Organizational Structure, Voluntary Disclosure, and Investment Efficiency

Hyun Hwang*

Carnegie Mellon University

Date: February 1, 2019

Abstract

Evidence on diversified firms that operate in multiple industries suggests that they often withhold information and appear to trade at a discount compared to a group of stand-alone firms. This raises the question of why firms choose to be diversified in the first place. To answer this question, I develop a model of voluntary disclosure that incorporates a key feature of diversified firms: internal capital allocation. My analysis demonstrates two main points. First, the ability to allocate capital internally can make a diversified firm more valuable than a group of stand-alone firms. This result holds true despite the fact that the diversified firm chooses to disclose less private information to the investors than stand-alone firms. Second, the first result holds true under a certain information environment a firm faces. Specifically, when it is moderately likely that the firm privately knows the profitability of its projects, the value of forming a diversified firm is higher than forming individual stand-alone ones. These results potentially explain why firms choose to be diversified even if the structure appears to be suboptimal.

Keyword: Organizational structure, internal capital allocation, voluntary disclosure, investment efficiency

*Email address: hyunh@andrew.cmu.edu. I am greatly indebted to Carlos Corona (Co-Chair), Pierre Jinghong Liang (Co-Chair), Tim Baldenius, Jonathan Glover, Austin Sudbury, and Erina Ytsma for their guidance and help. I also thank Aysa Dordzhieva, Steven Kachelmeier, Volker Laux, Lillian Mills, Ronghuo Zheng, and workshop participants at Carnegie Mellon University and the University of Texas at Austin. All errors are my own.
I. Introduction

In corporate policy, disclosure is an important factor that affects investment efficiency and firm value. One effect of disclosure is that it reduces the information asymmetry and “lemons” problem (i.e., Akerlof, 1970) between firms with investment projects and investors, thereby increasing firm values. Thus, firms compare the benefits of more disclosure against its costs such as proprietary costs to determine their optimal disclosure policy.

A diversified firm is one that has investment projects in multiple industries. Evidence shows that the allocation of capital across projects within the same firm (i.e., internal capital allocation) affects both the value of a firm and its disclosure behavior. For example, compared to a group of stand-alone firms, a diversified firm, on average, appears to suffer from inefficient internal capital allocation (e.g., Gertner and Scharfstein, 2012), and firms with inefficient internal capital allocation tend to withhold information (e.g., Berger and Hann, 2007). This is related to an empirical regularity that diversified firms appear to trade at a discount (e.g., Berger and Ofek, 1995). However, this raises the question of why firms choose to operate in multiple industries in the first place (e.g, Campa and Kedia, 2002). Moreover, few studies have theoretically studied how internal capital allocation affects the value of a firm and its disclosure decisions. My paper provides a theoretical explanation that sheds light on these concerns by analyzing how diversified firms make both disclosure and internal capital allocation decisions.

In this paper, I address two questions. First, how does internal capital allocation across projects in multiple industries affect the value of a firm and its disclosure decisions? Second, under what conditions does internal capital allocation increase the value of a diversified firm relative to the value of a group of stand-alone firms?

To answer the research questions, I develop an analytical model based on three assumptions. First, I follow the common assumptions from the corporate finance literature (e.g., Tirole, 2006), which posits that firms are at least as much informed about the profitability of their projects as investors, and that firms seek funding from investors. Second, I follow the assumptions of Williamson (1975) about internal capital allocation, which states that firms use their information to reallocate raised capital across projects to maximize the expected cash flows
from their projects. Last, I follow the assumptions of Dye (1985) and Jung and Kwon (1988) to capture firms’ disclosure behavior. If firms are informed of the profitability of their projects, they can credibly disclose them; if firms are not informed, they cannot credibly reveal that they do not have information and thus remain silent about the profitability of their projects.

The model delivers two main results. First, the value of a diversified firm can be higher than a corresponding group of stand-alone firms, even though the diversified firm chooses to withhold more information. This can potentially explain why diversified firms continue to exist despite their seemingly inefficient disclosure behavior. Second, the first result holds true when both stand-alone and diversified firms are moderately likely to learn the profitability of their projects. If instead both stand-alone and diversified firms are highly likely to be informed, they make more disclosures and both share the same firm value. This is because stand-alone firms already enjoy high firm value thanks to their abundant disclosure about their projects’ profitability, and thus the hypothetical shift to a diversified structure would have no effect on its value. If firms choose the most efficient organizational structure, the result implies that firms that are moderately likely to be informed choose to be diversified. Thus, the likelihood of being informed about projects’ profitability in the context of voluntary disclosure can potentially explain different firm values across different organizational structures.

