarters ahead

Financial conditions 8 quarters ahead
U.S.: Marginal quantiles, 8 quarters ahead

Economic conditions 8 quarters ahead

Financial conditions 8 quarters ahead
U.S.: Marginal quantiles, 8 quarters ahead

Economic conditions 8 quarters ahead

Financial conditions 8 quarters ahead
U.S.: Marginal distribution of CFNAI and NFCI
U.S.: Marginal distribution of CFNAI and NFCI
U.S.: Marginal distribution of CFNAI and NFCI
U.S.: Marginal distribution of CFNAI and NFCI
U.S.: Marginal distribution of CFNAI and NFCI
U.S.: Joint distribution of CFNAI and NFCI

1 quarter ahead

Diagram showing the joint distribution of CFNAI and NFCI over time, with the x-axis representing NFCI from 1975 to 2018, the y-axis representing CFNAI, and the z-axis representing time in quarters. The colors indicate the density of data points at different combinations of CFNAI and NFCI values.
U.S.: Joint distribution of CFNAI and NFCI

2 quarters ahead
U.S.: Joint distribution of CFNAI and NFCI
U.S.: Joint distribution of CFNAI and NFCI
U.S.: Joint/marginal distribution, 1 quarter ahead
U.S.: Joint/marginal distribution, 1 quarter ahead
U.S.: Joint/marginal distribution, 1 quarter ahead
U.S.: Joint/marginal distribution, 1 quarter ahead
U.S.: Joint/marginal distribution, 1 quarter ahead
U.S.: Joint/marginal distribution, 1 quarter ahead
U.S.: Joint distribution, 1-8 quarters ahead
U.S.: Joint distribution, 1-8 quarters ahead

1983:Q4 + h | 1983:Q4

CFNAI

NFCI

h
U.S.: Joint distribution, 1-8 quarters ahead
U.S.: Joint distribution, 1-8 quarters ahead
U.S.: Joint distribution, 1-8 quarters ahead
U.S.: Joint distribution, 1-8 quarters ahead
U.S.: Joint distribution, 1-8 quarters ahead
Forecasting the Great Recession
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Scores (Joint)
Log scores, difference from NL-VAR(1)

<table>
<thead>
<tr>
<th>Model</th>
<th>h=1</th>
<th>h=2</th>
<th>h=3</th>
<th>h=4</th>
<th>h=8</th>
</tr>
</thead>
<tbody>
<tr>
<td>NL-VAR(1)</td>
<td>-2.27</td>
<td>-1.66</td>
<td>-1.61</td>
<td>-1.45</td>
<td>-1.55</td>
</tr>
<tr>
<td>(L-)VAR(1) - NL-VAR(1)</td>
<td>0.935</td>
<td>-0.225</td>
<td>-0.733</td>
<td>-0.979</td>
<td>-1.11</td>
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<tr>
<td>SE</td>
<td>(0.983)</td>
<td>(0.146)</td>
<td>(0.223)</td>
<td>(0.287)</td>
<td>(0.281)</td>
</tr>
<tr>
<td>(L-)VAR(2) - NL-VAR(1)</td>
<td>1.10</td>
<td>-0.168</td>
<td>-0.571</td>
<td>-0.907</td>
<td>-1.02</td>
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<tr>
<td>SE</td>
<td>(1.092)</td>
<td>(0.131)</td>
<td>(0.126)</td>
<td>(0.273)</td>
<td>(0.236)</td>
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<tr>
<td>NL-VAR(2) - NL-VAR(1)</td>
<td>1.08</td>
<td>0.053</td>
<td>0.031</td>
<td>-0.363</td>
<td>-0.209</td>
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<tr>
<td>SE</td>
<td>(1.276)</td>
<td>(0.133)</td>
<td>(0.106)</td>
<td>(0.309)</td>
<td>(0.133)</td>
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</table>
Scores (CFNAI)
Scores (NFCI)
Joint CRPS
# Mean joint CRPS, difference from NL-VAR(1)

<table>
<thead>
<tr>
<th>Model</th>
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</thead>
<tbody>
<tr>
<td>NL-VAR(1)</td>
<td>0.328</td>
<td>0.412</td>
<td>0.439</td>
<td>0.454</td>
<td>0.488</td>
</tr>
<tr>
<td>(L-)VAR(1)</td>
<td>-0.003</td>
<td>0.033</td>
<td>0.075</td>
<td>0.098</td>
<td>0.127</td>
</tr>
<tr>
<td>SE</td>
<td>(0.02)</td>
<td>(0.017)</td>
<td>(0.008)</td>
<td>(0.01)</td>
<td>(0.023)</td>
</tr>
<tr>
<td>(L-)VAR(2)</td>
<td>-0.028</td>
<td>0.019</td>
<td>0.063</td>
<td>0.082</td>
<td>0.102</td>
</tr>
<tr>
<td>SE</td>
<td>(0.031)</td>
<td>(0.019)</td>
<td>(0.008)</td>
<td>(0.009)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>NL-VAR(2)</td>
<td>-0.003</td>
<td>0.005</td>
<td>0.004</td>
<td>-0.001</td>
<td>-0.012</td>
</tr>
<tr>
<td>SE</td>
<td>(0.017)</td>
<td>(0.013)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.007)</td>
</tr>
</tbody>
</table>
CRPS (CFNAI)
CRPS (NFCI)
PITS (NFCl)
Median forecasts (CFNAI)
Median forecasts (NFCI)
Squared errors (CFNAI)
Squared errors (NFCI)
### MSFE of median forecast (CFNAI), difference from NL-VAR(1)

