A Q-Theory Of Banks

Juliane Begenau† Saki Bigio‡

Jeremy Majerovitz§ Matias Vieyra¶

October 2021

Abstract

We propose a dynamic bank theory with a delayed loss recognition mechanism and a regulatory capital constraint at its core. The estimated model matches four facts about banks’ Tobin’s Q that summarize bank leverage dynamics. (1) Book and market equity values diverge, especially during crises; (2) Tobin’s Q predicts future bank profitability; (3) neither book nor market leverage constraints are binding for most banks; (4) bank leverage and Tobin’s Q are mean reverting but highly persistent. We examine a counterfactual experiment where different accounting rules produce a novel policy tradeoff.

Keywords: Banks, Leverage Dynamics, Market vs. Book Values, Delayed Accounting

JEL: G21, G32, G33, E44

---

*We would like to thank Youngmin Kim, Adam Su, and Mengbo Zhang for excellent research assistance. We are grateful for our discussants Dean Corbae, Miguel Faria-e-Castro, Elena Loutsksina, Thorsten Koeppl, David Martinez-Miera and Adrián Pardo. We thank Andrew Atkeson, Haelim Anderson, Patrick Bolton, Pablo D’Erasmo, Andrea Eisfeldt, Francesco Ferrante, Mark Gertler, Kinda Hachem, Valentin Haddad, Benjamin Hebert, Arvind Krishnamurthy, Anton Korinek, Hanno Lustig, Ken Miyahara, Gaston Navarro, Andrea Prestipino, Jose-Victor Rios-Rull, Pari Sastry, Jesse Schreger, Jeremy Stein, Robert Townsend and Pierre-Olivier Weill for their many useful comments. We thank the conference and seminar participants of the Barcelona GSE 2021, UofT Finance Conference 2021, MFM Winter 2019 Meeting, Columbia 2020 Junior Finance Conference, Banco de Mexico, Philadelphia Fed, Federal Reserve Board, UCSD, UCSC, Michigan Ross, Northwestern University, and the University of Virginia. Previous versions of the paper circulated under the title “Data Lessons on Bank Behavior” and “Banks Adjust Slowly”. These are our views and not necessarily those of the Bank of Canada nor the Federal Reserve Bank of San Francisco.

†Begenau: Stanford GSB and NBER and CEPR. E-mail: begenau@stanford.edu
‡Bigio: UCLA, Visiting Scholar SF Fed, & NBER. E-mail: sbigio@econ.ucla.edu
§Majerovitz: MIT. E-mail: jmajic@mit.edu
¶Vieyra: Bank of Canada. E-mail: mvieyra@bankofcanada.ca
1 Introduction

Banking theory shapes financial regulation and, through regulation, macroeconomic outcomes. However, developing frameworks suitable for policy analysis is a fine-tuning process in which regulators and academics constantly reassess models. In fact, bank regulation was entirely redesigned in the aftermath of the Great Recession, only to meet a wave of regulatory forbearance in the Covid-19 crisis. Hence, the continuous development of banking models is a macroeconomic priority and understanding the dynamics of bank leverage is critical to that process.

This paper presents a novel theory of banks and bank leverage dynamics with slow recognition of loan losses on bank accounting statements and a regulatory capital constraint at its core. Our theory is motivated by four stylized facts about banks’ leverage characteristics. Tobin’s Q, the ratio of market-to-book equity, is a unifying theme among these facts. Slow loan-loss recognition induces a distinction between the fundamental value of equity, which fully incorporates information on losses, and the book value of equity, which does not. We label the ratio of fundamental-to-book value “little q”, which is the part of Tobin’s Q ("big Q") that can be attributed to delayed accounting.\(^1\) The motivating facts for our Q-theory are:

1. **On the decomposition of Tobin’s Q:** Tobin’s Q fluctuations are primarily driven by market equity. Book equity is very stable, even during the 2008/2009 financial crisis.

2. **On the informational content of Tobin’s Q:** Market equity captures information that book equity does not. In the cross-section, Tobin’s Q predicts loan charge-offs and bank profits over a two-year horizon.

3. **On the cross-section of Tobin’s Q and bank leverage constraint:** The cross-sectional dispersion of Tobin’s Q varies over time. The dispersion of market leverage increased during the financial crisis of 2008/2009. By contrast, the distribution of book leverage is very stable. Nearly all banks keep a capital buffer above the regulatory minimum; only a minor fraction of banks violated their regulatory constraints during 2008/2009.

4. **On the dynamic response of Tobin’s Q to shocks:** Tobin’s Q and market-leverage are slowly mean-reverting after a shock that proxies net-worth losses. By contrast, the response of book leverage is protracted.

Collectively, these facts suggest that incorporating a distinction between market and book equity values driven by delayed accounting into banking models would improve macroprudential policy analysis. Fact 1 emphasizes the time-series difference between the book- and the market-value of equity.\(^2\) Understanding this difference is essential for taking a stance on whether the book or the

---

\(^1\)We formulate the facts in terms of Tobin’s Q because it is observable, while \(q\) is not.

\(^2\)The dichotomy between book and market values prompted policy discussions even in earlier banking crises (see the survey by Berger et al., 1995).
market value of equity is the relevant state variable for bank models. Empirically, book leverage and market leverage lead to different inferences about the time-series properties of leverage and the price of risk (see the debate between Adrian et al. 2014 and He et al. 2017). Typically, both measures co-move in models, but not in the data. Fact 2 implies that the market contains more information about book losses, consistent with findings in Blattner et al. (Forthcoming) and much of the accounting literature.

Fact 3 is about leverage constraints and the cross-sectional dynamics of market and book leverage. Before the 2008 crisis, market leverage already featured substantial cross-sectional dispersion. Yet, this dispersion was particularly high during 2008 and 2009 as well as at the beginning of our sample, the end of the savings and loan crisis. The distribution of book leverage is much more concentrated and near an equity buffer below the regulatory requirement. Amid a severe banking crisis (2008-2009), most banks kept their capital ratios above the regulatory limit, even among those whose market valuations eroded significantly. Together, these facts suggest that leverage constraints are unlikely to be strictly binding. Although market values of bank equity are more informative about bank losses compared to book values, market leverage does not appear to be constrained as it increased dramatically for most banks during different crises. Book equity values are not a timely predictor of bank health. Since book values incorporate information on losses with a delay, banks have time to adjust book their leverage in order to avoid hitting the regulatory limit. These patterns are inconsistent with the formulation of financial constraints in earlier models.

Fact 4 summarizes the slow adjustment dynamics of market and book leverage to a net-worth shock. It is challenging to empirically identify net-worth shocks because, as we note in fact 2, accounting measures do not convey all the information on bank losses. Unfortunately, while market values capture information not contained in books, variations in risk premia also affect market valuations. Hence, we cannot directly exploit variation in market valuations. However, we can exploit cross-sectional variation in market returns, which builds on the efficient-market hypothesis. The idea is that once we partial out variation driven by adequately chosen risk factors, the idiosyncratic variation in excess stock returns contains information about changes to the effective net-worth of banks. In conjunction with several robustness checks, we argue that these idiosyncratic-return shocks are valid proxies for net-worth shocks. We then construct a time series of return shocks for each bank and estimate an average impulse responses of market and book leverage, liabilities, dividends, and book equity from a panel of US bank data. These impulse responses inform us about the adjustment process after a shock to banks’ net-worth. We find that banks adjust very slowly.

---

3Papers that study the asset pricing implications of intermediary net-worth (e.g., He and Krishnamurthy, 2013 and Brunnermeier and Sannikov 2014) use market equity as a state variable, whereas papers that focus on the effects of regulation use book measures of equity (e.g., Adrian and Boyarchenko 2013; Begenau 2020; Adrian and Shin 2013; Corbae and D’Erasmo 2021; Begenau and Landvoigt Forthcoming).

4Laux and Leuz (2010) explains how banks have flexibility in accounting for losses. In fact, this was an issue raised by the United States Congress after the Savings and Loans crisis (General Accounting Office, 1990).

5A noteworthy example is Citibank, a bank that experienced market-based losses of up to 90% with only minor changes in its book equity.
slowly to net-worth shocks.\textsuperscript{6} Namely, in response to a negative return shock that mechanically increases market leverage, banks reduce market leverage back to the initial level by selling assets and slowly rebuild capital.\textsuperscript{7} The gradualism of the adjustment suggests an economic mechanism behind this slow adjustment process.

We demonstrate that a model with slow loan-loss recognition in conjunction with a regulatory constraint can qualitatively and quantitatively explain the four stylized facts about bank leverage dynamics. Our partial-equilibrium model features a cross-section of banks. Book values differ from fundamental values due to delayed loan loss accounting. Market values differ from fundamental values due to a valuation difference between equity inside and outside the bank. Banks are owned by diversified risk-neutral shareholders. Banks maximize the discounted stream of dividends under risk neutrality (i.e., they are risk-neutral firms) but prefer smoothed dividends. They fund risky loans with deposits and internal equity (no new outside equity issuance). The exogenous expected return spread between loans and deposits is positive and constant, such that the return on equity increases with leverage. However, when leverage is too high, loan default shocks increase the risk of costly liquidation. This trade-off induces an optimal fundamental leverage.\textsuperscript{8} Smooth dividends prevent market leverage from adjusting immediately via retained earnings and drive a wedge between a dollar inside and outside the bank, a source of variation in Tobin’s Q. Our accounting mechanism drives a wedge between the fundamental value and the book value of banks. Specifically, there are two types of loan default shocks. With some probability, the loan loss has to be recognized at once, while with 1 minus that probability losses are only gradually recognized on the books. Our model generates variation in Tobin’s Q, predictability of profits, a cross-section of market and book leverage with banks keeping an equity buffer, and slow responses to a negative net-worth shock.

We take the model to the data by estimating six important model parameters: the parameters that govern the discount rates of investors and bankers, the intertemporal elasticity of substitution, the size of the loan default shock, the loan loss recognition rate on books, and the probability that the loan default shock is recognized immediately. We estimate them using US bank data on the average equity growth rate, the market-to-book ratio of equity, the book leverage ratio, and the impulse responses of market leverage and liabilities to net-worth shocks proxied by excess-return shocks.

Our model accounts for the four facts without asset adjustment costs that generate the slow adjustment of balance sheet variables in other models. Because banks do not immediately recognize losses and prefer to stay levered, they do not immediately delever when hit by a loan default shock.

\textsuperscript{6}This is consistent with the literature on slow moving capital, see Darrell Duffie’s presidential address to the American Finance Society in 2010 (Duffie, 2010).

\textsuperscript{7}Gropp and Heider (2010) document that banks’ capital ratios follow a target leverage ratio.

\textsuperscript{8}In corporate finance, it is standard to explain a leverage target through either the tradeoff theory or a risk-return tradeoff (e.g., Kraus and Litzenberger, 1973, Leland and Pyle, 1977a, Myers, 1984, Hennessy and Whited, 2005, Frank and Goyal, 2011). In O’Hara (1983), a leverage target follows from the undiversified ownership that induces risk-averse behavior.
While there is an immediate market leverage response, book leverage does not respond on impact and reacts very slowly. Over time, banks delever gradually as they slowly recognize past losses, which tightens their regulatory constraints. As a result, banks reduce liabilities at the pace at which they recognize losses. Hence, delayed accounting explains the dynamics of market leverage, book leverage, and liabilities. As a result of these dynamics, the model explains the time-series and cross-sectional properties of Tobin’s Q in addition to its predictive power over future losses. With an aggregate shock of 2.5%, the model explains about 50% of the observed decline in Tobin’s Q during the crisis, exclusively attributed to changes in little $q$.

Our Q-theory also has a novel policy implication. It reveals a trade-off: more lenient accounting rules allow banks to increase their fundamental leverage beyond regulatory limits. That is, banks are more fragile than their book leverage levels make them appear. A severe shock would then lead to more defaults in the banking sector. However, more lenient accounting rules also help banks to recover faster after a significant shock. Thus, our model shows that accounting rules imply a trade-off between higher financial fragility and a speedier recovery after a severe financial shock.

Related Literature. The financial crisis of 2008 renewed interest in banking theories (e.g., Adrian and Shin, 2010; Bianchi, 2010; Rampini and Viswanathan, 2019; Jermann and Quadrini, 2012; Gertler et al., 2012; Adrian and Shin, 2013; He and Krishnamurthy, 2012, 2013; Brunnermeier and Sannikov, 2014; Gertler et al., 2016; Christiano and Ikeda, 2016; Lenel, 2017; Piazzesi and Schneider, 2018; Li, 2019). Some theories place constraints on market values and others on book values. Most do not differentiate between market and book values, or both measures move closely together. Models with market-based constraints include Gertler and Kiyotaki (2010); Jermann and Quadrini (2012); Brunnermeier and Sannikov (2014); He and Krishnamurthy (2013); Gertler et al. (2016); Nuño and Thomas (2017) are motivated by agency frictions. Models with book-based constraints emphasize the fact that regulatory constraints are based on book values (Adrian and Boyarchenko, 2013; Begenau, 2020; Bianchi and Bigio, Forthcoming; Martinez-Miera and Suarez, 2011; Corbae and D’Erasmo, 2021; Elenev et al., 2021) and do not differentiate between book and market values.

In this paper, we propose a theory that jointly speaks to economic differences in market and book equity valuation of banks. Our paper makes two contributions. First, it organizes facts about Tobin’s Q that shed light on the relevant constraints on banks. Second, it presents a Q-theory based on delayed loan loss accounting. This theory reconciles the behavior of book and market values and sheds light on the economics of bank leverage constraints. We argue that existing models cannot explain all of these facts without relying on economically implausible loan adjustment costs.

---

Examples of these frictions include costly verification (Townsend, 1979; Bernanke and Gertler, 1989), lack of commitment (Hart and Moore, 1994), or moral hazard (Holmstrom and Tirole, 1997, 1998). Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) were the first to model the connection between firm equity and aggregate outcomes. A different perspective is taken by Diamond and Rajan (2000), who argue that deposits through bank runs (a là Diamond and Dybvig, 1983) act as a disciplining device in the presence of agency frictions.
Our facts suggest that leverage constraints operate in more complex ways than conceived in banking models. Take the case of models with market-leverage constraints. Whereas these models generate the countercyclical movements in market leverage (fact 1), they cannot account for the lack of response in book values. Also, without economically large portfolio adjustment costs these models cannot generate an increase in the cross-sectional dispersion of market-leverage during crises (fact 3). Upon an aggregate shock, these models would predict a compression of market-leverage distribution at some higher level, the opposite of what we observe. Models with book-based constraints assume that book losses are immediately registered. Hence, Tobin’s Q would not predict future bank profits and losses. More importantly, the delayed accounting of losses alters how the regulatory constraint operates.

Models built on He and Krishnamurthy (2013), Brunnermeier and Sannikov (2014), or Gertler et al. (2016) also produce a slow response in bank variables (fact 4). The differences is that in those models, the slow leverage dynamics follow from portfolio adjustment costs. This is akin to the neo-classical adjustment cost models of investment built on Hayashi (1982). In finance, adjustment costs are rationalized through asymmetric information—e.g., Myers and Majluf (1984), Leland and Pyle (1977b); Diamond (1984); Williamson (1986); Dang, Gorton, Holmström and Ordonez (2017); Hachem (2011). In recent work—e.g., DeMarzo and He (2016) and Gomes et al. (2016)—slow-moving leverage emerges as the result of debt dilution. A novel feature of our Q-theory is that slow moving leverage dynamics arise through delayed accounting alone.

There is substantial evidence for delayed loss accounting (e.g., Caballero et al., 2008; Behn et al., 2016; Blattner et al., Forthcoming; Plosser and Santos, 2018; Flanagan and Purnanandam, 2019; Acharya et al., 2019). Laux and Leuz (2010) argue that delayed accounting was prevalent during the Great Recession. In the macro-finance literature, the effects of accounting rules on bank decisions and regulatory policies are understudied. A notable exception is Milbradt (2012) who studies theoretically the effect of accounting rules on banks’ trading behaviors in risky assets.

Our paper relates to the literature on accounting regulation. Some authors argue that marking assets to market can amplify a crisis by worsening financial frictions (Shleifer and Vishny, 2011; Laux and Leuz, 2010; Plantin and Tirole, 2018). Related to the Covid-19 crisis, questions such as whether banks should mark down their assets and the extent of regulatory forbearance are at the center stage of macro-prudential policy discussions (Blank, Hanson, Stein and Sunderam, 2020). Our paper is the first to study delayed accounting quantitatively and highlights a novel tradeoff between ex-ante leverage and ex-post adjustments.

Finally, our paper relates to Corbae and D’Erasmo (2021) and Rios-Rull et al. (2020) in the sense that we also study a cross-section of banks. However, our economic question and mechanism differs from this prior work as we focus on a novel mechanism—delayed loan loss accounting—to shed light on the balance sheet and leverage dynamics of banks.

\[^{10}\text{In Appendix A.3, we further detail the bank accounting literature. Bushman, 2016 and Acharya and Ryan, 2016 provide a good overview.}\]
2 Four Stylized Facts

This section presents four stylized facts about banks’ Tobin’s Q and bank leverage dynamics. We are of course by no means the first to highlight that market and book values of banks behave very differently, nor that market data is forward looking. Rather, we present this collection of facts as critical empirical guideposts for our model.

Data. We use panel level accounting data (balance sheet and income statements) on United States Bank Holding Companies (BHCs) from the FR-Y-9C regulatory reports filed with the Federal Reserve.\textsuperscript{11} We merge the accounting data with market data from the Center for Research in Security Prices (CRSP). We focus on the sample period from 1990 Q1 to 2021 Q1.\textsuperscript{12} BHCs file FR-Y-9C forms if they have assets above one billion dollars.\textsuperscript{13} This sample is highly representative of the banking sector. Appendix A.2 reports the time series of balance sheet variables for all BHCs and the largest four: Bank of America; J.P. Morgan, Citigroup, and Wells Fargo.

2.1 Tobin’s Q and the Time Series of Market versus Book Equity

In most banking models, bank equity is a key state variable. We can measure a bank’s equity using either accounting values (book equity) or market values (market equity). We refer to the ratio of market equity to book equity as Tobin’s Q.\textsuperscript{14} The left panel of Figure 1 presents the time series of Tobin’s Q of the aggregate banking sector. Market valuations of banks relative to their book value fluctuate a lot. We measure book equity alternatively as total book equity and total Tier 1 equity. While the former is available for our entire sample period, Tier 1 capital is a key variable for book regulatory constraints. We choose to focus the stylized facts on the total book equity based definition simply because both variables are highly correlated and the book based definition is available for a longer time.\textsuperscript{15}

After low market valuations relative to banks’ book values in the years after the Savings and Loan crisis, Tobin’s Q reached their highest levels in the late 1990s and early 2000s, but then fell dramatically during the financial crisis and has not yet recovered. The right panel shows the components of Tobin’s Q, i.e., the market equity and book equity aggregated across all BHCs. Most of the fluctuations in Tobin’s Q are driven by fluctuations in market valuations. There is

\begin{itemize}
\item \textsuperscript{11}A bank holding company is an umbrella company that holds banks and other financial institutions. A commercial bank is a single bank that provides traditional banking services such as deposits and loans. For example, Citibank, a commercial bank, is held by Citigroup, a BHC that holds Citibank and other banks, including non-commercial banks.
\item \textsuperscript{12}When constructing aggregate time series, we drop entrants to correct for the entry of major financial institutions such as Goldman Sachs and Morgan Stanley. Without this correction, aggregate bank assets increase due to the reclassification of large actors such as Morgan Stanley and Goldman Sachs into bank holding companies.
\item \textsuperscript{13}Prior to 2015 Q1, this threshold was $500 million and prior to 2006 Q1, this threshold was $100 million.
\item \textsuperscript{14}We are referring to the market-to-book equity ratio as Tobin’s Q, as opposed to the market-to-book assets ratio.
\item \textsuperscript{15}A regression of book equity on Tier 1 equity results in a $R^2$ of 0.99 and a coefficient of 1.2.
\end{itemize}
Figure 1: Tobin’s Q and the Book Equity and Market Equity for Bank Holding Companies.

Notes: These figures show data on Tobin’s Q in the left panel and book equity, market capitalization, and preferred equity in the right panel for an aggregate sample of BHCs. Tobin’s Q is measured as the ratio of market equity to book equity and the ratio of market equity to Tier 1 equity capital (only available since 1996). Book equity and preferred equity data come from the FR Y-9C. Market equity is from CRSP. All level variables are converted to 2012 Q1 dollars using the seasonally-adjusted GDP deflator. We exclude new entrants in 2009 such as Goldman Sachs and Morgan Stanley from these aggregate time series. “Equity (Public BHCs)” refers to the publicly-traded BHCs that we can match to CRSP data. “Tier 1 Equity” refers to the Tier 1 Equity Capital of publicly traded BHCs. The figure reports aggregate preferred equity and market equity, i.e. shares outstanding times the share price, of the publicly traded BHCs.

often a stark discrepancy between market and book valuations of banks, in particular during crises. For instance, just by looking at book equity over the years between 2008 and 2009, it is hard to detect any trouble in the financial sector. The sharp drop in market equity, on the other hand, clearly indicates a crisis. At the end of 2008, the market equity of banks fell by more than 54% relative to 2007 Q3 (see Table 3 in the Appendix). For comparison, the S&P500 index fell by 42% relative to 2007 Q3 and partially recovered whereas bank equity remained depressed. In contrast to market valuations, banks’ book equity actually increased by almost 12% over the same horizon.

At the beginning of the Covid-19 crisis, market valuations fell again sharply in the first quarter of 2021 without any movement in book equity.16 To summarize, the first fact for our theory is:

**Fact 1.** Tobin’s Q of the aggregate banking sector fluctuates widely over time, primarily driven by banks’ market valuations. Book equity is very stable, even during the 2008/2009 financial crisis.

Other studies have also documented the divergence between book and market equity during the crisis (see for example Adrian and Shin, 2010; He et al., 2017). However, this paper proposes

---

16This difference between market and book equity is not the result of the composition of public equity injections. Although these injections are counted as preferred equity in accounting books and market equity is measured relative to common equity, Figure 1 shows that preferred equity cannot explain the discrepancy between market and book equity. Preferred equity rose temporarily during the crisis due to the Troubled Asset Relief Program (TARP). Note that preferred equity is included in book equity, but not in market equity.
delayed accounting as a mechanism to reconcile book and market value dynamics. We next turn to cross-sectional evidence to provide suggestive evidence of the delayed accounting mechanism.

2.2 Tobin’s Q and Predictability

The differences between market and book equity values naturally reflect differences in the informational content of these measures when book values take time to recognize loan losses. Conceptually, book measures are *backward-looking* in that they register historical losses. By contrast, market equity measures are *forward-looking* in that they price future expected cash flows. Still, this conceptual difference does not imply that both measures contain different information: In principle, we can write a model where the history of events is encoded in a bank’s balance sheets and the information contained in the books is enough to predict future cash flows. In such a case, the informational content of market values would be the same while the time-series and cross-sectional variation of Tobin’s $Q$ would only reflect changes in risk premia.

However, there are at least two reasons to expect different information content in market and book value measures. One reason is the delayed acknowledgment of known losses, which is a widely documented fact in the accounting literature. As long as banks delay the recognition of losses, or refinance non-performing loans to avoid registering losses (evergreening), book values will not reflect banks’ actual losses. If market participants can update their valuations more quickly, detecting these losses, differences in informational content will emerge. This alone can produce differences in the informational content of market and book equity. The other reason is that changes in the underlying market value of loans reflect default expectations while the book value of loans does not (see filing instructions for FR-Y-9C BHCs regulatory reports) at least until January 2020. Before 2020, loan loss expectations were not updated in loan accounting books and loans were only written off once the loss had occurred. Publicly traded banks were supposed to change their accounting system to the new “current expected credit loss” (CECL) accounting system in January 2020.\(^\text{17}\) Note that our model can capture these accounting system changes (see Section 4.2) and study their effects on bank lending.

Figure 2 suggests that indeed market values contain more information than book values of equity. Loan loss provisions, denoted as PLL in the figure, peak in early 2009 when market values had already tanked. Loan net charge-offs peaked even later in 2010 when the economy was no longer officially in a recession.\(^\text{18}\) The decomposition of net charge-offs shows that these losses were heavily driven by real estate, which is consistent with the nature of the crisis. Loan loss provisions


\(^\text{18}\)When a bank has a loss that is estimable and probable, it first provisions for loan losses on the income statement, which shows up as PLL in the figure. Later when the loss has realized, the asset is charged off and thus taken off the books, which shows up as charge-offs. Occasionally, the bank can recover the asset later.
lead net-charge-offs, which can be best seen for the 2008/2009 crisis and in the beginning for the Covid crisis (note we present data until 2020 Q1).