I make the following modeling choices. A diversified firm, or two-division firm, has two divisions that operate in different industries, and it seeks funding from investors. A division in each industry has an investment project that generates cash flows with a success probability, which is initially unknown. I assume that the unconditional expected NPV of each project is positive. Then, the firm is randomly informed of the success probability of each project, all of which are independent. If informed of the success probability of a project, the firm can credibly disclose it to investors. However, if uninformed, the firm cannot disclose to investors that it is uninformed about the project, although the disclosure would signal to the investors that the expected NPV of the project remains positive. After the disclosure decision by the firm, investors decide whether to provide capital to the firm. If capital is raised, the firm decides which projects to implement. Finally, the cash flows from each project are generated with the corresponding success probability and distributed to both the firm and investors.
The intuition of the first result is as follows. As in Dye (1985) and Jung and Kwon (1988), the firm discloses the success probability of a project if it is high. However, the firm remains silent about the project if it is either uninformed or informed that the success probability of the project is low. Recall that, with no news about a project, its expected NPV remains positive, but the firm cannot reveal that it is uninformed. Then, if there is no disclosure, the firm with two independent projects is more likely to have at least one positive-NPV project than the stand-alone firm. In addition, the two-division firm allocates capital to the best project to maximize its expected payoff. Thus, even if there is no disclosure, there exist conditions under which investors provide capital for one project at the two-division firm but not for the single project at the stand-alone firm, thereby mitigating the *under-investment* problem of the diversified firm. However, if informed of two low success probabilities, the diversified firm can *afford* to withhold them and still raise capital for one project. As a result, more information is withheld, leading to an *over-investment* problem of the diversified firm. The benefits of mitigating the under-investment problem outweigh the costs of the over-investment problem since the firm has a lower likelihood of having two low success probabilities.

The intuition of the second result is as follows. Suppose that it is common knowledge that the two-division firm is moderately likely to be informed of the success probability of each project. Then, upon no disclosure, investors know that the firm is somewhat likely to allocate capital to at least one positive-NPV project and provide capital for one project at the two-division firm. However, the benefit of having two divisions under the same roof is not realized if a firm is either highly likely or not likely to be informed of each success probability. For example, if the two-division firm is highly likely to be informed, no disclosure is interpreted as the firm hiding low success probabilities. As a result, no capital is raised, rendering internal capital allocation unviable. Since no disclosure leads to no investment, the firm discloses more information to avoid not receiving capital. In contrast, if the firm is not likely to be informed, no disclosure is interpreted as the firm having positive-NPV projects but no information. Thus, capital is raised for each project, rendering internal capital allocation irrelevant.

The model delivers additional results. First, I show that there is no added value for firms to be diversified if their projects generate either low or high cash flows upon success. For
example, if the projects generate high cash flows upon success, the lack of disclosure does not
deter investors from providing capital, making internal capital allocation irrelevant. Conversely,
if the projects generate low cash flows upon success, no disclosure makes investors skeptical,
leading to no investment. This makes internal capital allocation unviable. If the magnitude of
cash flows upon success can be interpreted as a measure of productivity, the result predicts that
firms that have moderately productive investment projects may choose to be diversified and
firms that have either high or low productivity may choose to be stand-alone. If the population of
low-productivity stand-alone firms is large compared to that of high-productivity stand-alone
firms, this result can potentially explain Schoar’s (2002) empirical evidence that diversified
firms can be more productive than stand-alone firms.

An additional result is that diversified firms may pass up some positive-NPV projects. For example, suppose that a two-division firm is informed that the NPV of each project is just
above zero. In this case, the two-division firm chooses to remain silent about its two projects and
implements only one project, instead of implementing the two positive-NPV projects with
disclosure. With no disclosure, the firm can pretend that it has a project that has higher expected
cash flows than the two mediocre projects. In addition, internal capital allocation makes the
pooling easier. This result sheds light on why large organizations may pass up some positive-
NPV projects (e.g., Berger et al., 1999). The result also implies that the disclosure strategies of
two independent projects become interdependent due to internal capital allocation.

Related Papers

The ideas in this paper build on several earlier works. Conceptually, I follow Coase
(1990), Sunder (1997), and Zingales (2000). Sunder (1997:13) emphasizes that “to understand
accounting, the firm itself must be understood.” I follow this call by investigating accounting
problems in the context of organizational structures. Specifically, my paper is related to four
strands of literature: internal capital markets, voluntary disclosure, corporate financing under
asymmetric information, and capital budgeting in the context of organizational structures.

The literature on internal capital markets focuses on whether internal capital markets
perform better than external capital markets (see Gertner and Scharfstein, 2012 for a recent
review). My paper contributes to the literature by investigating the roles of disclosure in explaining the relative efficiencies of internal capital markets. My paper extends Stein (1997)’s model of internal capital allocation by endogenizing firms’ disclosure decisions. Laux (2001) shows that organizing multiple projects under the same roof is beneficial because it becomes easier to incentivize the agent to exert effort. This effect comes from imperfect correlation among multiple projects, a diversification effect. My paper also depends on a diversification effect, but my focus is on how the diversification effect affects firms’ disclosure behavior.

The voluntary disclosure literature focuses on how firms strategically manage disclosure in the presence of rational investors (i.e., Stocken, 2012). I follow Dye (1985) and Jung and Kwon (1988) to capture disclosure friction in my model. Kirschenheiter (1997) and Pae (2005) consider a setting in which there are two signals to be disclosed. My contribution is that, despite the independence between the two signals, the disclosure strategy of each signal depends on the other due to internal capital allocation. Einhorn and Ziv (2007) investigates disclosure strategy of a diversified firm. They show that, if different activities cannot be measured with the same level of precision, the diversified firm discloses less information. My paper does not assume that different activities are measured with different precision. In addition, I compare both stand-alone and diversified firms, whereas their paper considers only diversified firms. The disclosure literature also considers how diversified firms can hide information through aggregation in various settings (see Arya and Glover 2014 for a recent review). My paper contributes to the literature by considering the effects of internal capital allocation on information withholding.