<table>
<thead>
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<th>h=4</th>
<th>h=8</th>
</tr>
</thead>
<tbody>
<tr>
<td>NL-VAR(1)</td>
<td>0.299</td>
<td>0.487</td>
<td>0.508</td>
<td>0.548</td>
<td>0.599</td>
</tr>
<tr>
<td>(L-)VAR(1) - NL-VAR(1)</td>
<td>-0.084</td>
<td>-0.035</td>
<td>0.076</td>
<td>0.085</td>
<td>0.035</td>
</tr>
<tr>
<td>SE</td>
<td>(0.081)</td>
<td>(0.081)</td>
<td>(0.03)</td>
<td>(0.037)</td>
<td>(0.029)</td>
</tr>
<tr>
<td>(L-)VAR(2) - NL-VAR(1)</td>
<td>-0.138</td>
<td>-0.106</td>
<td>0.019</td>
<td>0.035</td>
<td>0.025</td>
</tr>
<tr>
<td>SE</td>
<td>(0.093)</td>
<td>(0.093)</td>
<td>(0.025)</td>
<td>(0.025)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>NL-VAR(2) - NL-VAR(1)</td>
<td>-0.111</td>
<td>-0.040</td>
<td>0.016</td>
<td>0.003</td>
<td>-0.014</td>
</tr>
<tr>
<td>SE</td>
<td>(0.076)</td>
<td>(0.074)</td>
<td>(0.034)</td>
<td>(0.011)</td>
<td>(0.013)</td>
</tr>
</tbody>
</table>
MSFE of median forecast (NFCI), difference from NL-VAR(1)

<table>
<thead>
<tr>
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<th>h=3</th>
<th>h=4</th>
<th>h=8</th>
</tr>
</thead>
<tbody>
<tr>
<td>NL-VAR(1)</td>
<td>0.138</td>
<td>0.201</td>
<td>0.223</td>
<td>0.246</td>
<td>0.275</td>
</tr>
<tr>
<td>(L-)VAR(1) - NL-VAR(1)</td>
<td>-0.063</td>
<td>-0.044</td>
<td>-0.009</td>
<td>0.007</td>
<td>0.135</td>
</tr>
<tr>
<td>SE</td>
<td>(0.042)</td>
<td>(0.034)</td>
<td>(0.028)</td>
<td>(0.047)</td>
<td>(0.095)</td>
</tr>
<tr>
<td>(L-)VAR(2) - NL-VAR(1)</td>
<td>-0.066</td>
<td>-0.041</td>
<td>-0.011</td>
<td>-0.001</td>
<td>0.081</td>
</tr>
<tr>
<td>SE</td>
<td>(0.044)</td>
<td>(0.029)</td>
<td>(0.023)</td>
<td>(0.033)</td>
<td>(0.049)</td>
</tr>
<tr>
<td>NL-VAR(2) - NL-VAR(1)</td>
<td>-0.007</td>
<td>0.012</td>
<td>0.002</td>
<td>-0.011</td>
<td>-0.013</td>
</tr>
<tr>
<td>SE</td>
<td>(0.018)</td>
<td>(0.02)</td>
<td>(0.011)</td>
<td>(0.009)</td>
<td>(0.007)</td>
</tr>
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</table>
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Measuring economic and financial conditions: Euro area

- Economic activity: GDP growth
- Financial Conditions: Composite Indicator of Systemic Stress (CISS)
- Quarterly data since 1999
Euro Area: Marginal quantiles, 1 quarter ahead

Economic conditions 1 quarter ahead

Financial conditions 1 quarter ahead
Euro Area: Marginal quantiles, 1 quarter ahead

Economic conditions 1 quarter ahead

Financial conditions 1 quarter ahead
Euro Area: Marginal quantiles, 1 quarter ahead

Economic conditions 1 quarter ahead

Financial conditions 1 quarter ahead
Euro Area: Marginal quantiles, 2 quarter ahead

Economic conditions 2 quarters ahead

Financial conditions 2 quarters ahead
Euro Area: Marginal quantiles, 2 quarter ahead

Economic conditions 2 quarters ahead

Financial conditions 2 quarters ahead
Euro Area: Marginal quantiles, 2 quarter ahead

**Economic conditions 2 quarters ahead**

**Financial conditions 2 quarters ahead**
Euro Area: Marginal quantiles, 4 quarter ahead

Economic conditions 4 quarters ahead

Financial conditions 4 quarters ahead
Euro Area: Marginal quantiles, 4 quarter ahead