Figure 2: Decomposition of Net Charge-offs

![Net Charge-Offs](image)

Notes: This figure shows aggregate net charge-offs for different categories (area chart) and aggregate loan loss provisions (solid black line) from 1990 Q1 to 2020 Q1. The data source are FR Y-9C reports. Net charge-offs for loans are defined as charge-offs minus recoveries. We decompose the net charge-offs into loans backed by real estate, commercial and industrial (C&I) loans, loans to individuals (e.g., such as credit card loans), and all other loans (e.g., inter-bank loans, agricultural loans, and loans to foreign governments).

Next, we formally show that variation in the cross-section of Tobin’s Q reflects differences in the information content of market and book equity values. If market equity values contain more information about bank profitability and credit losses than book equity values, then we expect for Tobin’s Q to predict future bank profits and loan losses even after controlling for book equity. In Figure 3, we show binned scatter plots of logged outcomes on the log market-to-book equity ratio. The plots control for time fixed effects, the Tier 1 regulatory capital ratio, and log book equity. The top left panel shows the log return on equity over the next year plotted against the log market-to-book ratio. Banks with higher market-to-book ratios earn higher future profits. A bank with a lower Tobin’s Q today is also more likely to have higher loan loss provisions even eight quarters ahead (top right panel). Banks with higher market-to-book ratios also have a lower share of delinquent loans (bottom left panel) and a lower future net charge-off rate on their loans (bottom right). Thus, Tobin’s Q predicts future book realized profits and actual loan losses beyond what is reflected in book values, suggesting that book values account for loan losses only very slowly. This is consistent with the fact that book equity did not decline during the crisis, despite widespread issues in credit markets. Note that discount rate variations affect most banks similarly and are therefore unlikely to drive these cross-sectional results. Indeed, our results suggest that banks

---

19To control for log book equity, the left and right-hand side variables are residualized on log book equity, and then the mean of each variable is added back to maintain the centering. It is important to control for log book equity to prevent spurious results due to ratio bias (see Kronmal, 1993).
Figure 3: Market equity contains more cash-flow relevant information than book equity

Notes: This figure presents cross-sectional binned scatter plots of log outcomes on the log Tobin’s Q for BHCs. All plots control for log book equity as a proxy for size, the Tier 1 capital ratio of each bank and a quarter-time fixed effect. Data on market equity are from CRSP. All other data are from the FR Y-9C reports. Return on equity over the next year is defined as book net income over the next four quarters divided by book equity in the current quarter. The two-year ahead loan provision rate is calculated as the ratio of eight quarter ahead quarterly loan provisions divided over total loans. The share of delinquent loans is the ratio of 30 days or more past due loans plus loans in non-accrual over total loans. The net charge-off rate is calculated as the difference between loan charge-offs over the next quarter and loan recoveries over the next quarter, divided by total loans this quarter.

with lower profitability and more delinquencies have lower Tobin’s Q, and that Tobin’s Q predicts future loan write-downs and future profitability. The second motivating fact for our theory is as follows:

Fact 2. Tobin’s Q predicts future cash flows in the cross-section of banks. Market equity values capture information that book equity values do not. In particular, in the cross-section, Tobin’s Q predicts loan charge-offs and profitability over a two-year horizon.

2.3 Tobin’s Q and Leverage Constraints

Models of banks typically impose constraints on either market or book leverage. In this section, we discuss the time series pattern of banks’ market and book leverage ratios and their cross-sectional differences. The left panel in Figure 4 presents the aggregate market and book leverage time series for the entire sample of public BHCs. Book leverage has been on a downward sloping trend from
Notes: This figure shows book and market leverage of the aggregate BHC banking sector, and the distribution of banks constrained by capital requirements. The left panel uses book data (book equity and liabilities) from the FR Y-9C and market equity data from CRSP data. We exclude new entrants in 2009 from the aggregate time series (e.g., Goldman Sachs and Morgan Stanley). Book leverage is computed as assets/book equity, and market leverage is computed as (liabilities + market equity)/market equity. The aggregate leverage ratios are computed as (aggregate liabilities + aggregate book equity)/aggregate book equity. The right panel shows data on the distribution of the Tier-1 capital ratio, defined as (Tier-1 capital)/(Risk-weighted assets). In particular, we plot the share of banks whose regulatory capital ratio falls below a given level, computed using the full, unweighted sample. The regulatory capital requirements are shown on the graph and described in the text.

During the financial crisis, book leverage fell. Market leverage, by contrast, spiked dramatically in 1990, during the financial crisis, and again at the beginning of the Covid-19 crisis. Market leverage also remained almost twice as high for at least four years after the onset of the financial crisis. This suggests that even in the midst of a financial crisis banks did not face a strictly binding ceiling on market leverage—else, we should have observed a concentration of banks with market leverage values close to a ceiling during crises.

Whereas constraints on market leverage are a theoretical possibility that has to be validated empirically, regulatory constraints are written in legislation. The right panel in Figure 4 presents the times series of the fraction of banks whose Tier 1 capital ratio falls below different values that are at or close to the regulatory constraint. The vast majority of banks have kept a capital

---

20Recall that we chose total book equity as opposed to Tier 1 equity for Tobin’s Q and leverage definitions simply because both are (a) highly correlated and (b) Tier 1 equity is only available since 1996.

21Under Basel II (the regulatory standard in place during the crisis), bank holding companies were subject to regulatory minimums on their total capital ratio and their tier-1 capital ratio. These capital ratios are computed as qualifying capital/risk-weighted assets and, thus, a bank with a higher capital ratio has lower leverage. Basel II required that banks hold a minimum tier-1 capital ratio of 4% and a minimum total capital ratio of 8%. In order to be categorized as “well-capitalized,” banks had to meet minimum capital ratios that were two percentage points higher (6% and 10%, respectively). Being categorized as well-capitalized is desirable because banks that are not well-capitalized are subject to additional regulatory scrutiny (Basel Committee on Banking Supervision, 1998, 2006). After the crisis, tighter capital requirements were phased in under Basel III. The minimum total
buffer above the regulatory minimum. Even during the financial crisis, less than 10% of banks were “under-capitalized” according to their Tier 1 capital ratio. Consistent with a delayed loan loss recognition mechanism, the share of banks that were near the regulatory limits peaked in the first quarter of 2010, at least 2 years after the first symptoms of a mortgage crisis.

Figure 5: Quantiles of Market Leverage, Book Leverage, and Tobin’s Q

Notes: This figure shows the quantiles of market leverage (Panel A), book leverage (Panel B), and Tobin’s Q (Panel C) for BHCs on a log scale. Book data (liabilities) comes from the FR Y-9C, and market equity data is from CRSP data. Market leverage is computed as \((\text{liabilities} + \text{market equity})/\text{market equity}\). Book leverage is computed as \((\text{liabilities} + \text{book equity})/\text{book equity}\). Tobin’s Q is computed as \(\text{market equity}/\text{book equity}\). The median value is plotted in maroon. Each tenth percentile is plotted in blue.

Figure 5 presents different sections of the distribution of market leverage (panel A), book leverage (panel B), and Tobin’s Q (panel C) over time. The median value is plotted in maroon and each 10th quantile is depicted in blue. In the early 1990s (after the savings and loan crisis) and during the financial crisis, market leverage increased across the entire bank distribution. Strikingly, the distribution of market leverage fans out, with a substantial 10% of banks sustaining market leverage ratios of nearly 80 during the financial crisis, while another 10% of banks had market leverage ratios of 8, implying a leverage difference factor of 10. Market leverage also took a long time to return to its pre-financial crisis levels. Panel A poses a challenge to many macro-finance models based on market leverage constraints. We should expect systemic loan defaults during a crisis like the one that occurred in 2008 and this should have led to an increase in market leverage for all banks, provided that assets were not immediately liquidated. At the same time, we would expect substantial reallocation of assets from highly levered banks toward banks with lower leverage. This would imply a compression in the dispersion of leverage in the cross-section. Panel A suggests that there was no such compression. Hence, at the very least the reallocation of assets across banks was delayed.

capital ratio stayed at 8% throughout our sample period, but the tier-1 capital ratio rose to 4.5% in 2013, 5.5% in 2014, and finallysettled at 6%starting in 2015. Also under Basel III, additional capital ratios (e.g., tier-1 leverage and common equity capital ratio) began being monitored (however these ratios are quite similar to the preexisting tier-1 and total capital ratios) and, starting in 2016, a “capital conservation buffer” and special requirements for systemically important financial institutions were introduced (Basel Committee on Banking Supervision, 2011). Kisin and Manela (2016) study whether banks violate different regulatory constraints and find that typically banks do not violate multiple regulatory constraints.
Panel B shows that the distribution of book leverage is much less dispersed relative to the market leverage distribution. In fact, it is also fairly stable: the top and bottom 10th percentile of the book leverage distribution differ by a factor of 2. The distributional differences between market and book leverage imply large cross-sectional and time-varying dispersion in Tobin’s Q (see panel C). We summarize these observations into our third motivating fact:

**Fact 3.** Most banks keep an equity buffer above the regulatory capital ratio minimum; fewer banks did so during the financial crisis of 2008/2009. The cross-section of Tobin’s Q shows large dispersion across banks. The dispersion in market leverage is by far greater than the dispersion in book leverage.

From the perspective of banking models, we contend that neither regulatory nor market constraints bind in a static way. Of course, we do not conclude that neither constraint affects banks. On the contrary, banks may worry that a constraint will bind in the future and will take steps to avoid that state, as we model in our Q-theory.

### 2.4 Tobin’s Q and the Dynamic Responses to Shocks

This section analyzes how Tobin’s Q and other variables of interest respond to a bank net-worth shock. To this end, we estimate the following panel regressions:

\[
\Delta \log(y_{i,t}) = \alpha_t + \sum_{h=0}^{k} \beta_h \cdot \log(1 + \varepsilon_{i,t-h}) + \psi_{i,t},
\]

where \(i\) indexes over banks, \(t\) indexes over quarters, \(y_{i,t}\) is the outcome of interest, \(\alpha_t\) is a time fixed effect, and \(\varepsilon_{i,t}\) denotes our measure of a cash flow shock to net-worth (i.e., the idiosyncratic excess stock return innovations over the past quarter for bank \(i\) in quarter \(t\); see detailed description in the next paragraph).\(^{22}\) These regressions allow us to construct impulse-response functions for Tobin’s Q and to better understand the response in several other bank variables such as liabilities, market leverage, market equity, and book equity.\(^{23}\) We include time fixed effects \(\alpha_t\) to absorb

---

\(^{22}\)One might favor an alternative specification that includes lags of the dependent variable in addition to contemporaneous and lagged returns. This poses two issues: Nickell bias and bad control. Including the dependent variable as a lag will induce bias, as documented by Nickell (1981). Dealing with this bias is challenging and may result in poor precision. Perhaps more importantly, the lagged dependent variable is a "bad control," in that it is endogenous to the regressor. We wish to back out the effect of a return shock in \(t - 3\) on the change in liabilities in \(t\): if we condition on liabilities in \(t - 1\), which is itself also affected by the past return shock, then we will not identify our parameter of interest.

\(^{23}\)Since market returns are changes in market equity valuations, taking first differences in logs provides a tight conceptual link between the outcome and the regressor. Using levels would mean that the outcome is highly correlated with bank size. This would raise concerns about stationarity. Using levels could also result in a regression that is heavily influenced by a few large banks, given the highly skewed bank size distribution. For the same reason, we do not weight our regressions: the bank size distribution is highly skewed and so a weighted regression would be equivalent to a regression with only a handful of the largest banks. If the variance of the residuals were lower for larger banks, then using weights would yield a more efficient estimator. Empirically however, the variance of the residuals does not appear to vary substantially by bank size.
aggregate shocks; e.g., the price of loans due to aggregate demand shocks. We thus recover a partial equilibrium supply-side impulse response, estimated from the cross-sectional variation in return shocks. In all specifications, we use $k = 21$. We cluster standard errors by bank. Finally, to report the impulse-response function, we sum the coefficients: the contemporaneous response is $\beta_0$, the next period is $\beta_0 + \beta_1$, and so on.

A critical component of our empirical design is the how we obtain $\varepsilon_{i,t}$, the cash flow shock to banks’ net-worth. We follow a similar procedure as Vuolteenaho (2002), who shows how to decompose an individual bank’s stock return into a cash flow shock and a discount rate shock component. To control for the discount rate shock component in banks’ stock return we use the same aggregate risk factors as in Gandhi and Lustig (2015). That is, we regress the excess stock returns $r_{i,t} - r_{f,t}$ of bank $i$ on a bank fixed effect $\alpha_i$ and a matrix of factors $X_t$ as follows:

$$
\begin{align*}
    r_{i,t} - r_{f,t} &= \alpha_i + X_t \beta_i + \varepsilon_{i,t} \\
    \text{Raw Return} &- \text{Risk-Free Rate} = \text{factors loadings} + \text{Idiosyncratic Component}
\end{align*}
$$

The vector of factor loadings, $\beta$, has dimension $K \times 1$ and the matrix of factors $X_t$ has dimension $T \times K$. We include the same factors as in Gandhi and Lustig (2015), namely the three Fama-French factors (Fama and French, 1993), a credit factor calculated as the excess return on an index of investment-grade corporate bonds, and an interest rate factor calculated as the excess return on an index of 10-year US Treasury bonds.24 (See Appendix Section B.1 for further details on the risk-adjustment process.) The idea behind risk-adjusting returns and using return innovations is that we want to isolate information about banks’ cash flows as opposed to variation in discount rates, since the latter are driven by aggregate movements in the factors.25 Our empirical design thus relies on the efficient-markets hypothesis according to which excess return variations should be unpredictable ex ante after adjusting for risk premia. By stripping out the predictable components of returns, the innovations $\varepsilon_{i,t}$ to the risk-adjusted returns are ex-ante unpredictable across banks. This forms the basis of our identification strategy: we treat the cross-sectional variation in $\varepsilon_{i,t}$ as unanticipated shocks that perturb bank equity. In the Appendix (see Figure 20), we show the time series of $\varepsilon_{i,t}$ for the largest four banks that indeed resemble white noise. In Section 2.4, we conduct a variety of robustness checks to validate our identification strategy and interpretation of $\varepsilon_{i,t}$ as cash flow shocks. For the rest of the paper, we refer to these innovations $\varepsilon_{i,t}$ as return shocks.

**Impulse responses.** We estimate impulse-response functions for Tobin’s Q, liabilities, market capitalization, book equity, market leverage, and the common dividend rates, and show the results

---

24 The three Fama-French factors are downloaded from Ken French’s website. The credit factor is the excess return on the Dow Jones Corporate Bond Return Index that we download from Global Financial Data. The interest rate factor is the U.S. 10-year Government Bond Total Return Index (ltg) that we also download from Global Financial Data. We use the one-month risk-free rate from Ken French’s website to calculate excess returns.

25 The results for simply adjusting returns with a time-fixed effect are qualitatively and quantitatively similar, and are also reported in the Appendix Section B.2.
in Figure 6. To normalize the effect, we report the response to a negative one percent return shock. The y-axis of our plots shows the contemporaneous response ($-\beta_0$) as quarter 1, the cumulative response one quarter later ($-\beta_0 - \beta_1$) as quarter 2, and so on.

The general take-away of the impulse response functions in Figure 6 is that banks adjust very slowly. Let us begin with the dynamic response in Tobin’s Q to a return shock presented in Panel A. A 1% negative return shock lowers Tobin’s Q by about 0.9% on impact (the inverse of the response in market leverage in Panel B), and slowly increases over four years to a new permanent lower level. To see why, note that the shock affects the components of Tobin’s Q, i.e., market equity and book equity, differently (see the impulse response functions in Panel D and F, respectively). Market equity falls immediately by roughly 1% in response to a 1% negative net-worth shock and increases subsequently only slightly to -0.8% after four years where it remains stable. Book equity only shows a very muted response on impact and slowly declines roughly to -0.5% after 10 quarters and remains at this level subsequently. That is, our estimation shows that the effect of the shock does not fully show up in book equity within the five years of our estimation window suggesting a very slow transmission of shocks onto the book balance sheet of banks and into book equity.\(^{26}\) Since book equity and market equity do not converge within the 5 years of our estimation window, Tobin’s Q does not recover to its pre-shock level. In other words, small cash flow shocks can drive a long lasting wedge between the market and book valuation of banks. Our quantitative results will test how much of this divergence between market and book equity can be explained by our delayed loan loss accounting mechanism.

Bank liabilities, Panel C, also adjust very slowly. In response to a return shock, banks slowly delever by paying off liabilities. On impact of the negative return shock, banks in fact increase dividend payout to shareholders though by a very small amount (see Panel E). Subsequently though, they reduce dividends to shareholders for a couple of years to rebuild their capital base (hence the small increase in banks’ market capitalization). If banks maintain a target market-leverage ratio, then we would expect banks to respond to a negative wealth shock (which mechanically increases market leverage) by moving back towards their target leverage. As we can see, from the impulse-response function of market leverage in Panel B of Figure 6, the data is consistent with a slow adjustment back to a leverage target. The impulse response of log market leverage, defined here as $\log(\text{liabilities}/\text{market capitalization})$, is the difference between the response of the log market capitalization (Panel D) and the log liabilities (Panel C).\(^{27}\) These observations lead us to our final motivating fact for our Q-theory:

**Fact 4.** Banks adjust very slowly to return shocks. Small shocks can drive a long-lasting wedge between the market and book equity valuations of banks, and thus Tobin’s Q. Market leverage takes

---

\(^{26}\)We estimated these impulse response functions also for up to lags of 50 quarters but the confidence intervals are then extremely large. That is, we just do not have enough power to test for convergence over the long horizon.

\(^{27}\)Estimating the impulse response function for 50 quarters, we can see that market leverage returns back to its initial state after about 50 quarters. The confidence intervals on those estimates are however extremely large.
a long time to return to its pre-shock level mainly driven by a slow delevering process.

**Identification and Robustness.** Our interpretation of the estimates relies on the assumption that bank-specific variations in risk-adjusted bank stock returns identify cash flow shocks on the existing portfolio, such as specific default shocks, as opposed to shocks to the profitability of future business opportunities. We conduct various analyses to alleviate identification concerns, including a narrative approach to validate our interpretation of the return shocks as being unanticipated and specific cash flow shocks.

One could be concerned that the return shocks capture idiosyncratic information about the relative profitability of a bank’s future portfolio (e.g., the default rate on this bank’s future mortgages) and, thus, affect the bank’s problem through channels other than perturbing equity. If a bank’s expected return on its future assets falls, then this bank would want to reduce its equity or lower its scale for a reason that would be unrelated to a target leverage ratio and adjustment costs.

**Figure 6: Estimated Impulse Responses**

![Graphs](image)

**Notes:** These figures show estimated impulse response functions for BHCs. The figures show the estimated percent impulse response to a 1% negative return shock. For example, in Panel b) we show that market capitalization decreases by roughly 1% in response to a 1% negative return shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. The panels display the impulse responses of log market leverage (Panel a), log market capitalization (Panel b), log liabilities (Panel c), log book equity (Panel d), and the common dividend rate (Panel e). Market leverage is defined as \( \frac{\text{Liabilities}}{\text{Market Capitalization}} \). The logged common dividend rate is defined as \( \log(1 + \text{Common Dividends/Market Capitalization}) \).

To investigate this concern, we study how banks’ liquid asset ratios respond to a negative return
shock. If negative return shocks indeed predict lower future investment opportunities rather than current cash flows, we would expect banks to respond to these shocks by moving their portfolio into liquid assets. We test this notion by looking at the impulse-response function of banks’ liquidity ratios, calculated as (cash + treasury bills) / total assets. The impulse-response function in the Appendix, Figure 25, shows no statistically significant response.\footnote{This is not a perfect test as perhaps banks would also want to raise liquidity in response to a cash flow shock on their current portfolio. A hypothetical example is the following. Suppose the bank was caught in unfair lending practices that causes a lawsuit. Banks might respond by increasing their cash holdings to prepare for the upcoming lawsuit.} In sum, banks do not tilt their portfolios towards safe and liquid assets in response to our return shock, which pushes against a story of worsening investment opportunities.

The lack of response of the liquid asset ratio is suggestive for our interpretation of the return shocks. However, we cannot fully rule out that the shock picks up information about the profitability of future assets. To provide additional corroborating evidence for our identification strategy, we use a narrative approach, which is detailed in Appendix B.3. To this end, we take the largest positive and negative values of the return shocks \( \varepsilon_{i,t} \) over the sample period for each of the four largest banks (J.P. Morgan Chase, Bank of America, Citigroup, and Wells Fargo). We then search various newspapers for articles that mention the name of any of the four banks in the quarter for which the absolute value of \( \varepsilon_{i,t} \) was high. Table 4 in the Appendix lists the results of our newspaper article search. In most cases, we can find supporting evidence for our \( \varepsilon_{i,t} \) estimates. For example, in the second quarter of 2009, Bank of America had a high and positive value of \( \varepsilon_{i,t} \). Our article search revealed that Bank of America fared better in the stress test and exceeded expectations. In 1999 Q1, Citigroup had a large positive \( \varepsilon_{i,t} \). This coincided with a Wall Street Journal article that stated that Citigroup had exceeded profit expectations even though profits fell. In 2001 Q1, a negative shock at Wells Fargo coincided with news reports that stated that Wells Fargo’s venture capital portfolios had incurred significant losses.

In Appendix Section B.3, we provide additional robustness checks. We verify that our results are not driven by mergers, or by specific events during the crisis, by excluding mergers and the crisis years 2008 and 2009 from our sample. For more details refer to Appendix Section B.3.\footnote{We also test for heterogeneous impulse responses by type of bank. We do not find evidence of sizable heterogeneity, but we have limited statistical power to detect these differences.}

### 2.5 Taking stock

Thus far, we established that book equity values and market equity values diverge during crises (fact 1). Moreover, from the cross-section, we show that variation in Tobin’s Q reflects current information about future book profitability and loan losses (fact 2). We also document that the cross-sectional dispersion of market leverage increases during crises, whereas the dispersion of book leverage—and book equity—remains relatively constant (fact 3). These facts challenge standard models: they indicate that most banks choose leverage levels away from a common constraint and
that constraints operate dynamically. In fact, we show that banks adjust very slowly to adverse net-worth shocks (fact 4).

Standard bank models also generate slow leverage dynamics and a wedge between book and market values through adjustment costs (Hayashi, 1982). However, fact 2 suggests that book values do not accurately reflect shocks to net-worth. We can generate predictability (fact 2) by adding delayed loan-loss accounting into a standard model as a measurement error. However, measurement error is innocuous to bank decisions. In reality, bank regulation is formulated in terms of book values, so delayed accounting does affect bank decisions. In fact, there is evidence that bank decisions are shaped by incentives to under report loan losses to avoid hitting regulatory constraints (e.g., Begley et al., 2017; Behn et al., 2016; Blattner et al., Forthcoming; Flanagan and Purnanandam, 2019; Plosser and Santos, 2018). In the next section, we develop a model where delayed loan-loss accounting and regulation, on their own, drive the bank leverage dynamics in a qualitatively and quantitative way consistent with the four facts. This model does not rely on adjustment costs. Section 4.3 compares our model with one that relies on adjustment costs.

3 Q-Theory

This sections presents our Q-theory of banks that is inspired by the facts presented in Section 2. The novel component of our theory is that economic decisions are affected by accounting rules. Proofs and derivations are contained in Appendix C.2.

3.1 The Model

Environment. Time is continuous, runs to infinity, and is indexed by $t$. There is a continuum of banks owned by diversified investors. Idiosyncratic loan defaults are the only source of risk. Each bank maximizes the expected discounted value of dividends.

