Rational Sentiments and Economic Cycles *

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Preliminary and incomplete  
February 23, 2020  

Abstract  

We propose a rational model of endogenous credit cycles generated by the two-way interaction of credit market sentiments and real outcomes. Sentiments are high when most lenders optimally choose lax lending standards. This leads to low interest rates and high output growth, but also to the deterioration of future credit application quality. When the quality is sufficiently low, lenders endogenously switch to tight standards, i.e. sentiments become low. This implies high credit spreads, low quantity of issued credit and a gradual improvement in the quality of applications, which eventually triggers a shift to lax lending standards. The equilibrium cycle might feature a long boom or a lengthy, possibly double-dip recession. It is generically different from the optimal cycle as atomistic lenders ignore their aggregate effect on the composition of borrowers. Carefully chosen risk-weighted capital requirements or countercyclical monetary policy can often improve the decentralized equilibrium cycle.

*We are grateful to John H. Moore whose insightful discussion to our previous work, Heterogeneous Global Cycles, was the main inspiration to write this paper. We also thank Vladimir Asriyan, Ricardo Caballero, Nobu Kiyotaki, Jonathan A. Parker, Jeremy Stein, and seminar and workshop participants at Bank of England, ECB, LSE, FTG, ESSFM Gerzensee, MIT, NBER EF&G. Kondor acknowledges financial support from the European Research Council (Starting Grant #336585).
1 Introduction

A growing body of empirical evidence suggests that periods of overheating in credit markets forecast recessions. Overheated, or high sentiment markets are characterized by increased total quantity of credit, low interest rates, and, importantly, deteriorating quality of newly issued credit. In the subsequent recessions, credit turns scarce and expensive even for ex-post high-value projects.\(^1\)

A major conundrum for policy makers and academics alike is how economic policy should respond to this phenomena. For this, it is essential to understand the mechanism which triggers overheating and then turns credit booms into recessions. That is, we need a framework where credit cycles arise endogenously. In this paper, we build a model where the interaction between credit market sentiments and real economic outcomes generate cycles. We also rank various policy instruments by their efficiency to push the economy towards higher welfare cycles.

We capture credit market sentiments as lenders rational choice of lending standards under imperfect information. In our model, entrepreneurs run projects and obtain credit from investors to scale up their operation. Only some entrepreneurs intend to pay back their credit. The majority of investors are not sufficiently skilled to distinguish these good entrepreneurs from the bad ones. These investors can run one of two types of imperfect tests to decide which entrepreneurs to grant credit to. A bold test represents lax lending standards. This test passes the credit application of all good entrepreneurs along with some bad ones. A cautious test represents tight lending standards as it rejects all applications from bad entrepreneurs along with some of the good ones. That is, tight lending standards improve the quality, but decreases the quantity of the credit issued by an investor.

It is rational to choose lax lending standards when there are few bad entrepreneurs among borrowers. In these periods, credit market exhibits the symptoms of overheating or high sentiments: a mixed quality of credit is issued at a low interest rate inducing high credit growth and high output. At the same time, the flow of credit towards bad types helps them

\(^1\)See (Greenwood and Hanson, 2013; López-Salido et al., 2017) on identifying overheated markets and their relationship with future bond excess returns and recessions. See also Morais et al. (2019) for US and international evidence on lax bank lending standards in booms, and Baron and Xiong (2017) for the negative relationship between banks’ credit expansion and banks’ equity returns. More generally, there is ample evidence of pro-cyclical volume and countercyclical value of investment in a wide range of contexts. For instance, Eisfeldt and Rampini (2006) demonstrates this for sales of property, plant and equipment, while Kaplan and Stromberg (2009) shows similar evidence on venture capital deals.
remain active, leading to the deterioration of borrowers’ quality in the following period. There is a point when borrowers’ average quality becomes so low that lenders rationally switch to tight standards implying high credit spreads, high quality and low quantity of issued credit. This leads to an improving pool of credit applications, eventually triggering a shift back to lax lending standards. As such, there is a two-way interaction between credit sentiment and the fundamentals of the economy.

We show that the model can generate a rich pattern of cycles featuring long booms and short recessions, or vice-versa. The economy might also feature a double-dip recession. We characterize how the properties of cycles change in response to changing parameters.

We also argue that the generated cycles are not constrained efficient. It is so because investors fail to internalize that their individual choice of lending standards, in the aggregate, affect the future quality of borrowers. Nevertheless, a planner often prefers a cycling economy to one with persistently high or persistently low sentiments. In a constrained optimal cycle, recessions induced by tight lending standards keep the fraction of bad projects at bay which makes the subsequent booms more beneficial.

We further connect the constrained planner’s solution to realistic monetary and macro-prudential policies. We show that both changing the risk-free rate and specifying capital requirements can be used to influence investors’ lending standards. Therefore, each of these policies affects the dynamics of the state distribution, and, consequently, welfare. However, the policy maker can improve the quality of loan applications only at the expense of increasing the average cost of capital. This trade-off determines the ranking across policies. Under our representation, we show that a risk-weighted capital requirements and a countercyclical risk-free rate both strongly dominate a non-state contingent risk-free rate policy. The countercyclical monetary policy can improve the welfare slightly more than the risk-weighted capital requirements, however, the former requires a sophisticated regulator since it is aggregate state contingent, while the latter is not.

Finally, we relate our results with a wide range of stylized facts on market segmentation, the fluctuation of credit market sentiment, output, the heterogeneity of returns and portfolios of investors and international spillovers of monetary policy.

**Literature.** To the best of our knowledge our paper is the first to provide a mechanism where economic cycles are endogenously generated by the interaction between the choice of lending standards and average borrower quality.
Our paper belongs to the growing literature on dynamic lending standards where lenders’ choice to acquire information on borrowers differs in booms and in recessions (Martin, 2005; Gorton and Ordonez, 2014; Hu, 2017; Asriyan et al., 2018; Fishman et al., 2019; Gorton and Ordonez, 2016). Gorton and Ordonez (2016) and the contemporaneous paper of Fishman et al. (2019) are the closest to our work. Similar to our model, the mechanism in Fishman et al. (2019) relies on the two-way interaction of lenders’ information choice and borrowers’ average quality. However, unlike our paper, their economy does not feature an endogenous cycle, and converges to a high or a low steady state depending on the parameters. In other words, Fishman et al. (2019) and most of the rest of the papers in this literature do not provide an endogenous mechanism repeatedly turning a boom into a recession and vice-versa. One exception is Gorton and Ordonez (2016). This paper has both an economy that converges to a good steady state, and one that cycles between multiple periods in the good state and one in the bad one. Unlike us, in this economy recessions and the corresponding tight lending standards have no welfare benefits. If possible, a planner prefers to force agents to always use lax lending standards. In our set up a planner often prefers a cycling economy to a persistent boom, because tight lending standards during the recession improve future borrowers’ quality which makes the subsequent boom more beneficial.2

Our paper also contributes to the literature on endogenous credit cycles (Azariadis and Smith, 1998; Matsuyama, 2007; Myerson, 2012; Gu et al., 2013). These papers present different mechanism that leads to endogenous fluctuations in granted credit quantity. However, none of them capture the interdependence of investors choice of lending standards and economic activity.

Our paper is also connected to the literature on collateral based credit cycles (Kiyotaki and Moore, 1997; Lorenzoni, 2008; Mendoza, 2010; Gorton and Ordonez, 2014). As in these papers, we are also interested in how a change in credit availability induces boom and busts. However, these papers focus on how exogenous shocks are amplified by the effect through the price of the collateral. In our model the price of collateral or exogenous shocks play no role.

There is a long tradition in economics starting with Keynes’ metaphor of animal spirits to associate boom-bust cycles with fluctuating investors’ sentiment.3 Recent papers in

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2The difference in welfare implications is a consequence of the different underlying mechanisms. Gorton and Ordonez (2016) argues that dynamic lending standards imply fluctuation in the perceived quality of the average borrowers. In our framework, they imply fluctuation in the realized quality of the average borrower.

3Angeletos and La’O (2013) provides a conceptually distinct approach to capture sentiment in a rational framework as rationally over weighted public information.
this literature (Bordalo et al., 2018; Greenwood et al., 2019) are based on extrapolative expectations. In contrast, we capture credit market sentiment as a rational choice of lending standards. Our model generates some of the leading facts of the empirical side of this literature; for instance the deterioration of credit quality in booms, or the strong correlation between high credit growth and low subsequent returns. However, as a rational model, our mechanism does not generate an exploitable anomaly under the least informed agent’s information set. That is, regarding evidence that points to such anomalies, our approach can only play a complementary role to behavioral models.

Finally, from a methodological perspective, the structure of our credit market builds on Kurlat (2016) which we further develop in the companion paper, Farboodi and Kondor (2018). Neither of these papers focus on endogenous economic cycles.

2 Example.

Lending Standards and Endogenous Cycles

We start with presenting an example which illustrates how the interaction between the state-dependent lending standards and average borrower quality leads to endogenous cycles.

In order to convey the intuition as simple and clear as possible, here we use a simplified version of the micro-foundation in the paper, and postulate a number of endogenous outcomes as exogenous.

Consider a continuum of entrepreneurs producing a consumption good and a continuum of investors providing credit for this production. Entrepreneurs are identified by the type of their projects. Each project is good or bad. The type distribution is the state of the economy, which can be summarized by the measure of bad entrepreneurs, \( \mu_t \). Aggregate output is decreasing in the measure of bad entrepreneurs.

In each period a fraction of entrepreneurs exit, either because they die with probability \( \delta \) or because they are not granted credit. That is, we assume that credit is essential for survival. When an entrepreneur exits, he is replaced with a newborn so as to keep the population fixed at 1. Exogenous fraction \( \lambda \) of the newborn entrepreneurs are bad.

Investors employ the following state-dependent lending standards. When the measure of bad entrepreneurs is larger than a constant \( \mu_t > K \), they impose tight lending standards
and extend no credit to bad entrepreneurs. When the measure of bad entrepreneurs is smaller than the same constant, \( \mu_t < K \), lending standards are lax implying that all bad entrepreneurs can obtain credit. In the next section, we derive investors’ credit allocation decision as an equilibrium outcome. To simplify the exposition we postulate the equilibrium outcome here.

When lending standards are tight, the law of motion for the state variable, measure of bad entrepreneurs, is governed by

\[
\mu_{t+1} = (\delta + (1 - \delta)\mu_t)\lambda
\]

which converges to the constant \( \bar{\mu}_C \equiv \frac{\delta \lambda}{1 - \lambda (1 - \delta)} \) regardless of the initial value of \( \mu_t \).

Alternatively, when lending standards are lax, the law of motion for the state variable is

\[
\mu_{t+1} = (1 - \delta)\mu_t + \delta \lambda
\]

which converges to the constant \( \bar{\mu}_B \equiv \lambda \).

Depending on the level of \( K \), this structure might or might not imply a cycling economy. First, consider the case when \( K \) is close to 0. Since \( K < \bar{\mu}_C \), the economy converges to one with permanent tight lending standards. Next, consider the case when \( K \) is close to 1. Since \( K > \bar{\mu}_B \), the economy converges to one with permanent lax lending standards.

However, when \( \bar{\mu}_C < K < \bar{\mu}_B \), the measure of bad entrepreneurs, \( \mu_t \), cycles through two or more values forever. Figure 2a illustrates one such case. To see the intuition, suppose the economy starts at \( \mu_0 < K \). That is, the measure of bad entrepreneurs is small and lending standards are lax. Therefore, \( \mu_t \) follows (2.1) and increases in each period converging towards \( \bar{\mu}_B \). However, at some point, before reaching \( \bar{\mu}_B = \lambda \), the measure of bad entrepreneurs crosses the critical threshold \( K \) and investors tighten their lending standards. Consequently, the law of motion switches to (2.2) and thus \( \mu_t \) decreases in each period approaching \( \bar{\mu}_C \). Again, at some point, before reaching \( \bar{\mu}_C \), the measure of bad entrepreneurs becomes sufficiently small that investors switch to lax lending standards and the cycle restarts.

As aggregate output decreases with the measure of bad entrepreneurs, the cycling entrepreneur-distribution leads to economic cycles. Low (high) output is associated with tight (lax) lending standards.

In the rest of the paper, we show that the interaction of rational investors and en-
entrepreneurs under asymmetric information naturally leads to state-dependent lending standards and, consequently, to endogenous cycles. We start by presenting the full model in the next section.

3 Set Up. Rational Sentiments and Economic Cycles

The economy consists of an infinite number of periods and is populated by entrepreneurs and investors. The type distribution of entrepreneurs serves as the state variable of the economy.

There is one type of consumption good which can be turned to investment or can be consumed. Each period is divided into two parts: morning and evening. Each agent, entrepreneur and investor, is risk-neutral and endowed with one unit of the good in the morning. Each agent can use a storage technology in the morning providing $1 + r_f$ return per unit of saving by the evening.\footnote{\(r_f\), can represent a physical return or a policy rate. In sections 4 and 5, we think of it as the rate of return on the storage technology, which can be normalized to zero. In section 6 we reintroduce \(r_f\) as the return on a risk-free asset provided by the policy maker.}

Here we explain the optimization problem of each type of agent. The formal optimization problems are stated in Appendix A.

Entrepreneurs. There is a unit mass of entrepreneurs present in the economy each day. Entrepreneurs are identified by the type of their projects. Each project has a two-dimensional type. It is good or bad, \(\tau = g, b\), and opaque or transparent, \(\omega = 0, 1\). Entrepreneurs know their own type. We denote the measure of opaque and transparent bad entrepreneurs at time \(t\) by \(\mu_{0,t}\) and \(\mu_{1,t}\), respectively.

Each entrepreneur \((\tau, \omega)\) chooses investment \(i_t(\tau, \omega)\) and obtains credit \(\ell_t(\tau, \omega)\) at the prevailing interest rate \(r_t(\tau, \omega)\) each morning, and produces and consumes in the evening.

Each unit of investment costs one unit of consumption and returns \(\rho > 1 + r_f\) consumption good by the evening.\footnote{We have also solved the model under the alternative assumption that good (bad) investment returns \(\rho_g > 1 + r_f\) (\(\rho_b < 1\)). The expressions are more complex without providing any new intuition, thus we have decided to stick to \(\rho = \rho_b = \rho_g > 1 + r_f\). The more general solution is available in the previous circulated versions of the paper, as well as available upon request.} The cost of investment should be covered by entrepreneur’s initial
endowment or credit, implying the following budget constraint

\[ i_t(\tau, \omega) = 1 + \ell_t(\tau, \omega). \]  

Credit is limited because debt has to be collateralized, as seizing the collateral is the only threat to enforce repayment. In principal, investment can be used as collateral because investors can turn each unit of investment that they can seize into a unit of consumption, which implies \((1 + r_t(\tau, \omega))\ell_t(\tau, \omega) \leq i_t(\tau, \omega)\). Using (3.3) this simplifies to

\[ \ell_t(\tau, \omega) \leq \frac{1}{r_t(\tau, \omega)}. \]  

The difference between good and bad projects is that investors cannot seize investment in bad projects. That is, if investors observed the type of entrepreneurs, they would only lend to good types as repayment from bad types cannot be enforced. However, as we explain below, investors might not have imperfect information on entrepreneurs’ type. This is the critical friction in the model.

At the end of each period a random fraction \(\delta\) of the entrepreneurs die and are replaced by an inflow of new entrepreneurs of the same measure, similar to the example. We continue to assume that credit is essential for survival. Therefore, any entrepreneur who is not able to raise financing is also replaced by a newborn entrepreneur. The type distribution of the new entrants is fixed. Assumption 3.1 below formalized the replacement rule for the entrepreneurs.

**Assumption 3.1** Randomly chosen \(\delta\) fraction of the living entrepreneurs, as well as any entrepreneur not financed by investors is replaced by a randomly chosen newborn from the outside pool of entrepreneurs in the next period. The type distribution of the newborns has \(\lambda\) fraction of bad entrepreneurs, and an independent \(\frac{1}{2}\) fraction of opaque ones.

Clearly, the law of motion of the state variable, the distribution of entrepreneurs, depends on credit market outcomes.

**Investors.** Each investor lives for one period, uses her unit endowment to grant credit to entrepreneurs or saves it by the storage technology. A small, \(w_1\), mass of investors are skilled,
while a large, $w_0$ mass of investors are unskilled. Skill is privately observable. All investors are born in the morning, provide loans in the afternoon, consume and die in the evening and are replaced by the same type investors. Let $h \in [0, w_0 + w_1]$ denote an individual investor.

Skilled investors can observe the type of each project. Unskilled investors instead can observe imperfect signals for the project sample they receive. These signals are generated by a test of investor choice. Each investor can opt for a **bold test** or a **cautious test**. The tests differ in the signal they generate for opaque projects. The bold test pools all opaque projects, good or bad, with good transparent ones (a false positive error). The cautious test pools all opaque projects with bad transparent ones (a false negative error). Intuitively, we can envision the bold test to reject bad transparent projects only and pass all other ones, while the cautious test passes only for good transparent projects.\(^6\) When an investor is indifferent between the two tests, we break the tie by assuming that she chooses the bold test.