My paper is built upon the literature on corporate finance with asymmetric information. De Meza and Webb (1987) investigate the over-investment problem in the presence of information asymmetry between capital providers and entrepreneurs. Accounting literature has investigated strategic disclosure in this setting. For example, Gox and Wagenhofer (2009) consider the optimal impairment rule and the optimal precision of accounting information in a setting in which a firm pledges its assets to raise capital for a risky project. Bertomeu et al. (2011) considers a model of financing that jointly investigates the capital structure, voluntary disclosure, and cost of capital of a firm. Cheynel (2013) considers the general equilibrium effect of voluntary disclosure on the cost of capital and investment efficiency. Laux and Stocken
(2018) considers how capital-raising and innovation activities are affected by the manipulation of accounting reports and the corresponding regulatory enforcement.

My paper is also related to the extensive literature on capital budgeting, which studies the capital investment decision within firms. Early papers that investigate the capital budgeting of diversified firms include Harris et al. (1982) and Arya et al. (1996). Other papers focus on the role of organizational structures in various investment settings. These papers include Melumad et al. (1992), Baiman and Rajan (1995), Ziv (2000), Arya et al. (2000), Arya et al. (2002), Baldenius et al (2002), Liang et al. (2008), and Dutta and Fan (2012). See Glover (2012) and Mookherjee (2013) for a recent review of related literature.

This paper is organized as follows. In Section 2, I describe the model setup for the stand-alone firm. In Section 3, I analyze the model of the stand-alone firm. In Section 4, I describe the model setup for the diversified firm and solve the model. Section 5 concludes the paper.

II. Model: Stand-alone Firm

Consider a stand-alone firm consisting of an entrepreneur and an investment project. The entrepreneur has no capital and must borrow capital $K$ from investors to implement the project. Every player is risk-neutral, and the entrepreneur is protected by limited liability. The market interest rate is normalized at zero. I refer to the entrepreneur as “she” for convenience.

The investment project requires a fixed investment $K$. If capital $K$ is invested in the project, it yields either cash flows $R$ with probability $p$ or zero cash flows with probability $1 - p$. Success probability $p$ is uniformly distributed over $[0,1]$. I assume that the unconditional expected Net Present Value (NPV) of the project is positive:

$$E[p]R - K = \frac{1}{2}R - K > 0.$$
Let $p_{BE}$ be defined such that

$$p_{BE}R - K = 0.$$ 

A project is positive expected NPV if $p \geq p_{BE}$ and negative expected NPV if $p < p_{BE}$. Note that $R > K$ implies that $\frac{1}{2} > p_{BE}$.

The investors break even in expectation from the investment, following the common assumption in corporate finance literature (i.e., Tirole 2006). This assumption implies that the entrepreneur receives the entire ex-ante social surplus. Thus, I use the ex-ante payoff of the entrepreneur, the ex-ante value of the firm and the project interchangeably. The game has four dates.

**Date 1 – Information Endowment.** Nature picks success probability $p$ from a uniform distribution over $[0,1]$. The entrepreneur privately observes a signal $s = p$ with probability $\gamma$. With probability $1 - \gamma$, she observes $s = \emptyset$ and remains uninformed about $p$. With $s = \emptyset$, the project is positive-NPV from the entrepreneur’s information set since $E[p]R - K > 0$.

**Date 2 – Voluntary Disclosure.** The entrepreneur chooses $d \in \{D, ND\}$, which is observable to the investors. If $s = p$, she chooses $d \in \{D, ND\}$. If $d = D$ is chosen, the investors observe signal $s = p$; if $d = ND$ is chosen, the investors do not observe signal $s = p$. If $s = \emptyset$, the entrepreneur always chooses $d = ND$ and the investors do not observe $s = \emptyset$.

**Date 3 – Investment.** The investors calculate the expected success probability $E[p|d]$ given $d \in \{ND, D\}$. If $k = 1$ is chosen, they invest capital $K$ in the project in return for repayment $R_\ell \in [0, R]$ upon cash flows $R$. If $k = 0$ is chosen, no investment is made, yielding zero return. The investors must be given enough repayment $R_\ell$ upon cash flows $R$ to break even in expectation. Repayment $R_\ell$ satisfies the following break-even condition given $d \in \{D, ND\}$:

$$E[p|d] \times R_\ell = K.$$ 

Thus, given $p_{BE} = K/R$, the investors choose $k = 1$ if $E[p|d]R \geq K$ or $E[p|d] \geq p_{BE}$ because repayment $R_\ell \in [0, R]$ can be arranged so that $E[p|d] \times R_\ell = K$ holds. However, if $E[p|d] <
there is no repayment \( R_\ell \in [0, R] \) satisfying the break-even condition. Thus, the investors choose \( k = 0 \).

**Date 4 – Outcome.** If cash flows \( R \) are generated from the project, repayment \( R_\ell \) is distributed to the investors and cash \( R - R_\ell \) is distributed to the entrepreneur. If cash flows \( R \) are not generated, all the players obtain zero cash flows.