We define the bank objective function recursively as

$$V_t = \mathbb{E}_t \left[ \int_t^\infty f(C_s, V_s) ds \right] \text{ where } f(C, V) \equiv \frac{\rho}{1-\theta} \left[ \frac{C^{1-\theta} - \{(1-\psi)V\}^{\frac{1-\theta}{1-\psi}}}{\{(1-\psi)V\}^{\frac{1-\theta}{1-\psi}-1}} \right].$$

$C_t$ denotes dividend payouts at time $t$ and $f$ is a Duffie-Epstein aggregator with a time discount rate $\rho > 0$, an intertemporal elasticity of substitution (IES) $1/\theta$, and a risk aversion parameter $\psi$. The Duffie-Epstein aggregator is the continuous-time analogue of Epstein-Zin preferences. We assume recursive preferences to generate smooth dividend payouts that are consistent with the

---

30We choose a continuous-time setup for analytic and computational reasons. An earlier version of this paper presented the same model in discrete time. Whereas the cross-sectional properties of both models are quantitatively close, the speed of computation is substantially faster in the continuous-time setup, something that facilitates the estimation.
empirical evidence; e.g., Lintner (1956); Dickens et al. (2002); Leary and Michaely (2011), while preserving risk-neutrality (by taking $\psi = 0$) as is standard in the theory of the firm.\textsuperscript{31}

**Equity.** We distinguish between three concepts of bank equity: the fundamental value of equity, $W$, the book (or accounting) value of equity, $\bar{W}$, and the market value of equity, $S$. Only the book value and the market value have data counterparts. The fundamental value of equity determines the future cash-flows. The book value differs from fundamental values because some equity losses generated by loan defaults are not written down immediately in the accounting books. Hence, book values do not entirely capture the cash-flow that the bank will generate over time.

Without regulation, the book value of equity would be irrelevant for any decision, and bank choices would only depend on fundamental values. However, since regulatory constraints are specified in terms of book values, these influence fundamental leverage. In turn, market values differ because the bank has a franchise value and cannot raise equity freely.

Tobin’s $Q$ is defined as,

$$Q \equiv \frac{S}{\bar{W}} = \frac{S}{W} \cdot q, \quad (2)$$

where $q \equiv W/\bar{W}$. Thus, relative to standard $Q$-theory models, here Tobin’s $Q$ is decomposed into two ratios, the market-to-fundamental value, $S/W$, and the fundamental to book value, little $q$. Without delayed loss recognition, $q = 1$.

Banks hold long-term loans that are funded with deposits and equity. Thus, at each instant, the bank chooses its leverage, $\lambda \geq 1$. Hence, loans are given by $L = \lambda \cdot W$. Deposits are denoted by $D$, so the bank’s balance sheet is given by $L = D + W$. Loans mature at an intensity $\delta$. Maturity is important only for taking the model to the data.

We work in partial equilibrium: the supply of deposits and the demand for loans are perfectly elastic, with exogenous rates $r^D$ and $r^L > r^D$, respectively. Loans are risky because they can be defaulted. In the event of a loan default, a fixed fraction $\varepsilon$ of loans is not repaid. We assume that default events are governed by a Poisson process $dN$ with intensity $\sigma$.

Below, we show that the bank’s problem is scale invariant. It is convenient to express the dividend rate as $c \equiv C/W$. Next, we present the laws of motion of key objects. The growth rate of bank equity satisfies the following stochastic differential equation:

$$dW = \left[ \underbrace{r^L \lambda - r^D (\lambda - 1)}_{\text{leveraged return}} - \underbrace{c}_{\text{dividend rate}} \right] \underbrace{W \, dt}_{\equiv \mu^W_W} - \underbrace{\varepsilon W}_{\equiv J^W_W} \cdot dN, \quad (3)$$

where $\mu^W$ denotes the drift and $J^W$ the jump in equity produced by a default, per dollar of equity.

\textsuperscript{31}Similar objective functions for banks are found in Bianchi and Bigio (Forthcoming) and Di Tella and Kurlat (Forthcoming).
The first term of Equation (3) represents leveraged returns: the interest income on loans, \( r^L \lambda \), net of the interest on deposits, \( r^D (\lambda - 1) \), per unit of equity. The second term is the dividend rate. The final term is the jump in equity after a default, which scales with leverage \( \lambda \).

There are two types of default events: defaults immediately recognized, and defaults slowly recognized. Upon a default, the loss is recognized immediately with probability \( 1 - p \) and slowly recognized with probability \( p \). We keep track of the stock of unrecognized loan losses. With abuse of terminology, we refer to this stock as zombie loans, because in the accounting books unrecognized losses appear as loans that will not yield returns. Zombie loans satisfy the following law of motion:

\[
dZ = -\alpha Z \cdot dt + \xi \lambda W \cdot dN^z,
\]

where \( dN^z \) is a Poisson process such that \( \Pr (dN = 1 \cap dN^z = 1) = \sigma p \) and \( \Pr (dN = 1 \cap dN^z = 0) = \sigma (1 - p) \). When a default event occurs and is not recognized immediately, \( dN^z = 1 \), then loans are added to the stock of zombie loans. If no additional zombie loans are added, zombie loans decrease at rate \( \alpha \) because past losses are recognized at that rate. We need both types of default events: Unrecognized defaults provide deviations between book and market values. In turn, recognized defaults are important to produce a book equity buffer.

**Fundamental and book variables.** We let \( \bar{X} \) denote the book value of fundamental variable \( X \). Book loans are the sum of fundamental loans, plus the amount of loans that no longer have value, \( Z, \bar{L} = L + Z \). Likewise, book equity also includes zombie loans \( \bar{W} = W + Z = \bar{L} - D \).

Figure 7 presents the balance sheet corresponding to the fundamental values (panel a) and the book values (panel b). As shown in panel (b), zombie loans add to the stock of book loans and book equity. Thereby, their presence means that measured book leverage looks less high compared to the bank’s fundamental leverage.

![Figure 7: Fundamental and Accounting Balance Sheet](image)

Since the laws of motion feature drifts and jumps produced by defaults, we introduce the following notation: We use \( \mu^x W \) to denote the drift of a variable \( x \) scaled by fundamental equity.
Similarly, we use $J^x W$ and $\bar{J}^x W$ to refer to a jump, scaled by $W$, in variable $x$ produced by the events $dN$ and $dN^z$, respectively.

**Leverage Ratio and Zombie Loan Ratio.** We define fundamental leverage as $\lambda \equiv L/W$. We also define the ratio of zombie loans to fundamental equity, the zombie loan ratio, as $z \equiv Z/W$. In turn, book leverage is given by $\bar{\lambda} \equiv \bar{L}/\bar{W} = (\lambda + z) / (1 + z)$. Thus, zombie loans hide a portion of fundamental leverage:

$$\lambda - \bar{\lambda} = \frac{z}{1 + z} (\lambda - 1) > 0.$$

Fundamental leverage follows the following law of motion:

$$d\lambda = (\iota - \mu^W) \lambda dt + \lambda \varepsilon \left[ \frac{\lambda - 1}{1 - \lambda \varepsilon} \right] \cdot dN - dF,$$

where $\iota$ is the continuous issuance rate of loans, a choice variable for banks, in excess of the depreciation rate $\delta$. Leverage jumps whenever there is a default event, $dN$. Finally, the term $dF$ corresponds to a countable process for a discrete jump in loan sales—which is controlled by the bank, but not measurable in $dN$.

The zombie loan ratio follows:

$$dz = -z \left( \alpha + \mu^W \right) dt + \lambda \varepsilon \left[ \frac{1 + z}{1 - \lambda \varepsilon} \right] dN^z + \lambda \varepsilon \left[ \frac{z}{1 - \lambda \varepsilon} \right] (dN - dN^z).$$

Note that the jump on $z$ depends on whether the default is unrecognized, $dN^z = 1$, or recognized, $dN - dN^z = 1$.

**Liquidation.** Banks remain operational if they satisfy two constraints. First, as occurs in practice, banks must satisfy a regulatory requirement. The regulation stipulates that book leverage cannot exceed a regulatory limit $\Xi > 1$. Thus, banks must satisfy the following regulatory limit $\bar{L} \leq \Xi \cdot \bar{W} \iff \bar{\lambda} \leq \Xi$.\(^{32}\) We express this regulatory requirement in terms of leverage and the zombie loan ratio:

$$\lambda \leq \Xi + (\Xi - 1) z.$$

\(^{32}\)Notice that we can equivalently, write the constraint as a limit to risk-weighted assets over Tier-1 capital. Let $\varrho$ be a risk-weight on loans and $\kappa$ a capital ratio restriction. Then, a Tier-1 capital constraint can be written as:

$$\varrho \bar{L} \leq \kappa (\varrho \bar{L} - D).$$

Equivalently, this can be re-interpreted as a constraint that stipulates that deposits cannot exceed a fraction $\xi$ of their book loans $D \leq \xi \bar{L}$. Re-arranging terms, we have that the three constraints are related through $(\kappa - 1) \varrho / \kappa = \xi = 1 - 1/\Xi < 1$. In the calibration section, we calibrate $\Xi$ directly.
Observe that regulation imposes a constraint on fundamental leverage, which depends on the zombie loan ratio. A lower $z$—a smaller gap between book and fundamental values—generates a tighter regulatory constraint. This constraint is meant to capture Tier 1 capital ratio requirements.

Second, banks are subject to a solvency condition imposed by markets, which states that the bank must remain with positive fundamental equity after a default shock: $W - \varepsilon L \geq 0$. We can express this constraint as an upper bound on fundamental leverage

$$\lambda \leq 1/\varepsilon. \quad (5)$$

The solvency constraint is introduced as a technical condition to prevent banks from accumulating losses as they increase their zombie loan ratio (to prevent a Ponzi scheme). The constraint is almost never binding in the simulations. On the contrary, the regulatory requirement is the main focus of interest.

We can combine both constraints into a single condition for leverage:

$$\lambda \leq \Gamma (z) \equiv \min \{1/\varepsilon, \Xi + (\Xi - 1) z\}, \quad (6)$$

where $\Gamma (z)$ indicates the liquidation boundary.

If either the regulatory or the solvency conditions are violated, the bank is liquidated. For simplicity, we assume that if the bank is liquidated, the bank is closed and its value is given by $v_0W$ where $v_0$ is an exogenous value. Because the bank earns a spread between the deposit and loans rate, it has a franchise value so liquidation is never desirable.

**Timing.** Liquidation is possible because banks cannot immediately offset the jump in leverage at the exact instant of the occurrence of a default. Thus, even though banks may, and will in equilibrium, offset the increase in leverage selling loans immediately after the shock, the assumption is that if they violate a constraint for a measure zero time interval, they are liquidated. In particular, given a choice of $\lambda$, the jump in leverage on the instant of a default event is given by $J^\lambda$. Thus, if $\lambda + J^\lambda > \Gamma (z + J^z)$ the bank is liquidated.

**Bank problem.** We now solve the bank’s problem. At each $t$, the bank has state variables $\{Z, W\}$ and chooses a dividend payout $C$ and leverage $\lambda$. Banks solve the following problem:

**Problem 1 [Bank’s Problem] The bank’s policy functions $\{C(Z, W), \lambda(Z, W)\}$ are the solutions to:**

$$0 = \max_{\{C, \lambda \in [1, \Gamma(Z/W)]\}} f(C, V(Z, W)) + V_Z(Z, W) \mu^Z W + V_W(Z, W) \mu^W W$$

$$+ \sigma p \left[ V(Z + J^Z, W + J^W) - V(Z, W) \right] + \sigma (1 - p) \left[ V(Z, W + J^W) - V(Z, W) \right]$$

unrecognized default

recognized default

22
subject to the laws of motion of fundamental equity, (3), and zombie loans, (4), and the liquidation constraint: $V(Z + J^Z, W + J^W) = v_0 W$ if $\lambda + J^Z > \Gamma (z + J^z)$, and $V(Z, W + J^W) = v_0 W$ if $\lambda + J^W > \Gamma (z + J^z)$.

This problem features a standard Hamilton-Jacobi-Bellman (HJB) equation, associated with Duffie-Epstein preferences. The last two terms represent the jumps in the value function after unrecognized and recognized default events, respectively.

**Market value of equity.** We need to price bank equity to produce a value for Tobin’s $Q$ and to obtain the market returns needed to reconstruct the cross-sectional impulse responses to return shocks. To price bank equity, we assume that banks are owned by outside investors who own the claims to bank dividends, but cannot directly make loans and cannot inject equity. Because of this friction, and the curvature on dividends, as we anticipated above, the market value of equity diverges from the fundamental value. Moreover, this frictions drive a wedge between an outside investor’s and an the bank’s value function. We endow the investor with a constant discount rate $\rho^I$ that differs from $\rho$, the discount rate of the bank, to have a parameter that controls Tobin’s $Q$. We further assume that investors are diversified so that a bank’s idiosyncratic risk does not affect their discount rate. Because we are interested only in the cross-sectional behavior of banks, our asset pricing abstracts from changes in investor risk premia. In principle, we could allow discount rates to vary with time which would not change the cross-sectional implications. For simplicity we set investors’ discount rate to a constant.

Next, we construct a pricing equation for the bank’s equity. We map the underlying default shocks to the return shocks, to be able to build the analogue impulse responses to those presented in Section 2. Investors price bank shares according to the net present value of discounted dividends. The market value of a bank, $S(Z, W)$, satisfies the following recursive representation:

$$
\rho^I S(Z, W) = C(Z, W) + S_Z (Z, W) \mu^Z W + S_W (Z, W) \mu^W W
+ \sigma p \left[ S(Z + J^Z, W + J^W) - S(Z, W) \right]
+ \sigma (1 - p) \left[ S(Z, W + J^W) - S(Z, W) \right],
$$

where $S(Z + J^Z, W + J^W) = 0$ if $\lambda + J^Z > \Gamma (z + J^z)$, and $S(Z, W + J^W) = 0$ if $\lambda + J^W > \Gamma (z + J^z)$.

This equation captures the dividend flow $C(Z, W)$ determined by the payout policy. Naturally, this recursive representation reflects the law of motion of the bank’s state variables. The valuation takes into account how changes in the state variables will affect future valuations, considering the effect of loan defaults. Upon liquidation, the equity of investors in the bank has a value of zero.

Implicitly, Equation (7) assumes that investors observe $\{dN, dN^z\}$. Hence, we assume that market prices contain information about loan losses not contained in their books. In general,
markets may not observe all information, but for the purposes of relating the model to the empirical section, this assumption captures that Tobin’s $Q$ has some predictive power.

**Discussion of Model Frictions.** As any model where financial frictions affect banks’ lending choice, we need frictions on equity and deposit financing that represent violations of Modigliani-Miller. In line with the intermediary asset pricing literature, we assume that banks cannot issue equity and, thus, must rely on retained earnings to grow equity. In addition, the objective of banks features dividend smoothing that also slows down the accumulation of equity. In turn, the leverage constraint places a limit on deposit financing.

Delayed accounting is a novel feature of our model and is critical to capturing banks’ ability to engage in evergreening (Caballero, Hoshi and Kashyap, 2008) and to avoid the immediate recognition of losses (e.g., Blattner, Farinha and Rebelo, Forthcoming; Flanagan and Purnanandam, 2019)). By not charging off losses, banks create zombie loans and avoid reductions in book equity, i.e., regulatory capital. Since rolling over a loan does not require new funds, evergreening allows a bank to inflate its accounting equity at no cost. The effect of delayed accounting is that the leverage constraint in is given by the zombie loan ratio.

We next proceed to explain the model’s mechanics, first working out the case with immediate accounting. Formal results are relegated to Appendix C.2.

**Immediate accounting.** To build intuition, we first solve for the polar case where all defaults are instantaneously recognized, so $p = 0$. In this case, $z = 0$ so $\alpha$ plays no role. Also, to simplify the algebra only for th, we assume that $v_0 = 0$. Thus, only the regulatory constraint binds and $\Gamma (z) = \min \{1/\varepsilon, \Xi\} = \Xi$. To characterize the solution, we impose the following assumption.

**Assumption 1** We assume the following:

a. Returns: $r^L - \sigma \varepsilon \geq r^D$.

b. No liquidation: $(r^L - r^D) \Xi - \sigma < \Xi \cdot \frac{r^L - \sigma \varepsilon - r^D}{1 - \varepsilon + \varepsilon \Xi}$.

Assumption 1.a is needed to have positive leverage. Without this assumption, the bank would lever up as much as possible and liquidate after the first loan default. Assumption 1.b guarantees banks avoid liquidation in all states. In general, we don’t need assumption 1.b, but we introduce it to avoid equilibrium default which is not central to our theory. Together, assumptions 1.a and 1.b produce a positive leverage and a capital buffer—banks set leverage below the constraint in 6.

Next, we solve the bank’s problem. A key object for our theory is what we call the shadow liquidation boundary, $\Lambda$. In the case of immediate accounting, the shadow boundary is the highest leverage ratio, $\Lambda$, such that leverage remains under the liquidation limit after a default event.

---

33The assumption of no equity finance can be relaxed easily by defining utility over net dividends.
Mathematically, the shadow boundary solves $\Lambda + J^\lambda(\Lambda) \equiv \Xi$. Solving explicitly for this value yields:

$$\Lambda = \Xi \cdot (1 + \varepsilon (\Xi - 1))^{-1}. $$

The next proposition shows that with immediate accounting, the bank maintains a constant leverage and dividend ratio, and banks only differ in their scale, $W$.

**Proposition 1** [Bank’s Problem] With immediate accounting, the dividend rate, the leverage ratio, and the bank’s value per unit of equity are constants $\{c^*, \lambda^*, v^*\}$. These constants solve the following HJB equation:

$$0 = \max_{\{c\}} f(c, v^*) - v^* \cdot c + v^* \cdot \Omega $$

(8)

where $\Omega$ is the expected leveraged bank return,

$$\Omega = r^D + \max_{\lambda \in [1, \Xi]} \left( r^L - r^D \right) \lambda + \sigma \left\{ (1 - \varepsilon \lambda) \mathbb{I}_{\{\lambda < \Lambda\}} - 1 \right\}. $$

Let Assumption 1 hold. Then, the solutions $\lambda^*$ and $c^*$ are given by:

$$\lambda^* = \Lambda \text{ and } c^*(v) = \rho^{1/\theta} v^{\gamma - 1/\theta}. $$

(9)

Then, $V(Z, W) = v^* \cdot W$, $C(Z, W) = c^* \cdot W$, $L = \bar{L} = \lambda^* W, D = (\lambda^* - 1)$.

When $dN = 0$, given $c^*$, the growth rate of loans $\iota^*$ is given by $\iota^* = \mu^W$. When $dN = 1$, the jump in leverage is immediately offset by a loan sale, such that $\lambda^* = \Lambda$ except when $dN = 1$; that is, $dF = J^\lambda$. Finally, the market value of equity, is $S(W) = s^* W$ where $s^* = c^*/(\rho^I - \gamma)$, where $\gamma$ is the expected fundamental equity growth rate, $\gamma = r^D + (r^L - r^D) (\lambda^* - 1)$.

A takeaway from the proposition is the scale invariance of the problem. The bank’s marginal value, $v$, transforms one unit of bank net-worth into the certainty-equivalent net present value of dividends. Furthermore, the proposition shows that the dividend and leverage choices satisfy a separation property: the leverage choice is independent of the dividend choice. Their solutions yield constant values per unit of equity.

The optimal leverage choice solves $\Omega$. Banks’ leverage choice results from a trade-off between intermediation profits and liquidation risk. Figure 8 graphs the objective function of $\Omega$ as a function of $\lambda$. There are three regions: if $\lambda > \Xi$ the bank is liquidated, triggering an immediate loss. Hence, the bank will only choose leverage $\lambda \leq \Xi$. The other two regions are given by the points above and below the shadow boundary. Below the shadow boundary, $\lambda < \Lambda$, the bank survives any default. There, leverage increases the levered returns, $(r^L - r^D) \lambda$, but also increases expected losses, $-\sigma \varepsilon$. However, the value function increases with leverage because the expected
levered return is positive, \( r^L - \sigma \varepsilon - r^D \geq 0 \), by Assumption 1. If leverage crosses the shadow boundary, \( \lambda > \Lambda \), the objective falls in level because a single default event triggers liquidation. For leverage above \( \Lambda \), leverage increases expected returns by \( r^L - r^D \). Thus, the objective in \( \Omega \) has two local maxima, one at the shadow boundary \( \Lambda \) and one at the liquidation boundary, \( \Xi \). By Assumption 1b, it is optimal for the bank to choose \( \Lambda \). The rest of the proposition explains that to guarantee that \( \lambda^* = \Lambda \) except when \( dN = 1 \), an asset sale must neutralize the jump in leverage triggered by a loan default immediately after a shock. Along the continuous portion of the equity path, the loan issuance rate must equal the equity growth rate to keep the leverage ratio constant.

Once we determine the optimal leverage \( \lambda^* \), the dividend choice is given by a standard intertemporal tradeoff. As we can see from Eq. 9, \( c^* \) is given by a formula that captures wealth and substitution effects. Here, \( v \) acts like a total return on equity. When \( \theta < 1 \) (\( \theta > 1 \)), the substitution (wealth) effect dominates and the bank retains (pays out) more dividends as \( v \) increases. With \( \theta = 1 \), we obtain the usual result that \( c^* = \rho \) because both effects cancel out.

Then, the marginal value \( v^* \) solves the equation:

\[
0 = f (c^*(v), v) - v \cdot c^*(v) + v \cdot \Omega.
\]

Finally, observe that because the problem is scale invariant, the market capitalization is also proportional to \( W \), the proportion \( s^* \). It is useful to return to our motivating facts: Immediate accounting produces a book leverage buffer, as desired. However, the impulse response of leverage to a return shock, under immediate accounting, would produce a single impulse that reverts immediately. In turn, the response of total liabilities looks like a one-time shock that tracks the path of \( W \). Furthermore, Tobin’s Q is a constant in this case, has no predictive power and features no cross-sectional variation. Thus, immediate accounting is inconsistent with any of the facts we highlight. Delayed accounting allows us to reproduce all four facts, as we show next.

**Delayed accounting.** We now study the case with delayed accounting where \( p > 0 \). The general model follows a similar logic to the case with immediate accounting. In this case, the shadow boundary depends on \( z \). Again, the shadow boundary is the value of leverage \( \Lambda \), for a given \( z \), such that the bank remains solvent after a recognized default event:

\[
\Lambda (z) + J^\lambda [\Lambda (z)] = \min \left\{ \frac{1}{\varepsilon}, \frac{1}{\Xi} (\Xi - 1) \left( z + \hat{J}^z \right) \right\}.
\]

We define the shadow boundary in terms of a recognized loan default because for an unrecognized loan default, there is also a jump in \( z \) such that book leverage is unaffected and the bank’s buffer with respect to the regulatory constraint is unaffected. Moreover, with an unrecognized loan
default, for the region where the solvency constraint is relevant, the shadow boundary is the same as the one that the recognized loan default derivation yields. Since $J^\lambda$ is a function of $\lambda$, we can solve for the shadow boundary:

$$\Lambda(z) = \min \left\{ \frac{1}{2 - \varepsilon} \frac{\Xi + (\Xi - 1)z}{1 - \varepsilon + \varepsilon\Xi} \right\}. $$

We make use of this expression to characterize the banks’ problem and market value:

**Proposition 2** [Bank’s Problem] With delayed accounting, the dividend rate, leverage, and the bank’s value are functions of $z$, $\{c^*(z), \lambda^*(z), v^*(z)\}$, that solve the following HJB equation:

$$0 = \max_{\{c\}} f(c, v^*(z)) - (v^*(z) - v_\ast(z)z) \cdot c - v_\ast(z) \cdot \alpha + (v^*(z) - v_\ast(z)z) \cdot \Omega(z) \quad (10)$$

where

$$\Omega(z) = r^D + \max_{\lambda \in [1, \Xi + (\Xi - 1)z]} (r^L - r^D) \lambda + \sigma \left\{ \frac{\mathbb{E} [\bar{v}(z + J^z)] \cdot (1 - \varepsilon \lambda) - v^*(z)}{v^*(z) - v_\ast(z)z} \right\} \quad (11)$$

where $\bar{v}(z + J^z) = v_0$ if the bank is liquidated and $\bar{v}(z + J^z) = v^*(z + J^z)$ otherwise.

For a sufficiently low liquidation cost, $v_0$, banks choose to avoid liquidation in all states. In that case, the solutions $\lambda^*$ and $c^*$ are given by $\lambda^*(z) = \Lambda(z)$ and

$$c^*(z) = \rho^{1/\theta} \frac{v(z)^{1-1/\theta}}{[1 + z \cdot v_\ast(z) / v(z)]^{1/\theta}}. \quad (11)$$
Then, $V(Z,W) = v(z) \cdot W$, $C(Z,W) = c^*(z) \cdot W$, $L = \bar{L} = \lambda^*(z) \cdot W$, $D = (\lambda^*(z) - 1) \cdot W$.