The size of the sample an investor tests is limited by the investor’s unit endowment; she cannot test more applications than the quantity she could finance if all pass her test. The cost of the test is $c \in (0, 1)$, and each unskilled investor runs exactly one test.

**Credit Market.** Credit market operates in the morning. After choosing the type of the test, each investor advertises an interest rate, $\bar{r}(h)$, at which he is willing to give loans to applications passing his test. Each entrepreneur chooses the measure of loan applications $\sigma(r; \tau, \omega) \in [0, \frac{1}{r}]$ she wishes to submit at each interest rate $r$. The credit market clears starting from the lowest interest rate and unskilled investors sample first. As we will see, due to the informational friction some types might be rationed in the credit market. This leads to the effective credit contraint

$$\ell_t(\tau, \omega) \leq \min \left( \bar{\ell}_t(\tau, \omega), \frac{1}{r_t(\tau, \omega)} \right) \quad (3.5)$$

where $\bar{\ell}_t(\tau, \omega)$ is the maximum credit available for the given type. We provide further details on the market clearing protocol and collateralization in Appendix A.

For simplicity, we assume that there is no credit history recorded for entrepreneurs. That

\(^6\)For simplicity we restrict investor’s choice set to these two tests. In appendix C we show that this restriction is not essential for our results. In particular, we enrich the investor choice set so that they are able to choose between the continuum of test lying between the bold and cautious extreme tests, and we show that the dominant choice is always one of the extreme.
is, investors cannot learn from the past. Also, there is no saving technology available across periods. Therefore, entrepreneurs consume their wealth at the end of each period, and if survived, they start the new period with their unit endowment received in the morning.

We refer to the interaction of entrepreneurs and investors within a period as the stage game. The decentralized equilibrium is defined as follows.

**Definition 3.1 (Equilibrium)** A decentralized dynamic equilibrium is a sequence of stage game equilibria. A stage game equilibrium is a set of entrepreneurs’ investment, $i_t(\tau, \omega)$, credit demand schedules, $\sigma_t(r, \tau, \omega)$, along with investors’ advertised interest rate schedule $\tilde{r}_t(h)$, unskilled investors’ choice of test, an equilibrium interest rate schedule $r_t(\tau, \omega)$, and credit allocation schedule $\ell_t(\tau, \omega)$ for each entrepreneur, and allocation of applications to investors such that

(i) each agent’s choice is optimal given the strategy profile of all other agents;

(ii) the implied interest rate schedule $r_t(\tau, \omega)$, and credit allocation schedule $\ell_t(\tau, \omega)$ for each entrepreneur, and allocation of applications to investors are consistent with agents’ choices and the market clearing process.

In each period, the stage game equilibrium is consistent with the realized type distribution, while the dynamics of the type distribution is consistent with Assumption 3.1.

We focus on the case where there are many unskilled investors, but few skilled investors in the following sense.

**Assumption 3.2** The mass of skilled and unskilled investors, $w_1, w_0$, satisfies the following criteria.

(i) Skilled investors capital is not sufficient to cover the credit demand of all opaque good entrepreneurs at any interest rate that any good entrepreneur is willing to borrow at.

(ii) Unskilled investors capital, $w_0$, is abundant. In particular, it is sufficiently large that it covers the credit demand of all entrepreneurs that unskilled investors are willing to lend to at any equilibrium interest rate.
Our structure allows for solving for the full equilibrium in steps. In section 4 we first solve for the credit market equilibrium in each period. In doing so, we take the entrepreneur type distribution as given. Then we characterize the credit market dynamics. In Section 5, we describe the real economy outcomes. To ease the notation, we omit the time-subscript in each object whenever it does not cause any confusion.

4 Credit Market Equilibrium

In this section, we characterize the stage game equilibrium in the credit market for a given measure of opaque and transparent bad entrepreneurs, \((\mu_0, \mu_1)\). We start the derivation of the equilibrium with a few basic properties of entrepreneurs’ credit demand.

Lemma 4.1 In any equilibrium entrepreneurs’ credit demand schedule, \(\sigma(r, \tau, \omega)\), simplifies as follows:

(i) Each type chooses a reservation interest rate \(r^{\text{max}}(\tau, \omega)\) and submits maximum demand to all weekly lower interest rates and 0 otherwise.

(ii) Good entrepreneurs never choose a higher reservation rate, while bad entrepreneurs never choose a lower reservation rate than \(\bar{r} \equiv \rho - 1\): \(r^{\text{max}}(g, \omega) \leq \bar{r}, r^{\text{max}}(b, \omega) \geq \bar{r}\).

This Lemma simplifies the analysis considerably. It shows that it is sufficient to find the equilibrium reservation interest rate of entrepreneurs instead of working out a full credit demand schedule. Observe that \(\bar{r}\) denotes the interest rate above which a good entrepreneur would never borrow as it would turn its project to negative net present value.

We next show that the unique equilibrium in the credit market is one of three distinct types, depending on the parameters. In order to do so, it is useful to first define three interest rate functions.
Definition 4.1 (Dynamic Interest Rates)

\[ r_B(\mu_0, \mu_1, c, r_f) \equiv \frac{\mu_0}{1 - \mu_1 - \mu_0} + \frac{1 - \mu_1}{1 - \mu_1 - \mu_0} r_f + \frac{1}{1 - \mu_1 - \mu_0} c \quad (4.1) \]

\[ r_C(\mu_0, \mu_1, c, r_f) \equiv r_f + \frac{2}{1 - \mu_1 - \mu_0} c \quad (4.2) \]

\[ r_I(\mu_0, \mu_1, c, r_f) \equiv \frac{2\mu_0}{1 - \mu_1 - \mu_0} + \frac{1 + \mu_0 - \mu_1}{1 - \mu_1 - \mu_0} r_f + \frac{1 + \mu_1 + \mu_0}{1 - \mu_1 - \mu_0} c. \quad (4.3) \]

Let \( \tilde{\mu}_0(\mu_1) \equiv \frac{\bar{r} - r_f - c - \mu_1(\bar{r} + c - r_f)}{2 + c + \bar{r} + r_f} \). The next proposition characterizes the three type of equilibria depending on \( \tilde{\mu}_0(\mu_1) \), and shows that in each of them, the entrepreneurs who can obtain credit face exactly one of the above three interest rates or the maximum interest rate \( \bar{r} \).

Proposition 4.1 When \( \min\{r_B(\mu_0, \mu_1, c, r_f), r_C(\mu_0, \mu_1, c, r_f)\} < \bar{r} \),

(i) \( \mu_0 \in [0, \frac{c}{1 + r_f}] \) is associated with a bold stage. In a bold stage the credit market has a pooling equilibrium where all entrepreneurs who obtain credit (all good and some bad), do so at interest rate \( r_B(\mu_0, \mu_1, c, r_f) \). Every unskilled investor chooses the bold test.

(ii) \( \mu_0 \in (\max\{\frac{c}{1 + r_f}, \tilde{\mu}_0(\mu_1)\}, 1] \) is associated with a cautious stage. In a cautious stage the credit market has a separating equilibrium, where opaque good entrepreneurs obtain credit at interest rate \( \bar{r} \), transparent good entrepreneurs obtain credit at \( r_C(\mu_0, \mu_1, c, r_f) \) and bad entrepreneurs don’t obtain any credit. Every unskilled investor chooses the cautious test.

(iii) \( \mu_0 \in (\frac{c}{1 + r_f}, \max\{\frac{c}{1 + r_f}, \tilde{\mu}_0(\mu_1)\}] \) is associated with a mix stage. In a mix stage the credit market has a semi-separating equilibrium, where opaque good and bad entrepreneurs obtain credit at interest rate \( r_I \). Good transparent entrepreneurs obtain credit at interest rate \( r_C(\mu_0, \mu_1, c, r_f) \). Some unskilled investors choose the bold test while others choose the cautious test.

Otherwise the economy is in autarky, where unskilled investors do not lend, bad entrepreneurs do not borrow, and good firms obtain credit at interest rate \( \bar{r} \) from skilled investors only.

When there are not too many bad projects around, investors are more concerned about losing out on good project by applying too tight lending standards. Thus lending standards
are lax, and many projects including bad ones are able to raise financing at the same relatively low rate. On the other hand, if there are many bad projects, lending standards are tightened and credit market becomes segmented. Not only bad projects are unable to raise financing, even some good projects are able to do so only at extremely high rates. Lastly, if the measure of bad projects are at some intermediate level, then some investors apply lax and some tight lending standards. Markets are still fragmented but still some bad projects are able to raise financing.

The intuition relies on the fact that abundant supply of unskilled capital implies a zero profit condition for unskilled investors. In fact, interest rates \( r_B (\mu_0, \mu_1, c, r_f) \), \( r_C (\mu_0, \mu_1, c, r_f) \) and \( r_I (\mu_0, \mu_1, c, r_f) \) are the rates at which an unskilled investor is indifferent between lending to entrepreneurs and earning the risk free rate \( r_f \) without running a test in the corresponding equilibrium. The indifference holds as long as all types apply for credit at that rate. The break-even interest rates, \( r_B, r_C \) and \( r_I \) depends on investors’ choice of test due to the following trade-off. The cautious test results in a loan portfolio of higher quality as unlike the bold test, only good entrepreneurs pass it. Thus investors are always paid back when they run a cautious test and end up extending a loan. However, their rejection rate is higher than with a bold test as a cautious test fails all the opaque good entrepreneurs. As running the test has a fixed cost, not lending to tested applications is costly.

The bold stage arises when the break-even rate for bold investors is smaller than that for cautious investors, \( r_B (\mu_0, \mu_1, c, r_f) \leq r_C (\mu_0, \mu_1, c, r_f) \). This is the case when \( \mu_0 \leq \frac{c}{1+r_f} \). This is the left region on the left panel of Figure 1, on the left of the vertical line. Here there are few bad opaque entrepreneurs and thus the rejection rate of cautious test is relatively high. Thus cautious investors cannot compete with bold investors and all active unskilled investors are bold. As the bold test passes all the good projects, opaque or transparent, skilled investors offering higher rate than \( r_B \) would end up with no applications. Therefore, there is a single prevailing market interest rate at which all good projects and some bad ones raise funding from both skilled and unskilled investors. Skilled investors obtain a rent as they finance only good projects.

The cautious stage arises if cautious investors are willing to enter at a lower interest rate than bold investors as long as all types apply for credit at that interest rate, \( r_B (\mu_0, \mu_1, c, r_f) > r_C (\mu_0, \mu_1, c, r_f) \). In this case, all unskilled investors are cautious in equilibrium. This is the

\[ \text{An additional requirement for a cautious stage is that the break-even interest rate for bold investors under the condition that transparent good entrepreneurs are not applying is not feasible. That is, } r_I (\mu_0, \mu_1, c, r_f) > \bar{r}, \text{ or, equivalently, } \bar{\mu}_0(\mu_1) < \frac{1}{1+r_f}. \text{ Otherwise we are in the mixed stage. We return to this distinction in} \]
Figure 1: Interest rates and output in the two-stage equilibrium as $\mu_0$ changes in a two-stage economy. The left panel displays the reservation interest rates $r_B$, $r_C$, (dashed curves), the maximum feasible rate $\bar{r}$ (dashed horizontal line), and the equilibrium interest rates (solid curves). The right panel displays output. The vertical line in both figures corresponds to $\mu_0 = \frac{c}{1+r_f}$, the threshold where the equilibrium changes between cautious and bold.

right region in the left panel of Figure 1 where different good projects raise financing at different interest rates. Bad projects cannot raise any financing. However, as cautious investors reject opaque good projects, in this stage, skilled investors can advertise a higher interest rate and attract applications from all opaque good entrepreneurs. Indeed, as skilled capital is in short supply, they are advertising the highest possible rate a good entrepreneur is willing to accept, $\bar{r}$.

A bold stage exhibits several features of an overheated, high sentiment credit market. Interest rates are uniformly low and most projects including some bad ones are financed. Thus the overall quality of initiated credit contracts is low with a significant share eventually defaulting. This is in contrast with the cautious stage which exhibits feature of a low sentiment credit market. Most importantly, this market is fragmented. Some good entrepreneurs (transparent ones) enjoy a lot of funding at low interest rates. However, aside from bad projects not being financed at all, some good entrepreneurs (opaque ones) can get only limited funding at very high rates. Therefore, the total loan quantity is relatively low, but its quality is high, which leads to high subsequent realized returns.

In economies where $\mu_1$ is such that $\tilde{\mu}_0(\mu_1) < \frac{c}{1+r_f}$, the equilibrium fluctuates only between the bold and the cautious stage. We refer this case as a two-stage economy. The left panel of Figure 1 displays the relevant zero profit interest rates along with $\bar{r}$, as a function of the proportion of opaque bad projects $\mu_0$.
The third part of the proposition shows that there might be an intermediate case which we refer to as the mix stage. When \( \tilde{\mu}_0(\mu_1) > \frac{c}{1+r_f} \), the mix stage arises for the intermediate range of \( \mu_0 \). In this equilibrium the credit market is segmented, but unskilled investors are some bold and some cautious so both bad and good projects get financing. We postpone the discussion of this equilibrium to section D.

4.1 Dynamics and Endogenous Cycles

Similar to our example, the key to the dynamics of the model is the interaction between the choice of lending standards and the quality composition of the population of borrowers. This quality deteriorates in the bold equilibrium when investors’ lending standards are lax, and improves in a cautious equilibrium when their lending standards are tight. At the same time, the changing type distribution induces rational shifts in investors choice of information test and implies fluctuations in sentiment. This endogenous interaction leads to deterministic economic cycles without any exogenous aggregate shock to the economy.

We first display with the general description of the evolution of the state variable. Then, we restrict our attention to a region of parameters where there is a two-stage economy. In appendix D we extend our discussion to a three-stage economy which can experience a double-dip recession before eventually recovering.

Evolution of State Variable. Let \( \mu'_0 \) and \( \mu'_1 \) denote the state variables next period.

When at least some lenders run the bold test, only bad transparent projects cannot raise financing. However, when lenders are all cautious, opaque bad projects are not financed either. In either stage who cannot raise financing exit the pool of projects and are replaced by a new draw from the outside pool. The next proposition summarizes the law of motion for measure of opaque and transparent bad entrepreneurs.

**Proposition 4.2** Assume \( \min\{r_B(\mu_0, \mu_1, c, r_f), r_C(\mu_0, \mu_1, c, r_f)\} < \bar{r} = \rho - 1 \) so that the economy is not in autarky.
(i) If \( \mu_0 \in \left[ 0, \max\{\frac{c}{1+r_f}, \bar{\mu}_0(\mu_1)\} \right] \), then law of motion for \( \mu_0 \) and \( \mu_1 \) follows

\[
\mu_{0B}(\delta, \lambda, \mu_0, \mu_1) = (1 - \delta)\mu_0 + (\delta + (1 - \delta)\mu_1) \frac{\lambda}{2},
\]

(4.4)

\[
\mu_{1B}(\delta, \lambda, \mu_0, \mu_1) = (\delta + (1 - \delta)\mu_1) \frac{\lambda}{2}.
\]

(4.5)

(ii) If \( \mu_0 \in (\max\{\frac{c}{1+r_f}, \bar{\mu}_0(\mu_1)\}, 1) \), then law of motion for \( \mu_0 \) and \( \mu_1 \) follows

\[
\mu_{0C}(\delta, \lambda, \mu_0, \mu_1) = (\delta + (1 - \delta)(\mu_0 + \mu_1)) \frac{\lambda}{2}.
\]

(4.6)

\[
\mu_{1C}(\delta, \lambda, \mu_0, \mu_1) = (\delta + (1 - \delta)(\mu_0 + \mu_1)) \frac{\lambda}{2}.
\]

(4.7)

These laws of motion are quite intuitive. For instance, consider the mass of opaque bad types \( \mu_0 \). When the economy is not in a cautious stage, function \( \mu_{0B}(\delta, \lambda, \phi, \mu_0, \mu_1) \) defines the evolution of \( \mu_0 \). It consists of survivals from this period, plus the replacements from the outside pool. From the existing bad opaque entrepreneurs, fraction \( (1 - \delta) \) survives. The replacements consists of two parts itself: \( \delta \) measure of all entrepreneurs are exogenously replaced. Furthermore, \( (1 - \delta)\mu_1 \) is the measure of the transparent bad types who have survived endogenously cannot raise funding and this are replaced. From the replacements, a fraction \( \lambda/2 \) enter as opaque bad. All the other cases follow a similar intuition.

Following the same logic, the opaque and transparent good types are subject to the same law of motion in both cases, given that their measures in the outside pool is the same. Thus in the long run both measures will be equal to \( \frac{1 - \mu_0 - \mu_1}{2} \). This validates that our \((\mu_0, \mu_1)\) are sufficient state variables for the economy despite four types of entrepreneurs.