<table>
<thead>
<tr>
<th>( t = 1 )</th>
<th>( t = 2 )</th>
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<tbody>
<tr>
<td>▪ Nature selects ( p ) from ( U[0,1] ).</td>
<td>▪ If ( s = p ), she makes disclosure decision</td>
<td>▪ Investors make investment</td>
<td>▪ Cash flows ( R ) generated with probability ( p \cdot R_\ell ) and</td>
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<tr>
<td>▪ Entrepreneur observes signal ( s = \emptyset, d = ND ). If ( s = p ) with probability ( \gamma ).</td>
<td>▪ Investors observe ( s = p ) if ( d = D ).</td>
<td>▪ Repayment ( R_\ell ) to investors is determined.</td>
<td>▪ No cash flows generated with probability ( 1 - p ).</td>
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**The Equilibrium Concept.** The equilibrium solution concept is Perfect Bayesian Equilibrium (PBE). A PBE is characterized by a set of decisions and repayment \( R_\ell \) such that

1. at date 2, \( d^* = \arg \max_{d \in \{D, ND\}} E[R - R_\ell | d, s] \) maximizes the entrepreneur’s expected payoff from the project given \( s \in \{p, \emptyset\} \);
2. at date 3, \( k^* = \arg \max_{k \in \{1,0\}} \max \{ (E[R_\ell | d] - K) \times k, 0 \} \) maximizes the investors’ expected return given \( d \in \{ND, D\} \);
3. at date 3, repayment \( R_\ell \in [0, R] \) satisfies the break-even condition of the investors given \( d \in \{ND, D\} \): \( K = E[p|d] \times R_\ell \);
4. the players have rational expectations at each date. The entrepreneur’s and investors’ beliefs about each other’s strategies are consistent with the Bayes rule, if possible.

I assume that \( d = ND \) is chosen if the entrepreneur expects not to raise capital from the investors upon the disclosure of signal \( s = p \) (e.g., if it is slightly costly to disclose, as in Cheynel 2013).
III. Analysis: Stand-alone Firm

I derive the ex-ante value of the stand-alone firm. This section serves as the benchmark for analysis of the two-division firm in section 4.

3.1. Benchmark – No Disclosure Friction

I consider two cases with no disclosure friction. First, signal $s \in \{p, \emptyset\}$ is public. Second, the entrepreneur observes signal $s \in \{p, \emptyset\}$ and can credibly disclose $s \in \{p, \emptyset\}$ to the investors. I show that signal $s \in \{p, \emptyset\}$ has productive use in increasing ex-ante firm value. In addition, both settings result in the same ex-ante firm value. All proofs are in the appendix.

**Lemma 1.** Suppose that the entrepreneur is endowed with one investment project.

(i) Suppose that both the entrepreneur and the investors observe signal $s \in \{p, \emptyset\}$. Then, the ex-ante value of the stand-alone firm is $rac{1}{2} R - K + \gamma \frac{K^2}{2R}$.

(ii) Suppose that the entrepreneur privately observes $s \in \{p, \emptyset\}$ and can credibly disclose both $s = p$ and $s = \emptyset$. Then, the ex-ante value of the stand-alone firm is $rac{1}{2} R - K + \gamma \frac{K^2}{2R}$.

Part (i) shows that the ex-ante firm value increases with probability $\gamma$ of observing signal $s = p$. This shows that information helps society to allocate capital to profitable projects, thereby increasing investment efficiency. For example, if $\gamma = 1$, the investors always observe success probability $p$ and invest only in a project with $p \geq p_{BE}$ (i.e., a positive-NPV project), thereby maximizing investment efficiency. If $\gamma = 0$, the investors invest capital in the project without observing $p$. Thus, with positive probability, capital is invested in a negative-NPV project. This is an *over-investment* problem with limited information (e.g., De Meza and Webb, 1987).

Part (ii) shows that the ex-ante firm value does not change if the entrepreneur privately observes signal $s$ and both $s = p$ and $s = \emptyset$ can be credibly disclosed. This is because private information is ultimately revealed by an unraveling argument (i.e., Grossman, 1981; Milgrom, 1981), and any positive-NPV projects are implemented as a result.
3.2. Main Analysis – Stand-alone Firm

Suppose that the entrepreneur of the stand-alone firm privately observes signal \( s \in \{ p, \emptyset \} \). I assume that the entrepreneur can credibly disclose \( s = p \) but cannot credibly disclose \( s = \emptyset \) and chooses \( d = ND \), following the assumptions of Dye (1985) and Jung and Kwon (1988). I use backward induction to derive the ex-ante value of the stand-alone firm.

**Date 4 – Cash Flows.** If \( k = 1 \) was chosen at date 3, the project generates cash flows \( R \) with probability \( p \). In this case, the investors receive repayment \( R_e \) and the entrepreneur receives the residual cash flows \( R - R_e \). In every other case, both players receive zero cash flows.

**Date 3 – Investment.** The investors calculate \( E[p|d] \), the conditional expected success probability given the disclosure decision \( d \in \{ D, ND \} \) by the entrepreneur. If \( E[p|d] \geq p_{BE} \), the investors believe that the project is positive-NPV and decides to invest by choosing \( k = 1 \). Then, \( R_e \) satisfies \( K = E[p|d]R_e \). If \( E[p|d] < p_{BE} \), the investors believe that the project is negative-NPV and decides not to invest by choosing \( k = 0 \). \( E[p|d] \) is calculated in lemma 2.