Whenever $dN = 0$, the loan growth rate $\iota^*$ induces a leverage drift given by:

$$\mu^\lambda = (\iota^* - \mu^W) \Lambda(z) \text{ such that } \mu^\lambda = \Lambda_z(z) \mu^z.$$  

(12)

The instant when $dN = 1$, the jump in leverage is immediately offset by loan sales, such that $\lambda^* = \Lambda(z + J^z)$. Finally, the market value of equity, is $S(W) = s^*(z)W$ and

$$\rho^I s^* = c^* (1 - (s^* - s^*_z)) - s^*_z \alpha z + (s^* - s^*_z) \gamma(z),$$

where $\gamma(z)$ is the expected equity growth:

$$\gamma(z) = r^D + (r^L - r^D) \cdot \Lambda(z) + \sigma \frac{E[s^*(z + J^z)] \cdot (1 - \varepsilon \Lambda(z)) - s^*(z)}{s^*(z) - s^*_z(z) z}.$$

Proposition 2 clarifies that two banks with the same value of $z$ but different $W$ behave as scaled replicas. We now discuss the leverage dynamics characterized by the proposition, illustrated by Figure 9. Panels (a) and (b) depict two curves, the shadow boundary characterized by the points $\{z, \Lambda(z)\}$ and the liquidation boundary by the points $\{z, \Gamma(z)\}$. Almost all the time, leverage and zombie loans live on a point of the shadow boundary. Default events dislocate leverage for the instants in which $dN = 1$, but leverage returns immediately to another point on the shadow boundary. When $dN = 0$, leverage drifts continuously along the shadow boundary. Panels (a) and (b) depict the trajectories of leverage after recognized and unrecognized default events, respectively.

Consider the trajectory in Panel (a). A bank experiences a default event starting from a point $\{z_0, \lambda_0\}$ in the shadow boundary. If the default is recognized, the stock of zombie loans remains the same, but equity falls. Thus, $\lambda$ and $z$ jump. The jump takes the bank to a point on the liquidation boundary. To offset the jump and to return to the shadow boundary, the bank sells loans in an amount $dF$ to delever. It then reaches the point $\{z_0 + J^z, \Lambda(z_0 + J^z)\}$ on the shadow boundary. After the jump, leverage drifts along the shadow boundary.
Panel (b) of 9 depicts a typical path after an unrecognized default. The logic is similar. Starting from a point \( \{z_0, \lambda_0\} \), both \( \lambda \) and \( z \) jump to a point between the shadow and liquidation boundaries—the default event does not take \( z \) all the way to the liquidation boundary because, although the default produces a fall in equity, the stock of zombie loans also increases.

It is important to understand that the choice of \( c \) and the recognition rate \( \alpha \) govern the dynamics of leverage along the shadow boundary. The points in the frontier are given by \( \lambda^* (z) = \Lambda (z) \). Thus, since \( z \) drifts toward zero, leverage must also drift downward within any time interval where \( dN = 0 \). The drift satisfies Eq. 12 in the proposition, which guarantees that leverage stays on the shadow boundary. Equation Eq. 12 projects the drift of \( z \) on the drift of \( \lambda \), using the shape of the shadow boundary \( \Lambda (z) \). Recall that the zombie loan ratio \( z \) drifts toward zero at rate \( \alpha \) and equity drifts at rate \( \mu W \), which is in turn influenced by \( c^* \). Hence, the bank chooses a loan issuance rate \( \iota^* \) to maintain leverage at the shadow boundary.

The optimal dividend choice, \( c^* \), is given by Eq. 11. This formula is governed by a race between wealth and substitution effects. However, different from the case with immediate accounting, the substitution effect is modified because on the margin the choice of dividends affects \( z \) through \( \mu W \). Because \( z \) determines the leverage consistent with the shadow boundary, the dividend choice affects the path of expected returns. The effect is encoded in the term \( z \cdot v_z (z) / v (z) \) in the denominator, which is an additional substitution effect.\(^{34}\)

Once we obtain \( c^* (z) \), we can solve for the loan growth rate along the continuous path:

\[
\iota \text{\ growth in loans} = -\alpha \frac{\Lambda_z (z)}{\Lambda (z)} + \frac{1 - \Lambda_z (z)}{\Lambda (z)} \mu W. \tag{13}
\]

The first term captures that losses are recognized slowly, at rate \( \alpha \). As losses are recognized, and \( z \)

\(^{34}\)The intuition is that by paying out more dividends, the bank lowers its fundamental equity, and thus increases its zombie loan ratio. A higher zombie loan ratio then allows the bank to operate with greater leverage than imposed by regulation. This raises returns, and as we increase \( \theta \), this substitution effect becomes smaller.
decreases, banks must gradually delever by reducing their loan issuance rates—hence the negative sign. The second term captures that leverage has a tendency to fall because retained earnings increase equity, $\mu^W(z)$—recall that $\Lambda_z(z)$ has a positive slope.

The model with delayed accounting is consistent with dynamics where, upon a default shock, the bank immediately returns to the shadow boundary. Once at the shadow boundary, the bank continues a slow deleveraging, which induces a slow response of market leverage and total liabilities, as we observe in the data. Thus, even though there are no adjustment costs, the leverage adjustment is gradual along the shadow boundary. This slow adjustment forms the basis of our $Q$-theory. In the next section, we calibrate the model and demonstrate how the model can reproduce all four motivating facts.

4 Calibration, Estimation and Quantitative Evaluation

We now describe the calibration and estimation procedures and then investigate the model’s ability to reproduce the four facts described in Section 2. We use quarterly data from 1990 Q3 to 2021 Q1 to produce the target moments. Thus, all corresponding model moments are also at the quarterly frequency. To keep the parametrization tractable, we calibrate $\{\psi, r^L, r^D, \delta, \sigma, \Xi\}$ independently, matching model moments to target moments in the data. Then, conditional on these calibrated parameters, we jointly estimate $\{\rho, \rho^I, \theta, \varepsilon, \alpha, \rho\}$, the parameters that govern the delay in the balance sheet responses. The parameter values are listed in Table 1. Table 2 presents both the targeted and untargeted moments in the data and the corresponding model moment.

Calibrated parameters. We set the utility parameter $\psi$ (risk-aversion) to a value of 0 to capture the idea that banks are risk-neutral, in line with the theory of the firm. The exogenous returns on loans and deposits, $r^L$ and $r^D$, are respectively set to 1.01% and 0.51%, consistent with the quarterly yield on loans (total interest income on loans divided by total loans) and the rate banks pay on their debt (total interest expenses divided by interest-bearing liabilities) in bank call reports. These values are also consistent with the calibration in Corbae and D’Erasmo (2021). Every period, a fraction of loans $\delta$ matures. The fraction is set to $1/(4 \times 3.5)$, so that the average duration of loans is three years and a half. Once we have estimated $\varepsilon$ (see below), the default intensity $\sigma$ is pinned down by the mean quarterly net-charge-off rate of $\sigma \times \varepsilon = 0.12\%$, resulting in $\sigma = 0.134$. We set the capital requirement parameter $\Xi$ to 12.5, reflecting a Tier-1 risk-based capital ratio requirement of 8% at which a bank is considered well capitalized. $^3^5$ Since bank liquidations are not a focus of this paper, we assume that $v_0$ is low enough such that banks never choose to expose themselves to liquidation risk.

Table 1: Parametrization

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Independently calibrated</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r^L = 1.01%$</td>
<td>Loan yield</td>
<td>BHC data: interest income / loans</td>
</tr>
<tr>
<td>$r^D = 0.51%$</td>
<td>Bank debt yield</td>
<td>BHC data: interest expense / debt</td>
</tr>
<tr>
<td>$\Xi = 12.5$</td>
<td>Regulatory maximum asset to equity ratio</td>
<td>Capital requirement of 8% to be well-capitalized</td>
</tr>
<tr>
<td>$\psi = 0$</td>
<td>Banker’s risk aversion</td>
<td>Standard value</td>
</tr>
<tr>
<td>$\delta = 0.071$</td>
<td>Rate at which loans mature</td>
<td>BHC data: average qrt. duration of loans</td>
</tr>
<tr>
<td>$\sigma = 0.134$</td>
<td>Arrival rate of default shocks</td>
<td>Mean quarterly net charge-off rate of 0.12%</td>
</tr>
<tr>
<td>Jointly estimated</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\varepsilon = 0.89%$</td>
<td>Loan loss rate in event of default</td>
<td>Mean book leverage</td>
</tr>
<tr>
<td>$\alpha = 4.30%$</td>
<td>Speed of loan loss recognition</td>
<td>Liabilities IRF</td>
</tr>
<tr>
<td>$p = 0.13%$</td>
<td>Fraction of slowly recognized loan losses shock</td>
<td>Initial liabilities IRF</td>
</tr>
<tr>
<td>$\theta = 1.98$</td>
<td>Banker’s inverse IES</td>
<td>Market leverage IRF</td>
</tr>
<tr>
<td>$\rho = 2.65%$</td>
<td>Banker’s discount rate</td>
<td>BHC data: book equity growth</td>
</tr>
<tr>
<td>$\rho^I = 4.33%$</td>
<td>Investor’s discount rate</td>
<td>CRSP and BHC: market to book ratio of equity</td>
</tr>
</tbody>
</table>

Notes: This table summarizes the parameter values, their role in the model, and the data target used to set or estimate the value. The text provides more details.

Estimated parameters. We estimate $\{\rho, \rho^I, \varepsilon, \theta, \alpha, p\}$ using simulated method of moments. The parameters $\{\alpha, p\}$ speak directly to our Q-theory: $p$ determines whether loan losses are recognized on impact and $\alpha$ governs the speed of loan-loss recognition over time. We estimate them together with the parameters that determine the severity of the loan shock, $\varepsilon$ and $\sigma$, and bankers’ $\rho$ and investors’ $\rho^I$ discount rate, and elasticity of substitution $\theta$.

We estimate these parameters to match the growth rate of book equity, the average market to book equity ratio, the average book leverage ratio, and the impulse response functions of market leverage and bank liabilities to a return shock in the data. To produce analogue estimated impulse responses to return shocks in the model, we solve and simulate the model. We run the same specification for the impulse responses of Section (2). We construct excess return shocks by first calculating the realized equity returns between adjacent quarters and the cross-sectional equity return. The bank specific return shock is then just the difference between a bank’s individual realized equity return and the cross-sectional average of the realized equity returns. The latter absorbs any potentially time varying aggregate effects on banks’ equity returns, similar to only using a time fixed-effect specification. Formally, the model is overidentified because each impulse response in the data contains effectively 21 moments, one for each $\beta_h$ in Equation (1). However, model generated moments such as these are highly correlated so the effective degree of over-identification is lower.\footnote{Each impulse response is well approximated by two moments, the jump on impact and the persistence.}

We simulate a panel of 10,000 banks. To compute the stationary distribution we simulate forward until the cross-sectional mean and standard deviation of $z$ stay approximately constant,
and discard these initial periods. We then take the last cross-section as initial condition and simulate the same number of quarters as in the data. From that sample, we calculate the moments and run the cross-sectional regressions using model simulated data, just as we did with the actual data. We discuss the values from this estimation and the resulting model fit in the next paragraph.

### Identification and Estimated Values.

We now describe what variation in the data identifies the parameters of the model. We estimate these parameters jointly since they jointly determine the targeted moments in the model. In our discussion below, we highlight which moment speaks particularly to which parameter. The parameter values are listed in Table 1.

The growth rate of a bank’s book equity is informative about bankers’ discount rate \( \rho \), because it says how much dividends the banker wants to pay out versus to keep inside the bank to accumulate equity. Since the ergodic means of the growth rates of fundamental and book equity move together but we only observe book equity we target the growth rate of book equity that equals 2.00\% to inform us about \( \rho \), which we estimate as 2.66\%.

We use the impulse response function of market leverage as a target for \( \theta \). This parameter governs how much bankers dislike variation in dividends from one period to the next. Since dividends affect the market value of the bank, the impulse response function of market leverage to a net-worth shock is informative about \( \theta \). We estimate an \( \theta = 1.98 \), which is very close to the standard value of 2 in the macro literature.

Investors are risk-neutral and hence their discount rate \( \rho^I \) approximately maps into the average market return on bank shares. Hence, we target a bank market-to-book ratio of 1.316 in the data, which gives a value of 4.33\%.

Banks choose a leverage ratio on the shadow boundary. The distance between the shadow boundary and the liquidation set is determined by the size of the idiosyncratic loan default shock \( \varepsilon \) and the likelihood of not having to acknowledge loan losses on impact \( p \). Thus, once \( \Xi \) is fixed, we estimate \( \varepsilon = 0.89\% \) to match the average book leverage ratio of 11.36.

We use the impulse response functions of liabilities and market leverage to net-worth shocks proxied by return shocks as targets for \( p \) and \( \alpha \). These impulse responses render a transparent identification: when shocks are not recognized on impact (\( p = 0 \)) nothing happens to the balance sheet size upon the arrival of a default shock. Hence \( p \) is pinned down by the initial response of liabilities to the net-worth shock. We estimate that \( p = 0.13\% \), which means that in almost all cases shocks to the loan books are initially unrecognized.

The rate at which losses are recognized on bank books, \( \alpha \), governs how fast book equity reverts to the fundamental value of equity \( q = W/\bar{W} = 1 \), along the shadow frontier. In response to a net-worth shock, banks’ book leverage jumps up. As \( z \) trends down to 0, the regulatory constraint becomes tighter and banks are forced to delever. The speed of reversal of the impulse response for book leverage then informs the value of \( \alpha \). We estimate that \( \alpha = 4.30\% \), which means that roughly 65\% of unrecognized losses are recognized within 10 quarters. This delay is consistent with Figure
2, where the net charge-offs taper off by the end of 2010, about two and a half years after the trough in bank market values.

We do not target second moments of the empirical counterparts. In the data, the dispersion of variables in the cross-section and over time is influenced by factors such as ex-ante heterogeneity (e.g., different business models) and aggregate shocks, that our model abstracts from for simplicity and tractability.

### Table 2: Model and Data Moments

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
<th>mean/(s.e.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log Market Returns</td>
<td>0.020</td>
<td>0.044</td>
<td>z 0.237</td>
</tr>
<tr>
<td></td>
<td>(0.176)</td>
<td>(0.000)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>Market leverage</td>
<td>8.596</td>
<td>8.470</td>
<td>q 0.821</td>
</tr>
<tr>
<td></td>
<td>(0.590)</td>
<td>(0.000)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>Book Leverage</td>
<td>11.361</td>
<td>11.361</td>
<td>λ 13.821</td>
</tr>
<tr>
<td></td>
<td>(0.361)</td>
<td>(0.000)</td>
<td>(0.018)</td>
</tr>
<tr>
<td>Market to Book Equity</td>
<td>1.316</td>
<td>1.320</td>
<td>c 0.041</td>
</tr>
<tr>
<td></td>
<td>(0.545)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>Growth Rate of Book Equity</td>
<td>0.020</td>
<td>0.019</td>
<td>ι 0.020</td>
</tr>
<tr>
<td></td>
<td>(0.112)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>Log Common Dividend Rate</td>
<td>0.006</td>
<td>0.033</td>
<td>dW/W 0.021</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>Charge-Off Rate</td>
<td>0.001</td>
<td>0.001</td>
<td>s(λ, q) 1.625</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.000)</td>
<td>(0.001)</td>
</tr>
</tbody>
</table>

**Notes:** The data uses the full sample from 1990 Q1 to 2021 Q1. The moments from the model are generated from a panel of 10,000 banks with the same number of quarters as in the data. We compute the stationary distribution by first simulating enough quarters so that the mean and standard deviation of the state variables (λ, z) are approximately constant, and then keeping the last one as the initial quarter of the simulated sample. The first row for each variable shows the mean. The second row shows standard error of the mean in parenthesis. For market leverage, book leverage and market-to-book equity, the mean and standard error are computed on the logs, but when reporting the mean we apply exponential to show the mean in levels.

**Model Fit and Interpretation.** Table 2 compares the moments generated by the model and those obtained from the data: our model fits most data moments well, with the exceptions of log market returns (which in the data includes other aggregate factors) and the common dividend rate. The model fits market leverage (8.596 in the model vs 8.806 in the data), book leverage (11.361 in the model vs 11.365 in the data), the growth rate of bank equity (2% in the model vs 1.8% in the data), and the net charge-off rate (0.1% in the model vs 0.1% in the data) very tightly. Note that the capital requirement constraint limits banks’ book leverage ratio to at most 12.5. Hence,
in our model banks keep an equity buffer over the capital requirement constraint as in the data.\footnote{In Appendix C.1, Figure 30 presents the stationary distribution of fundamental leverage $\lambda$ and the zombie loan to equity ratio $z$ together with the liquidation set. It also shows that banks keep an equity buffer over the liquidation boundary determined by the regulatory constraint.} The model also hits the market-to-book equity ratio target, 1.316 in the model vs 1.262 in the data. The model substantially overshoots the dividend rate (0.6% in the data vs 3% in the model) and the market return of equity (2% in the data vs 4.2% in the model). In the data, banks can also repurchase shares to return cash to their shareholders, leading to a higher dividend rate in the model. We also abstract from operating expenses, which leads to a higher market return in the model.

Table 2 presents unobservable model variables, such as fundamental leverage $\lambda$ and $q$ (fundamental equity/accounting value of equity). Fundamental leverage is 13.8, substantially higher than the book leverage value of 11.4. The average value for $q = \bar{W}/\bar{W}$ is 0.82, implying that the fundamental value and the accounting value of equity differs by 18 percentage points.

Figure 13 presents the impulse response function of the model to a net-worth shock and compares it to the data, including the 95% confidence interval in gray. Note that each impulse response function consists of several estimates, which means that our model is overidentified. As in the data section, all impulse responses are calculated based on a 1% negative net-worth shock. Panel (a) presents the impulse response function of Tobin’s Q. Our model reproduces the slow return to pre-shock levels by slowly incorporating the default shock on the books. Panel (b) presents the impulse response function of market leverage. While the initial jump in market leverage is slightly smaller compared to the data, the subsequent slow reversal to the initial market leverage level is very similar. Panel (c) displays the impulse response of bank liabilities that shows the slow delevering process. Finally, panel (d) presents the impulse response function of market equity. Our model generally replicates the impulse response of market equity in the data, the initial 1% decline in response to a 1% shock and the very slow recapitalization process. After 20 quarters, market equity is still below 90% the initial value in the data and 60% in the model. Thus, the model fits the impulse response in the data with only delayed accounting, standard dividend smoothing incentives and a capital requirement constraint. In sum, our parsimonious model generates slow movements in banks’ leverage and balance sheet dynamics.

\subsection{Matching Facts}

In this section, we evaluate the model’s ability to reproduce the four facts on Tobin’s Q from Section 2. We focus on the period between 2006Q4 and 2014Q3, where banks experienced a large credit shock.

\textbf{Aggregate shocks.} With idiosyncratic default events and a continuum of banks, the law of large numbers guarantees that the aggregate time series generated by the model are deterministic.
In this section, we want to evaluate the model’s ability to match cross-sectional and time series facts and use the model to run counterfactuals. We want to do so in the most parsimonious way possible. To that end, we assume that the individual default events are of two types: there are idiosyncratic events with intensity $\sigma^i$, and aggregate events with intensity $\sigma^a$. We maintain the assumption that $\sigma = \sigma^i + \sigma^a$ so that individual bank decisions remain the same. Yet, with this simple extension, we can calibrate a sequence of realized aggregate defaults to match a particular time series from the data.

**Inferring aggregate default shocks.** We infer the series of aggregate default shocks from the quarterly aggregate provision for loan loss series. That is, we choose the shocks such that the losses in the model match that of the data (see panel A of Figure 10). With this shock series, we simulate the period from 2006Q4 to 2014Q3 and aggregate over banks in each quarter.

**Fact 1. Market and book equity value divergence.** Fact 1 describes the divergence of market and book equity, especially during crises. Panel B of Figure 10 shows the time series of Tobin’s Q in the model and the data. The delayed loan loss recognition mechanism in our model generates a sustained and pronounced decline in Tobin’s Q of about 40%, while in the data Tobin’s Q fell by more than 60%. In our decomposition of Tobin’s Q, Eq. 2, we establish that $Q$ can vary through two channels: changes in the price per unit of equity $s$ and the discrepancy between book and fundamental values, $q$, attributed to accounting. We are interested in knowing what fraction of the drop in $Q$ can be attributed to $q$. We exploit a back-of-the-envelope calculation to answer this question. We distinguish aggregate from idiosyncratic variables and denote by $x$ the aggregate version of a variable $x$. Consider a default shock to loans of 1% that lasts one year, and assume that aggregate leverage is $\lambda = 14$. In this case, we obtain:

$$J^W = -J^Z \approx -0.01\lambda W = -0.14W.$$

This means that the average $z$ is 0.24. As a result, the jump in $q$ satisfies

$$J^q = \frac{W + J^W}{W + J^W + Z + J^Z} - 1 \approx \frac{1 - 0.14}{1 + 0.24} - 1 = -30\%.$$

Thus, a one time shock can generate a quantitatively significant movement in Tobin’s Q.

---

38 The model is restricted to generate high losses from slow recognition whenever the fraction of immediately recognized shocks, $p$, is low. The restriction comes from the drift in the process for $Z$, $-\alpha Z$. Since $\alpha$ is small, the losses have to be very large for the model to be able to match the PLLs observed during the 2008 crisis. An alternative specification with curvature in $Z$ would improve the model’s fit. Therefore, to allow the model to generate the high losses observed during this time period, we assume that $p$ increases unexpectedly to 0.4, which does not change the policy functions. As a result, banks have to recognize a larger fraction of the default losses immediately, increasing the loan provision rate in the simulated data.
Notes: This figure compares aggregate series of model generated data (blue line) to the data (black line). These series are based on a simulation that feeds in shock that are chosen to match the times series of aggregate loan loss provisions (Panel A). Panel B presents Tobin’s Q. Panel C the evolution of book loans, as deviation from a trend that is based on the 10 years before 2006 Q4. The units for Panel C are in decimals, hence we document a 70% decline in the model relative to trend.

Fact 2. Predictive power. Our second fact of interest is the predictive power of Tobin’s Q in terms of book-equity returns and loan charge-off rates up to even two years. Our model captures this effect because market values capture losses that are unrecognized in books. We replicate fact 2 with model generated data (see Figure 11). As in the data, market equity contains predictive power for loan losses beyond the information contained in book equity. Recall that without considering the changes in the price per unit of fundamental equity $W$, upon a default shock returns fall approximately by $J^W$. A bank’s charge-off rate per unit of equity is approximately $\alpha \cdot z$. Since the jump $J^z$ is correlated with the jump $J^W$, the model generates the predictability of loan losses with the market-to-book equity ratio as in the data.

Fact 3. Equity buffer. The third fact states that (a) banks keep an equity buffer over the regulatory capital constraint, and (b) that the cross-sectional dispersion in market leverage increases a lot during recessions, especially compared to a very stable cross-sectional distribution of book leverage. We show in Figure 30 in the Appendix that banks indeed keep an equity buffer over
the regulatory minimum.\textsuperscript{39} With the fitted series of losses, Figure 12 shows that the model also replicates the increase in the cross-sectional dispersion of market leverage (Panel A) without barely any change in the cross-sectional dispersion of book leverage (Panel B) —note the differences in the scale. The model does not explain the full extent of the increase in the cross-section of leverage because we abstract away from features that in the data could generate greater dispersion such as ex-ante heterogeneity, a fat tail in default shocks, or different risk exposures.

\textsuperscript{39}In particular, the capital buffer is approximately: $\Xi - \frac{A + z}{1 + z} = 12.5 - \frac{13.8 + 0.24}{1 + 0.24} = 1.2\%$, which is close to the value in the simulations.
Fact 4. Slow mean reversion of leverage. The main takeaway from fact 4 states that the leverage ratio of banks is mean reverting, but the reversion rate is low—see the impulse response estimation described in Section 2. Figure 13 compares the impulse responses to return shocks of Tobin’s Q (Panel a), total liabilities (Panel b), market leverage (Panel c), and market equity (Panel d) of the model with the data. The black lines are the estimated response from the data, the shaded areas their 95% confidence interval, and the blue lines represent the model.

Figure 13

The figures present the impulse response functions of model simulated data (blue) for the benchmark calibration and compares it to the data (gray line represents the point estimates and the shaded area the 95% confidence interval). We show the impulse response function of Tobin’s Q (Panel a), liabilities (Panel b), market leverage (Panel c), and market equity (Panel d).

Notes: The figures presents the impulse response functions of model simulated data (blue) for the benchmark calibration and compares it to the data (gray line represents the point estimates and the shaded area the 95% confidence interval). We show the impulse response function of Tobin’s Q (Panel a), liabilities (Panel b), market leverage (Panel c), and market equity (Panel d).