We next characterize the endogenous cycle in a two-stage economy where \( \bar{\mu}_0(\mu_1) < \frac{c}{1+r_f} \) along the equilibrium path. We will provide a characterization of the more general case in section D.

**Two-Stage Economy**  To ensure \( \bar{\mu}_0(\mu_1) < \frac{c}{1+r_f} \) in the long run, for the rest of this section we maintain the following assumption to ensure the investors are all bold or cautious.

**Assumption 4.3**

\[
\frac{\bar{r} - r_f - c - \frac{2c}{1+r_f}}{\bar{r} + c} - \frac{c}{1 + r_f} \leq \frac{\delta \lambda}{2 - \lambda(1 - \delta)}.
\]

(4.8)
In the two-stage economy, the dynamic economy is characterized by a single state variable, \( \mu_0 \). The following four constant levels of \( \mu_0 \) are very useful in our credit cycle characterization:

- \( \bar{\mu}_0^B(\delta, \lambda) \): value of \( \mu_0 \) in a steady state where every lender is bold and remains bold forever.

- \( \bar{\mu}_0^C(\delta, \lambda) \): value of \( \mu_0 \) in a steady state where every lender is cautious and remains cautious forever.

- \( \mu_0^*B(\delta, \lambda) \) and \( \mu_0^*C(\delta, \lambda) \): values of \( \mu \) if the economy fluctuates between two states in the long run, one in which every one is bold (\( \mu_0^*B(\delta, \lambda) \)), and one in which everyone is cautious (\( \mu_0^*C(\delta, \lambda) \)).

The first two values, \( \bar{\mu}_0^B \) and \( \bar{\mu}_0^C \), correspond to steady states that are not cycles. \( \mu_0^*B \) and \( \mu_0^*C \) on the other hand correspond to a cycle of length 2. The appendix provides detail for derivation of these levels. It also shows that \( \bar{\mu}_0^B(\delta, \lambda) > \mu_0^*C(\delta, \lambda) > \mu_0^*B(\delta, \lambda) > \bar{\mu}_0^C(\delta, \lambda) \).

Using these values, the following proposition characterizes the steady state dynamic cycles of the economy.

**Proposition 4.3** Given \( \bar{\mu}_0^B(\delta, \lambda) > \mu_0^*C(\delta, \lambda) > \mu_0^*B(\delta, \lambda) > \bar{\mu}_0^C(\delta, \lambda) \) above, the ergodic distribution of the economy is characterized as follows.

(i) \( \frac{c}{1+r_f} \geq \bar{\mu}_0^B \): the ergodic distribution is degenerate, \( \mu_0 \rightarrow \bar{\mu}_0^B \). The economy converges to a permanent bold stage.

(ii) \( \frac{c}{1+r_f} < \bar{\mu}_0^C \): the ergodic distribution is degenerate, \( \mu_0 \rightarrow \bar{\mu}_0^C \). The economy converges to a permanent cautious stage.

(iii) \( \mu_0^*B \leq \frac{c}{1+r_f} \leq \mu_0^*C \): the ergodic distribution has a two-point support, \( \mu_0^*C, \mu_0^*B \). The economy oscillates between a one-period bold stage and a one-period cautious stage for ever. Thus the economy has a credit cycle of length 2.

(iv) \( \mu_0^*C < \frac{c}{1+r_f} < \mu_0^*B \): the ergodic distribution has more than two points of support. The credit cycle consists of a multi-period bold stage (while \( \mu_0 \) increases), followed by a one-period cautious stage when \( \mu_0 \) declines to that of the bold stage with the lowest \( \mu_0 \).
(v) $\bar{\mu}_0 C \leq \frac{c}{1+r_f} < \mu_{0B}^*$: the ergodic distribution has more than two points of support. The credit cycle consists of a multi-period cautious stage (while $\mu_0$ decreases), followed by a one-period bold stage when $\mu_0$ rises to that of the cautious stage with the highest $\mu_0$.

One can think of $\frac{c}{1+r_f}$ as investor opportunity cost of time, or opportunity cost of giving up on good investment. When it is large, investors have a tendency to use the bold test over cautious test since the opportunity cost of giving up on good investment is high. When this value is very large, then the economy is doomed to end in a permanent overheated bold stage. Basically the measure of opaque bad projects never reaches a high enough value where it would be wise to be cautious. As a mirror image when $\frac{c}{1+r_f}$ is very low, there is a permanent low-sentiment cautious stage.

In contrast when investors has an intermediate opportunity cost $\frac{c}{1+r_f}$, the economy features endogenous, deterministic cycles of various types. We refer to this set of parameters as the cyclicality region. Within the cyclicality region, when investor opportunity cost of giving up good investment is relatively high, the cycle features multi periods of boom and a one period recession. In this case, a short recession is enough to improve the quality of loan applications sufficiently such that investors are happy to be bold again so they do not risk losing good investment, at the cost of financing some bad investment. A lower $\frac{c}{1+r_f}$ implies a symmetric cycle, where one-period booms are followed by one period recessions. An even lower investor opportunity cost of time implies multi-period recessions followed by a one period booms.

The top panel of Figure 2 helps clarify the intuition behind deterministic cycles. Given the evolution of entrepreneurs’ type distribution, under a fixed information choice of investors, the proportion of bad opaque types would converge to a single steady state. The upper (lower) dashed horizontal line denotes this steady state when investors are bold (cautious). The measure of opaque bad entrepreneurs ($\mu_0$) in the bold steady state has to be higher that of the cautious one, as the exit rate of opaque bad entrepreneurs is higher when investors are cautious. If investors test switching threshold lies between these two steady states, the economy must exhibit deterministic cycles of one type or another. For instance, consider starting at a low $\mu_0$, below the threshold $\frac{c}{1+r_f}$. When measure of opaque bad firms is low, investors are bold and hence $\mu_0$ moves up, towards the higher $\mu_0$, bold steady state. Therefore, there must be a point when $\mu_0$ surpasses the threshold $\frac{c}{1+r_f}$ and triggers a switch to being cautious. But then, $\mu_0$ immediately moves towards the lower $\mu_0$, cautious steady state. Intuitively, the length of booms and recessions depend on how many steps the system
Figure 2: Panel (a) depicts the law of motions of state variables. Panel (b) shows the interest rates, and Panel (c) depicts the total gross output and welfare in a two-stage economy with a multi-period boom and a one period recession cycle.
needs in any of these stages to cross the threshold. The Figure depicts the case when booms are long and recessions are short.\(^8\)

The next panel of Figure 2 plot the corresponding ergodic distribution of interest rates. Consistently with Proposition 4.1, we see that there is no credit spread across different entrepreneurs in the bold stage. However, the credit market is fragmented in the cautious stage. As unskilled investors stop lending to opaque good firms, their interest rate, and the observed credit spread spikes.

Note that cycles are an outcome of the two-way interaction between investor sentiment and the fundamentals of the economy. In booms investors are bold because the opportunity cost of losing a good project is high for them, since the measure of opaque and bad applicants are relatively low. Thus lending standards are lax and there is a lot of credit. However, as a result the quality of the credit pool starts to deteriorate. At some point, the measure of opaque bad applicants becomes so high that investors prefer to turn cautious. Being cautious implies tight lending standards, high interest rate and little credit for opaque projects, which stops opaque bad entrepreneurs from raising funding. Hence, they are replaced by newborns which improves the quality of the credit pool. Therefore, the cycle continues.

The last panel plots the corresponding ergodic distribution output and welfare which we study in the next sections.

5 Investment and Output

In section 4 we described when the economy is in a bold, cautious or mix stage depending on the state \(\mu_0, \mu_1\) and we characterized the corresponding interest rates \(r(\tau, \omega)\) for each type. We have also pointed out that in a bold stage, skilled and unskilled investors lend to all the good types, while bad opaque types served by unskilled investors by mistake. In contrast, in a cautious stage, skilled investors lend only the opaque good types, while unskilled investors lend the transparent good types only. In this section, we conclude the

\(^8\)Is it possible to construct an equilibrium where the economy is permanently at the threshold \(\mu_0 = \frac{c}{1+r_f}\) by disposing the assumption that indifferent investors choose to be bold? At that point investors are indifferent, hence, one could require any given measure of them to be bold. However, under our market clearing protocol, as long as any positive measure of investors choose to be bold, the law of motion is given by (4.4)-(4.5). The reason is that bold investors capital is distributed pro rata among passed applicants, and Assumption 3.1 implies that even minimal credit is enough for survival. This implies that the economy cannot be stuck at \(\mu_0 = \frac{c}{1+r_f}\) even if investors mix between bold and cautious.
characterization by deriving the implied quantity of credit, investment and output of each type in each of these stages. We also characterize the implications to the path of aggregate output along equilibrium cycles.

Recall that as $\bar{r} \geq r(\tau, \omega) > r_f$ in all stages for all types, all entrepreneur prefer to borrow as much as the collateral constraint, (3.5), allows and invest all to her project. That is, $\ell(\tau, \omega) = \min(\frac{1}{r(\tau, \omega)}, \bar{\ell}(\tau, \omega))$. In particular, some entrepreneurs are unconstrained by the informational friction, because the capital rich unskilled investor group is lending to them willingly. These types borrow $\ell(\tau, \omega) = \frac{1}{r(\tau, \omega)}$. This is the case with all the good entrepreneurs when unskilled investors are bold, and with transparent good entrepreneurs when unskilled investors are cautiuos. The last step is to work out $\bar{\ell}(\tau, \omega)$ for each type by market clearing in all market segments which unskilled investors are not willingly serve. The summarize the result in the next proposition in each type of equilibria. We spell out the derivation in the Appendix.

**Proposition 5.1**

(i) In any equilibrium transparent bad entrepreneurs are not financed by any investors, hence $i(b, 1) = 1$.

(ii) In the bold stage, all entrepreneurs face interest rate $r_B$. All good entrepreneurs invest $i(g, \omega) = \frac{1}{r_B} + 1$, while bad opaque entrepreneurs’ investment plan is limited by unskilled investors’ mistakes at interest rate $r_B$, implying $\ell(b, 0) = \frac{1}{r_B} - \frac{w_1}{1 - \mu_0 - \mu_1}$.

(iii) In a cautious stage, all transparent good entrepreneurs face interest rate $r_C$ thus invest at $i(g, 1) = \frac{1}{r_C} + 1$, while good opaque ones face $\bar{r}$ and their investment $i(g, 0)$ is limited by the capital of skilled investors, implying $i(g, 0) = \frac{2w_1}{1 - \mu_0 - \mu_1} + 1$. Bad opaque entrepreneurs are not financed by investor, hence $i(b, 0) = 1$.

(iv) In a mix stage, all transparent good entrepreneurs face $r_C$ while opaque good face $r_I$, and they both implement the maximum scale, $i(g, 1) = \frac{1}{r_C} + 1$ and $i(g, 0) = \frac{1}{r_I} + 1$. Opaque bad investment $i(b, 0)$ is limited by unskilled investor’s mistake at $r_I$, $\ell(b, 0) = \frac{1}{2r_I} - \frac{w_1}{1 - \mu_0 - \mu_1}$.

Note that the output of entrepreneur $(\tau, \omega)$ is given by $y(\tau, \omega) \equiv \rho i(\tau, \omega) = \rho(1 + \ell(\tau, \omega))$.

In a bold stage all entrepreneurs apply for loans at interest rate $r_B$. All unskilled investors use the bold test and they have abundant capital, thus every good entrepreneurs can obtain
all the credit they are willing to absorb at interest rate $r_B$. Among the bad entrepreneurs, transparent ones cannot obtain credit as they are rejected by the bold test. On the other hand, opaque bad entrepreneurs can obtain some credit because unskilled investors cannot distinguish them from good entrepreneurs using the bold test. However, their credit and thus investment is limited by the mistakes made by unskilled investors who choose to participate in the credit market. Since all good entrepreneurs and even some bad ones invest, the output is high in a bold stage, thus it corresponds to a “boom”.

In a cautious stage, good transparent entrepreneurs obtain credit from unskilled investors using the cautious test at interest rate $r_C$. Unskilled capital is in large supply, therefore good transparent entrepreneurs can implement $i = \frac{1}{r_C} + 1$. Good opaque entrepreneurs instead are obtain credit only from skilled investors at the maximum feasible interest rate $\bar{r}$. As the capital of skilled investors is in short supply, their capital limits the credit of these entrepreneurs implying low low credit quantities. None of the bad entrepreneurs can raise any financing from investors. Thus investment is low in a cautious stage and it corresponds to a “recession”.

We discuss the mix stage (iv) in section D in detail. Here we just note that is in between the other two regimes of equilibria.

The aggregate output in state $(\mu_0, \mu_1)$ is given by

$$Y(\mu_0, \mu_1) \equiv \frac{1 - \mu_0 - \mu_1}{2} \left( y \left( g, 1 \right) + y \left( g, 0 \right) \right) + \rho \left( \mu_1 y \left( b, 1 \right) + \mu_0 y \left( b, 0 \right) \right) = \rho \left( 1 + \frac{1 - \mu_0 - \mu_1}{2} \left( \ell \left( g, 1 \right) + \ell \left( g, 0 \right) \right) + \mu_0 \ell \left( b, 0 \right) \right). \quad (5.1)$$

The right panel of Figure 1 illustrates aggregate output conditional on state state $\mu_0$ (for a fixed state $\mu_1$) in a two-state economy. A natural observation is that aggregate output is smoothly monotonically decreasing in the measure of bad opaque entrepreneurs within any range of parameters where the type of the equilibrium is not changing. This is so because, as Figure 1 illustrates, the equilibrium interest rates are (weakly) increasing in $\mu_0$. A larger proportion of bad entrepreneurs increases the equilibrium interest rates, because of adverse selection. This increases the cost of capital for production, which decreases investment and total output according to Proposition 5.1. Also, as we state in the next Lemma, the change in total output is not smooth when the economy switches between the bold and cautious stage. Continuous changes in $\mu_0$ can lead to discontinuous jumps in $Y(\mu_0, \mu_1)$. 

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In this sense, the economy crashes around the thresholds where agents switch from bold to cautious strategy. This is the consequence of the discontinuous drop in credit when unskilled investors stop lending to all opaque entrepreneurs good or bad. This leads to less investment and discontinuously smaller output.

**Lemma 5.1** Consider a set of parameters for which \( r_B \left( \frac{c}{1+r_f}, \mu_1, c, r_f \right) < \bar{r} \). Total output, \( Y(\mu_0, \mu_1) \), jumps downward at \( \mu_0 = \frac{c}{1+r_f} \), when the economy switches from the bold stage to the cautious stage in a two-stage economy.

The bottom panel on Figure 2 illustrate the cyclicality of output, \( Y(\mu_0, \mu_1) \), the crash when sentiment switches, and its co-movement with the spread between opaque and transparent rates in the corresponding two-stage economy. Comparison with the top panel shows the co-movement with the measure of opaque bad entrepreneurs \( \mu_0 \). Unsurprisingly, larger measure of bad opaque entrepreneurs implies smaller output. However, the effect of a change from an overheated credit market to a low-sentiment credit market is very pronounced. The top panel of Figure 2 shows that this switch occurs in periods 4, 11 and 18 in our example. While \( \mu_0 \) increases only slightly in those periods, the bottom panel of Figure 2 shows a sizeable drop in output. This is the result of the switch in sentiment. In these periods, the deterioration of the pool of credit applications triggers investors to become cautious. Therefore all bad projects lose financing, and opaque good projects are significantly squeezed. As the middle panel on Figure 2 shows, the fragmentation in the credit market means the opaque good entrepreneurs face a significantly larger interest rate than before. On the bright side though, this crash has a “purification effect” on the economy. Bad entrepreneurs exit the economy and are replaced by an average entrepreneur from the outside pool. This leads to a sufficient improvement in the credit application quality for the next period that increases the opportunity cost of giving up on good investments, and triggers investors to switch to bold test. Over the next couple of periods, the credit market becomes overheated again.

### 6 Welfare, Optimal Cycles and Economic Policy

In the previous sections, we demonstrated how fluctuations of sentiment and that of fundamentals feed onto each other, creating endogenous cycles. As we explicitly model the mechanism which turns booms to busts and vice-versa, our framework is well suited to explore how certain economic policies could influence these economic cycles.
First, we define welfare and show that, similarly to total output, it discontinuously drops when the economy moves from the bold, high sentiment stage, to the cautious, low sentiment stage.

Then, we explore the rational for policy makers to intervene in this economy. In particular, we study the problem of a constrained planner who chooses which test investors use in each aggregate state. We argue that the planner can often improve on the decentralized outcome, because investors do not internalize how their individual choice of lending standards affects the long run dynamics evolution of the state. In particular, the planner often prefers a cycling economy where low sentiment stages keep the measure of bad projects in the economy at bay, which in turn makes the high sentiment stages more beneficial.