**Date 2 – Disclosure.** If \( s = p \), the entrepreneur chooses \( d \in \{ D, ND \} \). If she observes \( s = \emptyset \), she always decides not to disclose by choosing \( d = ND \). Lemma 2 summarizes both the disclosure strategy by the entrepreneur and the investment strategy by the investors.

**Lemma 2.** Suppose the entrepreneur of the stand-alone firm decides \( d \in \{ ND, D \} \) at date 2 and that the investors decide \( k \in \{ 0,1 \} \) at date 3. \( p^*(\gamma) \) is defined in Lemma A1 in the appendix.

1. Suppose that \( p^*(\gamma) \geq p_{BE} \). The investors’ belief is as follows: \( E[p|d = D] = p \) and \( E[p|d = ND] = p^*(\gamma) \). At date 3, the investors choose \( k = 1 \). The entrepreneur chooses \( d = D \) for \( s \geq p^*(\gamma) \) and \( d = ND \) for \( s < p^*(\gamma) \) at date 2.

2. Suppose that \( p^*(\gamma) < p_{BE} \). The investors’ belief is as follows: \( E[p|d = D] = p \) and \( E[p|d = ND] = \frac{p_{BE}}{2} \). At date 3, the investors choose \( k = 1 \) if signal \( s \geq p_{BE} \) is disclosed and \( k = 0 \) if either \( s < p_{BE} \) is disclosed or \( d = ND \). The entrepreneur chooses \( d = D \) for \( s \geq p_{BE} \) and \( d = ND \) for \( s < p_{BE} \) at date 2.
Disclosure of Signal $s = p$. Lemma 2 shows that the entrepreneur prefers to disclose a good signal (i.e., high $s = p$) so that she can raise capital with a lower repayment $R_\ell$ upon success to the investors. Recall the break-even condition of the investors given $d \in \{ND, D\}$:

$$E[p|d] \times R_\ell = K.$$  

Thus, if $d = D$ and $s = p$ is disclosed, the break-even condition becomes $pR_\ell = K$. Thus, the disclosure of a high signal $s = p$ increases the entrepreneur’s payoff upon success, $R - R_\ell$, by lowering repayment $R_\ell = K/p$ to the investors.

Withholding of Signal $s = p$. Part (i) of lemma 2 shows that the entrepreneur who is less likely to observe $s = p$ (i.e., a lower $\gamma$) can raise capital upon $d = ND$. If the entrepreneur is unlikely to observe $s = p$, the investors believe that $d = ND$ is most likely due to the arrival of $s = \emptyset$. Since the project is positive-NPV with $s = \emptyset$ (i.e., $E[p]R - K > 0$), the conditional NPV of the project upon $d = ND$ is close to $E[p]R - K > 0$ and positive. Thus, the investors are willing to provide capital to the entrepreneur upon $d = ND$. If the entrepreneur learns that the project is negative-NPV (i.e., $s < p_{BE}$), the entrepreneur can raise capital by choosing $d = ND$. This is an over-investment problem for the stand-alone firm.

Part (ii) of lemma 2 holds if $\gamma$ is high, and the entrepreneur cannot raise capital upon $d = ND$. If the entrepreneur is highly likely to be informed of $s = p$, the investors believe that $d = ND$ is most likely due to the arrival of a low signal $s < p_{BE}$ rather than $s = \emptyset$. Thus, the investors become skeptical about the profitability of the project upon $d = ND$ and choose not to invest. As a result, if the entrepreneur observes $s = \emptyset$ and chooses $d = ND$, she cannot raise capital $K$ for the positive-NPV project, thereby lowering investment efficiency. This is an under-investment problem for the stand-alone firm. However, if the entrepreneur observes a signal $s < p_{BE}$ and chooses $d = ND$, the negative-NPV project is efficiently liquidated, thus increasing investment efficiency.

Date 1 – Information Endowment. Probability $p$ is drawn from a uniform distribution over $[0,1]$, and the entrepreneur observes $s = p$ with probability $\gamma$ and $s = \emptyset$ with $1 - \gamma$. Based on Lemma 2, Proposition 1 derives the ex-ante value of the stand-alone firm.
**Proposition 1.** Suppose the entrepreneur of the stand-alone firm chooses \( d \in \{D, ND\} \) if \( s = p \) and \( d = ND \) if \( s = \emptyset \).

(i) Define \( \gamma^*(R, K) \equiv \frac{R(R - 2K)}{(R - K)^2} \). \( \gamma^*(R, K) \) increases in cash flows \( R \) and decreases in investment costs \( K \).

(ii) If \( \gamma \leq \gamma^*(R, K) \), the project is always implemented and the ex-ante value of the stand-alone firm is the same as the unconditional expected NPV of the project, \( \frac{1}{2}R - K \).

(iii) For \( \gamma > \gamma^*(R, K) \), the project is implemented only if signal \( s \geq p_{BE} \) is disclosed. The ex-ante value of the stand-alone firm is \( \gamma \frac{(R - K)^2}{2R} > \frac{1}{2}R - K \).

(iv) \( \gamma \frac{(R - K)^2}{2R} - \left( \frac{1}{2}R - K \right) \) increases in \( \gamma \). It decreases in \( R \) and increases in \( K \).