The figure shows that the model generates an initial (mechanical) jump in market equity, as well as the slow adjustments of market leverage, Tobin’s Q, and liabilities to return shocks as in the data. Note that our $Q$ theory does not rely on loan adjustment costs once we allow for accounting values to differ from fundamental values, i.e., $p > 0$. We also show in Appendix C.1 that the slow adjustment is not driven by $\theta$ (the intertemporal smoothing incentives) as they look virtually identical with $\theta = 1$. In contrast, solving the model with $p = 0$ and keeping $\theta$ at the benchmark calibration level counterfactually delivers no response in market leverage and Tobin’s Q and an immediate response in market equity and liabilities. Panels b) and c) are targets of the estimation—although with a minimal of only two parameters. Panel d) is a consequence of fitting these two panels. However, we can observe that Tobin’s Q which is not a target, has a dynamic
response identical to what we observe from the estimation. All in all, our model reproduces all four facts.

**Explaining the aggregate loan series.** With the fitted series for aggregate shocks, we reproduce the decline in lending consistent with the dynamics of leverage at the micro level. Panel c) of Figure 10 shows the time series of book loans in the model and the data. Because book loans are growing, we de-trend book loans in the model and the data using the exponential trend of the 10 years prior 2006 Q4. The shock causes banks to shrink book loans by 70% in the model and by 40% in the data. The difference between model and data is possibly due to the lack of equity issuance in our model. Likewise, it is possible that loan-loss recognitions were slower during the crisis than implied by our estimate.

### 4.2 Effects of Accounting Rules

**Effects of Accounting Rules.** In the previous sections, we argued that delayed accounting is key to explain the four facts this paper highlights. In our model, accounting rules shape bank decisions. In this section, we show that a reform toward a quicker recognition of loan losses involves a trade-off: namely, between the fragility of the banking sector and the speed of adjustment after loan losses.

To highlight this trade-off, we solve the model for different values of $\alpha$. Recall that a lower $\alpha$ means that losses are more slowly recognized. Lower values of $\alpha$ reflect accounting rules that make it easier for banks to hide losses. In the model, lowering $\alpha$ changes the distribution of $z$ and $\lambda$: on the one hand it leads to more zombie loans (higher average $z$) and on the other hand it increases banks’ fundamental leverage, $\lambda$. Indeed, lower values of $\alpha$ provide banks with more slack to circumvent regulatory constraints and this manifests in a higher fundamental leverage and a greater discrepancy between the fundamental value and the book value of loans.

Figure 15a reports cross-sectional means of $z$ and $\lambda$, as we vary $\alpha$ keeping all other parameters unchanged. The negative relationship between $z$ and $\lambda$ is evident from the figure. Strikingly, although the fundamental leverage ratio $\lambda$ differs for different accounting rules, average book leverage is basically identical in each of these equilibria: it goes from 11.36 with $\alpha = 3\%$ to 11.35 with $\alpha = 10\%$. According to the model, all of these economies will look the same to a regulator that uses book leverage to gauge the health of the financial system even though the fundamental leverage of banks and therefore risk differs significantly. This feature highlights one side of the trade-off: a more delayed loan loss recognition process can lead to greater potential losses.

The other side of the trade-off is that the slower recognition of loan losses allows banks to recover more quickly from default events and to sustain higher levels of lending. To see this, consider how a large default event affects economies with different loan loss recognition speeds $\alpha$. We study how aggregate lending responds to a negative default shock, given different values of $\alpha$.  

39
The shock is a 50 fold increase in the arrival rate $\sigma$ of the default event for one quarter, leading to an increase of the average loan default rate from 0.12% to 6.39% in the quarter of the shock. Figure 15b presents the results. Two effects are at play. Since a lower $\alpha$ means that banks are more fundamentally levered, the shock pushes more banks closer to the liquidation boundary. The decline in capitalization forces these banks to sell more loans on impact. This is consistent with Figure 15a: the impact of the default shock is stronger in economies with a low $\alpha$ that feature more financially fragile banks.\footnote{See Eq. 3 where a lower $\alpha$ leads to a higher $\lambda$ increasing $J^W$.}

![Figure 14: Counterfactual Exercise](image)

Notes: The figures show the results from our counterfactual exercise. Each dot in the top figure is a pair of cross-sectional means of $\lambda$ and $z$ in the stationary allocation, for a given value of $\alpha$ as well as for the case of immediate loan loss recognition $p = 0$ (black dot). The gray area represents the regulatory liquidation set. The bottom panel shows the path for aggregate loans measured as percentage deviation from trend after an unanticipated increase in the default arrival rate $\sigma$ to 6.56 for one quarter, for various values of $\alpha$ as well as $p = 0$ (in dashed black). The values of $\alpha$ in Figure (15b) match the colors of Figure (15a).

In the first quarter after the shock, aggregate lending falls by 38% in the low $\alpha$ economy with $\alpha = 3\%$ and by 24% in the high $\alpha$ economy with $\alpha = 10\%$. At the same time, the low $\alpha$ economies also feature faster recoveries and higher equilibrium lending levels after the initial negative default event. After 20 quarters, the $\alpha = 3\%$ economy increased aggregate lending by 12 percentage points, while $\alpha = 10\%$ economy decreased lending by 16 percentage points relative to each path’s pre-trend.\footnote{Note that our model is stationary in growth rates, not levels.} The difference in these two recovery trajectories is because of two effects. First, low $\alpha$ economies allow banks to sustain higher fundamental leverage ratios, which allows them to lend more profitably and recapitalize.\footnote{Lending growth $\iota$ is increasing in leverage $\lambda$ from Eqs. (13) and (3).} Second, low $\alpha$ economies provide banks with more flexibility to move along the shadow frontier (i.e., sell or make loans).\footnote{Lending growth $\iota$ is decreasing in $\alpha$ from Eq. (13).} Hence, while the initial impact of the shock is more severe, the low $\alpha$ economies lead to higher lending and faster recoveries. This is
the other side of the trade-off.

**Discussion: more market-based accounting rules?** The trade-off result connects our model with the debate on whether accounting rules should incorporate more market-based information to improve macro-prudential regulation. Fact 2 shows that market values incorporate information on losses much faster than book values. Incorporating market values into regulatory constraints would therefore increase $\alpha$. This literature describes the trade-off from including more market information as follows. On the one hand, it would exacerbate fire sale dynamics (Laux and Leuz, 2010; Ellul et al., 2011; Shleifer and Vishny, 2011). If market values are mainly driven by risk premia, regulation should mostly ignore valuation changes that are not germane to the health of the banking industry. On the other hand, the discretion in accounting rules opens the door to evergreening that contributes to the creation of zombie loans and lowers economic efficiency (Caballero, Hoshi and Kashyap, 2008; Huizinga and Laeven, 2012; Blattner, Farinha and Rebelo, Forthcoming). While a welfare analysis of this additional trade-off is beyond the scope of this paper, we identified the race between the size and smoothing out of equity losses discussed above as another trade-off to be considered for policy discussions.

**Discussion: role of accounting rules for macro-prudential policies** Our analysis suggests that regulators should consider exploiting the loan-loss recognition mechanism as an additional policy tool to countercyclical capital buffers. Stricter accounting rules (i.e., faster loan loss recognition) could achieve lower financial fragility and thus mitigate the impact of shocks. Countercyclical capital buffers achieve the same by relaxing the capital requirement during the bad state of the world. Yet, countercyclical buffers require constant monitoring by regulators of the entire banking system. Suppose only half of the banking system is affected by a shock. In that case, regulators would face a trade-off between relaxing constraints for banks affected by shocks against allowing greater leverage for healthier banks. By contrast, delayed accounting provides an automatic countercyclical regulation. It automatically helps banks that are affected by the shock. The issue is that delayed accounting does induce greater fundamental leverage (i.e., risk) of the banking system. We hence suggest studying countercyclical buffers and delayed accounting in a comprehensive framework, an important analysis we leave to future work. A general equilibrium extension of the model could be used to evaluate this important tradeoff.

Another matter of discussion should be regulatory forbearance. An application of the model could treat $\alpha$ as a policy parameter that is a function of aggregate conditions. This exercise could be useful to evaluate the tradeoff between the benefit of more relaxed accounting after an aggregate default event against the costs of greater risk-taking prior to an aggregate default event.
4.3 Discussion: Alternative Adjustment Cost Model

As we described in the introduction, banking models operate with different constraints on leverage. By incorporating loan adjustment costs, these models can also generate variation in Tobin’s $Q$ and the slow adjustment of bank variables. However, there will not be a distinction between fundamental equity and book equity because loss accounting is immediate. For that reason, Tobin’s $Q$ would not predict future loan losses (fact 2). To give models with adjustment costs the best shot at reproducing our four facts without our mechanism, we augment them with delayed accounting without a regulatory constraint. Then book valuations become irrelevant for bank decisions. With loan adjustment costs, leverage becomes a state variable and wealth evolves as:

$$dW = \left[ (r^L - r^D) \lambda + r^D - c - \frac{\gamma}{2} (\iota)^2 \lambda \right] W dt - \varepsilon \lambda W \cdot dN,$$

where $\iota$ is the loan issuance rate and $\gamma$ is the adjustment cost parameter—see Appendix C.6 for a description of the model with adjustment costs. With $\Xi > 1/\varepsilon$ accounting becomes irrelevant for bank decisions because the regulatory liquidation never happens.

To provide an appropriate comparison, we estimate $\gamma$, the adjustment cost parameter, by targeting the impulse response function of liabilities to a 1% net-worth shock in the data. We match fact 4 with an adjustment cost parameter of 15. This strikes us as an unreasonably large number. To see that, consider an example where the bank sells 6% of its loans to decrease its leverage from 16 to 15. A value $\gamma = 15$ then means that the bank would lose more than 40% of its equity as the loan adjustment cost implies that equity declines by $= -\frac{15}{2} \times 6\%^2 \times 15 \approx -40\%$. However, with this high loan adjustment cost parameter, the model explains facts 1-3 and by construction fact 4. It produces leverage dispersion because loan adjustment costs make leverage adjustments very costly as the back on the envelope calculation highlights. With a market leverage distribution in place, a shock to net-worth increases market leverage dispersion further. Given that we assumed delayed accounting (but without capital requirement and hence without economic bite), the shock will only slowly dissipate to book values and therefore generate the increase in the cross-sectional dispersion of Tobin’s $Q$ during banking crises.

In sum, economically large adjustment costs in conjunction with delayed accounting deliver our four facts. Our model rationalizes the same facts without any adjustment costs. It requires just two primitives: a regulatory capital requirement and delayed accounting. Further, while $\gamma$ is difficult to interpret and outside the purview of a regulator, $\alpha$ and $p$ are policy parameters.

5 Conclusion

This paper presents four empirical facts about banks’ Tobin’s $Q$. Motivated by these facts, the paper presents a heterogeneous-bank model that distinguishes between accounting, fundamental,
and market values. In our model, all three measures of equity matter for banking decisions. The novel feature of our theory is that banks delay the recognition of loan losses on their books. This allows the model to reproduce the four facts.

We also demonstrate that regulatory reforms designed to accelerate loss recognition introduce a trade-off between the scale of equity losses and subsequent lending and financing choices of banks. As part of the continuous fine-tuning process of banking models, future work can use our model as a building block to study the macroeconomic effects of delayed accounting and policies designed to change the speed of loan-loss recognition in general equilibrium.

A clear limitation of the model is that banks are treated in isolation, but there are many dimensions in which they interact. For starters, banks lend funds to each other, a feature that creates credit exposure among themselves. Second, we have assumed that the demand for loans is perfectly elastic. With a finite elasticity, aggregate deleveraging carries additional effects through fire-sale externalities—if accounting values partially recognize market values. Third, the increase in lending by a group of banks can increase macroeconomic risks that increase the default likelihood of other banks. This broader set of interactions have been already studied by the literature, but the effects of delayed accounting have not been studied in these richer environments. These are tasks left for the future.
References


Behn, Markus, Rainer FH Haselmann, and Vikrant Vig, “The limits of model-based regulation,” 2016. 1, 2.5


Blank, Michael, Samuel G Hanson, Jeremy C Stein, and Aditya Vikram Sunderam, “How Should US Bank Regulators Respond to the COVID-19 Crisis?,” 2020. 1

Blattner, Laura, Luisa Farinha, and Francisca Rebelo, “When losses turn into loans: The cost of undercapitalized banks,” American Economic Review, Forthcoming. 1, 1, 2.5, 3.1, 4.2


Corbae, Dean and Pablo D’Erasmo, “Capital buffers in a quantitative model of banking industry dynamics,” Econometrica, 2021. 3, 1, 4


DeMarzo, Peter and Zhiguo He, “Leverage Dynamics without Commitment,” 2016. NBER Working Paper No. 22799. 1


Flanagan, Thomas and Amiyatosh Purnanandam, “Why do banks hide losses?,” Available at SSRN 3329953, 2019. 1, 2.5, 3.1


Goh, Beng Wee, Dan Li, Jeffrey Ng, and Kevin Ow Yong, “Market pricing of banks’ fair value assets reported under SFAS 157 since the 2008 financial crisis,” *Journal of Accounting and Public Policy*, jan 2015.  46


Hachem, Kinda, “Relationship lending and the transmission of monetary policy,” *Journal of Monetary Economics*, 2011, 58 (6), 590 – 600.  1

Harris, Trevor, Urooj Khan, and Doron Nissim, “The Expected Rate of Credit Losses on Banks’ Loan Portfolios,” 2013.  A.3


Lintner, John, “Distribution of incomes of corporations among dividends, retained earnings, and taxes,” *The American economic review*, 1956, 46 (2), 97–113. 3.1


Myers, Stewart C. and Nicholas S. Majluf, “Corporate financing and investment decisions when firms have information that investors do not have,” Journal of Financial Economics, 1984, 13 (2), 187 – 221. 1


Piazzesi, Monika and Martin Schneider, “Payments, credit and asset prices,” 2018. 1

Plantin, Guillaume and Jean Tirole, “Marking to market versus taking to market,” American Economic Review, 2018, 108 (8), 2246–76. 1

Plosser, Matthew C and Joao AC Santos, “Banks’ incentives and inconsistent risk models,” The Review of Financial Studies, 2018, 31 (6), 2080–2112. 1, 2.5


Rios-Rull, Jose Victor, Tamon Takamura, and Yaz Terajima, “Banking Dynamics, Market Discipline and Capital,” March 2020. 1


Tella, Sebastian Di and Pablo Kurlat, “Why are banks exposed to monetary policy?,” American Economic Journal: Macroeconomics, Forthcoming. 31


For Online Publication

Online Appendix for
“A Q-Theory of Banks”

Juliane Begenau      Saki Bigio
Jeremy Majerovitz   Matias Vieyra
A Data Appendix

A.1 Sample Selection

We analyze bank holding companies (BHCs), drawing data from multiple sources. We focus on top-tier bank holding companies that are headquartered in the 50 states or in Washington D.C. In most of our analyses, we analyze data from 2000 Q1 to 2015 Q4. For the analysis of impulse response functions, we extend the sample back to 1990 Q3 (this is the first year for which we can identify whether a BHC is top-tier). For book variables, we use data from the FR Y-9C, downloaded through Wharton Research Data Services (WRDS). We match this to data on market capitalization and returns from the Center for Research in Securities Prices (CRSP) using the PERMCO-RSSD links data set provided by the New York Fed (https://www.newyorkfed.org/research/banking_research/datasets.html). For analyses that use solely book data, we use data for those BHCs that we find in our sample in the FR Y-9C; for analyses that use market data, we use only the observations which we observe in both FR Y-9C and CRSP. In one robustness check, we use information on the dates of, and participants in, bank mergers and acquisitions; we obtain data on bank mergers from the Chicago Fed (https://www.chicagofed.org/banking/financial-institution-reports/merger-data). In an additional robustness check, we drop all banks that were ever stress-tested (CCAR and DFAST). We obtain information on whether banks were ever stress tested from the Federal Reserve (The main website is https://www.federalreserve.gov/supervisionreg/stress-tests-capital-planning.htm, and the specific data sets can be found at https://www.federalreserve.gov/supervisionreg/ccar.htm and https://www.federalreserve.gov/supervisionreg/dfa-stress-tests.htm).

A.2 Evolution of main balance sheet variables

To get a sense of how the crisis affected banks, we report the evolution of key balance sheet components in Figure 15. This figure shows total assets, liabilities, and loans—not netting out the allowance for loan losses—for the aggregate banking sector (left panel) and the four largest BHCs in terms of assets. The banking industry is highly concentrated: the “Big Four” largest BHCs account for roughly 50 percent of aggregate assets. At the onset of the crisis, the growth of bank assets, loans, and liabilities slowed down, but never dropping as dramatically as bank equity market valuations (see below). The amount of outstanding loans, the largest component of bank assets, stagnated during the crisis and eventually fell. By 2009 Q4, loans net of the allowance for loan losses had fallen by $361 billion, a drop of only 6.84%.\(^4\) This number is driven only in part by losses as banks also slowed down the issuances of new loans.

A.3 Difference between Market and Book Data

Difference between market and book data. To get a quantitative sense of how much book and market equity differed during the crisis, in Table 3 we present the percentage change in bank market equity valuations (top two rows) and book equity valuations (middle two rows) together with the change in the S&P 500 stock return index from the beginning of the crisis in 2007 Q3 to the end of each of 2008, 2009, and 2010, respectively. We report simple percentage changes in the real value (columns titled “real change”) as well as the changes in fitted log-linear trends (columns entitled “log linear”). Between 2007 Q3 and 2008 Q4, the market capitalization of the banking sector dropped by 54% compared to a 42% drop in the S&P 500. By 2010 Q4, market equity was still down 30% from its value in 2007 Q3. Much of this rebound followed from new equity issuances. By contrast, book equity did not fall during the crisis and actually increased substantially post-crisis. In fact, recorded book equity losses were entirely made up

\(^4\)The allowance for loan losses is an estimate of likely loan losses for the outstanding loans on the balance sheet. The next subsection provides more detail on how bank accountants calculate this number.
Notes: These figures show data on assets, liabilities, loans, and loans net of ALL for BHCs. Data come from the FR Y-9C. Loans net of ALL refers to loans minus the allowance for loan losses (this subtracts out “probable and estimable” future losses on the current stock of loans). All variables converted to 2012 Q1 dollars using the seasonally-adjusted GDP deflator. The left panel shows aggregate time series, excluding new entrants to the sample BHC such as Goldman Sachs. The right panel shows the same data for the “Big Four”, i.e., the four largest BHCs. Note that the spike in the data of Wells Fargo is due to its acquisition of Wachovia. Likewise, JP Morgan took on Bear Stearns and WaMu, while Bank of America took on Merrill Lynch and what was left of CountryWide.

for by new equity issuance. This large discrepancy implies that banks’ average Tobin’s $Q$, defined as the market–to–book equity ratio, drastically declined during the crisis and remained much lower thereafter.

Bank Accounting Practices. The discrepancy between book and market equity reflects bank accounting practices. Banks can delay acknowledging losses on their books (e.g. Laux and Leuz 2010), because banks are not required to mark-to-market the majority of their assets. There are many incentives to delay book losses. In practice, a key metric for measuring success of a bank is the book return on equity (ROE).\footnote{For example, JP Morgan’s 2016 annual report states “the Firm will continue to establish internal ROE targets for its business segments, against which they will be measured” (on page 83 of the report).} Given that ROE is a measure of success, manager compensation is linked to book value performance. Moreover, shareholders and other stakeholders may base their valuations on information from book data. Finally, banks are required to meet capital standards based on book values.

The flexibility of accounting their accounts is studied extensively in the accounting literature (Bushman, 2016 and Acharya and Ryan, 2016 review the literature on this issue, Francis et al., 1996 studies the same issue for non-financial firms). In practice, banks can record securities on the books using two methodologies: either amortized historical cost (the security is worth what it cost the bank to buy it with appropriate amortization) or fair value accounting.\footnote{Fair value accounting can be done at three levels: Level 1 accounting uses quoted prices in active markets. Level 2 uses prices of similar assets as a benchmark to value assets that trade infrequently. Level 3 is based on models that do not involve market prices (e.g. a discounted cash flow model). Banks are required to use the lowest level possible for each asset. In practice, most assets are recorded at historical cost. The majority of fair value measurements are Level 2 (Goh et al. 2015; Laux and Leuz 2010). Recent work has shown that the stock market values fair value assets less if they are measured using a higher level of fair value accounting. This leaves room to mis-price assets on books. Particularly during 2008, Level 2 and Level 3 measures of assets were valued substantially below one (Goh et al. 2015; Kolev 2009; Song et al. 2010). Laux and Leuz (2010) document sizable reclassifications from Levels 1 and 2 to Level 3 during this period. They highlight the case of Citigroup, which moved $53 billion}
Table 3: Aggregate Descriptive Statistics

<table>
<thead>
<tr>
<th></th>
<th>Real Change</th>
<th></th>
<th></th>
<th>Log-Linear</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2008</td>
<td>2009</td>
<td>2010</td>
<td>2008</td>
<td>2009</td>
</tr>
<tr>
<td>Market Equity</td>
<td>-54.08%</td>
<td>-39.35%</td>
<td>-29.03%</td>
<td>-61.21%</td>
<td>-49.98%</td>
</tr>
<tr>
<td></td>
<td>(-$705B)</td>
<td>(-$513B)</td>
<td>(-$378B)</td>
<td>(-$945B)</td>
<td>(-$790B)</td>
</tr>
<tr>
<td>Book Equity</td>
<td>11.83%</td>
<td>21.70%</td>
<td>25.97%</td>
<td>-3.46%</td>
<td>-1.50%</td>
</tr>
<tr>
<td></td>
<td>($94B)</td>
<td>($172B)</td>
<td>($206B)</td>
<td>(-$32B)</td>
<td>(-$15B)</td>
</tr>
<tr>
<td>S&amp;P 500</td>
<td>-42.08%</td>
<td>-28.83%</td>
<td>-21.20%</td>
<td>-25.55%</td>
<td>-7.01%</td>
</tr>
</tbody>
</table>

Notes: The columns headed with the “Real Change” label show the percentage change from the raw variables. The columns headed with the “Log-Linear” label show the cyclical deviations from a log-linear trend in percentage points since 2007 Q3. Market Equity refers to shares outstanding times share price aggregated across all publicly traded BHC. Book equity is the book equity of publicly traded BHCs. All variables deflated using the seasonally-adjusted GDP deflator and converted to 2012 Q1 dollars. The dollar values are obtained by multiplying the cumulative percentage point deviation by real market capitalization and real book equity at the end of 2007 Q3, respectively. The last row shows the percentage change in the return on the S&P 500 in the first three columns, while the last three columns show the change relative to a linear log-linear trend.

degree of freedom is the extent to which banks can acknowledge impairments: banks have the right to delay acknowledging impairments on assets held at historical cost, if they deem those impairments as temporary (i.e. they believe the asset will return to its previous price). This gives banks substantial leeway, and led banks to overvalue assets on the books during the crisis. Huizinga and Laeven (2012) find that banks used discretion to hold real-estate related assets at values higher than their market value. (Laux and Leuz, 2010) note some notable cases of inflated books during the crisis: Merrill Lynch sold $30.6 billion dollars of CDOs for 22 cents on the dollar while the book value was 65 percent higher than its sale price. Similarly, Lehman Brothers wrote down its portfolio of commercial MBS by only three percent, even when an index of commercial MBS was falling by ten percent in the first quarter of 2008. Laux and Leuz (2010) also document substantial underestimation of loan losses in comparison to external estimates.

This shows up in our own analysis as well: Figure 2 shows that provisions for loan losses and net charge-offs only reached their peak in 2009 and 2010 respectively, and remained quite elevated at least through 2011, well after the recession had ended. The decomposition of net charge-offs shows that these losses were heavily driven by real estate, suggesting they were associated with the housing crisis.47 Banks’ books were only acknowledging in 2011 losses that the market had already predicted when the crisis hit.

Harris et al. (2013) construct an index, based on information available in the given time period, that predicts future losses substantially better than the allowance for loan losses.48 This implies that the

---

47 When a bank has a loss that is estimable and probable, it first provisions for loan losses, which shows up as PLL. Later when the loss occurs, the asset is charged off and thus taken off the books, which shows up as charge-offs, although occasionally the bank can recover the asset later. Net charge-offs is charge-offs minus recoveries. We show a decomposition by category for net charge-offs but not for PLL because the FR Y-9C does not provide information on PLL by loan category.

48 The ALL is the stock variable corresponding to the PLL.
allowance for loan losses is not capturing all of the available information to estimate losses. This may in part be strategic manipulation, but there may also be a required delay in acknowledging loan losses. Under the “incurred loss model” that was the regulatory standard during the crisis, banks are only allowed to provision for loan losses when a loss is “estimable and probable” (Harris et al., 2013). Thus, even if banks know that many of their loans will eventually suffer losses, they were not supposed to update their books until the loss was imminent.