Finally, we consider the implementation of the constraint optimal outcome with realistic monetary and macro-prudential policies. We show that both changing the risk-free rate and specifying capital requirements can be used to influence investors’ lending standards. Therefore, each of these policies affects the long run cyclical behaviour of the economy, i.e. the dynamic evolution of the aggregate state, and consequently, welfare. In general, tighter monetary or macro prudential policy leads to an economy spending less time in the high sentiment stage. At the same time, tighter policies in a given state lead to higher cost of capital for entrepreneurs which decreases welfare in that state. By characterizing this trade-off for different policy instruments, we rank their efficiency.

### 6.1 Welfare

A natural welfare measure is the aggregate consumption of all entrepreneurs and investors.

\[
W(\mu_0, \mu_1) \equiv \rho (1 + \mu_0 \ell(b, 0) + \mu_1 \ell(b, 1)) + \frac{1 - \mu_0 - \mu_1}{2} \sum_{\omega=0,1} \ell(g, \omega) \left[ \rho - (1 + r(g, \omega)) \right] \\
+ w_0 (1 + r_f) + w_1 \left( 1 + \max_{\tau} r(\tau, \omega) \right) 
\]  

(6.1)

The next proposition states that welfare decreases in the measure of bad entrepreneurs, \(\mu_0\), within any segment of the state space where the type of equilibrium does not change. Moreover, welfare discontinuously drops when the economy switches to the cautious stage from a bold stage.

**Proposition 6.1** Consider a two-stage economy. Welfare is decreasing in the measure of
bad projects, \( \mu_0 \). There is a discontinuous drop in \( W(\mu_0, \mu_1) \) around the threshold \( \mu_0 = \frac{c}{1+r_f} \).

An increase in the measure of bad entrepreneurs decreases welfare, similar to total output, because this group imposes higher cost of capital on all firms through adverse selection. While the higher cost of capital can increase skilled investors consumption, this effect is always dominated by the smaller consumption of entrepreneurs. When the economy switches from bold to cautious, bad opaque entrepreneurs lose all of their financing, while good opaque entrepreneurs experience both a jump in their interest rate from \( r_B \) to \( \bar{r} \) and a drop in the available funding since unskilled investors stop financing them. Again the resulting upward jump in the consumption for skilled investors is dominated by those adverse effects.

In the dynamic economy the state distribution is endogenous as we described in section 4.1. In a cycling economy, just as output, welfare is higher in the bold stage and lower in the cautious stage, re-enforcing our interpretation of these stages as booms and busts. Figure 2c depicts the dynamics of welfare and output under our baseline parametrization.

In the previous section, we illustrated that the state variable \( \mu_0 \) (and \( \mu_1 \)) cycles through a fixed set of points in a two-stage economy. Suppose that the equilibrium cycle has \( \tau \) periods and the economy has already reached its ergodic set by period \( t_0 \). Thus form \( t_0 \) onward, the expected welfare along the equilibrium path is

\[
EW \equiv \frac{1}{\tau} \sum_{i=0}^{\tau} 1_{\mu_0(t+i) \leq \tilde{\mu}_0} W_B (\mu_0(t+i), \mu_1(t+i)) + 1_{1 - 1_{\mu_0(t+i) \leq \tilde{\mu}_0}} W_C (\mu_0(t+i), \mu_1(t+i)).
\]

where \( W_B (\mu_0t, \mu_1t) \equiv W(\mu_0t, \mu_1t)|_{\mu_0t \leq \tilde{\mu}_0} \) and \( W_C (\mu_0t, \mu_1t) \equiv W(\mu_0t, \mu_1t)|_{\mu_0t > \tilde{\mu}_0} \) denote welfare in the bold and cautious stages, respectively. In the rest of this section, the effect of policy on average welfare is our main object of interest.

### 6.2 Optimal Cycles

As the focus of our analysis is the relationship between the choice of investors’ lending standards and that of fundamentals, it is instructive to study the following constrained planner’s problem.

**Definition 6.1** The constrained planner’s solution is a dynamic equilibrium where the planner chooses which test is available in any given state \((\mu_0, \mu_1)\) for all investors, in order to
maximize the average welfare in the ergodic state distribution.

As the definition shows, limited tools are available to the planner to influence the economic outcome. He can only partition the state space into two parts, in the first part all investors choose the bold test, while in the second part all investors choose the cautious test. Given the implied information structure, the equilibrium interest rates, quantities and the law of motion for the state are determined as before. For instance, if the planner chooses all agents to use the bold test in all states, the economy will feature only bold stages with no cycles and a degenerate ergodic distribution of \((\mu_0 = \bar{\mu}_0B, \mu_1 = \bar{\mu}_1B)\). Similarly, the planner can implement an only-cautious economy with a degenerate ergodic distribution of \((\bar{\mu}_0C, \bar{\mu}_1C)\). Following the intuition in Proposition 4.3, the planner can also implement various cyclical economies, for instance, by forcing agents to be cautious if and only if \(\mu_0 < \hat{\mu}_0^P\) for some \(\hat{\mu}_0^P \in [\bar{\mu}_0C, \bar{\mu}_0B]\).

The next proposition provides a sufficient condition for the constrained planner’s solution to be a cyclical economy.

**Proposition 6.2** Let \(\lambda_{\text{min}} \equiv \frac{2c+2r_f}{3c+3r_f+1} < \lambda_{\text{max}} \equiv \frac{2c-\rho-c-r_f-1}{2\rho-c-r_f-1}\), and consider \(\lambda \in [\lambda_{\text{min}}, \lambda_{\text{max}}]\). Then there exists a \(\delta\) such that for \(\delta < \tilde{\delta}\), the constrained planner’s solution features endogenous cycles.

The proposition states conditions where rational sentiment driven cycles are the choice of a welfare maximizing planner. Intuitively, the choice of the test is planner’s instrument to influence the ergodic state distribution. Tight lending standards keep the measure of bad firms at bay. However, if the planner forces agents to be always cautious, opaque good firms are always squeezed. Therefore, to maximize average welfare, the planner periodically allows investors to be bold when the measure of entrepreneurs not paying back their loans is sufficiently low.

**Externality.** The type of cyclical in the constrained planner’s solution is determined very differently from the decentralized outcome. The decentralized equilibrium features an externality because investors do not internalize that their individual choice of test affects the ergodic state distribution. This is the case even if investors were long lived. The reason is that from an atomistic individual investor point of view, a unilateral deviation to another test would not affect the ergodic distribution.
The solid curve in Figure 3a illustrates the mean welfare over the cycle corresponding to different levels of planner choice of threshold $\hat{\mu}_0^P$. The blue dot represents the welfare in the decentralized economy. The vertical dashed lines partition the figure according to the cyclicality of the implied economy in line with Proposition 4.3. The leftmost region corresponds to a cautious-only economy, the middle region to a cyclical economy which can feature a short boom/long recession, a short boom/short recession, or a long boom/short recession, and the rightmost region is a bold-only economy. Welfare changes discontinuously wherever the choice of the planner changes the cyclicality of the economy and it is flat otherwise. Within the long boom/short recession region, the drops correspond to the points where the long boom becomes a period longer.

Instead of the decentralized economy with a long boom and a short recession, the planner prefers to push the economy to short booms and short recessions. Panels 3b and 3c illustrate the intuition. Panel 3b compares the path of the state variable $\mu_0$ in the planner’s optimal choice and the decentralized solution. The planner, by choosing a lower threshold, forces the economy to a purifying cautious stage more often. This keeps the measure of bad types in the applicant pool lower on average. Panel 3c compares the welfare paths between the planner’s choice and the decentralized economy. Because of the lower measure of bad types, both the booms and the recessions lead to a higher welfare under the planner’s solution compared to the decentralized economy.

### 6.3 Economic Policy

In the previous section we established that the constraint optimal economy is often still cyclical. In this section, we connect the constrained planner’s solution to realistic monetary and macro-prudential policies. We show that both changing the risk-free rate and specifying capital requirements can be used to influence investors’ lending standards. As such both policies affects the state transition dynamics, and consequently, welfare. However, the policy maker can improve the quality of loan applications only at the expense of increasing the average cost of capital. This trade-off determines the ranking of these policies.

For the rest of this section, we normalize the physical return to storage technology to zero. However, the policy maker introduces a risk-free asset for saving within each period. This asset is available in perfectly elastic supply for entrepreneurs and investors alike. The monetary policy rate, $r_f$, is the net return on this saving. To ensure that the budget con-
straint of the policy maker is satisfied, assume that a lump-sum tax is imposed on investors each period which exactly covers the aggregate expenditure of providing return $r_f$. Potentially, the policy maker might set a different risk-free rate when the economy is in the cautious or bold stages, $r^{C}_f$ and $r^{B}_f$ (or mix stage, $r^{M}_f$).

As a macro-prudential tool, we model risk-weighted capital requirements as follows. Consider the capital a bold investor invests in loans which have passed the bold test. Assume that the regulator imposes a risk weight $x \geq 1$ on this investment as it is risky. The macro-prudential policy is permanent and non-state-contingent.\footnote{Introducing a different risk-weight for cautious investors’ loans would be straightforward. For simplicity, and because they are investing only in loans which certainly pay back, we omit that treatment.} Out differently, this policy is only a function of individual investor choice and is independent of the resulting aggregate state.

Let $v_g$ and $v_r$ be the bold investor’s investment in the above risky and risk-free asset, respectively. As risk free asset is safe investment, its risk-weight is 1, thus $v_g x + v_r = 1$. If $x = 1$, this reduces to the budget constraint of the investor in our baseline economy. When $x > 1$ the capital constraint forces an investor running the bold test to forego investing $v_g(x-1)$ units of her endowment. We assume that the investor consumes this excess capital at the end of the period.

The tuple $\pi = (x, r^{C}_f, r^{B}_f)$ is a policy profile. In the next proposition, we characterize the effect of any policy profile on the equilibrium outcome.

**Proposition 6.3** Under the policy profile $\pi$, the equilibrium is characterized by Propositions 4.1-5.1 with the modification that the interest rate functions (4.1) and (4.3) are replaced by

\[
\begin{align*}
    r_B^{\pi}(\mu_0, \mu_1, c, \pi) &\equiv r_B(\mu_0, \mu_1, c, r^{B}_f) + \frac{(x-1)(c + r^{B}_f + 1)(1 - \mu_1)}{1 - \mu_1 - \mu_0} \\
    r_C^{\pi}(\mu_0, \mu_1, c, \pi) &\equiv r_C(\mu_0, \mu_1, c, r^{C}_f) \\
    r_I^{\pi}(\mu_0, \mu_1, c, \pi) &\equiv xr_I(\mu_0, \mu_1, c, r^{M}_f) + (x-1)(1 - \frac{2\mu_1}{1 - \mu_0 - \mu_1})
\end{align*}
\]  

respectively, and thresholds $\frac{c}{1+r_f}$ and $\tilde{\mu}_0(\mu_1, c, r_f, \rho_g)$ are replaced by

\[
\begin{align*}
    \tilde{\mu}_0^{c}(\mu_1, c, \pi) &\equiv \frac{c}{1 + r^{c}_f} - \frac{(1 - \mu_1)}{1 + r^{c}_f} \left( (x-1) (1 + r^{B}_f + c) + (r^{B}_f - r^{C}_f) \right) \\
    \tilde{\mu}_0^{\pi}(\mu_1, c, \rho, \pi) &\equiv \frac{(1 - \mu_1)(\rho - (1 + r^{M}_f) - (x-1)(c + r^{M}_f + 1)) - (1 + \mu_1)c}{\rho + x(1 + c + r^{M}_f)}
\end{align*}
\]
respectively.

The next Corollary summarizes the effect of the policy instruments on the economic cycle, as well as the cost of capital in a two-stage economy.

**Corollary 6.1** Consider a two-stage economy. Keeping the state \((\mu_0, \mu_1)\) fixed, an increase in the risk-free rate in the bold stage, \(r_f^B\), or the risk-weight, \(x\), increases the cost of capital for all entrepreneurs in the bold stage only. Similarly, an increase in the risk-free rate in the cautious stage, \(r_f^C\), (weakly) increases the cost of capital for all entrepreneurs in the cautious stage only. A sufficiently large increase in \(r_f^B\) or \(x\) pushes a high sentiment boom to a low sentiment recession. If \(r_f^B = r_f^C = r_f\), then a sufficiently large increase in \(r_f\) has the same effect.

Furthermore, an increase in \(r_f^B\) or \(x\), as well as \(r_f\) when \(r_f^B = r_f^C = r_f\), results in the economy spending more time in low sentiment stages where lending standards are tight, and less time in high sentiment stages where lending standards are lax.

Any of these three instruments imply a tighter policy regime compared to the baseline economy. A higher risk-free rate in the bold stage implies higher opportunity cost to provide loans to entrepreneurs using a bold test. Higher risk-weight has the same effect by directly decreasing the amount of capital an investor can lend using a bold test. Therefore, both of these instruments increase the break-even interest rate at which an investor is willing to lend using a bold test. As the cost of cautious lending is not influenced by these two instruments, investors will use a cautious test more often along the equilibrium path.

When both states have the same risk-free rate the effect is similar. The reason is that \(r_f(\cdot)\) is more sensitive to the prevalent interest rate than \(r_C(\cdot)\). Thus in either case lending standards are tighter more often, and the economy spends more time in the low sentiment stage.

Formally, the statement is a direct consequence of Propositions 4.1-5.1 and 6.3, using the comparative statics of expressions (6.2) and (6.3) with respect to the elements of \(\pi\).

To gain some insight about the relative efficiency of the available policy instruments, we compare three specific policy profiles. A **simple monetary policy** is non-state-contingent, specifying the same interest rate \(r_f\) regardless of the state of the economy, \(\pi_{rf} = (1, r_f, r_f)\). A **counter-cyclical monetary policy** sets a positive risk-free rate in bold stages, \(r_f^B\), and a zero
risk-free rate in cautious stage, $\pi_{rf} = (1, 0, r^{B}_f)$. Finally, a macroprudential policy specifies a risk-weight for bold investment, $x$, without providing risk-free assets with positive returns, $\pi_x = (x, 0, 0)$.

Finally, to make the welfare effects of these policies comparable, we introduce the concepts of equivalent policies.

**Definition 6.2** Two policy profiles $\pi$ and $\pi'$ are equivalent to each other or to the planner’s choice $\mu^P_0$ if they imply the same ergodic distribution for the states $(\mu_0, \mu_1)$.

The next Lemma illustrates usefulness of the concept of equivalent policies. It shows that any cycle that arises under planner’s choice can be implemented with all the three policies.

**Lemma 6.1** Fix a planner’s choice $\mu^P_0 \in [0, c]$ implying a two-stage economy. There are always $r_f, r^{B}_f$ and $x$ values defining equivalent policies $\pi_{rf}, \pi^{B}_f$ and $\pi_x$, respectively.

However, equivalent policies do not imply the same welfare. Our main result below ranks the three policy instruments according to their relative efficiency in achieving the constraint optimal cycle.

**Proposition 6.4** Fix a planner’s choice $\mu^P_0$ implying a two-stage economy with longer or more frequent cautious stages than the baseline economy. Consider the equivalent policies $\pi_{rf}, \pi^{B}_f$ and $\pi_x$ with the smallest implied interest rate $r^{B}_f(\cdot)$.

(i) Average welfare under policies $\pi_{rf}, \pi^{B}_f$ and $\pi_x$ is strictly lower than under the planner’s choice $\mu^P_0$.

(ii) Equivalent policies $\pi_{rf}$ and $\pi_x$ imply the same equilibrium interest rate for any entrepreneur in every stage. However, the countercyclical monetary policy entails a higher average welfare than the equivalent macroprudential policy.

(iii) Suppose that $\lambda \leq \frac{8}{9}$. Then

(a) $\pi^{B}_f$ is more efficient than $\pi_{rf}$,

(b) $\pi_x$ is more efficient than $\pi_{rf}$ as long as the cost of testing, $c$, is not too high.
Any of these policies are costly compared to the planner’s choice. This is so, because
the planner can affect lending standards directly. The policy maker instead, has to change
investors’ incentives to choose among the available tests which results in higher equilibrium
interest rates. In other words, any of these policies imply a strictly higher equilibrium interest
rate \( r^\pi_B(\cdot) \) and a weakly higher equilibrium interest rate \( r^\pi_C(\cdot) \). That is, affecting cycles
with monetary or macroprudential policy is costly. Because of the higher cost of capital,
entrepreneurs borrow and invest less, leading to smaller total output and consumption.

Interestingly, a countercyclical monetary policy has the same effect on the cost of capital
as an equivalent macro-prudential policy. However, the quantity constraint implied by lend-
ing under the macro-prudential policy implies that more investors have to enter to provide
the same amount of capital to entrepreneurs. As all of them has to pay the fixed cost \( c \),
the macroprudential policy is dominated by the countercyclical monetary policy. When this
cost is not too high, the macroprudential policy still dominates the simple monetary policy.
It is so, because the latter increases the cost of capital in cautious stages as well. That is, it
is a more costly tool to reduce lax lending than the countercyclical policy.