If the entrepreneur cannot disclose signal \( s \) (i.e., \( d = ND \) for any \( s \in \{p, \emptyset\} \)), the ex-ante firm value is the same as the unconditional expected NPV of the project, which is \( \frac{R}{2} - K \).

**Likelihood \( \gamma \) of Observing \( s = p \).** Part (ii) of proposition 1 shows that, with a low \( \gamma \), the project is always implemented, regardless of \( d \in \{ND, D\} \). In this case, voluntary disclosure does not increase the ex-ante firm value and only determines how cash flows \( R \) are shared between the entrepreneur and the investors. In contrast, part (iii) shows that a higher \( \gamma \) leads to an increase in the ex-ante firm value. Upon \( d = ND \), the investors believe that the firm with a higher \( \gamma \) is more likely to have observed a signal \( s < p_{BE} \) than \( s = \emptyset \) and refuse to invest. No investment upon \( d = ND \) is more likely to result in the efficient liquidation of a negative-NPV project than the inefficient liquidation of a positive-NPV project, thereby increasing the ex-ante firm value.

**Cash Flows \( R \) and Investment Costs \( K \).** Part (i) and (ii) show that a higher profit margin (i.e., higher cash flows or lower investment costs) renders voluntary disclosure ineffective in increasing ex-ante firm value. With a higher profit margin, the investors always choose to invest because the higher profit margin makes the investors optimistic about the project upon \( d = ND \). In contrast, part (i) and part (iii) show that, with a lower profit margin, the investors become
skeptical about the project upon \( d = ND \) and choose not to invest. This is more likely to result in an efficient liquidation of a negative-NPV project, increasing the ex-ante firm value.

Part (iv) shows that the benefit of voluntary disclosure relative to the case in which the entrepreneur cannot disclose any signal \( s \) (i.e., \( d = ND \) for any \( s \in \{p, \emptyset\} \)) is higher with either a higher \( \gamma \) or a lower profit margin (i.e., lower cash flows \( R \) or higher investment costs \( K \)).

**Empirical Implications of Proposition 1.** High cash flows \( R \) upon success can be interpreted as good economic or industry conditions. It may also reflect a project with high productivity. Likelihood \( \gamma \) of observing signal \( s = p \) can be interpreted as either overall quality of internal information system or overall knowledge of the firm about projects.

Suppose the entrepreneur cannot disclose signal \( s \) (i.e., \( d = ND \) for any \( s \in \{p, \emptyset\} \)). Suppose also that she decides to incur some fixed costs so that she can credibly disclose \( s = p \), as in Proposition 1. The results suggest that firms may exhibit more voluntary disclosure if they (i) suffer from adverse market conditions or lower productivity (i.e., low cash flows \( R \) or high investment costs \( K \)), or (ii) have access to better project knowledge or higher quality of internal information system (i.e., a high \( \gamma \)). This is because the relative benefit of making disclosure decision outweighs the fixed costs for these firms. In contrast, firms may exhibit less voluntary disclosure if they enjoy (i) favorable market conditions or high productivity (i.e., a high \( R \) or a low \( K \)), or (ii) poor internal information system or knowledge about projects (i.e., a low \( \gamma \)).

**IV. Model and Analysis: Two-Division Firm**

Thus far, we have focused on the investment for a single project. In this section, I consider an entrepreneur with project 1 and 2. Like the project in the stand-alone firm, project
\( i \in \{1,2\} \) requires a fixed investment \( K \), and project \( i \) generates cash flows \( R \) with success probability \( p_i \) and zero cash flows with probability \( 1 - p_i \). \( p_1 \) and \( p_2 \) are independent, and each follows a uniform distribution over \([0,1]\). The entrepreneur observes signal \( s_i = p_i \) with probability \( \gamma \) and \( s_i = \emptyset \) with probability \( 1 - \gamma \) for \( i \in \{1,2\} \).

If \( s_i = p_i \), the entrepreneur decides to disclose \( s_i \) by choosing \( d_i \in \{D, ND\} \); if \( s_i = \emptyset \), \( d_i = ND \) is chosen. After \( d_1 \) and \( d_2 \) are chosen, the investors decide to provide capital \( k \times K \) to the entrepreneur by choosing \( k \in \{0,1,2\} \). If \( k = 1 \), the investors invest capital \( K \) in return for repayment \( R_\ell \) upon cash flows \( R \) from one project. If \( k = 2 \), the investors invest capital \( K \) in each project in return for repayment \( R_\ell \in [0, R] \) upon cash flows \( R \) from each project.

The two-division firm has two features that are absent in the stand-alone firm. First, the entrepreneur has a higher likelihood of having at least one positive-NPV project with \( s_i = \emptyset \), which is an example of the diversification effect (e.g., Adams and Yellen, 1976). That is, the entrepreneur is uninformed of both success probabilities with probability \((1 - \gamma)^2\), informed of one success probability with \(2\gamma(1 - \gamma)\), and informed of both success probabilities with \(\gamma^2\).

Then, the probability of observing at least one \( s_i = \emptyset \) is \((1 - \gamma)^2 + 2\gamma(1 - \gamma) = 1 - \gamma^2\), which is greater than \(1 - \gamma\) with which the entrepreneur of the stand-alone firm observes \( s = \emptyset \).