**Information content** We test whether book equity captures all available information about bank cash flows using cross-sectional regressions of market equity on book equity and several other profitability measures. We are motivated by the efficient markets hypothesis that suggests that market values reflect all available information about future dividends, and, by extension, about banks future profits and net-worth today: If market equity indeed contains additional information about bank profitability not captured by their book values, then market equity will be correlated with variables capturing profitability, even after conditioning on book equity. To help us visualize the additional information content of market values over and above book values, consider the following cross-sectional regression

$$\log (\text{Market Equity}_i) = \alpha + \beta \log (\text{Book Equity}_i) + f(X_i) + \epsilon_i,$$

where $f(X)$ represents polynomials in our variables of interest, and $i$ indexes banks.\(^{49}\) We then construct the partial residual $\log (\text{Market Equity}) - \alpha - f(X)$ and plot this on the vertical axis of Figure 16. We plot the regressor of interest, $X$, on the horizontal axis. By construction, the polynomial $f(X)$ that best fits the outcome variable $\log (\text{Market Equity})$ will also be the polynomial that best fits the partial residual. Thus, Figure 16 allows us to plot $f(X)$ and assess the goodness of its fit. We consider a quartic in log RoE over the past year (controlling for log book equity) and a quartic in log RoE over the next year (controlling for log book equity and a quartic in log RoE over the past year) as $f(X)$.\(^{50}\) These graphs confirm that market capitalization, controlling for book equity, is increasing in both RoE over the past year and in RoE over the next year. Hence even after controlling for book equity, market capitalization captures information content of net-income from the past and upcoming year. Note that the non-linear regression specification is important. For example, in the post-crisis period, there is a left tail of banks with very negative RoE. In this region the marginal effect of RoE on market capitalization is much smaller.

\(^{49}\) Appendix Section A.3 shows the partial $R^2$ for this regression for a range of variables capturing profitability measure such as loan charge offs and ROE over various time horizons.

\(^{50}\) For improved visibility, we exclude outliers from the graph window by limiting the graph’s horizontal axis to values within $\pm 3$ standard deviations from the mean.
Notes: These figures show results from a cross-sectional regression of log market equity on assorted variables. The top row shows results from a regression of log market capitalization on log book equity and a quartic in log RoE over the past year. The bottom row shows results from a regression of log market capitalization on log book equity, a quartic in log RoE over the past year, and a quartic in log RoE over the next year. The horizontal axis shows the regressor of interest, and the vertical axis shows the outcome minus the effect of the controls (for the top row, the controls are a constant and log book equity, for the bottom row, the controls are a constant, log book equity, and a quartic in log RoE over the past year). The left column shows results for 2006 Q1, the right column shows results for 2009 Q1. Regressions are run on the cross-section of banks with all variables available, but the horizontal axis of the graph window is restricted to ±3 standard deviations from the mean to improve visibility. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. Log RoE is defined as log (1 + RoE). RoE over the past year is defined as book net income over the last four quarters divided by book equity four quarters ago; RoE over the next year is defined as the one year lead of this variable.
A.4 Equity and Leverage of the four largest Bank Holding Companies

Figure 17 presents the time series of book equity, market equity, and preferred equity. As with the aggregate data, market equity and book equity are often not aligned. The discontinuities in the individual bank series reflect mergers and acquisitions, e.g. the acquisition of Wachovia by Wells Fargo during the crisis. Citigroup is an extreme example of the discrepancy between book and market values: Citigroup lost 90% of its market capitalization but its book equity remained intact.\footnote{Citigroup suffered heavy losses during the crisis and did not undergo any major mergers or acquisitions, making it a particularly clean example case.}

Figure 4 presents the market and book leverage time series for the four largest banks in the US.
Figure 17: Book Equity and Market Equity for the four largest Bank Holding Companies.

Notes: These figures show data on book equity, market capitalization, and preferred equity for the four largest BHCs. Book equity and preferred equity data come from the FR Y-9C, and market capitalization data is based on CRSP data. All variables converted to 2012 Q1 dollars using the seasonally-adjusted GDP deflator. We show the book equity (Equity), preferred equity, and the aggregate market capitalization, i.e. shares outstanding times the share price, of Bank of America, Citigroup, J.P. Morgan, and Wells Fargo.

Figure 18: Book and Market Leverage of Bank Holding Companies

Notes: These figures show data on book and market leverage for the four largest BHCs. Book leverage is computed as assets/book equity, and market leverage is computed as (liabilities + market equity)/market equity. The aggregate leverage ratios are computed as (aggregate liabilities + aggregate book equity)/aggregate book equity.
B  Additional Impulse Responses

B.1  Risk Adjustment

For our main impulse response results, we wish to use risk-adjusted returns, rather than raw returns. More formally, we assume that the market returns of bank \( i \) at time \( t \) are given by

\[
\frac{r_{it}}{r_{it}^f} = \alpha_i + X_t \beta_i + \varepsilon_{i,t}
\]

Raw Return  Risk-Free Rate  factors loadings  Idiosyncratic Component

All returns are logged, e.g. \( r_{it} \) refers to \( \log(1 + \text{Raw Bank Return}) \). We wish to isolate variation in the idiosyncratic shocks, \( \varepsilon_{i,t} \), and use this variation to estimate the impulse responses.

A natural, but naive, approach would be to estimate the above model for each bank \( i \) using OLS, and then use the estimated residuals, \( \hat{\varepsilon}_{i,t} \), as the regressors in the impulse response estimation. The problem here is that it induces bias: \( \hat{\varepsilon}_{i,t} \) is a noisy measure of the true regressor \( \varepsilon_{i,t} \), which leads to bias as long as \( T \) is finite (the bias will shrink as \( T \) grows large, because \( \hat{\varepsilon}_{i,t} \) will converge to the true \( \varepsilon_{i,t} \)).

Fortunately, there is a simple solution: we estimate \( \hat{\varepsilon}_{i,t} \) using OLS, and then we use \( \hat{\varepsilon}_{i,t} \) as an instrument for the unadjusted return. Since our main regressions use contemporaneous returns, twenty lags, and their interaction with a post-crisis dummy, this means we use contemporaneous \( \hat{\varepsilon}_{i,t} \), twenty lags of \( \hat{\varepsilon}_{i,t} \), and their interaction with a post-crisis dummy as instruments. Instrumental variables does not suffer from the same problem of bias under classical measurement error. Instead, to get identification under the assumed model for returns, we need our instrument to be correlated with the “good variation”, \( \varepsilon_{i,t} \), and uncorrelated with the “bad variation,” \( \alpha_i + X_t \beta_i \). This is mechanically what we are doing when we run OLS at the bank level, and if the assumed model for returns is correct, then we have

\[
\mathbb{E} [\hat{\eta}_t (\alpha_i + X_t \beta_i)] = \alpha_i \mathbb{E} [\hat{\eta}_t] + \mathbb{E} [\hat{\eta}_t X_t] \beta_i = 0 + 0.
\]

Thus, our instrumental variables strategy will give us a consistent estimator of the true impulse response, under the assumption that we have the correct model of returns. Since the OLS regression estimating \( \hat{\varepsilon}_{i,t} \) is conducted at the bank level, we cluster our standard errors at the bank level (clustering at the bank level is already a good idea).

B.2  Results without factor risk adjustment

While we favor the risk-adjusted results, we also have computed “unadjusted results” for the impulse responses, which we report here for completeness. The results are qualitatively and quantitatively similar across the methods. Compared to the risk-adjusted results, however, the unadjusted results, suggest a smaller response of liabilities in the pre-crisis period, and thus also suggest a slower pre-crisis adjustment of leverage.
Figure 19: Estimated Impulse Responses for Stock Variables (No Risk Adjustment)

Notes: These figures show estimated impulse response functions for BHCs. The figures show the estimated impulse response to a one unit negative returns shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The “post-crisis” period begins in 2007 Q4. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. The panels display the impulse responses of log liabilities, log market capitalization, log market leverage, and log book equity. Market leverage is defined as log(Liabilities/Market Capitalization), so that it represents the difference between the response of log liabilities and log market capitalization (results using log(Liabilities+Market Capitalization)/Market Capitalization) are extremely similar).
Notes: This figure plots the idiosyncratic shocks (for the Big Four BHCs) used to estimate the impulse response functions. First, we isolate the idiosyncratic component of returns using the factor model, and then we residualize this on time fixed effects.

B.3 Robustness and Validity of Identification Strategy

In this section, we conduct various tests to check the validity of our identification strategy and robustness of our results.

A narrative approach to corroborate the idiosyncratic shocks To provide corroborating evidence of the validity of our identification strategy, we first show that the estimated return shocks do indeed look like idiosyncratic shocks for the four largest banks (Bank of America, J.P. Morgan Chase, Wells Fargo, Citigroup). To construct the idiosyncratic shocks, we regress each bank’s market return on the Fama-French three-factor returns and regress the residual further on time fixed effects. The residuals from this regression represent our idiosyncratic shocks. Figure 20 presents our estimates of the idiosyncratic shocks. They indeed look like white noise and do not seem to be substantially autocorrelated. Note that the time series for Citigroup starts a little later because Citigroup did not exist until 1998 when Traveler’s merged with Citicorp.

We also provide narrative support for the idiosyncratic nature of our estimated shocks using an extensive search of newspaper articles for large idiosyncratic shock value estimates.

\footnote{We are controlling for the time fixed effects, because they are included in the regression we actually run to get the impulse response function.}
<table>
<thead>
<tr>
<th>Bank Name</th>
<th>Year-Qtr</th>
<th>idiosyncratic shock</th>
<th>Bank specific events</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank of America</td>
<td>2000q4</td>
<td>-0.200</td>
<td>Sunbeam (which BofA lended to) posted $86M loss. BofA said net charge-offs in Q4 will double. BofA issues warning on $1B uncollectible debt, may miss the December quarter profit forecast by as much as 27%.</td>
</tr>
<tr>
<td></td>
<td>2003q4</td>
<td>-0.218</td>
<td>BofA agrees to pay $47 to buy FleetBoston Financial &quot;hefty premium&quot; &amp; &quot;could dilute earnings.&quot;</td>
</tr>
<tr>
<td></td>
<td>2008q3</td>
<td>0.288</td>
<td>BofA to buy Merrill for $50B (Sept 15)</td>
</tr>
<tr>
<td></td>
<td>2009q2</td>
<td>0.452</td>
<td>Stress test: BofA needs to address $34B capital shortfall, better than expectation.</td>
</tr>
<tr>
<td></td>
<td>2011q4</td>
<td>-0.275</td>
<td>Merrill Lynch has agreed to pay $315 million to end a mortgage-securities lawsuit (Dec 7)</td>
</tr>
<tr>
<td></td>
<td>2012q4</td>
<td>0.248</td>
<td>BofA considered better buy after increase in house prices that (given its portfolio composition) particularly benefited BofA.</td>
</tr>
<tr>
<td>Citigroup</td>
<td>1999q1</td>
<td>0.319</td>
<td>Citigroup Profit Fell 53% in 4th period, but still topped analysts’ expectations</td>
</tr>
<tr>
<td></td>
<td>1999q3</td>
<td>0.205</td>
<td>Citigroup posts an unexpected increase of 9.3% in net income for second quarter (July 20)</td>
</tr>
<tr>
<td></td>
<td>1999q4</td>
<td>0.250</td>
<td>Citigroup’s citibank unit is marketing credit card for the internet to millions</td>
</tr>
<tr>
<td></td>
<td>2000q1</td>
<td>0.226</td>
<td>Citi Intelligent Technology Receives Investment; Dividends increase from $1.05 to $1.20</td>
</tr>
<tr>
<td></td>
<td>2000q3</td>
<td>0.280</td>
<td>Citi to repay certain funds $16 mln plus interest; Citigroup Asset Management faces federal probe.</td>
</tr>
<tr>
<td></td>
<td>2000q4</td>
<td>0.356</td>
<td>Citigroup had $2B in direct gross exposure to LyondellBasell Industries, who filed for bankruptcy protection last week. Fitch cuts Citi preferred to junk</td>
</tr>
<tr>
<td></td>
<td>2000q5</td>
<td>0.199</td>
<td>Citi reports profit after gain from Smith Barney. Citigroup’s mortgage mitigation rises 29% in second quarter.</td>
</tr>
<tr>
<td></td>
<td>2000q4</td>
<td>0.267</td>
<td>Citi fined in tax crackdown. Abu Dhabi’s sovereign wealth fund is demanding that Citigroup scraps a deal that would see the fund make a heavy loss on a $7.5 billion investment in the bank.</td>
</tr>
<tr>
<td></td>
<td>2010q2</td>
<td>0.285</td>
<td>Citi reported quarterly earnings of $4.4B exceeding expectations</td>
</tr>
<tr>
<td></td>
<td>2000q1</td>
<td>0.169</td>
<td>J.P. Morgan told investors on Monday that January and February had topped performance levels seen in the fourth quarter. Dividends increase from $0.2733 to $0.3200 on March 21.</td>
</tr>
<tr>
<td></td>
<td>2000q3</td>
<td>0.357</td>
<td>Chase buying J.P. Morgan.</td>
</tr>
<tr>
<td></td>
<td>2001q4</td>
<td>-0.185</td>
<td>J.P. Morgan Chase disclosed this week that their venture capital portfolios had incurred significant losses.</td>
</tr>
<tr>
<td></td>
<td>2002q3</td>
<td>-0.322</td>
<td>JPMorgan Partners Reports $165M Operating Loss for Q2. J.P. Morgan sees third-quarter shortfall.</td>
</tr>
<tr>
<td></td>
<td>2003q4</td>
<td>-0.198</td>
<td>JPMorgan Chase profit falls 13%.</td>
</tr>
<tr>
<td></td>
<td>2008q3</td>
<td>0.234</td>
<td>J.P. Morgan profit falls 53%, but tops Wall Street target.</td>
</tr>
<tr>
<td></td>
<td>2009q1</td>
<td>0.249</td>
<td>J.P. Morgan net falls sharply, but tops Wall Street view. J.P. Morgan to sell Bear Wagner to Barclays Capital: WSJ</td>
</tr>
<tr>
<td></td>
<td>2012q2</td>
<td>-0.207</td>
<td>J.P. Morgan: London Whales $2 Billion Losses. Two Shareholder Suits Filed Against J.P. Morgan</td>
</tr>
<tr>
<td>Wells Fargo</td>
<td>2001q4</td>
<td>-0.161</td>
<td>Wells Fargo disclosed that their venture capital portfolios had incurred significant losses. Wells Fargo to take $1.1 billion charge</td>
</tr>
<tr>
<td></td>
<td>2008q3</td>
<td>0.338</td>
<td>Wells Fargo’s net dropped 21% as it set aside $3 billion for loan losses, better than expected. Earnings declined but beat estimates.</td>
</tr>
<tr>
<td></td>
<td>2009q1</td>
<td>-0.315</td>
<td>Wells Fargo posted a surprise $2.55B Q1 loss, later revised to $2.77B. Wells Fargo added a pretax $328.4M impairment of perpetual preferred securities to its fourth-quarter loss.</td>
</tr>
<tr>
<td></td>
<td>2009q2</td>
<td>0.405</td>
<td>Wells Fargo sees record Q1 profit, projections easily exceed expectations (expects earnings of $3 billion).</td>
</tr>
</tbody>
</table>
Table 4 shows that large absolute idiosyncratic shock values are consistent with good or bad bank specific events, such as “Wells Fargo sees record Q1 profit, projections easily exceed expectations,” or “Citi fined in tax crackdown.” The table shows that large positive or negative idiosyncratic shocks can be corroborated with specific events that appear bank specific, which supports the validity of our identification strategy.

Placebo Tests To test the validity of our identification strategy, we conduct placebo tests where we include ten leads of returns (in addition to the contemporary value and twenty lags as before). If the returns really are unanticipated shocks, then the leading values should not affect current behavior. This is similar to testing for pre-trends. We are testing whether the banks that will experience higher returns in the future are already acting differently today. Overall, the placebo test are encouraging, and suggest that our results are not driven by prior differences in the behavior of banks which experience return shocks.

Identification robustness We provide a few additional pieces of evidence that corroborate the validity and robustness of our identification strategy.

First, we verify that our results are robust to excluding the crisis years 2008 and 2009 from our sample. The idea is to rule out a lot of stories related to specific events during the crisis (e.g. the realization that the government might not guarantee that a bank wouldn’t fail, or that this was somehow about exposure to Lehman). The results are below (for our main outcomes: Liabilities, Market Cap, and Market Leverage). It makes no noticeable difference to the results.

Second, we check whether bank mergers drive the results. To this end, we drop the quarter of the merger as well as the quarter before and after the merger. The results for our main outcomes: liabilities, market equity, and market leverage are in Figure 23. Again, it makes no noticeable difference to the results.

Similarly, we check whether the results are driven by the stress tests performed by banks: these stress tests were implemented after the onset of the crisis, and encouraged or mandated that banks raise additional capital. To show that the stress tests do not drive the results, we drop all banks that ever participated in a stress test (e.g. Bank of America participated in the stress tests, and so we drop Bank of America from our sample in all periods). The results for our main outcomes are in Figure 24. Again, it makes no noticeable difference to the results.

Another potential concern is that the return shocks could be picking up shocks to future investment opportunities, rather than default shocks. To test this concern, we check the response of the liquid assets ratio: if negative return shocks indeed predict lower future investment opportunities rather than current cash flows, we would expect banks to respond to these shocks by moving their portfolio into liquid assets. The results, shown in Figure 25, show no statistically significant response of liquid assets pre-crisis, and a small temporary response post crisis that is reversed within a few quarters. We take this as evidence against the hypothesis that return shocks reflect shocks to investment opportunities.

Am alternative, broader version of the liquidity ratio test calculates the liquidity ratio as the ratio of (Cash + Federal Funds Sold + Securities Purchased Under Agreement to Resell + Securities)/Total Assets. We display the impulse response function for this version of the liquidity ratio in Figure 26. The impulse response function has no significant response pre-crisis, and a significant but quantitatively small response post-crisis.

To put the size of the post-crisis response in perspective, the graph is saying that if there is a 10% negative shock to market returns, then the liquid asset ratio would rise by 0.02 over the course of two years. This is off of a base of 0.25-0.30, depending on whether we are taking the mean of log(1+ratio) or of the raw liquid assets ratio.
Figure 21: Estimated Impulse Responses for Stock Variables (Risk-Adjusted, with Placebo)

Notes: These figures show estimated impulse response functions for BHCs. The figures show the estimated impulse response to a one unit negative returns shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The “post-crisis” period begins in 2007 Q4. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. The panels display the impulse responses of log liabilities, log market capitalization, log market leverage, and log book equity. Market leverage is defined as log(Liabilities/Market Capitalization), so that it represents the difference between the response of log liabilities and log market capitalization (results using log(Liabilities+Market Capitalization)/Market Capitalization) are extremely similar).
Figure 22: Estimated Impulse Responses: Dropping 2007 and 2008

Notes: These figures show estimated impulse response functions for BHCs, dropping observations from the years 2007 and 2008. The figures show the estimated impulse response to a one unit negative returns shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The “post-crisis” period begins in 2007 Q4. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. The panels display the impulse responses of log liabilities, log market capitalization, log market leverage, and log book equity. Market leverage is defined as log(Liabilities/Market Capitalization), so that it represents the difference between the response of log liabilities and log market capitalization.
Notes: These figures show estimated impulse response functions for BHCs, dropping observations from quarters in which the bank is recorded as taking part in a merger, as well as dropping the quarter before and the quarter after. The figures show the estimated impulse response to a one unit negative returns shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The “post-crisis” period begins in 2007 Q4. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. The panels display the impulse responses of log liabilities, log market capitalization, log market leverage, and log book equity. Market leverage is defined as log(Liabilities/Market Capitalization), so that it represents the difference between the response of log liabilities and log market capitalization.
Notes: These figures show estimated impulse response functions for BHCs, dropping observations from quarters in which the bank is recorded as taking part in a merger, as well as dropping the quarter before and the quarter after. The figures show the estimated impulse response to a one unit negative returns shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The “post-crisis” period begins in 2007 Q4. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. The panels display the impulse responses of log liabilities, log market capitalization, log market leverage, and log book equity. Market leverage is defined as log(Liabilities/Market Capitalization), so that it represents the difference between the response of log liabilities and log market capitalization.
Figure 25: Estimated Impulse Responses of the Liquidity Ratio

Notes: This figure shows the estimated impulse response function for BHCs to a 1% negative return shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The “post-crisis” period begins in 2007 Q4. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. The liquid assets ratio is defined as $\log((\text{Cash} + \text{Treasury Bills})/\text{Total Assets})$. Within the regression sample, the average liquid assets ratio is 0.057.

Figure 26: Estimated Impulse Responses of Liquidity Ratios (Alternative Formula)

Notes: This figure shows estimated impulse response functions for BHCs. The figure shows the estimated impulse response to a one unit negative returns shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The “post-crisis” period begins in 2007 Q4. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. The liquid assets ratio is defined as $\log((\text{Cash} + \text{Fed Funds Sold} + \text{Securities Purchased Under Agreement to Resell} + \text{Securities})/\text{Total Assets})$. 
B.4 Heterogeneity

We explore heterogeneity in impulse response functions by dividing banks into two groups based on a variable, and estimating impulse responses separately for each group. We divide banks by size (total assets), by trading assets ratio (trading assets as a share of total assets), by the risk-weighted asset ratio (risk-weighted assets as a share of total assets), and by the mortgage ratio (real estate loans as a share of total assets). We use the value of the variable in 2000 Q1 to sort banks into two groups: above-median and below-median. We report the results in this section. Broadly, we do not find strong evidence of differential responses, but we lack statistical power to rule out some meaningful differences.

Since bank size is among the most important differences across different banks, we begin by discussing the results for heterogeneity by size. The results are shown in Figures 27 and 28. Visually, these impulse responses look remarkably similar to each other. However, the standard errors are sufficiently large that we cannot rule out meaningful differences in the impulse responses.