The three non-solid curves in Figure 3a illustrate the results in the proposition. Each
curve corresponds to the average welfare for a given policy equivalent, with the threshold \( \hat{\mu}_0^P \)
on the horizontal axis. One can see that the macroprudential and countercyclical monetary
policies lead to similar welfare, while the simple monetary policy performs considerably
worse.

An important observation is that the countercyclical monetary policy assumes a relatively
high degree of sophistication for the policy maker. She essentially has to solve a fixed point
problem to compute the endogenous aggregate state. When increasing the interest rate, she
has to foresee whether the chosen interest rate keeps the economy in a boom or moves it
to a downturn. In contrast, the macroprudential policy conditions only on the individual
lending choice of investors, and delivers a very similar performance. The simple monetary
policy requires minimal sophistication from the policy maker but does not perform as well.

7 Model and Facts

Despite its simple structure, our model generates a rich set of empirical predictions. When
mapping the model outcomes to the data, a critical question is the empirical counterpart of
Figure 3: Mean welfare for different levels of planner choice of threshold $\mu_0^P$, as well as the comparison between the implied paths for the measure of bad opaque entrepreneurs $\mu_0$, and welfare, along the optimal versus the decentralized cycle.
the distribution of credit flow to different firm types in the model. The most conservative approach is to treat the heterogeneity across firms as unobservable. As such, the econometrician can observe only aggregate credit flows, without being able to identify flows to different firm types. We start with the predictions under this approach.

**Tightness of credit, interest rates, and economic cycles** Treating firms’ types as fundamentally unobservable, our model predicts that for any group of borrowers, (1) conditions of credit supply are more favourable in booms than in recessions, (2) the average quality of issued credit is deteriorating in booms, and (3) within group, credit is granted at less dispersed interest rates in booms compared to recessions. Regarding the first prediction, Becker and Ivashina (2014) presents various measures to argue that the cyclicality of aggregate credit is mostly due to the cyclicality in credit supply, at least for small firms in the US. Regarding the second statement, Morais et al. (2019) finds both US and international evidence for lax lending standards in booms in the bank loan market. In a different context, Demyanyk and Van Hemert (2009) documents that the quality of sub-prime loans deteriorated for six consecutive years before the 2007 crisis. We are not aware of any work focusing on the cyclicality of interest rate dispersion within a group of borrowers.

A less conservative approach is to assume that, at least ex-post, it is possible to partition firms according to their transparency. Consider the following thought experiment building on the example of the commercial paper market around the European debt crisis in 2010. When global fundamentals are strong, an investor might choose to be bold, lending to all major banks based in developed countries. She understands that some have less healthy balance sheets than the others, but she does not have the expertise to distinguish them. Instead, when global fundamentals are weak, the investor might choose to be cautious lending only to major US banks as the safest strategy. If our mechanism captures main determinants of the European debt crisis, we should be able to identify a large group of investors following the earlier strategy before 2010, but switching to the latter after the Greek default. By observing the difference between these two strategies, the econometrician would conclude that credit to European banks maps to opaque credit in our model.

**Market fragmentation and heterogeneous portfolio rebalancing** With this interpretation our model provides crucial insights about the market fragmentation in recessions. As a bold stage turns into a cautious stage, skilled and unskilled investors rebalance their portfolio very differently. Unskilled investors rebalance from low-quality bonds (to opaque
entrepreneurs) to high-quality ones (to transparent entrepreneurs), while skilled investors do the opposite. This implies that good entrepreneurs face heterogeneous experiences. Transparent good entrepreneurs enjoy abundant credit supply while opaque good entrepreneurs are squeezed, although in the bold stage they faced the same market conditions. This market fragmentation and the implied heterogeneous effect of a downturn is a unique feature of our model.

Indeed, our suggested thought experiment was inspired by Ivashina et al. (2015) and Gallagher et al. (2018) who find that in 2011 a group of US money market funds stopped lending to all European banks but not to other banks which had similar fundamentals. These predictions are consistent with our mechanism when considering these funds as low skilled investors. Moreover, Ivashina et al. (2015) finds evidence that this process led to a significant disruption in the syndicated loan market, a possible channel for the real effects that our model predicts.10

Credit composition, the quality spread and credit market sentiment In a similar spirit, one can map junk bond issuance as credit to firms rejected by a cautious test, i.e., credit to opaque firms. In this case, loans to transparent good firms map to high-grade bond issuance.

With this interpretation, our model is consistent with the well-known fact that the quality spread, i.e. the spread between AAA and BAA corporate borrowers, is counter-cyclical. As such, our paper provides an information based alternative explanation for time-varying risk-premium.

More specifically, we can interpret our predictions within the context of the growing body of evidence suggesting that periods of overheating in credit markets forecasts low excess bond or loan returns. This is not a tautology if credit market overheating is measured ex-ante by the quantity or composition of credit. Importantly, Greenwood and Hanson (2013) show that the share of junk bond issuance out of total issuance inversely predicts the excess return on these bonds.11 Panel (a) in Figure 4 is the illustration of this fact using the reproduction

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10 See also our companion paper Farboodi and Kondor (2018) providing a substantially richer picture on market fragmentation, by treating sentiment switches as exogenous.

11 The inverse relationship between credit expansion and subsequent returns is remarkably widespread across various financial markets. For instance, Baron and Xiong (2017) documents the negative relationship between bank's credit expansion and banks' equity returns, Kaplan and Stromberg (2009) finds a similar inverse relationship between venture capitalists aggregate flow to new investments and their subsequent returns. A related early work is Eisfeldt and Rampini (2006), who shows that volume of transactions are
Figure 4: Opaque credit share and its realized excess return. Panel (a) is reproduced from Stein (2013). High-yield (HY) share is the ratio of speculative-grade issuance to total rated corporate bond issuance. Excess returns are calculated as the difference between the log return on a HY index and a corresponding treasury index, between \( t \) and \( t + 2 \). They are plotted alongside the HY share at time \( t \) on an inverted scale, so that the negative correlation appears positive. Panel (b) shows the share of issued credit to opaque projects relative to all credit in a given period (solid blue line, right scale), and the realized excess return on opaque credit one period later (dashed red line, left scale, invested).

of Stein (2013). The blue curve is the share of junk bonds issued in a given period as a measure of all issuance, measured on the right axis. The black curve is the excess return on those low-grade bonds in the subsequent two years, measured on the left axis on a reverse scale. Thus years when both curves are high correspond to overheated periods with low subsequent returns. Periods where both curves are low tend to correspond to recessions: low sentiment credit markets with high subsequent returns.

Panel (b) of Figure 4 plots the model-equivalent time-series, for the two-stage economy simulated on Figure 2. We proxy the high-yield share as the share of credit to opaque firms, \( S(\mu_0, \mu_1) \), and calculate the net excess realized return on a portfolio of these loans, \( R(\mu_0, \mu_1) \).\(^{12}\) Note the strong co-movement between \( S(\mu_0, \mu_1) \) and \( R(\mu_0, \mu_1) \) on a reverse scale, pro-cyclical while return on transactions is counter-cyclical in the sales of property, plant and equipment.

\(^{12}\)Formally, \( S(\mu_0, \mu_1) = \frac{\mu_0 \ell(b, 0) + \frac{1 - \mu_0 - \mu_1}{2} \ell(g, 0)}{\mu_0 \ell(b, 0) + \frac{1 - \mu_0 - \mu_1}{2} (\ell(g, 0) + \ell(g, 1))} \quad R(\mu_0, \mu_1) = \frac{1 - \mu_0 - \mu_1}{\mu_0 \ell(b, 0) + \frac{1 - \mu_0 - \mu_1}{2} \ell(g, 0)} - 1 + r_f \).
both within the bold stage and across periods. It is important to note that although our model generates a strong positive correlation between these variables, this does not amounts to an exploitable anomaly based on the information set of unskilled investors.\textsuperscript{13}

**International spillovers of US monetary policy**  Our model is applicable beyond a developed economy such as the US. As an example, consider our set up as a description of an emerging market economy where the banking sector is sensitive to changes in an external risk-free rate, as this determines the opportunity cost of lending domestically. In particular, assume the return on risk-free storage in our model is the US policy rate, controlled by the FED. This rate is effectively exogenous for the emerging market banking sector, but clearly influences its activity. Corollary 6.1 argues that a small increase in the risk-free rate can have a significant adverse effect both in the short and long run. Even a small increase in the risk-free rate can push a bold, overheated economy into a low-sentiment, recessionary one. A permanent change in the rate can change the nature of the cycle in the economy. In general, higher opportunity cost of lending leads to longer recessions and shorter booms. This is in line with several emerging market officials including Raghuram Rajan’s warning, who was the Chair of India’s central bank around 2013-14, when the emerging markets were expected to be adversely affected when US starts to raise rates.\textsuperscript{14}

## 8 Conclusion

The idea that economic fluctuations can be captured by models with endogenous cycles is not new. In fact, the earliest business cycle models, by John Hicks and Nicolas Kaldor, followed this approach. However, as Boldrin and Woodford (1990) explain, these models fell out of favour by the late 1950’s because they had been empirically rejected: actual business cycles were found not to show regular cycling behaviour.\textsuperscript{15}

In this paper, we argue that despite real world cycles being stochastic and difficult to forecast, simple models with endogenous cycles are a useful apparatus for macroeconomic theory as indispensable analytical tools for policy analysis. To asses the effect of various

\textsuperscript{13}See Bordalo et al. (2018) and Greenwood et al. (2019) for a discussion of empirical facts pointing towards such anomalies to exist and on boundedly rational models which can target those.\textsuperscript{14}See Financial Times, September 3, 2013 and January 31, 2014.\textsuperscript{15}See the recent work of Beaudry et al. (2020) for the argument that modern statistical techniques might refute this statement.
policies on the length and depths of booms and recessions, it is essential to understand what predictably turns booms into recessions and vice-versa.

We propose a model where endogenous cycles are generated by the interaction of lenders’ choice of lending standards and economic fundamentals. Tight credit standards forces low quality entrepreneurs to exit at a higher rate implying an improving quality of the borrowing pool in the future. At some point, a switch to lax standards are triggered. In turn, this leads to the deterioration of fundamentals. We show that the planner, with carefully chosen capital requirements for investors or with counter-cyclical monetary policy can change the cyclicity of the economy, using recessions to keep the borrower pool quality at bay. We further demonstrate that the predictions of the model matches numerous stylized facts related to credit cycles.

References


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A Appendix: Agent Optimization Problem and the Market Clearing Protocol

In this Appendix, we define the problem of each agent along with the market clearing protocol. We also introduce a robustness criterium. The structure of our credit market is a
modified version of Kurlat (2016). The entrepreneur and investor problems are simplified versions of those in Farboodi and Kondor (2018).

A.1 Entrepreneur and Investor Problem

Let $R$ be the a set of trading posts, each of them identified by an interest rate $r$. Then, the formal problem for an entrepreneur $(\tau, \omega)$ is

$$\max_{\{\sigma(r;\tau,\omega), r \in R\}} \rho \cdot i(\tau, \omega) - 1_{r=g} \cdot \ell(\tau, \omega) \cdot (1 + r(\tau, \omega))$$

(A.1)

s.t.

$$0 \leq \sigma(r;\tau,\omega) \leq \frac{1}{r} \quad \forall r \in R \quad \rho i(\tau, \omega) - 1_{r=g} \cdot \ell(\tau, \omega) \cdot (1 + r(\tau, \omega))$$

(A.2)

$$\ell(\tau, \omega) = \int_{R} \sigma(r;\tau,\omega) d\eta(r;\tau,\omega)$$

(A.3)

$$r(\tau, \omega) = \int_{R} r \cdot \sigma(r;\tau,\omega) d\eta(r;\tau,\omega)$$

(A.4)

$$\ell(\tau, \omega) \leq \frac{1}{r(\tau, \omega)}$$

(A.5)

where $\eta$ is a rationing function assigning $\eta(R_0;\tau,\omega)$ measure of credit, per unit application, for an entrepreneur of type $(\tau, \omega)$ submitting applications to the subset of trading posts $R_0 \in R$. $\eta$ is an equilibrium object, determined by the choices of the agents and the market clearing protocol as we explain below. The entrepreneur takes $\eta$ as given.

We obtain a more intuitive form of this problem by defining the maximum available credit for a given type,

$$\bar{\ell}(\tau, \omega) \equiv \int_{R} \frac{1}{r} d\eta(r;\tau,\omega).$$

Then, we can rewrite the entrepreneur’s problem as

$$\max_{\ell(\tau, \omega)} \rho \cdot \ell(\tau, \omega) \cdot (\rho - 1_{r=g}(1 + \tau(\tau, \omega)))$$

(A.5)

s.t.

$$\ell(\tau, \omega) \leq \min \left(\bar{\ell}(\tau, \omega), \frac{1}{r(\tau, \omega)}\right).$$

(A.6)

This form suppresses the choice over credit applications, $\sigma(\cdot)$, and focuses on the total obtained credit $\ell(\cdot)$. The implied $\ell(\cdot)$ in the two forms are the same, because for any obtained credit $\ell(\tau, \omega)$ satisfying (A.6), there is a collection of $\sigma(r;\tau,\omega)$ implementing that $\ell(\tau, \omega)$. 
The problem of an unskilled investor $h \in [0, w_0]$ is
\[
\max_{\chi(h), \tilde{r}(h)} \left( 1 + \tilde{r}(h) \right) (S_u(r; g, 1) + 1_{\chi(h) = B} S_u(r; g, 0)) + (1 + r_f) \left( S_u(r; b, 1) + 1_{\chi(h) = C} (S_u(r; b, 0) + S_u(r; g, 0)) \right)
\]
and that of a skilled investor $h \in (w_0, w_0 + w_1]$ is
\[
\max_{\tilde{r}(h)} ((1 + \tilde{r}(h)) (S_s(r; g, 1) + S_s(r; g, 0))).
\]
\(\chi(h)\) is the unskilled agent’s choice of test, while \(S_u\) and \(S_s\) are the sampling functions for unskilled and skilled investors respectively. A sampling function, \(S_u\), assigns a measure \(S_u(r; \tau, \omega)\) to any interest rate advertised by an unskilled investor. This describes the measure of credit applications submitted by entrepreneurs of type \((\tau, \omega)\) in the sample of the unskilled investor advertising that rate. \(S_s(r; \tau, \omega)\) is the analogous object for skilled investors. Each investor takes the sampling functions as given. Just as the rationing function, the sampling functions are endogenous objects determined by the market clearing protocol and the choices of agents as described next.

A.2 Market Clearing Protocol

Let \(r_{\text{max}}(\tau, \omega)\) be the maximum interest rate at which \(\sigma(r; \tau, \omega)\) is positive for some given type of entrepreneur. If \(\max_{(\tau, \omega)} r_{\text{max}}(\tau, \omega) \leq \min_h \tilde{r}(h)\), then no applications are financed. Otherwise the market clearing process starts from the smallest advertised interest rate \(\tilde{r}(h)\), say, \(r'\). First, each entrepreneurs submitting applications at that rate has to post \(r'\) per unit of application from her endowment. We refer to this as down-payment. Applications without a down-payment are automatically discarded. Then, each unskilled investor advertising rate \(r'\) runs his chosen test and grants credit for all applications which pass the test in the sample he receives. He obtains a sample of the (non-discarded) applications submitted at that rate. The applications are allocated pro rata. That is, the proportion of each type \((\tau, \omega)\) in each sample is identical to their proportion in the (non-discarded) application pool at that interest rate. If there are not enough applications to fill up every present investor’s capacity limit, then all applications have been sampled and the sampling process stops. Otherwise, all unskilled investors received a sample up to their sampling capacity. This defines the measure \(S_u\). Their remaining endowment is invested in the risk-free asset. Entrepreneurs have to invest the credit amount along with the down-payment and the invested units are posted as collateral for the loan. These invested units enter into a public registry, so they cannot serve as collateral to other loan applications. Applications which enter into a sample, but do not pass the test are discarded, and the down-payment is returned to the entrepreneur who can invest it in the risk free asset.

If all unskilled investors reached their sampling capacity and there are remaining good projects, then they are distributed pro rata across skilled investors up to their capacity given by their endowment. This defines \(S_s\). Skilled investors grant credit to these projects and
the investment and collateralization process are as before. Any remaining applications are discarded and the down-payment is returned. Then, the process is repeated at the next lowest advertised interest rate, etc. If there are no more advertised rates, or there are no more applications for which \( r_{\text{max}}(\tau, \omega) \) is larger than a remaining advertised rate, the process stops. Aggregating over investors granting credit to a given entrepreneur at a given interest rate defines \( \eta(r; \tau, \omega) \).

### A.3 Robustness

Following Kurlat (2016), we make a robustness assumption implying tie-braking rules and avoiding multiple equilibria.