Economic conditions 4 quarters ahead

Financial conditions 4 quarters ahead
Euro Area: Marginal quantiles, 4 quarter ahead

Economic conditions 4 quarters ahead

Financial conditions 4 quarters ahead
Euro Area: Marginal quantiles, 8 quarter ahead

Economic conditions 8 quarters ahead

Financial conditions 8 quarters ahead
Euro Area: Marginal quantiles, 8 quarter ahead

**Economic conditions 8 quarters ahead**

**Financial conditions 8 quarters ahead**
Euro Area: Marginal quantiles, 8 quarter ahead

Economic conditions 8 quarters ahead

Financial conditions 8 quarters ahead
Euro Area: Joint distribution over time
Euro Area: Joint distribution over time
Euro Area: Joint distribution over time

3 quarters ahead

Graph showing joint distribution over time with axes labeled 'Economic' and 'Financial'.
Euro Area: Joint distribution over time

4 quarters ahead
Euro Area: Joint distribution over time
Euro Area: Joint/marginal distribution, 1 quarter ahead

![Graph showing joint/marginal distribution](image)
Euro Area: Joint/marginal distribution, 1 quarter ahead
Euro Area: Joint distribution, 1-8 quarters ahead
Euro Area: Joint distribution, 1-8 quarters ahead
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Multiplicity in Macroeconomics

- Much research in the 80s and 90s
  - Diamond (1982): thick market externalities
  - Bryant (1983): technological complementarities
  - Diamond and Dybvig (1983): bank runs
  - Murphy, Shleifer, and Vishny (1989): demand spillovers

- But multiplicity has never been shown empirically
  - Morris and Shin (2000, 2003) propose unique equilibrium via imperfect knowledge while preserving amplification
  - hugely impactful literature

- Our evidence suggests a reconsideration of multiple equilibria
Is Bimodality Evidence of Multiplicity?

- Recently, policy has aggressively counteracted “bad” equilibria
- But bad policy can lead to the bad equilibrium, as history has shown
- However, bimodality could be an expression of nonlinearity
Macro-Finance Literature and Nonlinearity

■ He Krishnamurthy (2013) and Brunnermeier Sannikov (2014)
  ▪ Present continuous time theories of macro-financial amplification with occasionally binding constraints
  ▪ All shocks are conditionally gaussian, but vol and drift depend on state variables in highly nonlinear ways

■ Adrian Boyarchenko (2016) and Adrian Duarte (2018)
  ▪ Present macro-financial theories with value-at-risk constraints
  ▪ VaR constraints directly tie conditional means and vols together, even when constraints always always bind

■ Gertler Kiyotaki Prestipino (2016, 2019)
  ▪ Present macro-finance models with multiplicity
  ▪ In a “run” equilibrium banks stop financial intermediation

■ All these theories produce multimodal distributions, but only the latter relies on multiplicity
Implications for Macro-Finance

- We only detect multiplicity when financial conditions are included.
- This suggests that macro-financial dynamics are key.
- However, the data says that multimodality only occurs in recessions, in normal times the conditional density is unimodal, and it is unimodal in the long run.
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Forecasting the Great Recession
Density Impulse Responses
Density Impulse Responses
Density Impulse Responses
Density Impulse Responses
Density Impulse Responses

counterfactual - benchmark (CFNAI)

counterfactual - benchmark (NFCI)
Density Impulse Responses

counterfactual - benchmark (CFNAI)

counterfactual - benchmark (NFCI)
CFNAI Density Impulse Responses: Selected quantiles
NFCI Density Impulse Responses: Selected Quantiles
Density Impulse Responses: Expected shortfall and longrise
Density Impulse Responses (shifted)
Density Impulse Responses (shifted)
Density Impulse Responses (shifted)
Density Impulse Responses (shifted)
Density Impulse Responses (shifted)
Density Impulse Responses (shifted)
CFNAI Density Impulse Responses: Selected quantiles (shifted)
NFCI Density Impulse Responses: Selected Quantiles (shifted)
Density Impulse Responses: Expected shortfall and longrise (shifted)
Drawbacks and further research

- Beyond two dimensions: Curses of dimensionality
  - Accuracy of the smoothing Kernel estimates deteriorates quickly with the dimension
  - Direct sampling becomes computationally too demanding

- Solutions
  - Bayesian estimation of conditional densities + Shrinkage priors
  - Penalized quantile regression or distributional regression + smoothing
    - Recursive models (marginal, joint)
    - Conditionally specified models combined with a “pseudo” Gibbs sampler
    - Combination/ensemble across different permutations
Conclusion

- We present a parsimonious and computationally straightforward method to estimate nonlinear system dynamics.
- We document that the joint density of economic growth and financial conditions features bimodality in bad times.
- We argue that economic theory has to consider macro-financial models that feature multiple equilibria in bad times.


References II


References III


References IV