We summarize the results of these impulse responses, as well as of the other potential groupings (by trading assets ratio, risk-weighted assets ratio, and mortgage ratio) in Tables 5, 6, 7, and 8 below. For each grouping, we report the cumulative impulse response for the high and low groups after 10 quarters and after 20 quarters, and we also report the p-value of a test of equality between the impulse responses of the two groups. In a table of 64 tests, only one of the tests rejects the null at the 5% level. As before, we take this to suggest that there is not strong evidence in favor of sizable heterogeneity, but we caution that the standard errors are too large to rule out meaningful heterogeneity.
Notes: These figures show estimated impulse response functions for BHCs. The figures show the estimated impulse response to a one unit negative returns shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The “post-crisis” period begins in 2007 Q4. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. The panels display the impulse responses of log liabilities, log market capitalization, log market leverage, and log book equity. Market leverage is defined as log(Liabilities/Market Capitalization), so that it represents the difference between the response of log liabilities and log market capitalization (results using log(Liabilities+Market Capitalization)/Market Capitalization) are extremely similar).
Figure 28: Impulse Responses for Large Banks

Notes: These figures show estimated impulse response functions for BHCs. The figures show the estimated impulse response to a one unit negative returns shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The “post-crisis” period begins in 2007 Q4. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. The panels display the impulse responses of log liabilities, log market capitalization, log market leverage, and log book equity. Market leverage is defined as log(Liabilities/Market Capitalization), so that it represents the difference between the response of log liabilities and log market capitalization (results using log(Liabilities+Market Capitalization)/Market Capitalization) are extremely similar).
<table>
<thead>
<tr>
<th></th>
<th>Response After 10 Quarters</th>
<th></th>
<th>Response After 20 Quarters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Small</td>
<td>Large</td>
<td>p-value on Equality</td>
<td>Small</td>
</tr>
<tr>
<td><strong>Market Equity</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-Crisis</td>
<td>-1.13</td>
<td>-1.09</td>
<td>0.75</td>
<td>-1.22</td>
</tr>
<tr>
<td></td>
<td>(0.08)</td>
<td>(0.07)</td>
<td></td>
<td>(0.12)</td>
</tr>
<tr>
<td>Post-Crisis</td>
<td>-0.71</td>
<td>-0.76</td>
<td>0.71</td>
<td>-0.58</td>
</tr>
<tr>
<td></td>
<td>(0.14)</td>
<td>(0.06)</td>
<td></td>
<td>(0.14)</td>
</tr>
<tr>
<td><strong>Liabilities</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-Crisis</td>
<td>-0.42</td>
<td>-0.33</td>
<td>0.39</td>
<td>-0.65</td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td>(0.07)</td>
<td></td>
<td>(0.13)</td>
</tr>
<tr>
<td>Post-Crisis</td>
<td>-0.13</td>
<td>-0.15</td>
<td>0.49</td>
<td>-0.23</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.02)</td>
<td></td>
<td>(0.05)</td>
</tr>
<tr>
<td><strong>Market Leverage</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-Crisis</td>
<td>0.71</td>
<td>0.76</td>
<td>0.59</td>
<td>0.57</td>
</tr>
<tr>
<td></td>
<td>(0.08)</td>
<td>(0.05)</td>
<td></td>
<td>(0.11)</td>
</tr>
<tr>
<td>Post-Crisis</td>
<td>0.58</td>
<td>0.61</td>
<td>0.81</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>(0.13)</td>
<td>(0.06)</td>
<td></td>
<td>(0.11)</td>
</tr>
<tr>
<td><strong>Book Equity</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-Crisis</td>
<td>-0.25</td>
<td>-0.31</td>
<td>0.68</td>
<td>-0.29</td>
</tr>
<tr>
<td></td>
<td>(0.12)</td>
<td>(0.08)</td>
<td></td>
<td>(0.23)</td>
</tr>
<tr>
<td>Post-Crisis</td>
<td>-0.78</td>
<td>-0.44</td>
<td>0.04</td>
<td>-0.73</td>
</tr>
<tr>
<td></td>
<td>(0.15)</td>
<td>(0.09)</td>
<td></td>
<td>(0.20)</td>
</tr>
</tbody>
</table>

*Notes:* The table compares impulse responses of small vs. large BHCs. BHCs are categorized into the small vs. large group based on their total assets in 2000 Q1, relative to the median for all banks in the IRF sample. The first column shows the cumulative impulse response after 10 quarters of each variable, pre-crisis and post-crisis, to a one unit negative return shock, for small banks. The second column shows the same results, but for large banks. Standard errors, clustered at the bank level, are in parentheses. The third column shows the p-value of a test of equality between the impulse response for small banks vs. large banks. The fourth through sixth columns mirror the first three columns, but examining the cumulative impulse response after 20 quarters.
<table>
<thead>
<tr>
<th></th>
<th>Response After 10 Quarters</th>
<th></th>
<th>Response After 20 Quarters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low</td>
<td>High</td>
<td>p-value on Equality</td>
<td>Low</td>
</tr>
<tr>
<td><strong>Market Equity</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-Crisis</td>
<td>-1.10</td>
<td>-1.20</td>
<td>0.60</td>
<td>-1.15</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.19)</td>
<td></td>
<td>(0.07)</td>
</tr>
<tr>
<td>Post-Crisis</td>
<td>-0.76</td>
<td>-0.57</td>
<td>0.11</td>
<td>-0.60</td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td>(0.08)</td>
<td></td>
<td>(0.10)</td>
</tr>
<tr>
<td><strong>Liabilities</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-Crisis</td>
<td>-0.36</td>
<td>-0.36</td>
<td>0.99</td>
<td>-0.56</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.14)</td>
<td></td>
<td>(0.08)</td>
</tr>
<tr>
<td>Post-Crisis</td>
<td>-0.14</td>
<td>-0.15</td>
<td>0.73</td>
<td>-0.24</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.04)</td>
<td></td>
<td>(0.03)</td>
</tr>
<tr>
<td><strong>Market Leverage</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-Crisis</td>
<td>0.74</td>
<td>0.84</td>
<td>0.35</td>
<td>0.59</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.10)</td>
<td></td>
<td>(0.07)</td>
</tr>
<tr>
<td>Post-Crisis</td>
<td>0.63</td>
<td>0.42</td>
<td>0.08</td>
<td>0.37</td>
</tr>
<tr>
<td></td>
<td>(0.08)</td>
<td>(0.09)</td>
<td></td>
<td>(0.08)</td>
</tr>
<tr>
<td><strong>Book Equity</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-Crisis</td>
<td>-0.25</td>
<td>-0.42</td>
<td>0.35</td>
<td>-0.28</td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td>(0.17)</td>
<td></td>
<td>(0.13)</td>
</tr>
<tr>
<td>Post-Crisis</td>
<td>-0.62</td>
<td>-0.45</td>
<td>0.39</td>
<td>-0.66</td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td>(0.16)</td>
<td></td>
<td>(0.13)</td>
</tr>
</tbody>
</table>

Notes: The table compares impulse responses of low vs. high trading asset ratio BHCs. BHCs are categorized into the low vs. high group based on their trading assets as a share of total assets in 2000 Q1, relative to the median for all banks in the IRF sample. The first column shows the cumulative impulse response after 10 quarters of each variable, pre-crisis and post-crisis, to a one unit negative return shock, for low banks. The second column shows the same results, but for high banks. Standard errors, clustered at the bank level, are in parentheses. The third column shows the p-value of a test of equality between the impulse response for low vs. high banks. The fourth through sixth columns mirror the first three columns, but examining the cumulative impulse response after 20 quarters.
Table 7: Heterogeneity in Impulse Responses: Low vs. High Risk-Weighted Asset Ratio

<table>
<thead>
<tr>
<th></th>
<th>Response After 10 Quarters</th>
<th>Response After 20 Quarters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td><strong>Market Equity</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-Crisis</td>
<td>-1.09</td>
<td>-1.15</td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>Post-Crisis</td>
<td>-0.72</td>
<td>-0.79</td>
</tr>
<tr>
<td></td>
<td>(0.12)</td>
<td>(0.09)</td>
</tr>
<tr>
<td><strong>Liabilities</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-Crisis</td>
<td>-0.29</td>
<td>-0.41</td>
</tr>
<tr>
<td></td>
<td>(0.06)</td>
<td>(0.07)</td>
</tr>
<tr>
<td>Post-Crisis</td>
<td>-0.13</td>
<td>-0.17</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.02)</td>
</tr>
<tr>
<td><strong>Market Leverage</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-Crisis</td>
<td>0.80</td>
<td>0.73</td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td>(0.06)</td>
</tr>
<tr>
<td>Post-Crisis</td>
<td>0.59</td>
<td>0.62</td>
</tr>
<tr>
<td></td>
<td>(0.11)</td>
<td>(0.09)</td>
</tr>
<tr>
<td><strong>Book Equity</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-Crisis</td>
<td>-0.19</td>
<td>-0.35</td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>Post-Crisis</td>
<td>-0.49</td>
<td>-0.74</td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td>(0.16)</td>
</tr>
</tbody>
</table>

Notes: The table compares impulse responses of low vs. high risk-weighted asset ratio BHCs. BHCs are categorized into the low vs. high group based on their risk-weighted assets as a share of total assets in 2000 Q1, relative to the median for all banks in the IRF sample. The first column shows the cumulative impulse response after 10 quarters of each variable, pre-crisis and post-crisis, to a one unit negative return shock, for low banks. The second column shows the same results, but for high banks. Standard errors, clustered at the bank level, are in parentheses. The third column shows the p-value of a test of equality between the impulse response for low vs. high banks. The fourth through sixth columns mirror the first three columns, but examining the cumulative impulse response after 20 quarters.
### Table 8: Heterogeneity in Impulse Responses: Low vs. High Mortgage Ratio

<table>
<thead>
<tr>
<th></th>
<th>Response After 10 Quarters</th>
<th></th>
<th>Response After 20 Quarters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low</td>
<td>High</td>
<td>p-value on Equality</td>
<td>Low</td>
</tr>
<tr>
<td>Market Equity</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-Crisis</td>
<td>-1.04</td>
<td>-1.21</td>
<td>0.17</td>
<td>-1.07</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.11)</td>
<td></td>
<td>(0.07)</td>
</tr>
<tr>
<td>Post-Crisis</td>
<td>-0.75</td>
<td>-0.75</td>
<td>0.98</td>
<td>-0.61</td>
</tr>
<tr>
<td></td>
<td>(0.13)</td>
<td>(0.08)</td>
<td></td>
<td>(0.17)</td>
</tr>
<tr>
<td>Liabilities</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-Crisis</td>
<td>-0.28</td>
<td>-0.46</td>
<td>0.11</td>
<td>-0.45</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.10)</td>
<td></td>
<td>(0.08)</td>
</tr>
<tr>
<td>Post-Crisis</td>
<td>-0.17</td>
<td>-0.11</td>
<td>0.09</td>
<td>-0.28</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.02)</td>
<td></td>
<td>(0.05)</td>
</tr>
<tr>
<td>Market Leverage</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-Crisis</td>
<td>0.76</td>
<td>0.75</td>
<td>0.92</td>
<td>0.62</td>
</tr>
<tr>
<td></td>
<td>(0.06)</td>
<td>(0.07)</td>
<td></td>
<td>(0.08)</td>
</tr>
<tr>
<td>Post-Crisis</td>
<td>0.59</td>
<td>0.64</td>
<td>0.72</td>
<td>0.34</td>
</tr>
<tr>
<td></td>
<td>(0.11)</td>
<td>(0.09)</td>
<td></td>
<td>(0.13)</td>
</tr>
<tr>
<td>Book Equity</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-Crisis</td>
<td>-0.20</td>
<td>-0.36</td>
<td>0.32</td>
<td>-0.27</td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td>(0.15)</td>
<td></td>
<td>(0.09)</td>
</tr>
<tr>
<td>Post-Crisis</td>
<td>-0.66</td>
<td>-0.56</td>
<td>0.57</td>
<td>-0.70</td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td>(0.14)</td>
<td></td>
<td>(0.13)</td>
</tr>
</tbody>
</table>

**Notes:** The table compares impulse responses of low vs. high mortgage ratio BHCs. BHCs are categorized into the low vs. high group based on their real estate loans as a share of total assets in 2000 Q1, relative to the median for all banks in the IRF sample. The first column shows the cumulative impulse response after 10 quarters of each variable, pre-crisis and post-crisis, to a one unit negative return shock, for low banks. The second column shows the same results, but for high banks. The fourth through sixth columns mirror the first three columns, but examining the cumulative impulse response after 20 quarters.
C Model Appendix

C.1 Additional Model discussion

Timing. To clarify the assumption, Figure 29 plots a sample path of \( dN \) and the right panel plots a path of \( \lambda \). Assume that the bank decides to set leverage to a constant, prior and after a default, and that it starts with \( z = 0 \). The Figure depicts a hypothetical scenario of a recognized default event that occurs at time \( t^* \). On the right panel, the solid line depicts the continuous path of \( \lambda \), but the discontinuity point represents the leverage ratio after the jump. In this example, \( \lambda + J^\lambda > \Xi \), so the bank is immediately liquidated even though it could have sold loans to return back to a constant leverage path. Hence, even though the violation is for an infinitesimal period of time, the bank is intervened.\(^{53}\)

Figure 30: Model Stationary Distribution of Banks Across the \( z \) and \( \lambda \) State Space

Notes: This figure presents a two dimensional histogram of the stationary distribution of banks across the \((\lambda, z)\) space along with the regulatory liquidation region in gray.

Model Stationary distribution. The distribution of the state variable \( z \) and fundamental leverage \( \lambda \) is reported in Figure 30. This figure shows how the cross-sectional distribution of \( \{\lambda, z\} \) traces out a shadow boundary. The grey area of the figure represents the liquidation region.

\(^{53}\)This assumption is equivalent to the discrete time assumption that the shock occurs between periods. It can also be obtained as a limit process, where we have adjustment costs to selling loans that are taken to zero.
The Role of the IES for the Quantitative Fit. In Section 4.1 of the main text, we argue that the delayed loan loss recognition mechanism is driving the slow adjustment of banks to net-worth shocks. Since the bankers’ preference imply an intertemporal smoothing incentive whenever $\theta > 0$, one might worry that instead what drives the slow adjustment is $\theta$. Hence, we investigate the role of the IES ($1/\theta$) for the quantitative results in two ways. First, we solve the model for the same parameter configuration as in the baseline model (see Section 4), except setting $\theta = 1$, and reestimate the IRF on the model generated data. This calibration implies significantly lower intertemporal smoothing incentives. Second, we solve the model with the benchmark calibration but shut down the delayed loan loss recognition by setting $p = 0$. The results are in Figure 31.

The blue line presents the impulse response functions of the model for the case when $\theta = 1$. Relative to Figure 13, the IRF still shows substantially slow adjustment to a negative net-worth shock. The red line presents the impulse response functions in the case of immediate loan loss recognition, $p = 0$, and all parameters as in the benchmark calibration, hence $\theta = 1.98$. Clearly, the IRFs feature immediate adjustment in market equity and liabilities. Tobin’s Q and market leverage do not respond at all. This version of the model shows that it is the delayed loan loss recognition mechanism and not $\theta$ that drives the impulse response functions of the model.

Figure 31: Data IRFs versus Model IRFs from two Versions ($\theta = 1$ and $p = 0$)

Notes: .

27
C.2 Derivations and Proofs

Summary Table. The summary table of the drift and jump terms as functions of the ratios \(\{\lambda, z, q\}\) is the following:

<table>
<thead>
<tr>
<th>Definition</th>
<th>Variable</th>
<th>Drift (\mu_x^x)</th>
<th>Jump (J^x_x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fundamental Equity</td>
<td>(W)</td>
<td>(r^L\lambda - r^D (\lambda - 1) - c) (W)</td>
<td>(-\varepsilon\lambda W)</td>
</tr>
<tr>
<td>Loans</td>
<td>(L)</td>
<td>(\iota\lambda W)</td>
<td>(-\varepsilon\lambda W)</td>
</tr>
<tr>
<td>Deposits</td>
<td>(D)</td>
<td>(r^D (\lambda - 1) - r^L\lambda + \iota\lambda + c) (W)</td>
<td>0</td>
</tr>
<tr>
<td>Book Loans</td>
<td>(L)</td>
<td>((\iota - \alpha z)\lambda W)</td>
<td>(-\tau\varepsilon\frac{1}{q} (\lambda + 1) W)</td>
</tr>
<tr>
<td>Zombie Loans</td>
<td>(Z)</td>
<td>(-\alpha Z)</td>
<td>(-\varepsilon\lambda W)</td>
</tr>
<tr>
<td>Leverage</td>
<td>(\lambda)</td>
<td>((\iota - \mu W)\lambda)</td>
<td>(\frac{2\lambda}{1-\varepsilon}) ((\lambda - 1))</td>
</tr>
<tr>
<td>Fundamental Equity</td>
<td>(z)</td>
<td>(-z (\alpha + \mu W))</td>
<td>(-\varepsilon\lambda q \frac{z}{1-\varepsilon\lambda + z} ) if recognized</td>
</tr>
<tr>
<td>Zombie Loans</td>
<td>(z)</td>
<td>(-z (\alpha + \mu W))</td>
<td>(\varepsilon\lambda q \frac{z}{1-\varepsilon\lambda + z} ) if unrecognized</td>
</tr>
<tr>
<td>Book Equity</td>
<td>(q)</td>
<td>(\frac{\alpha z}{(1+z)^2})</td>
<td>(-\varepsilon\lambda q ) if unrecognized</td>
</tr>
</tbody>
</table>

Notation and Definitions. We begin by presenting some definitions and deriving the laws of motion of the state variables. We use \(\mu^x\) and \(J^x\) to refer to the drift and jump components of the path of a variable \(x\) scaled by wealth \(W\), respectively. Along a continuous path, the net investment rate of the bank is:

\[ \iota \equiv \frac{I}{L} - \delta \]

and we express the dividend-to-equity ratio as:

\[ c \equiv \frac{C}{W}. \]

Note that the following identities allow us to recover the original state variables \(\{L, \bar{L}, D\}\) from the triplet \(\{\lambda, z, W\}\):

\[ L = \lambda \cdot W \]  
(14)
\[ D = (\lambda - 1) \cdot W \]  
(15)
\[ \bar{L} = \lambda W + zW. \]  
(16)
\[ \bar{W} = W + zW. \]  
(17)

We present some observations that aid the proof of the proposition.

Observation 1: homogeneity in \(W\) of constraints. We want to express the regulatory capital requirement in terms of the end-of-period choices \((\lambda, z)\). The regulatory constraint is

\[ \bar{L} \leq \Xi \cdot \bar{W} \iff \lambda \leq \Xi, \]  
(18)

as we noted in the main body of the text, where

\[ \bar{\lambda} \equiv \frac{\bar{L}}{\bar{W}} = \frac{\lambda W + Z}{W + Z}. \]  
(19)
Dividing both sides by \( W \), we obtain that:

\[
\bar{\lambda} = \frac{\lambda + z}{1 + z}.
\]

Therefore, combining (19) with (18), we obtain:

\[
\frac{\lambda + z}{1 + z} \leq \Xi \Rightarrow \lambda \leq \Xi + (\Xi - 1) z.
\]

Next, consider the market based constraint:

\[
W - \varepsilon L \geq 0 \quad (20)
\]

which can be written as:

\[
\lambda \leq \frac{1}{\varepsilon}. \quad (21)
\]

Hence, we summarize the set of states where the bank is not liquidated by:

\[
\lambda \leq \Gamma (z) = \min \left\{ \frac{1}{\varepsilon}, \Xi + (\Xi - 1) z \right\}. \quad (22)
\]

**Observation 2: derivations of laws of motion.** Here, we provide an explicit derivation of the law of motion of bank equity, starting from a discrete time formulation. With probability \( \sigma \) over interval \( \Delta \), the bank receives deterministic default shock \( \varepsilon < 1 \). Let:

\[
dN = \begin{cases} 
0 & \text{with prob } 1 - \sigma dt \\
1 & \text{with prob } \sigma dt
\end{cases}
\]

denote a default event process. Recall that \( dN \) is a Poisson process.

Now consider a time interval of length \( \Delta \). The law of motion for fundamental loans satisfies:

\[
L_{t+\Delta} = (1 - \delta \Delta) L_t + I_t \Delta - \varepsilon L_t (N_{t+\Delta} - N_t),
\]

with the interpretation that the first term is the non-maturing fraction of loans, the second are loan issuances, and the third are losses in a time interval. Taking \( \Delta \to 0 \), we obtain the following law of motion:

\[
dL = (I - \delta L) dt - \varepsilon L dN.
\]

We express this law of motion in terms of net-worth, replacing (14), to obtain:

\[
dL = \iota \lambda W dt - \varepsilon \lambda W dN. \quad (23)
\]

To ease the notation, we define the growth rate of fundamental loans and the jump relative to net-worth:

\[
\mu^L \equiv \iota \lambda \text{ and } J^L \equiv -\varepsilon \lambda.
\]

Similarly, for deposits we have that:

\[
D_{t+\Delta} = \left(1 + r^D \Delta \right) D_t - \left( r^L \Delta + \delta \Delta \right) L_t + I_t \Delta + C_t \Delta
\]

with the interpretation that the first term is the increase in deposits that results from paying interest with deposits; the second term is the reduction in deposits by the interest and principal payments on outstanding loans; the third term is the increase in deposits as a result of loan issuances; and the final
term is dividend payments, all paid with deposits. Taking $\Delta \to 0$, we obtain the following law of motion:

$$dD = [r^D D - (r^L + \delta) L + I + C] \, dt.$$  

We express this law of motion in terms of wealth, by using (15), to obtain:

$$dD = [r^D (\lambda - 1) - r^L \lambda + \iota \lambda + c] \, W \, dt. \quad (24)$$

We define the growth rate of deposits relative to net-worth:

$$\mu^D \equiv r^D (\lambda - 1) - r^L \lambda + \iota \lambda + c \quad \text{and} \quad J^D = 0.$$  

**Observation 3: growth independence.** Next, we present the evolution of fundamental equity:

$$dW = dL - dD = \left[ \mu^L - \mu^D \right] W \, dt + J^L \, dN \quad (25)$$

$$= \left[ \rho^L \lambda - r^D (\lambda - 1) - c \right] \left[ \text{levered returns} \right] W \, dt - \frac{\varepsilon \lambda}{\text{dividend rate}} W \, dN. \quad (26)$$

where the second line uses the laws of motion in (23) and (24). The interpretation of this expression is  

natural: the terms multiplying rates represent the net interest margin on the bank, which are the banks  

levered return; the second term are the capital gains that are accounted immediately as the bank creates  

an asset that can be worth more or less than a liability; the third term is the banks’ dividend rate; and  

the final term is the loss rate, which scales with leverage.

Define the drift of the growth rate of bank equity as:

$$\mu^W \equiv r^L \lambda - r^D (\lambda - 1) - c$$

and denote the jump component of wealth as:

$$J^W \equiv -\varepsilon \lambda W = J^L W.$$  

Also, note that:

$$\mu^W = dZ = -\alpha Z \, dt - \varepsilon L dN^z = -\alpha z W \, dt - \varepsilon \lambda W dN^z \cdot \mu^L - \mu^D.$$  

**Observation 4: book and zombie loans.** A default event increases the stock of zombie loans for  

with probability $(1 - p) \sigma$ over interval $\Delta$, the bank receives deterministic default shock $\varepsilon < 1$. Let:

$$dN^z = \begin{cases} 
0 & \text{with prob } 1 - \sigma dt \\
1 & \text{with prob } (1 - p) \sigma dt 
\end{cases}$$

denote a default event process. Recall that $dN^z$ is a Poisson process for the unrecognized loan default  

events with $Pr\left(dN^z = 1, dN = 1\right) = (1 - p) Pr\left(dN = 1\right)$ and $Pr\left(dN^z = 0, dN = 0\right) = 1$.  

The law of motion for zombie loans satisfies:

$$dZ = -\alpha \Delta Z - \varepsilon L_t \left( N^z_{t+\Delta} - N^z_t \right)$$

Taking $\Delta \to 0$, we obtain the following law of motion:

$$dZ = -\alpha Z \, dt - \varepsilon L dN^z = -\alpha z W \, dt - \varepsilon \lambda W dN^z. \quad (27)$$
Thus,
\[ \mu^Z \equiv -\alpha Z \text{ and } J^Z \equiv -\varepsilon \lambda W = J^L W. \]

**Observation 5: law of motion for leverage.** Next, we derive the law of motion for leverage \( \lambda \) given any choice of \( \iota \) and \( c \), along the continuous path variables. Employing the formula for the differential of a ratio we get:

\[ \mu^\lambda = \lambda \left( \frac{\mu^L W}{L} - \frac{\mu^W W}{W} \right) = \lambda \left( \frac{\iota \lambda W}{L} - \frac{\mu^W W}{W} \right) = \lambda (\iota - \mu^W). \]

Upon a default shock, the discontinuous jump in leverage is given by:

\[ J^\lambda = \frac{L - \varepsilon \lambda W}{W - \varepsilon \lambda W} - \frac{L}{W} = \left( \frac{1 - \varepsilon}{1 - \varepsilon \lambda} \right) = \varepsilon \lambda \cdot \frac{\lambda - 1}{1 - \varepsilon \lambda}. \]

Therefore, combining the drift and jump portions of the law of motion, we obtain:

\[ d\lambda = (\iota - \mu^W) \lambda dt - \varepsilon \lambda \cdot \frac{\lambda - 1}{1 - \varepsilon \lambda} dN. \]

The interpretation of this law of motion is that leverage increases with the issuance rate, falls as loans mature and falls as the bank makes earns income on its current portfolio, \( \mu^W \). We thus have:

\[ \mu^\lambda = (\iota - \mu^W) \lambda, \]

and for the jump term, we obtain

\[ J^\lambda = \varepsilon \lambda \cdot \frac{\lambda - 1}{1 - \varepsilon \lambda} = -J^W \frac{\lambda - 1}{1 - \varepsilon \lambda}. \]

Naturally, leverage jumps with defaults, and more so the more levered the bank is.

**Observation 6: law of motion of zombie ratio.** Employing the formula for the differential of a ratio we get:

\[ \mu^z = z \left( \frac{\mu^Z W}{Z} - \frac{\mu^W W}{W} \right) = z \left( -\alpha z W \frac{W}{Z} - \frac{\mu^W W}{W} \right) = -z \left( \alpha + \mu^W \right). \]

Next, we derive the two possible jumps for \( z \). We have that for a recognized default, the jump is given by:

\[ j^z = \frac{Z - \varepsilon L}{W - \varepsilon L} - z = z \left( \frac{1}{1 - \varepsilon \lambda} - 1 \right) = \varepsilon \lambda \left( \frac{z}{1 - \varepsilon \lambda} \right) = -J^W \left( \frac{z}{1 - \varepsilon \lambda} \right). \]

For an unrecognized default event, we have that:

\[ j^z = \frac{Z - \varepsilon L}{W - \varepsilon L} - z = \frac{z + \varepsilon \lambda}{1 - \varepsilon \lambda} - z = \varepsilon \lambda \left( \frac{z + 1}{1 - \varepsilon \lambda} \right) = -J^W \left( \frac{z + 1}{1 - \varepsilon \lambda} \right). \]
Observation 7: law of motion for \( q \). Next, we produce the law of motion for leverage \( q \). Recall that \( q = \frac{W}{W+Z} = \frac{1}{1+z} \).