**Assumption A.1** Suppose that \( \epsilon \) fraction of applications submitted at an advertised interest rate are granted unconditionally. We require that the equilibrium strategy of each entrepreneur is the limit of equilibrium strategies as \( \epsilon \) goes to 0.

Apart from avoiding multiplicity of equilibria, as we will see, an additional consequence of this assumption is that each type who chooses to submit loan applications at a given interest rate submits the same, maximal amount. That is, \( \sigma(r; \tau, \omega) > 0 \) implies \( \sigma(r; \tau, \omega) = \frac{1}{r} \). This feature has the convenient consequence that the application pool at any given interest rate is independent of cross-sectional distribution of choices \( i(\tau, \omega) \), therefore, we can solve the credit market equilibrium independently of choices \( i(\tau, \omega) \). This simplifies the analysis considerably.

### B Appendix: Proofs

**Proof of Lemma 4.1**

The market clearing mechanism and Assumption A.1 implies that if any agent would like to raise credit at an interest rate \( r_{\text{max}} \), she would want to submit a maximum measure of applications, \( \sigma(r; \tau, \omega) = \frac{1}{r} \) at every interest rate smaller than \( r_{\text{max}} \) too. The reason is that it makes it possible that they are receiving a fraction of their credit at a lower rate (as markets clear from the lowest interest rate), and potentially even without the requirement to invest the received amount (Assumption A.1). This latter possibility is attractive for bad entrepreneurs. Because applications with no down-payment are discarded, there is no possibility of having more credit granted as intended. Agents also want to submit the maximum measure of applications at \( r_{\text{max}} \). Given the linear structure, if, at a given interest rate an agent would like borrow to invest, she also would like to borrow up to the limit \( \frac{1}{r} \) and invest at that rate. This concludes the first part of the Lemma.
For the second part, observe that the objective function (A.1) implies that a good entrepreneur does not apply for credit at any interest rate \( r(g, \omega) > \rho - 1 \) as that would imply negative return on her investment. As we noted before, Assumption A.1 and (A.1) implies that bad entrepreneurs instead apply for maximum credit at any interest rate as they do not plan to pay back.

**Proof of Proposition 4.1**

The main steps of the proof are explained in the text. Here, we just have to specify the details.

First, we show that if all entrepreneurs submit the maximum demand to an advertised rate \( r_B^p \) than bold, unskilled investors are indifferent to stay out or enter. The superscript refers to the fact that it is a pooled market where all entrepreneurs submit. In fact, \( r_B^p \) is defined by the indifference condition

\[
(1 - \mu_1 - \mu_0)(1 + r_B^p) + \mu_1(1 + r_f) - c = 1 + r_f \quad (A.7)
\]

Note that \((1 - \mu_1 - \mu_0)(1 - \mu_1 - \mu_0)\) is the probability of ending up financing a good project with a bold test \((Pr(green\ signal) \times Pr(good\ project|green\ signal))\), while \(\mu_1\) is the probability that a entrepreneur in the sample will not pass the bold test, hence the investor invests in the risk-free asset instead. Therefore, the left hand side is the expected utility of running the bold test on a proportional sample of applications. Note that we are using the assumption that unskilled investors sample first.

Similarly, a cautious investor is indifferent to enter to a pooled market at interest rate \( r_C^p \), which is defined as:

\[
\frac{(1 - \mu_1 - \mu_0)}{2}(1 + r_C^p) + (\frac{(1 - \mu_1 - \mu_0)}{2} + (\mu_1 + \mu_0))(1 + r_f) - c = 1 + r_f \quad (A.8)
\]

We claim that if and only if \( r_B^p \leq r_C^p \) holds, \( r_B^p \) supports a bold equilibrium where the entering mass of unskilled investors is determined by the following market clearing condition. Given the fraction of bold investors’ capital financing good projects, together with the capital of skilled investors (which all finance good projects) all good projects, opaque or transparent, have all their credit demand satisfied. (This market clearing condition is spelled out in the proof of Proposition 5.1). Then, following the intuition in the text, it is easy to check that no one has a profitable deviation: skilled or unskilled investors do not want to change their interest rate from \( r_B^p \), and none of the entrepreneurs want to demand less than \( \bar{L} \) at that rate. While, if the condition above did not hold, investors would be motivated to choose to be cautious advertising a rate \( \tilde{r} \in (r_C^p, r_B^p) \).

Now consider a cautious equilibrium where all unskilled are cautious and advertise \( r_C^p \). This implies that opaque good projects can be financed only by skilled investors. As skilled
capital is scarce, they will advertise the maximum feasible rate \( \bar{r} \). As unskilled capital is abundant, therefore \( r_C^s \) has to make cautious unskilled indifferent whether to enter. As all entrepreneurs demand credit at all advertised rate which is lower than their reservation rate, the pool of applicants in that low interest rate post is identical to the one in the pooled equilibrium at \( r_p^S \). That is, \( r_C^s \) solves (A.8) and \( r_C^s = r_C^p = r_C \) holds. If an unskilled investors is to deviate to a bold test, she has two options. She can advertise an interest rate \( \bar{r} \leq r_C^s \) attracting the pool of all type of entrepreneurs or it can advertise a high rate \( \tilde{r} \in [r_C^s, \bar{r}] \) attracting all, but the transparent good ones. The earlier is a profitable deviation if and only if \( r_B^s \leq r_C^s \) where \( r_B^s \) solves (A.7). That is, a necessary condition for a cautious equilibrium is \( r_B = r_B^p > r_C \). The latter option is a profitable deviation if and only if \( r_I \leq \bar{r} \) where \( r_I \) is determined by the indifference condition

\[
\frac{(1-\mu_1-\mu_0)}{2} (1 + r_I) + \frac{\mu_1}{2} + (\mu_1 + \mu_0) (1 + r_f) - c = (1 + r_f).
\]

Note that \( r_I > r_B \) because it refers to an adversely selected pool of applicants. Checking that neither skilled investors nor any type of entrepreneurs want to deviate from the assigned strategies concludes the construction of the cautious equilibrium.

Finally, if \( r_I < \bar{r} \) \( r_B > r_C \), then there is a mix equilibrium. In this case, skilled investors cannot offer \( \bar{r} \) as they would be undercut by bold unskilled ones. Instead, skilled and bold unskilled investors advertise \( r_I \). This high interest rate post is cleared similarly to the one at the bold equilibrium: the fraction of entering bold unskilled investors have to be sufficient to satisfy, together with skilled investors, all the credit demand of good opaque projects. At the same time, a group of unskilled investors choose to be cautious and advertise \( r_C \) to serve good transparent projects. Note that the two groups of unskilled investors make the same expected profit of \( 1 + r_f \) by the definition of \( r_I \) and \( r_C \). Again, we can check that none of the agents prefer to deviate from the assigned strategies. Given that the conditions for each type of equilibria are mutually exclusive, we have uniqueness.

Observe that the static reasoning can be applied in each period of the dynamic set up, and express the equilibrium criteria in terms of \( \mu_0 \).

**Proof of Propositions 4.2 and 4.3**

Since the switch between the two regimes is only a function of \( \mu_0 \) in a two-stage economy, it is sufficient to compare \( \mu_0 \) across different regimes. In other words, what determines whether the economy is in a boom or a recession is measure of bad entrepreneurs who are opaque.

**Step 1.** The first step is to find the single point steady states in the dynamic model, i.e. the measures that correspond to being in a cautious market, and end up in the same cautious market, and similarly for bold market. One can use the system of equations (4.7-4.4) to get
these fixed points.

\[
\begin{align*}
\bar{\mu}_0 B(\delta, \lambda) &= \frac{\lambda}{2 - (1 - \delta)\lambda} \tag{A.9} \\
\bar{\mu}_1 B(\delta, \lambda) &= \frac{\delta \lambda}{2 - (1 - \delta)\lambda} \tag{A.10} \\
\bar{\mu}_0 C(\delta, \lambda) &= \frac{\delta \lambda}{2((1 - \delta)(1 - \lambda) + \delta)} \tag{A.11} \\
\bar{\mu}_1 C(\delta, \lambda) &= \frac{\delta \lambda}{2((1 - \delta)(1 - \lambda) + \delta)} \tag{A.12}
\end{align*}
\]

Note that \(\bar{\mu}_0 C = \bar{\mu}_1 C\), and \(\bar{\mu}_0 B > \bar{\mu}_0 C\). Since \(\bar{\mu}_0\) is exogenous to these steady states (using \(c\) and \(r_t\), we will focus on the case where \(\bar{\mu}_0\) is in the middle, i.e. \(\bar{\mu}_0 B > \bar{\mu}_0 > \bar{\mu}_0 C\).

\textbf{Step 2.} Two point oscillating distribution. Using the law of motions (4.7-4.4) we get the two point oscillating distribution by conjecturing that if the lower of those two points, \(\mu_0^*\) is a boom, then the implied next period value is \(\mu_0^* C\) and it corresponds to a recession. Two two points has to be such that given the recession, the implied next period value is \(\mu_0^* B\) again. The implied points are

\[
\begin{align*}
\mu_0^* B(\delta, \lambda) &= \frac{\delta \lambda((\delta - 1)\lambda - 1)}{(\delta - 1)^2 \lambda(\lambda + 1) - 2} \tag{A.13} \\
\mu_1^* B(\delta, \lambda) &= \frac{\delta \lambda((\delta - 1)\lambda - 1)}{(\delta - 1)^2 \lambda(\lambda + 1) - 2} \tag{A.14} \\
\mu_0^* C(\delta, \lambda) &= -\frac{(\delta - 2)\delta \lambda((\delta - 1)\lambda - 2)}{2(\delta - 1)^2 \lambda(\lambda + 1) - 4} \tag{A.15} \\
\mu_1^* C(\delta, \lambda) &= \frac{\delta \lambda((\delta - 1)\delta \lambda - 2)}{2(\delta - 1)^2 \lambda(\lambda + 1) - 4}. \tag{A.16}
\end{align*}
\]

It is clear that \(\mu_0^* C > \mu_0^* B\). Thus, the statement follows.

\textbf{Proof of Proposition 5.1}

We described in the main text how entrepreneurs' decide on investment \(i\) and borrowing \(\ell\) taking the interest rate \(r(\tau, \omega)\) and the borrowing limit \(\bar{\ell}(\tau, \omega)\) as given. Then, expressions in Proposition 5.1 follow from the determination of \(r(\tau, \omega)\) in Proposition 4.1 and the borrowing limits \(\bar{\ell}(\tau, \omega)\) which we derive here. We also derive here \(k(\mu_0, \mu_1)\), the equilibrium fraction of unskilled investors who decide to not to enter the credit market in a given state. Consider the bold stage first. The market clearing condition for credit to good transparent and opaque
entrepreneurs is
\[ w_1 + (1 - b_p) w_0 (1 - \mu_0 - \mu_1) = (1 - \mu_0 - \mu_1) \frac{1}{r_B} \]
where \( k(\mu_0, \mu_1) = k_B \) in a bold stage. Then, \( \tilde{\ell}(b, 0) \) is determined by the endowment of unskilled investors which is allocated to bad, opaque credit by mistake:
\[ \mu_0 \tilde{\ell}(b, 0) = (1 - k_B) w_0 \mu_0 \]
implying
\[ \tilde{\ell}(b, 0) = \frac{1}{r_B} - \frac{w_1}{(1 - \mu_0 - \mu_1)} \]
and
\[ i(b, 0) = \tilde{\ell}(b, 0) (1 + r_B) = \frac{(1 + r_B)}{r_B} - \frac{(1 + r_B) w_1}{(1 - \mu_0 - \mu_1)}. \]

Assumption 3.2 requires \( \frac{w_1}{(1 - \mu_0 - \mu_1)} < \frac{1}{r_B} \), validating that bold entrepreneurs are indeed constrained.

In the cautious stage market clearing for opaque good firms gives
\[ \frac{1}{2} (1 - \mu_0 - \mu_1) \bar{\ell}(g, 0) = w_1 \]
implying
\[ \bar{\ell}(g, 0) = \frac{2 w_1}{(1 - \mu_0 - \mu_1)} \]
and investment
\[ i(g, 0) = 1 + \frac{2 w_1}{(1 - \mu_0 - \mu_1)}. \]

Assumption 3.2 requires \( \frac{w_1}{(1 - \mu_0 - \mu_1)} < \frac{1}{2r} \) implying that good opaque entrepreneurs are indeed constrained in this stage. The fraction of entering unskilled investors in a cautious stage, \( (1 - k_C) \), is determined by the market clearing condition for the low interest rate market,
\[ \frac{1}{2} (1 - \mu_0 - \mu_1) \frac{1}{r_C} = (1 - k_C) w_0 \frac{1}{2} (1 - \mu_0 - \mu_1). \]

Turning to the mix stage recall from the proof of Proposition 4.1 that \( \frac{1 - \mu_0 - \mu_1}{\mu_0 + \mu_1 + 1} \) fraction of invested unskilled capital finances good, opaque projects at the high interest rate market, \( 2 \frac{\mu_0}{\mu_0 + \mu_1 + 1} \) finances bad opaque projects and \( 2 \frac{\mu_1}{\mu_0 + \mu_1 + 1} \) ends up at risk-free storage. Then
market clearing for good opaque firms then is
\[
\frac{(1 - \mu_1 - \mu_0)}{2} \ell(g, 0) = (1 - k_I) w_0 \frac{(1 - \mu_1 - \mu_0)}{1 + (\mu_1 + \mu_0)} + w_1
\]
as good opaque entrepreneurs are not constrained, this implies
\[
\frac{1}{2} r_I - \frac{w_1}{1 - \mu_0 - \mu_1} = (1 - k_I) w_0 \frac{1}{1 + (\mu_1 + \mu_0)}
\]
Then market clearing for bad, opaque entrepreneurs gives
\[
\mu_0 \bar{\ell}(b, 0) = (1 - b_I) w_02 \frac{\mu_0}{\mu_0 + \mu_1 + 1}
\]
Substituting back \((1 - b_I)\) implies
\[
\ell(b, 0) = \left( \frac{1}{2} r_I - \frac{w_1}{1 - \mu_0 - \mu_1} \right)
\]
and
\[
i(b, 0) = (1 + r_I) \left( \frac{1}{2} r_I - \frac{w_1}{1 - \mu_0 - \mu_1} \right).
\]
Assumption 3.2 requires \(\frac{w_1}{1 - \mu_0 - \mu_1} < \frac{1}{2r_I}\). Also, \(w_0\) has to be sufficiently large that \(k_I, k_B, k_C \in [0, 1]\). We can summarize the requirements on \(w_1\) for later use as:
\[
\frac{w_1}{1 - \mu_0 - \mu_1} < \min \left( \frac{1}{2r_I}, \frac{1}{2r_I}, \frac{1}{r_B} \right) = \frac{1}{2r_I}.
\]

**Proof of Propositions 5.1 and 6.1**

Proposition 5.1 follows as described in the text. Proposition 6.1 follows from the following three Lemmas.

**Lemma B.1** *Within the pooling region, welfare is decreasing in \(\mu_0\).*

**Proof.** Welfare in the bold stage is
\[
W_B = (1 - \mu_0 - \mu_1) (\rho - 1) \left( 1 + \frac{1}{r_B} \right) + \mu_0 \rho \left( 1 + \frac{1}{r_B} - \frac{w_1}{1 - \mu_0 - \mu_1} \right) + \mu_1 \rho \\
+ w_0 (1 + r_f) + w_1 (1 + r_B)
\]
which we rewrite as

\[ W_B = \rho + w_0(1 + r_f) + w_1\rho + \left( \rho(1 - \mu_1) - (1 + r_B)(1 - \mu_0 - \mu_1) \right) \left( \frac{1}{r_B} - \frac{w_1}{1 - \mu_0 - \mu_1} \right) \]

Note that

\[ d \left( \frac{1}{r_B} - \frac{w_1}{1 - \mu_0 - \mu_1} \right) / d\mu_0 = \left( -\frac{1}{r_B^2} \frac{dr_B}{d\mu_0} - \frac{w_1}{(1 - \mu_0 - \mu_1)^2} \right) < 0 \]

also

\[ (1 - \mu_1) (\rho - (1 + r_f)) - c = (1 - \mu_1) (\bar{\rho} - r_B) + \mu_0 (1 + r_B) > 0, \]

implying the result. ■

**Lemma B.2** Within the separating region, welfare is decreasing in \( \mu_0 \).

**Proof.** Welfare in the cautious stage is

\[ W_C = \frac{1 - \mu_0 - \mu_1}{2} \left( \rho + \frac{1}{r_C} \left( \rho(1 + \frac{1}{r_C}) - \frac{1}{r_C} (1 + r_C) + \rho(1 + \frac{2w_1}{1 - \mu_0 - \mu_1}) - \frac{2w_1}{1 - \mu_0 - \mu_1} \rho \right) \right) + \mu_0\rho + \mu_1\rho + w_0(1 + r_f) + w_1\frac{(1 + r)}{2} \]

which we rewrite as

\[ W_C = \rho + \frac{1 - \mu_0 - \mu_1}{2} \left( \rho - \frac{1 - r_C}{r_C} \right) + w_0(1 + r_f) + w_1\rho \]

Then

\[ \partial \left( \frac{(1 - \mu_0 - \mu_1) (\rho - 1 - r_C)}{r_C} \right) / \partial \mu_0 = \frac{1 - \mu_0 - \mu_1}{2} \left( -\rho - \frac{1}{r_C^2} \right) \frac{\partial r_C}{\partial \mu_0} - \frac{1}{2} \frac{1 - r_C}{r_C} < 0 \]

where we used \( \frac{\partial r_C}{\partial \mu_0} > 0 \). This implies the Lemma. ■

**Lemma B.3** Let us fix \( \mu_1 \) and \( \mu_0 \) at any level \( \mu_0 \leq \frac{c}{1 + r_f} \). Welfare is strictly larger in a pooling equilibrium than it would be in a counterfactual separating equilibrium, \( W_B(\mu_0, \mu_1) - W_C(\mu_0, \mu_1) > 0 \), as long as \( \mu_0 \leq \frac{c}{1 + r_f} \).