Second, the entrepreneur can allocate raised capital across the two projects after raising capital (e.g., Williamson, 1975; Stein, 1997). After \( k \) is chosen, the entrepreneur decides to allocate capital \( I_i \times K \) to project \( i \) by choosing \( I_i \in \{0,1\} \). The new timeline is as follows.

<table>
<thead>
<tr>
<th>( t = 1 )</th>
<th>( t = 2 )</th>
<th>( t = 3 )</th>
<th>( t = 4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nature independently selects ( p_1 ) and ( p_2 ) from ( U[0,1] ).</td>
<td>The entrepreneur makes disclosure decision ( d_1, d_2 \in {D, ND} ).</td>
<td>Investors make investment decision ( k \in {0,1,2} ).</td>
<td>Cash flows generated, distributed to entrepreneur</td>
</tr>
<tr>
<td>Entrepreneur observes ( s_1 ) and ( s_2 ).</td>
<td>Investors observe ( s_i = p_i ) if ( d_i = D ).</td>
<td>Entrepreneur chooses ( I_1, I_2 \in {0,1} ).</td>
<td>and investors.</td>
</tr>
</tbody>
</table>

Figure 4: Timeline – Two-division Firm
4.1. No Credit Rationing

Suppose that $\gamma \leq \gamma^*(R, K)$ holds for project $i \in \{1, 2\}$, where $\gamma^*(R, K)$ is defined in proposition 1. The entrepreneur is less likely to learn $s_i = p_i$, and part (ii) of proposition 1 implies that capital is always raised, leading to no credit rationing. Therefore, the ex-ante value of the two-division firm is twice as much as the ex-ante value of the stand-alone firm. That is,

$$2 \times \left(\frac{1}{2}R - K\right).$$

4.2. Internal Capital Allocation

Suppose $\gamma > \gamma^*(R, K)$ holds so that the entrepreneur of the two-division firm is highly likely to observe signal $s_i = p_i$. Part (iii) of proposition 1 suggests that the stand-alone firm cannot implement the positive-NPV project with $s = \emptyset$ and suffers from the under-investment problem. I investigate whether the two-division firm can mitigate the under-investment problem through internal capital allocation. I use backward induction to solve the model.

**Date 4 – Cash Flows.** If $k = 2$, the investors and the entrepreneur receive $R_\ell$ and $R - R_\ell$, respectively, upon cash flows $R$ from each project at the two-division firm. If $k = 1$, the investors and the entrepreneur receive $R_\ell$ and $R - R_\ell$ upon cash flows $R$ from one project.

**Date 3 – Capital Allocation.** After the investors make their investment decision, the entrepreneur decides to allocate capital across the two projects. The following lemma summarizes the entrepreneur’s optimal capital allocation strategy.

**Lemma 3.** Suppose that the entrepreneur of the two-division firm has two symmetric projects with independent success probabilities $p_1$ and $p_2$.

(i) Suppose $k = 1$. Then, the entrepreneur chooses $I_i = 1$ and $I_j = 0$ if one of the following is satisfied: (a) $s_i > s_j$, (b) $s_i = \emptyset$, and $s_j < \frac{1}{2}$. If $s_i = s_j$, the entrepreneur chooses $I_i = 1$ with an arbitrary probability.

(ii) If $k = 2$, $I_1 = I_2 = 1$. If $k = 0$, $I_1 = I_2 = 0.$
Lemma 3 summarizes the optimal capital allocation strategy of the entrepreneur if the investors provide capital $K$ to the entrepreneur. Intuitively, the entrepreneur allocates capital to the most profitable project to maximize her expected payoff. This is a formalization of the bright side of internal capital markets in the spirit of Williamson (1975:148): “In many respects, this assignment of cash flows to high yield uses is the most fundamental attribute of the M-form enterprise…” M-form refers to multidivisional or diversified form.

**Date 3 – Investment.** The investors calculate $E[p_i|d_1,d_2]$ for $i \in \{1,2\}$, the conditional expected success probability $p_i$ given $d_1,d_2 \in \{D,ND\}$. $k = 2$ is chosen if the two conditional expected probabilities are greater than $p_{BE}$. $k = 1$ is chosen if $E[p_i|d_1,d_2] \geq p_{BE}$ and $E[p_j|d_1,d_2] < p_{BE}$ for $i \neq j$. $k = 0$ is chosen if $E[p_1|d_1,d_2]$ and $E[p_2|d_1,d_2]$ are less than $p_{BE}$.

**Date 2 – Disclosure.** The entrepreneur decides whether to disclose signal $s_i = p_i$ to the investors. If she observes $s_i = \emptyset$, she chooses $d_i = ND$. Recall that $\gamma > \gamma^* (R,K)$ implies that the entrepreneur of the stand-alone firm cannot raise capital for the positive-NPV project with $s = \emptyset$. This is the under-investment problem for the stand-alone firm discussed in section 3.

However, with two projects under the same roof, the entrepreneur of the two-division firm can raise capital $K$ upon $d_1 = d_2 = ND$ for two reasons: the diversification effect and internal capital allocation. Intuitively, upon $d_1 = d_2 = ND$, the entrepreneur of the two-division has a higher likelihood of having at least one positive-NPV project with $s_i = \emptyset$. In addition, the entrepreneur maximizes her expected payoff by allocating raised capital to the best project, making investors more confident about investing capital $K$ in the two-division firm.