Thus, the continuous portion of \( q \) satisfies:

\[
\mu^q = \frac{1}{1+z} \left( -\frac{\mu^z}{1+z} \right) = \frac{\alpha z}{(1+z)^2}.
\]

The jump upon a recognized default event is:

\[
\hat{J}^1 = \frac{W - \varepsilon L}{W - \varepsilon L + Z} - q = \frac{1 - \varepsilon \lambda}{1 - \varepsilon \lambda + z} - q = \frac{(1 - \varepsilon \lambda)(1 - q) - qz}{1 - \varepsilon \lambda + z} = -\varepsilon \lambda q \frac{z}{1 - \varepsilon \lambda + z} = J^w q.
\]

The jump upon an unrecognized default event is:

\[
\tilde{J}^q = \frac{W - \varepsilon L}{W - \varepsilon L + Z + \varepsilon L} - q = \frac{1 - \varepsilon \lambda}{1 + z} - q = q (1 - \varepsilon \lambda - 1) = -\varepsilon \lambda q = J^w q.
\]

Duffie-Epstein. The value function of the Duffie-Epstein satisfies:

\[
V_t = E_t \int_t^\infty f(C_s, V_s) \, ds,
\]

where the \( f \) is given by:

\[
f(C, V) \equiv \frac{\rho}{1 - \theta} \left[ \frac{C^{1-\theta} - \{(1 - \psi) V + \psi\}^{1-\theta}}{\{(1 - \psi) V + \psi\}^{1-\theta} - 1} \right].
\]

We have some limits of interest. First, the limit as risk-aversion vanishes:

\[
\lim_{\psi \to 0} f(C, V) = \frac{\rho}{1 - \theta} V \left[ \frac{C^{1-\theta}}{V^{1-\theta}} - 1 \right].
\]

and for the derivative with respect to dividends, we obtain:

\[
\lim_{\psi \to 0} f_c(C, V) = \rho C^{-\theta} V^\theta.
\]
C.3 Proof of Proposition 2

In this Appendix we prove the following detailed version of Proposition 2:

**Proposition 3** [Bank’s Problem] Given \{z\}, \(V (Z,W) = v(z) W\), where \(v\) is the solution to the following HJB equation:

\[
0 = \max_{\{c,d\}} f(c,v) + v_z \mu_z + v_W + \sigma \mathbb{E}[J^v] \tag{32}
\]

where \(\mathbb{E}[J^v]\) is the expected jump in the bank’s value given a default event:

\[
J^v = p \left( v \left( z + \tilde{j}_z \right) \left( 1 + J^W \right) - v \right) \mathbb{I}_{\left[ \lambda + \tilde{j}_\lambda \leq \Gamma(z + \tilde{j}_\lambda) \right]} + \left[ \mathbb{I}_{\left[ \lambda + \tilde{j}_\lambda > \Gamma(z + \tilde{j}_\lambda) \right]} \right] \cdot \mathbb{E}[v^o - v].
\]

\[
(1 - p) \left( v \left( z + \tilde{j}_z \right) \left( 1 + J^W \right) - v \right) \mathbb{I}_{\left[ \lambda + \tilde{j}_\lambda \leq \Gamma(z + \tilde{j}_\lambda) \right]} + \left[ \mathbb{I}_{\left[ \lambda + \tilde{j}_\lambda > \Gamma(z + \tilde{j}_\lambda) \right]} \right] \cdot \mathbb{E}[v^o - v].
\]

The optimal policies are given by: \(C (Z,W) = c(z) \cdot W\) and \(I (Z,W) = (i(z) + \delta) \cdot L\). The bank’s market value satisfies \(S (Z,W) \equiv s(z) \cdot W\), where \(s\) solves:

\[
p^s = c(z) + s_z \mu_z + s_W + \sigma J^s, \tag{33}
\]

where \(J^s\) is given by:

\[
J^s = p \left( s \left( z + \tilde{j}_z \right) \left( 1 + J^W \right) - s \right) \mathbb{I}_{\left[ \lambda + \tilde{j}_\lambda \leq \Gamma(z + \tilde{j}_\lambda) \right]} + \left[ \mathbb{I}_{\left[ \lambda + \tilde{j}_\lambda > \Gamma(z + \tilde{j}_\lambda) \right]} \right] \cdot \mathbb{E}[s^o - s].
\]

\[
(1 - p) \left( s \left( z + \tilde{j}_z \right) \left( 1 + J^W \right) - s \right) \mathbb{I}_{\left[ \lambda + \tilde{j}_\lambda \leq \Gamma(z + \tilde{j}_\lambda) \right]} + \left[ \mathbb{I}_{\left[ \lambda + \tilde{j}_\lambda > \Gamma(z + \tilde{j}_\lambda) \right]} \right] \cdot \mathbb{E}[s^o - s].
\]

Finally, Tobin’s \(Q\) is given by:

\[
Q (z) = s(z) \times q(z,\lambda(z)). \tag{34}
\]

We can re-arrange the terms in the objective and obtain the proposition as shown in the body of the text. The modified proposition is:

**Proposition 4** [Bank’s Problem] Given \{z\}, \(V (Z,W) = v(z) \cdot W\), where \(v\) is the solution to the following HJB equation:

\[
0 = -\alpha v_z + \max_{\{c\}} f(c,v) - (v - v_z) c - v_z \alpha z + (v - v_z) \Omega^v \tag{35}
\]

where

\[
\Omega^v = \frac{r^d + \max_{\{\lambda\}} \left( r^d - r^d \right) \lambda + \frac{J^v}{v - v_z}}{v - v_z}.
\]
Formulation. We next prove Proposition 3. The primitive bank value HJB equation is given by:

$$0 = \max_{(C,\lambda)} \left\{ C, \lambda \right\} f(C, V(Z,W)) + \frac{E[dV(Z,W)]}{dt}$$

subject to the laws of motion (23), (24), (27), and the boundary $V = V_0$ when (18) and (21) are not satisfied. In the objective, the differential form is:

$$\frac{E[dV(Z,W)]}{dt} = V_Z(Z,W) \mu^Z W + V_W(Z,W) \mu^W W + \sigma E[J^V],$$

where $J^V$ is given by:

$$E[J^V] = p \hat{J}^V + (1 - p) \tilde{J}^V,$$

where $\hat{J}^V$ is the jump in the value after an unrecognized default event,

$$\hat{J}^V = \left[ V \left( Z + \hat{J}^Z, W + J^W \right) - V(Z,W) \right] \hat{1} - V(Z,W) \left( 1 - \hat{1} \right)$$

where

$$\hat{1} = \begin{cases} 
1 & \text{if } \lambda + J^\lambda = \Gamma \left( z + \hat{z} \right) \\
0 & \text{otherwise}
\end{cases}$$

and $\tilde{J}^V$ the jump in the value after a recognized default event,

$$\tilde{J}^V = \left[ V \left( Z + \tilde{J}^Z, W + J^W \right) - V(Z,W) \right] \tilde{1} - V(Z,W) \left( 1 - \tilde{1} \right)$$

where

$$\tilde{1} = \begin{cases} 
1 & \text{if } \lambda + J^\lambda = \Gamma \left( z + \tilde{z} \right) \\
0 & \text{otherwise.}
\end{cases}$$

Conjecture. We conjecture a solution to the value function and verify that it satisfies the HJB equation. The conjecture is:

$$V(Z,W) = v(z) W,$$

for a suitable candidate $v(z)$. Under this conjecture, we verify that $C(Z,W) = c(z) \cdot W$ and $I = (\iota(z) + \delta) \lambda W$. 

34
Factorization. We perform some useful calculations on the guess (37). In particular, we factorize equity from every term in the HJB equation. Under the conjecture,

\[ f(C, V) = f(c(z) W, v(z) W) \]

\[ = \frac{\rho}{1 - \theta} v(z) W \left[ \frac{c(z)^{1-\theta} W^{1-\theta}}{(v(z) W)^{1-\theta}} - 1 \right] \]

\[ = \frac{\rho}{1 - \theta} v(z) W \left[ \frac{c(z)^{1-\theta}}{v(z)^{1-\theta}} - 1 \right] \]

\[ = f(c(z), v) W. \]

(38)

The change in the value function with respect to zombie loans is:

\[ V_Z = \frac{\partial [v(Z/W) W]}{\partial Z} = v_z. \]

Finally, the derivative of the value function with respect to \( W \) is given by:

\[ V_W = \frac{\partial [v(Z/W) W]}{\partial W} = -v_z \frac{Z}{W} + v(z). \]

(39)

Next, we collect terms to construct a modified drift for the value function:

\[ V_Z \mu Z W + V_W \mu W W = \]

\[ = v_z \cdot \left( -\alpha \frac{Z}{W} \right) W + \left( -v_z \frac{Z}{W} + v(z) \right) \mu W W \]

\[ = -v_z (\alpha + \mu W) zW + v(z) \cdot \mu W W \]

\[ = \left( v_z \mu z + v(z) \cdot \mu W \right) W. \]

Finally, under the conjecture, the jump in the value function after an unrecognized default event is:

\[ \hat{J}V = \left[ v \left( \frac{Z + \hat{J}Z}{W + \hat{J}W} \right) (W + \hat{J}W) - v(z) W \right] \hat{I} - v(z) \left[ 1 - \hat{I} \right] W \]

\[ = \left[ v \left( z + \hat{J}z \right) (1 - \varepsilon \lambda) W - v(z) W \right] \hat{I} - v(z) \left[ 1 - \hat{I} \right] W, \]

\[ = \left[ \left[ v \left( z + \hat{J}z \right) (1 - \varepsilon \lambda) - v(z) \right] \hat{I} - v(z) \left[ 1 - \hat{I} \right] \right] W; \]

\[ \equiv \hat{J}^v \]

where the indicator is \( \hat{I} \) is scale invariant. Likewise, for the recognized jump we obtain:

\[ \tilde{J}V = \left( \left[ v \left( z + \tilde{J}z \right) (1 - \varepsilon \lambda) \right] \hat{I} - v(z) \right) W. \]

\[ \equiv \tilde{J}^v \]
**Verification.** We verify that the conjecture satisfies its HJB equation. With the factorization above, (36) can be written as:

\[
0 = \max_{\{c, \lambda\}} f(c, v) W + [v_z(z) \ v(z)] \times \left[ \begin{array}{c} \mu^z \\ \mu W \end{array} \right] \cdot W + \sigma \left[ p \hat{J}^v + (1 - p) \tilde{J}^v \right] W.
\]

where we used the fact that any choice of \(C\) and \(I\), can be expressed as a choice of \(c(z)W\) as there is a one to one map from the \(\{z, W\}\) space to the original space—by change of coordinates. Then, we can factor wealth from this HJB equation to express it as:

\[
0 = \left[ \max_{\{c, \lambda\}} f(c, v) + \mu^v + J^v \right] \cdot W,
\]

and since the maximization is independent of net-worth, this verifies the conjecture.

Collecting terms the HJB solution to the HJB equation:

\[
\alpha v_z z = \max_{\{c\}} f(c, v) - (v - v_z z) c + (v - v_z z) \Omega(z),
\]

where

\[
\Omega(z) = r^D + \max_{\lambda \in [1, \Xi + (\Xi - 1) z]} (r^L - r^D) \lambda - \sigma J^v.
\]

we verify the conjecture that the formula (37) satisfies the HJB equation (32).

### C.4 Policy Functions

We derive the first-order conditions of this problem.

**Optimal Dividend.** The first-order condition for dividends is given by:

\[
f_c(c, v) = v - v_z z,
\]

we can solve this to obtain:

\[
c = \rho^{1/\theta} \left[ \frac{v}{(v - v_z z)^{1/\theta}} \right].
\]

**Optimal Leverage.** Now consider the optimal leverage choice given by:

\[
\Omega(z) = r^D + \max_{\lambda \in [1, \Xi + (\Xi - 1) z]} (r^L - r^D) \lambda - \sigma \left\{ p \left[ v \left( z + \hat{J}^z \right) (1 - \varepsilon \lambda) \right] \right\} + (1 - p) \left\{ v \left( z + \tilde{J}^z \right) (1 - \varepsilon \lambda) \right\} - v(z)
\]

We verify the conjecture that the formula (37) satisfies the HJB equation (32).
where $\hat{I}$ and $\tilde{I}$ are given by

$$\hat{I} = \begin{cases} 1 & \text{if } \lambda + J^\lambda = \Gamma (z + \hat{J}z) \\ 0 & \text{otherwise} \end{cases}$$

and

$$\tilde{I} = \begin{cases} 1 & \text{if } \lambda + J^\lambda = \Gamma (z + \tilde{J}z) \\ 0 & \text{otherwise} \end{cases}.$$

Next, we construct the shadow boundary to express $\hat{I}$ and $\tilde{I}$, explicitly in terms of a threshold $\lambda$.

In particular, we find the function $\tilde{\Lambda}(z)$ that satisfies the boundary constraint in the case of a recognized default,

$$\tilde{\Lambda}(z) + J^\lambda \left[ \tilde{\Lambda}(z) \right] = \min \left\{ 1/\varepsilon, \Xi + (\Xi - 1) (z + \tilde{J}z [z]) \right\}.$$

The solution to this equation is:

$$\tilde{\Lambda}(z) = \min \left\{ \frac{1}{(2 - \varepsilon) \varepsilon}, \frac{\Xi + (\Xi - 1) z}{1 - \varepsilon + \varepsilon \Xi} \right\}.$$

Likewise, we solve for $\hat{\Lambda}(z)$ that satisfies the boundary constraint in the case of a recognized default,

$$\hat{\Lambda}(z) + J^\lambda \left[ \hat{\Lambda}(z) \right] = \Xi + (\Xi - 1) (z + \hat{J}z [z]).$$

The solution is given by:

$$\hat{\Lambda}(z) = \Xi + (\Xi - 1) \cdot z = \Gamma(z).$$

Thus, the shadow boundary of the unrecognized jump coincides with the liquidation boundary. Thus, only the first boundary is relevant. Hence, we can write the objective as:

$$r^d + \max_{\lambda \in [1, \Gamma(z)]} \left( r^L - r^D \right) \lambda + \sigma \left( \left( \frac{v(z + \hat{J}z)}{v(z) - v_z(z) z} + (1 - p) \frac{v(z + \tilde{J}z)}{v(z) - v_z(z) z} \right) (1 - \varepsilon \lambda) - \frac{(p + (1 - p) I_{[\lambda \leq \tilde{\Lambda}(z)]}) v(z)}{v(z) - v_z(z) z} \right).$$

We now investigate the solution to $\lambda$. Let $\lambda \leq \tilde{\Lambda}(z)$. The derivative in that region of the state space is given by:

$$\left( r^L - r^D \right) - \varepsilon \sigma \left( \frac{v(z + \hat{J}z)}{v(z) - v_z(z) z} + (1 - p) \frac{v(z + \tilde{J}z)}{v(z) - v_z(z) z} \right) +$$

$$\sigma \left( \frac{v_z(z + \hat{J}z)}{v(z) - v_z(z) z} \hat{J}^\lambda + (1 - p) \frac{v_z(z + \tilde{J}z)}{v(z) - v_z(z) z} \right) (1 - \varepsilon \lambda),$$
and thus:

\[(r^L - r^D) - \sigma \varepsilon \left( \frac{\mathbb{E}[v(z + J^z)] - (1/\varepsilon - \lambda) \mathbb{E}[v_z(z + J^z) J^z_\lambda]}{v(z) - v_z(z) z} \right).\]

Now, let \( \lambda > \hat{\Lambda}(z) \). Condition in this case is:

\[(r^L - r^D) - \sigma \varepsilon (v \left( z + \tilde{J}^z \right) - v_z \left( z + \tilde{J}^z \right) \left( 1/\varepsilon - \lambda \right)) \frac{v(z) - v_z(z) z}{v(z) - v_z(z) z}.

Assume that

\[\frac{(r^L - r^D)}{\sigma \varepsilon} > \frac{\mathbb{E}[v(z + J^z)] - (1/\varepsilon - \lambda) \mathbb{E}[v_z(z + J^z) J^z_\lambda]}{v(z) - v_z(z) z},\]

such that,

\[\frac{(r^L - r^D)}{\sigma \varepsilon} > p \left( \frac{v \left( z + \tilde{J}^z \right) - v_z \left( z + \tilde{J}^z \right) \left( 1/\varepsilon - \lambda \right))}{v(z) - v_z(z) z} \right).\]

We observe that leverage is increasing in this region. Hence the solution must fall in a corner. Either \( \lambda = \hat{\Lambda}(z) \) or \( \lambda = \Gamma(z) \). The solution falls at the shadow boundary if:

\[(r^L - r^D) \Lambda(z) + \sigma \left( \frac{\mathbb{E}[v(z + J^z)] (1 - \varepsilon \Lambda(z)) - v(z)}{v(z) - v_z(z) z} \right)\]

is greater than:

\[(r^L - r^D) \Gamma(z) + \sigma \left( p \cdot v \left( z + \tilde{J}^z \right) (1 - \varepsilon \Gamma(z)) - v(z) \frac{v(z) - v_z(z) z}{v(z) - v_z(z) z} \right).\]

Recall that:

\[\frac{\mathbb{E}[v(z + J^z)] (1 - \varepsilon \Lambda(z))}{v(z)} < 1.\]

Then,

\[\sigma \left( \frac{\mathbb{E}[v(z + J^z)] (1 - \varepsilon \Lambda(z)) - v(z)}{v(z) - v_z(z) z} \right) = \sigma \cdot \frac{v(z)}{v(z) - v_z(z) z} \cdot \left( \frac{\mathbb{E}[v(z + J^z)] (1 - \varepsilon \Lambda(z))}{v(z)} - 1 \right) < 0.\]
C.5 Proofs for $p = 0$ case

Main Result. In this Appendix we prove the following result:

**Proposition 5 [Bank’s Problem]** The bank’s value function when $p = 0$ solves

$$0 = \max_{\{c\}} f(c, v^*) - v^* \cdot c + v^* \cdot \Omega \tag{42}$$

where $\Omega$ is the expected leveraged bank return,

$$\Omega = r^D + \max_{\lambda \in [1, \Xi]} \left( r^L - r^D \right) \lambda + \sigma \left\{ (1 - \varepsilon \lambda) \mathbb{I}_{[\lambda < \Lambda]} - 1 \right\}. \tag{43}$$

**Derivation of the Main Result.** The solution is obtained as the solution for the case where $v_z = 0$, and $z = 0$ always. Thus, 40 solves:

$$\alpha v_z z = \max_{\{c\}} f(c, v) - vc + v \Omega(z), \tag{44}$$

where

$$\Omega(z) = r^D + \max_{\lambda \in [1, \Xi]} \left( r^L - r^D \right) \lambda - \sigma J^v.$$

we verify the conjecture that the formula (37) satisfies the HJB equation (32), for $v$, a solution to 32.

Thus, (41), is modified to:

$$c = \rho^{1/\theta} \frac{v}{v^{1/\theta}}. \tag{45}$$

In turn, the optimal leverage condition is given by

$$r^d + \max_{\lambda \in [1, \Xi]} \left( r^L - r^D \right) \lambda + \sigma \left( \mathbb{I}_{[\lambda \leq \tilde{\Lambda}(z)]} (1 - \varepsilon \lambda) - 1 \right). \tag{46}$$

Thus, we solve for $\lambda$. Taking first order conditions, we have that if $\lambda \leq \tilde{\Lambda}(z)$, leverage increases the objective if:

$$r^L - \varepsilon \sigma > r^D.$$ 

If the condition holds, the leverage increases the objective in the region where $\lambda > \tilde{\Lambda}(z)$ since:

$$r^L - r^D > 0.$$

Thus, we have two possible solutions. Either $\lambda = \Lambda = \frac{\Xi}{1 - \varepsilon + \varepsilon \Xi}$ or $\lambda = \Gamma = \Xi$.

Then, the shadow boundary is the optimal solution if:

$$(r^L - r^D - \varepsilon \sigma) \frac{\Xi}{1 - \varepsilon + \varepsilon \Xi} > (r^L - r^D) \Xi - \sigma.$$
C.6 Version with Adjustment Costs

In this section, we derive a version of the model with adjustment costs on loans,
\[ \Phi (I, L) = I + \frac{\gamma}{2} \left( \frac{I}{L} - \delta \right)^2 L. \]

We can factor out \( L \) and employing the definition of \( \iota \) to obtain:
\[ \Phi (I, L) = \left( \iota + \delta + \frac{\gamma}{2} \iota^2 \right) L = \Phi (\iota, 1) L + \delta L. \]

Thus, we can express the funding cost relative to equity as:
\[ \frac{\Phi (I, L)}{W} = (\Phi (\iota, 1) + \delta) \lambda, \]
which is a function independent of the bank’s size and depends on leverage and the investment rate.

**Observation 3: Derivations of Laws of Motion.** Now consider a time interval of length \( \Delta \). The law of motion for fundamental loans satisfies:
\[ L_{t+\Delta} = (1 - \delta \Delta) L_t + I_t \Delta - \varepsilon L_t (N_{t+\Delta} - N_t), \]
with the interpretation that the first term is the non-maturing fraction of loans, the second are loan issuances, and the third are losses in a time interval. Taking \( \Delta \to 0 \), we obtain the following law of motion:
\[ dL = (I - \delta L) dt - \varepsilon L dN. \]
We express this law of motion in terms of net-worth to obtain:
\[ dL = \iota \lambda W dt - \varepsilon \lambda W dN. \]
(46)
To ease the notation, we define the growth rate of fundamental loans and the jump relative to net-worth:
\[ \mu^L \equiv \iota \lambda \text{ and } J^L \equiv -\varepsilon \lambda. \]

Similarly, for deposits we have that:
\[ D_{t+\Delta} = (1 + r^D \Delta) D_t - (r^L \Delta + \delta \Delta) L_t + \Phi (I, L) \Delta + C_t \Delta \]
with the interpretation that the first term is the increase in deposits that results from paying interest with deposits; the second term is the reduction in deposits by the interest and principal payments on outstanding loans; the third term is the increase in deposits as a result of loan issuances; and the final term is dividend payments, all paid with deposits. Taking \( \Delta \to 0 \), we obtain the following law of motion:
\[ dD = [r^D D - (r^L + \delta) L + \Phi (I, L) + C] dt. \]
We express this law of motion in terms of wealth to obtain:
\[ dD = [r^D (\lambda - 1) - (r^L + \delta) \lambda + (\Phi (\iota, 1) + \delta) \lambda + C] W dt. \]
(47)
We define the growth rate of deposits relative to net-worth:

\[ \mu^D \equiv r^D (\lambda - 1) - (r^L + \delta) \lambda + (\Phi(\iota, 1) + \delta) \lambda + c. \]

The evolution of \( Z \) is identical.

**Observation 3: growth independence.** Next, we present the evolution of net-worth with adjustment costs:

\[
dW = dL - dD = \left[ \begin{array}{ccc}
(r^L + \delta) \lambda - r^D (\lambda - 1) + (\iota - (\Phi(\iota, 1) + \delta)) \lambda - c \\
\text{levered returns} & \text{capital loss from adjustment} & \text{dividend rate}
\end{array} \right] Wdt
\]

\[
= \begin{cases}
\varepsilon \lambda WdN, & \text{loss rate}
\end{cases}
\]

Where the second line uses the laws of motion in (46) and (47), and employed observation 1.
D Model Appendix: Numerical Solution

We solve the model using the finite-differences method with an upwind scheme for the choice of forward or backward differences. Specifically, we compute the numerical derivatives of the value function \( v(z) \) using finite differences and use the first order conditions to solve for policies \((c, \iota)\), and iterate on the HJB equation. A detailed description of this algorithm for a general class of models known as mean-field games can be found in Achdou et al. (2020).

Our model, which belongs to this class, is simpler to solve because we keep prices constant, but presents an added complication in that the size of the jump depends on the endogenous state variable. In particular, starting from a point \( z \), upon receiving a Poisson shock the bank jumps to \( z + J_z \). We use linear interpolation to get the value function off the grid.

To compute the stationary distribution, we simulate the model for enough periods such that the mean and standard deviation of \( \lambda \) and \( z \) are approximately constant. Finally, to aggregate variables to a quarterly frequency, we set time steps \( dt = 1/90 \) and for every 90 time steps we use the last value for stocks and the mean for flows.