**Proof.** As welfare is aggregate consumption, we can decompose \( W_B(\mu_0, \mu_1) - W_C(\mu_0, \mu_1) \) as follows. The difference in transparent good entrepreneurs’ consumption is

\[ \frac{(1 - \mu_0 - \mu_1)}{2} (\rho - 1) \left( \frac{1}{r_B} + 1 \right) - \frac{(1 - \mu_0 - \mu_1)}{2} (\rho - 1) \left( \frac{1}{r_C} + 1 \right) \]
which is non-negative in any point when \( r_B \leq r_C \), that is, in the pooling region. The difference in opaque good plus skilled consumption is

\[
\left[ \frac{1 - \mu_0 - \mu_1}{2} (\rho - 1) \left( \frac{1}{r_B} + 1 \right) + w_1 (1 + r_B) \right] - \left[ \frac{1 - \mu_0 - \mu_1}{2} \rho + w_1 (1 + \bar{r}) \right]
\] (A.18)

note that the term in the first squared bracket is decreasing in \( r_B \) as

\[
\frac{\partial}{\partial r_B} \left( \frac{1 - \mu_0 - \mu_1}{2} (\rho - 1) \left( \frac{1}{r_B} + 1 \right) + w_1 (1 + r_B) \right) =
\]

\[
= - \frac{1}{r_B^2} \frac{1 - \mu_0 - \mu_1}{2} (\rho - 1) + w_1 \leq - \frac{1}{r_B^2} \frac{1 - \mu_0 - \mu_1}{2} (\rho - 1) + \frac{1 - \mu_0 - \mu_1}{r_B} =
\]

\[
= \frac{1 - \mu_0 - \mu_1}{r_B^2} (1 - \rho - 1) < 0
\]

where we used (A.17), and equals to the term in the second left bracket when \( r_B = \bar{r} \). That is, (A.18) is non-negative at any point as long as \( r_B \leq \bar{r} \). Unskilled consumption is equal under the two regimes, while the difference in bad consumption is equal to

\[
\mu_0 \left( \frac{1}{r_B} - \frac{w_1}{1 - \mu_0 - \mu_1} \right) > 0.
\]

\[\blacksquare\]

Proof of Proposition 6.2

We will show that under the conditions of the proposition, there is at least one cyclical economy (the one with short-booms and short recessions) which is preferred by the planner compared to both the always bold and always cautious economies. We will argue that for this conclusion, it is sufficient to show that \( \lambda \in \left[ \lambda_{\min}, \lambda_{\max} \right] \) implies

\[
\max(\lim_{\delta \to 0} W_C (\bar{\mu}_0C, \bar{\mu}_1C), \lim_{\delta \to 0} W_B (\bar{\mu}_0B, \bar{\mu}_1B)) < \lim_{\delta \to 0} \frac{W_B (\mu^*_0B, \mu^*_1B) + W_C (\mu^*_0C, \mu^*_1C)}{2}.
\]

Note that \( \lim_{\delta \to 0} \bar{\mu}_0B = \frac{\lambda}{2^\lambda} \) and

\[
\lim_{\delta \to 0} \bar{\mu}_1B, \bar{\mu}_1C, \mu^*_1C, \mu^*_1B, \bar{\mu}_0C, \mu^*_0C, \mu^*_0B = 0.
\]

In an economy where investors are always bold or always cautious, welfare converges to
$W_B(\mu_{0B}, \mu_{1B})$ and $W_C(\bar{\mu}_0C, \bar{\mu}_1C)$ by definition. First, note that

$$\lim_{\delta \to 0} W_C(\bar{\mu}_0C, \bar{\mu}_1C) = W_C(0, 0) < \lim_{\delta \to 0} \frac{W_B(\mu_{0B}, \mu_{1B}) + W_C(\mu_{0C}, \mu_{1C})}{2} = \frac{W_B(0, 0) + W_C(0, 0)}{2}.$$ 

This is implied by Lemma B.3. Then, we show that $\lambda \in [\lambda^{\min}, \lambda^{\max}]$ is a sufficient condition that

$$\lim_{\delta \to 0} W_C(\mu_{0C}, \mu_{1C}) > \lim_{\delta \to 0} W_B(\bar{\mu}_0B, \bar{\mu}_1B).$$ \hspace{1cm} (A.19)$$

or

$$W_C(0, 0) > W_B\left(\frac{\lambda}{2 - \lambda}, 0\right)$$

which we can rewrite as

$$(\rho - 1 - (r_f + c)) \frac{1}{2 r_f + c} > (\rho - 1 - (r_f + c)) \left(\frac{1}{r_B(\frac{\lambda}{2 - \lambda}, 0, r_f)} - \frac{w_1}{(1 - \frac{\lambda}{2 - \lambda})}\right).$$

This holds when $\lambda \in [\lambda^{\min}, \lambda^{\max}]$, because by (4.1) $\lambda \in [\lambda^{\min}, \lambda^{\max}]$ is the condition for

$$\frac{1}{2 r_f + c} > \frac{1}{r_B(\frac{\lambda}{2 - \lambda}, 0, r_f)}$$

and $r_B(\frac{\lambda}{2 - \lambda}, 0, r_f) < \bar{r}$ to hold simultaneously. As all inequalities are strict and all relevant functions are continuous from the left in $(\mu_0, \mu_1)$, for any $\lambda \in [\lambda^{\min}, \lambda^{\max}]$ we can pick a $\delta(\lambda)$ that if $\delta < \delta(\lambda)$ then our statement holds. Picking

$$\delta = \max_{\lambda \in [\lambda^{\min}, \lambda^{\max}]} \delta(\lambda)$$

defines the threshold for $\delta$.

**Proof of Proposition 6.3**

Clearly, a risk weight of $x > 1$ does not influence the interest rate in a cautious stage as investors are lending to projects which they all pay back.

In a bold stage, we require

$$v_g x + v_r = 1$$
but still assume that the technology of a bold test did not change implying
\[
\frac{v_g}{v_g + v_r} = (1 - \mu_1).
\]

Therefore,
\[
v_g = \frac{1 - \mu_1}{x(1 - \mu_1) + \mu_1}, v_r = \frac{\mu_1}{x(1 - \mu_1) + \mu_1}
\]

which modifies the indifference condition determining the zero profit rate \( r_B^x \) as follows
\[
\frac{1 - \mu_1}{x(1 - \mu_1) + \mu_1} (1 + r_B^x) \frac{(1 - \mu_1 - \mu_0)}{1 - \mu_1} + \frac{\mu_1}{x(1 - \mu_1) + \mu_1} (1 + r_f) - c = 1 + r_f
\]
implying the expression for \( r_B^x \) in the proposition.

In the mix stage, the bold test on the high interest rate market (at which transparent good entrepreneurs do not apply for credit) implies
\[
\frac{v_g}{v_g + v_r} = \frac{\frac{(1-\mu_1-\mu_0)}{2} + \mu_0}{\frac{(1-\mu_1-\mu_0)}{2} + (\mu_1 + \mu_0)}.
\]

Therefore
\[
v_g = \frac{\mu_0 - \mu_1 + 1}{x + 2\mu_1 + x\mu_0 - x\mu_1}, v_r = \frac{2\mu_1}{x + 2\mu_1 + x\mu_0 - x\mu_1}
\]
in the mix stage. This implies that the indifference condition determining the zero profit rate \( r_I^x \) is modified as follows:
\[
\frac{1 - \mu_0 - \mu_1}{x + 2\mu_1 + x\mu_0 - x\mu_1} (1 + r_I^x) + \frac{2\mu_1}{x + 2\mu_1 + x\mu_0 - x\mu_1} (1 + r_f) - c = 1 + r_f
\]
which gives the expression of \( r_I^x \) in the proposition. Finally, by analogous arguments to the baseline case, the threshold between the bold and cautious stages is given by identity
\[
r_B^x (\hat{\mu}_0^x (\mu_1, c, r_f), \mu_1, c, r_f, x) \equiv r_C (\hat{\mu}_0^x (\mu_1, c, r_f), \mu_1, c, r_f)
\]
while the threshold \( \hat{\mu}_x (\cdot) \) is given by identity
\[
r_I^x (\hat{\mu}_0^x (\mu_1, \rho_g, c, r_f), \mu_1, c, r_f, x) \equiv \rho_g - 1.
\]
Proof of Lemma 6.1

From (6.3)
\[ \hat{\mu}_0^\pi(\mu_1, c, \pi_{r_f}) = \frac{c}{1 + r_f} \]  
(A.20)
\[ \hat{\mu}_0(\mu_1, c, \pi_x) = c - (c + 1)(1 - \mu_1)(x - 1) \]  
(A.21)
\[ \hat{\mu}_0(\mu_1, c, \pi_{r_B}) = c - (1 - \mu_1) r_B. \]  
(A.22)

As \( \hat{\mu}_0^\pi(\mu_1, c, \pi_{r_f}) \) is independent of \( \mu_1 \), it is trivial that \( r_f = \frac{c - \hat{\mu}_0^\pi}{\hat{\mu}_0} \) defines an equivalent \( \pi_{r_f} \) policy to \( \hat{\mu}_0^P \). However, as we will see below, the other two policies require more work because they depend non-trivially to \( \mu_1 \).

Note first that \( \pi_x \) and \( \pi_{r_B} \) are equivalent for any \( \mu_1 \) if
\[ r_B = (c + 1)(x - 1). \]  
(A.23)

Therefore, we only have to prove that for any \( \hat{\mu}_0^P \) there is an \( r_B \) implying the same ergodic distribution.

Let the increasing series \( (\mu_{0i}, \mu'_{1i}) \), \( i = 1,..\tau \), the support of the implied ergodic state distribution by \( \hat{\mu}_0^P \) and \( t_0 + 1 \) is a time period at which the system reached the ergodic distribution. The following cases describe the proof for possible cyclical patterns.

(i) Suppose that \( \hat{\mu}_0^P \) implies long booms and short recessions. That is, for any integer \( k \) and \( t = t_0 + k\tau - 1 \), we have
\[ \mu_{0t+2} < \mu_{0t} < \hat{\mu}_0^P < \mu_{0t+1} \]
where \( \mu_{0t+1} = \mu'_{0c} \) is the single state in the cautious stage. Then,
\[ r_B = \frac{c - \hat{\mu}_0^P}{\left(1 - (\delta + (1 - \delta)\mu_{1(\tau-1)})\frac{\lambda}{2}\right)} \]
defines an equivalent \( \pi_{r_f}^\pi \) policy to \( \hat{\mu}_0^P \) as it implies
\[ \hat{\mu}_0^\pi(\mu_{1t+1}, c, \pi_{r_f}) = c - \left(1 - (\delta + (1 - \delta)\mu_{1t})\frac{\lambda}{2}\right) r_f < \mu_{0t+1} \]
\[ \mu_{0t} < c - (1 - \mu_{1t}) r_f \]
as required, where we used that \( c - (1 - \mu_{1t}) r_B > c - (1 - (\delta + (1 - \delta)\mu_{1t})\frac{\lambda}{2}) r_B \) by \( \mu_{1t} \in [\bar{\mu}_{1B}, \bar{\mu}_{1C}] \) in the ergodic distribution.
ii) Suppose that \( \hat{\mu}_0^P \) implies short booms and long recessions. That is, for any integer \( k \) and \( t = t_0 + k \tau \)

\[
\mu_{0t+1} < \hat{\mu}_0^P < \mu_{0t} < \mu_{0t+2}
\]

where \( \mu_{0t+1} = \mu_{01} \) is the single state in the bold stage. We show that

\[
\frac{c - \hat{\mu}_0^P}{1 - \hat{\mu}_0^P} = r_f^B
\]

defines equivalent \( \pi_f^B \) policy to \( \hat{\mu}_0^P \). Note that the dynamics of \( \mu_0 \) and \( \mu_1 \) is the same within the cautious stage, that is, if \( t \) is a cautious date, then in \( t + 1, \mu_{0t+1} = \mu_{1t+1} \)

Hence, we need to find an \( r_B \) for which

\[
c - (1 - \mu_1) r_B = c - (1 - \mu_{0t}) r_f^B < \mu_{0t}
\]

or

\[
\frac{c - r_f^B}{1 - r_f^B} < \mu_{0t}
\]

and

\[
c - (1 - \mu_{1t+1}) r_f^B > \mu_{0t+1}
\]

or

\[
\frac{c - r_f^B}{1 - r_f^B} > \mu_{0t+1}.
\]

Clearly, given (A.24), (A.25) implies that this holds.

(iii) Finally, if \( \hat{\mu}_0^P \) implies short booms and short recessions, or .

\[
\mu_{0B}^* < \hat{\mu}_0^P < \mu_{0C}^*
\]

we need that

\[
\mu_{0B}^* < c - (1 - \mu_{1B}^*) r_f^B
\]

\[
c - (1 - \mu_{1C}^*) r_f^B < \mu_{0C}^*.
\]

We show that picking

\[
\frac{c - \mu_{0C}^* + \varepsilon}{1 - \mu_{1C}^*} = r_B
\]
for an arbitrary small $\varepsilon$ satisfies both these equations. We need to show that

$$\mu^*_0 r < c - (1 - \mu^*_1 r) \frac{c - \mu^*_0}{1 - \mu^*_1},$$

which we can rewrite as

$$\mu^*_0 < \frac{\mu^*_0 c - c \mu^*_1}{1 - \mu^*_1 C - c + \mu^*_0 C}.$$

Given that

$$\frac{\partial}{\partial c} \frac{\mu^*_0 c - c \mu^*_1}{(1 - \mu^*_1 c - c + \mu^*_0 C)} = (1 - \mu^*_1) \frac{\mu^*_0 c - \mu^*_1 c}{(c - \mu^*_0 c + \mu^*_1 C - 1)^2} > 0$$

it is sufficient to show that

$$\mu^*_0 < \frac{\mu^*_0 C}{1 + \mu^*_0 C - \mu^*_1 C}$$

which always holds by the definition of $\mu^*_0 C, \mu^*_0 B$ and $\mu^*_1 C$.

**Proof of Proposition 6.4**

Suppose that $\hat{\mu}_P^0$ imply a cycle of $\tau$ periods and by period $t_0 + 1$ the system reached its ergodic state. Then, expected welfare implied by an equivalent policy $\pi$ is

$$EW(\pi) = \frac{1}{\tau} \sum_{i=1}^{\tau} \begin{cases} 1_{\mu_0(0) \leq \hat{\mu}_0(\mu_1, c, \pi)} W_B^\pi(\mu_0, \mu_1, \pi) + \\ 1_{\mu_0(0) > \hat{\mu}_0(\mu_1, c, \pi)} W_C^\pi(\mu_0, \mu_1, \pi) \end{cases}$$

where

$$W_B^\pi(\mu_0, \mu_1; \pi) =$$

$$\rho + (\rho - 1) \left( (1 - \mu_0 - \mu_1) \frac{1}{r_B(\mu_0, \mu_1, c, \pi)} + \mu_0 \left( \frac{1}{r_B(\mu_0, \mu_1, c, \pi)} - \frac{w_1}{1 - \mu_0 - \mu_1} \right) \right)$$

$$+ (w_0 + w_1) - c \left( \frac{1}{r_B(\mu_0, \mu_1, c, \pi)} - \frac{w_1}{1 - \mu_0 - \mu_1} \right) - (1 - \mu_1) (x - 1) c \left( \frac{1}{r_B(\mu_0, \mu_1, c, \pi)} - \frac{w_1}{1 - \mu_1 - \mu_0} \right)$$
and

\[ W^\pi_C = \rho + \frac{1}{r^\pi_C(\mu_0, \mu_1, c, \pi)} \left( \frac{1 - \mu_0 - \mu_1}{2} (\rho - 1) - c \right) + w_0 + w_1 \rho. \]

These formulas follow the calculation in the baseline case with the additional adjustment in the last line of (13). For that last term, the market clearing condition in a bold stage is

\[ w_1 + (1 - k_P) w_0 v_g \frac{(1 - \mu_1 - \mu_0)}{((1 - \mu_1 - \mu_0) + \mu_0)} = (1 - \mu_0 - \mu_1) \frac{1}{r_B} \]

where \( v_g \) is the amount of money going to green signal firms per unskilled investor, while \( \frac{(1 - \mu_1 - \mu_0)}{((1 - \mu_1 - \mu_0) + \mu_0)} \) is the fraction of the green-signal firms who are good. Then the fraction of entering unskilled investors \((1 - k_P)\) has to satisfy

\[ (1 - k_P) w_0 = x (1 - \mu_1) + \mu_1 \left( (1 - \mu_0 - \mu_1) \frac{1}{r_B} - w_1 \right). \]

This implies that the total cost paid by these entrants is

\[ -c (1 - k_P) w_0 = - (x (1 - \mu_1) + \mu_1) c \left( \frac{1}{r_B} - \frac{w_1}{(1 - \mu_1 - \mu_0)} \right). \]

As \((x (1 - \mu_1) + \mu_1) > 1\), this implies an adjustment of

\[ - ((x (1 - \mu_1) + \mu_1) - 1) c \left( \frac{1}{r_B} - \frac{w_1}{(1 - \mu_1 - \mu_0)} \right) \]

which is the last term in (13).

For the first statement, note that monetary policy effects welfare only through the cost of capital \( r^\pi_B \) and \( r^\pi_C \). As

\[ \frac{\partial W^\pi_B}{\partial r^\pi_B} = - \frac{1}{(r^\pi_B)^2} ((\rho - 1) (1 - \mu_1) - c (\mu_1 + (1 - \mu_1) x)) = \]

\[ = - \frac{1}{(r^\pi_B)^2} ((\rho - 1) (1 - \mu_1) - (1 - \mu_0 - \mu_1) (1 + r^\pi_B) + (r^B_f + 1) x (1 - \mu_1)) = \]

\[ = - \frac{1}{(r^\pi_B)^2} ((\bar{r} - r^\pi_B) (1 - \mu_1) + (r^B_f x + (x - 1)) (1 - \mu_1) + \mu_0 (1 + r^\pi_B)) < 0 \]

and

\[ \frac{\partial W^\pi_C}{\partial r^\pi_C} < 0 \]
and $\frac{\partial r_B}{\partial r_f}$, $\frac{\partial r_C}{\partial r_f} > 0$, any of our monetary policies lead to smaller welfare than the equivalent $\hat{\mu}_0^P$.

The macroprudential policy has a similar negative effect through cost of capital as $\frac{\partial r_B}{\partial x} > 0$, along with an additional direct negative effect

$$\frac{\partial W_B^\pi}{\partial x} = -(1 - \mu_1) c \left( \frac{1}{r_B^\pi(\mu_0, \mu_1, c, \pi)} - \frac{w_1}{(1 - \mu_1 - \mu_0)} \right) < 0$$

The term in the bracket is the loan amount of an opaque bad borrower, hence it is positive.

The additional argument for the second statement is to show that equivalent $\pi_x$ and $\pi_r^B$ implies the same $r_B^\pi$ and $r_C^\pi$. None of them have an effect on $r_C^\pi$ and

$$r_B^\pi(\mu_0, \mu_1, c, \pi_r^B) - r_B^\pi(\mu_0, \mu_1, c, \pi_x) =$$

$$= \frac{1 - \mu_1}{1 - \mu_1 - \mu_0} r_f^B - \frac{(x - 1)(c + 1)(1 - \mu_1)}{1 - \mu_1 - \mu_0} =$$

$$= \frac{1 - \mu_1}{1 - \mu_1 - \mu_0} (x - 1)(c + 1) - \frac{(x - 1)(c + 1)(1 - \mu_1)}{1 - \mu_1 - \mu_0} = 0,$$

where we used (A.23) in the second step.

For the first part of the third statement, consider first the simple monetary policy which is equivalent to $\hat{\mu}_0^P$ and implies the smallest interest rate $r_B^\pi$. Using the notation of the proof of Lemma 6.1, for that $r_f$ has to be arbitrarily close to

$$\frac{c - \mu_0'}{\mu_0'}, \frac{c - \mu_0'}{\mu_0}, \frac{c - \bar{\mu}_0 B}{\bar{\mu}_0 B}$$

in a long boom short recession, short-boom long recession, and in a short-boom long recession cycle respectively. Following the logic of Lemma 6.1, the equivalent countercyclical monetary policy $r_f^B$ has to be arbitrarily close to

$$\frac{c - \mu_0'}{1 - \mu_1'}, \frac{c - \mu_0}{1 - \mu_1'}, \frac{c - \mu_C}{1 - \mu_C} = r_B^f$$

in the three cases. Also, if $r_f \geq r_f^B$ in the two equivalent policies than welfare is weakly smaller in the bold stage and strictly smaller in the cautious stage under the simple monetary policy. Hence, it is sufficient to show that

$$\frac{c - \mu_0}{1 - \mu_1} < \frac{c - \mu_0}{\mu_0}$$

if $\mu_0$ implies a bold stage, that is, $\mu_0 < \hat{\mu}_0^P(\mu_1, c, \pi)$, and $\mu_1$ is within the support of the ergodic distribution of $\mu_1, \mu_1 \in [\bar{\mu}_1 B, \bar{\mu}_1 C]$. As $\hat{\mu}_0^P(\mu_1, c, \pi) < \bar{\mu}_0 B$ in any cyclical economy, it
is sufficient that
\[ 1 - \max (\bar{\mu}_{1C}) = 1 - \frac{\delta \lambda}{2((1 - \delta)(1 - \lambda) + \delta)} \geq \bar{\mu}_B \]
or
\[ \frac{\lambda}{2 - (1 - \delta)\lambda} \leq 1 - \frac{\delta \lambda}{2((1 - \delta)(1 - \lambda) + \delta)}, \]
for which \( \lambda \leq \frac{8}{9} \) is a sufficient condition.

Finally, the last result and the second statement implies the final statement: if \( c \) is sufficiently small than average welfare under \( \pi_x \) and \( \pi_{rf} \) is sufficiently close. As \( \pi_{rf} \) strictly dominates \( \pi_{rf} \), \( \pi_x \) is also more efficient than \( \pi_{rf} \).

\section*{C Appendix: Continuum of Tests}

Assume there is a continuum of tests, indexed by \( s \in [0, 1] \). Every test \( s \) passes all \( \frac{1 - \mu_0 - \mu_1}{2} \) transparent good projects and rejects all \( \mu_1 \) transparent bad projects. Furthermore, test \( s \) passes \( s \) fraction of the opaque projects, i.e. \( s \frac{1 - \mu_0 - \mu_1}{2} \) good projects and \( s \mu_0 \) bad opaque projects. Thus, \( s = 0 \) corresponds to the cautious test, and \( s = 1 \) corresponds to the bold test. Tests with \( s \in (0, 1) \) cover everything in between. We follow the logic as in proof of Proposition 4.1 to show that both the bold and the cautious equilibrium are robust to this modification. In particular, investors strictly prefer to choose the bold test when \( \mu_0 < \frac{c}{1 + r_{rf}} \) and the cautious test when \( \mu_0 > \frac{c}{1 + r_{rf}} \) even if the intermediate choices are also available.

Recall that the unskilled investors choose a test which allows them to advertise the lowest break even interest rate under the conjecture that at that interest rate all types will submit an application. If that were not true, unskilled investors not entering in equilibrium could choose a test and advertise an interest rate which leads to higher profit than staying outside. (We rely here on Lemma 4.1 (i) ensuring that if an entrepreneur applies for a given rate in equilibrium, he also applies for all lower rates, advertised or not.) The break even interest rate for any test characterized by \( s \) is

\[
\left( \frac{1 - \mu_0 - \mu_1}{2} + s \frac{1 - \mu_0 - \mu_1}{2} \right) (1 + r(s)) + \left( \mu_1 + (1 - s) \mu_0 + (1 - s) \frac{1 - \mu_0 - \mu_1}{2} \right) (1 + r_{rf}) - c = 1 + r_{rf},
\]
which in turn implies
\[
(1 + r_f) \left( 1 - \left( \mu_1 + (1 - s) \mu_0 + (1 - s) \frac{1 - \mu_0 - \mu_1}{2} \right) \right) + c - 1 = r(s).
\]

Note that
\[
\frac{\partial r(s)}{\partial s} = -2 \frac{c - \mu_0 - \mu_0 r_f}{(s + 1)^2 (1 - \mu_0 - \mu_1)},
\]
implying that whenever \( \mu_0 < \frac{c}{1 + r_f} \), the smallest interest rate is implied by the test \( s = 1 \), while in the opposite case it is \( s = 0 \). Thus, by the same argument as in the main text, if \( \mu_0 < \frac{c}{1 + r_f} \), the equilibrium advertised interest rate by unskilled investors corresponds to the test \( s = 1 \) (bold test), and in the opposite case they choose \( s = 0 \) (cautious test). In this sense, the continuum of intermediate tests are always dominated by either the bold or the cautious test, and restricting investor choice to these two tests is without loss of generality.

D Appendix: The Three Stage Economy and Double-dip Recessions

When \( \bar{\mu}_0(\mu_1) < \frac{c}{1 + r_f} \) does not hold everywhere along the equilibrium path in the long run, the economy features more elaborate dynamics. Importantly, the economy not only can be in bold and cautious stages, but also in a mix stage. In this case, we need two state variables to characterize the dynamic economy, \((\mu_0, \mu_1)\).

We first explain the mix stage in credit market and real economy, and then proceed to considering the dynamics associated with a three-stage economy.

**Mix stage in credit market.** If the maximum feasible interest rate \( \bar{r} \) is sufficiently high, the interest rate \( r_I(\mu_0, \mu_1, c, r_f) \) in proposition 4.1.(iii) becomes feasible. In this case, some unskilled investors choose to be bold and some cautious. A semi-separating equilibrium arises where opaque good entrepreneurs are financed by bold unskilled investors and skilled investors at a relatively high interest rate, \( r_I(\mu_0, \mu_1, c, r_f) \). Furthermore, the bold unskilled investors mistake opaque bad entrepreneurs with good opaque ones and finance them at the same interest rate. Alternatively, cautious unskilled investors finance all transparent good entrepreneurs at the lower interest rate \( r_C(\mu_0, \mu_1, c, r_f) \). Interest rates \( r_I(\mu_0, \mu_1, c, r_f) \) and \( r_C(\mu_0, \mu_1, c, r_f) \) are such that unskilled investors are indifferent whether to be bold and finance a worse quality portfolio for a higher interest rate, or to be cautious and finance a high quality portfolio for a lower interest rate.
Real economy. In a mix stage, the masses of entering cautious and bold unskilled investors are such that the first group can satisfy the credit demand of transparent good investors at the low interest rate $r_C$ while the second group, together with skilled capital, can satisfy the credit demand at the higher interest rate $r_I$. Therefore, the investment of both of good entrepreneurs are given by $i = \frac{1}{c} + 1$ with the relevant different interest rates. Similar to the bold stage, the credit to opaque bad entrepreneurs is given by the share of capital of bold unskilled investors who cannot identify their loan applications from good opaque ones. The credit of of bad opaque entrepreneurs is lower in mix relative to bold stage since they face a higher interest rate $r_I > r_B$ in the mix stage.

When $\tilde{\mu}_0(\mu_1) < \frac{c}{1+r_f}$ does not always hold in the long run equilibrium the economy can cycle through all three stages, going from bold to mix and then to cautious, and then jumping back to bold stages. Such a three-stage economy features two different type of downturns. The mix stage has signs of low credit market sentiment as it leads to a similar fragmentation of the market as the cautious stage. However, the mixed stage corresponds to a recession which is not sufficiently deep to trigger the “purification effect” on the entrepreneur pool we observed in the cautious stage. This is so, because even a small mass of bold investors is sufficient to mistakenly provide credit to bad opaque entrepreneurs and prevent them from exiting. Therefore, in a mix stage the fraction of bad entrepreneurs keeps increasing to the point when the cautious stage is triggered. In a cautious stage the sufficiently tight credit conditions reverses the direction of the economy.

The following assumption ensures a three-stage cycle for the dynamic economy.

**Assumption A.2** Assume

\[
(i) \quad \frac{c}{1+r_f} < \frac{\mu_0(\mu)}{\mu_1} \quad \text{and} \quad \frac{\mu_0(\mu_1)}{\mu_1} < \frac{\lambda}{2-\lambda(1-\delta)}
\]

\[
(ii) \quad \frac{c}{1+r_f} > \frac{\mu_0(\mu_1)}{\mu_1} < \frac{\lambda}{2(2-\delta)(2-\lambda)}
\]

Assumption A.2 ensures that $\mu_0(\mu_1) < \frac{c}{1+r_f} < \tilde{\mu}_0(\mu_1) < \hat{\mu}_0$. This is sufficient to make sure that in the long run, the economy goes through all the three stages, as formalized in the following proposition.

**Proposition D.1** Consider $\tilde{\mu}_0(\delta, \lambda) > \mu_0(\delta, \lambda) > \mu_0(\delta, \lambda) > \tilde{\mu}_0(\delta, \lambda)$, and assume assumption A.2 holds. Then the ergodic distribution has more than two points of support. The credit cycle consists of a multi-period bold stage when $\mu_0$ increases, followed by a multi-period mix stage when $\mu_0$ still increases, followed by a one-period cautious stage when $\mu_0$ declines to that of the bold stage with the lowest $\mu_0$.

Figure ?? portraits an example of the dynamic cycle of the economy. The top panel illustrates the dynamics of the state variables. In the beginning of each upward segment,
μ₀ is sufficiently small and the composition of borrowers sufficiently good, μ₀ < \frac{c}{1+rf}. In this region (below the dotted blue line) all investors are bold. Between the dotted blue line and the yellow curve, some unskilled investors turn cautious but some are still bold (semi-separating equilibrium). In this region, opaque bad entrepreneurs still get funded (although at worse interest rates), so μ₀ is still growing. When μ₀ becomes sufficiently large, i.e. it crosses the yellow threshold, then the composition of borrowers become sufficiently bad that no investor chooses to be bold. Thus all investors turn cautious and a one period crash happens.

Observe that although the dynamics of the state variables are qualitatively similar to the two-stage economy depicted in Figure 2, the implied outcomes are quite different. The second panel of Figure ?? depicts the interest rates in the three-stage economy. In the mix stage there is already a considerable spread between the interest rates faced by opaque and transparent entrepreneurs although μ₀ is still increasing. This spread drops to zero again, only when the economy reverses to the bold stage. Output dynamics in the bottom panel shows that in the three-stage economy output crashes twice in each cycle, creating a double dip recession.

Further comparison of Figure 2 and Figure ?? shows that although the interest rate in the mix stage and in the cautious stage are at similar levels, the output effect of switching to the cautious stage is significant in the three-stage economy. In fact, the output dynamics shows a double-dip recession. Despite the significant drop in output around period 3, the recession is not deep enough for the economy to experience the “purification effects” of a cautious equilibrium. Therefore, output drops further until a second drop in output occurs in period 6. In this period finally the fraction of opaque bad entrepreneurs drops, and corresponds to a deep downturn which then triggers a boom in the following period.

A few observations are worth mentioning. First, if μ₂B < \frac{c}{1+rf} ≤ \tilde{μ}_0(μ₁) < μ₂C (assumption 4.3 and A.2 both violated), the ergodic set consists of the same two point distribution as described in Proposition 4.3.(iii), i.e. a two-stage economy with a cycle of length two which

Figure D.1: Interest rates (left panel) and output (right panel) in a three-stage economy as the fraction of bad opaque entrepreneurs μ₀ changes.
Figure D.2: Panel (a) depicts the law of motions of state variables. Panel (b) shows the interest rates, and Panel (c) depicts the total gross output in a three-stage economy with a multi-period bold, a multi-period mix, and a one period cautious stage cycle.
fluctuates between all lenders being bold or cautious.

Second, it is not possible to have a three-stage economy with a long downturn. In other words, whenever there is a cycle which consists of multiple consecutive cautious stages, it ends either by a single bold stage (if $\tilde{\mu}_0(\mu_1) < \frac{c}{1+r_f}$), or by a single mix stage (if $\tilde{\mu}_0(\mu_1) > \frac{c}{1+r_f}$).

The key to this observation is that whenever $\mu_0$ falls below $\max\{\frac{c}{1+r_f}, \tilde{\mu}_0(\mu_1)\}$, the dynamics is dictated by $\mu_B(\delta, \lambda, \mu_0, \mu_1)$ function and is upward sloping, so it cannot enter a third (lower) stage.

Finally, note that the threshold $\tilde{\mu}_0(\mu_1)$ depends on one of the state variable $\mu_1$. This implies that the dynamic economy might even fluctuate between a two-stage and a three-stage economy. This can lead to cycles with varying length.