Let $V_{ND}$ denote the conditional expected NPV of one project at the two-division firm if $d_1 = d_2 = ND$. That is,

$$V_{ND} = E[p|d_1 = d_2 = ND,k = 1]R - K.$$ 

If $V_{ND} > 0$, the entrepreneur can raise capital $K$ for one project. For now, I assume that $V_{ND} > 0$ and investigate the effects of $V_{ND} > 0$ on disclosure and investment behavior in Lemma 4. In Proposition 2, I will derive the condition under which $V_{ND} > 0$. 

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Lemma 4. Suppose that $\gamma > \gamma^*(R, K)$. Let $p^* \in [p_{BE}, 1/2]$ and $V_{ND}$ be given such that $V_{ND} = p^{**}R - K > 0$. The disclosure strategy of the entrepreneur of the two-division firm and the investment strategy of the investors are as follows.

(i) If the entrepreneur observes $s_i = p_i$ and $s_j = \emptyset$, she chooses $d_i = ND$ for $s_i < p^{**}$, $d_i = D$ for $s_i \geq p^{**}$, and $d_j = ND$.

(ii) If the entrepreneur observes $s_1 = p_1$ and $s_2 = p_2$,

a. she chooses $d_1 = d_2 = ND$ for $s_1, s_2 < p^{**}$ and $s_1 + s_2 < p^{**} + p_{BE}$.

b. she chooses $d_i = D$ and $d_j = ND$ for $s_i \geq p^{**}$ and $s_j < p_{BE}$.

c. she chooses $d_1 = d_2 = D$ for $s_1, s_2 \geq p_{BE}$ and $s_1 + s_2 \geq p^{**} + p_{BE}$.

(iii) The investors choose $k = 2$ if $d_1 = d_2 = D$ and $k = 1$ otherwise.

![Figure 5: Nondisclosure set upon $s_1 = p_1$ and $s_2 = p_2$](image)

Part (i) shows that if the entrepreneur has observed $s_i = p_i$ and $s_j = \emptyset$, her disclosure strategy of signal $s_i$ is a threshold strategy $p^{**}$ under which signal $s_i = p_i$ is withheld. Threshold $p^{**}$ is chosen to satisfy $p^{**}R - K = V_{ND} > 0$. By assumption, $s_j = \emptyset$ leads to $d_j = ND$.

Part (ii) summarizes the disclosure strategies of the entrepreneur upon $s_1 = p_1$ and $s_2 = p_2$. She chooses $d_i = D$ and $d_j = ND$ if

$$s_iR - K \geq V_{ND} \text{ and } s_j < p_{BE}.$$
The intuition is as follows. $d_i = D$ is chosen if the disclosure of signal $s_i$ results in a higher expected net present value than $V_{ND}$, that is, $s_i \geq p^{**}$. With the disclosure of $s_i$, the investors treat project $j$ as if it is a stand-alone firm. Thus, the threshold for signal $s_j$ is the same as $p_{BE}$ in the stand-alone firm case. This is described in region $Q$ of Figure 5.

The entrepreneur chooses $d_1 = d_2 = ND$ if the following two conditions are satisfied. First, $s_i < p^{**}$ for $i \in \{1, 2\}$. Second, the entrepreneur’s expected payoff of implementing both projects upon $d_1 = d_2 = D$ must be less than expected NPV of one project at the two-division firm upon $d_1 = d_2 = ND$. That is,

$$(s_1 + s_2)R - 2K < V_{ND} = p^{**}R - K \Rightarrow s_1 + s_2 < p^{**} + p_{BE}. \quad (1)$$

This inequality implies that the sum of the two signals must be high enough so that implementing the two projects with $d_1 = d_2 = D$ is more worthwhile than implementing one project with $d_1 = d_2 = ND$. It suggests that, although signals $s_1$ and $s_2$ are independent, the disclosure decisions of both signals are interdependent. This is described in region $W$ in Figure 5. In addition, region $W$ implies that, even if the entrepreneur learns that she has two positive-NPV projects (i.e., $s_1, s_2 > p_{BE}$), she chooses $d_1 = d_2 = ND$ and implements only one positive expected-NPV project with signal $s_i > s_j$. By doing so, the entrepreneur enjoys even higher expected payoff. The following corollary shows that the entrepreneur of the two-division firm withholds more information than the entrepreneurs of the two stand-alone firms, and this result is mainly due to $p^{**} \geq p_{BE}$.

**Corollary 1.** Suppose that $\gamma > \gamma^*(R, K)$. Let $p^{**} \in [p_{BE}, 1/2]$ and $V_{ND}$ be given such that $V_{ND} = p^{**}R - K > 0$. The entrepreneur of the two-division firm withholds more signals than the entrepreneurs of two stand-alone firms.

**Empirical Implications of Lemma 4 and Corollary 1.** The result suggests that the entrepreneur with two projects withholds more bad news than the two entrepreneurs with a single project, if $V_{ND} > 0$. Empirical papers have shown that the withholding of information by diversified firms can be an indicative of unresolved agency conflicts (e.g., Berger and Hann, 2007; Bens et al., 2011). My analysis suggests that there is another side of the story. Withholding
more information stems from agency conflicts with respect to information asymmetry between firms and investors. However, the entrepreneur can afford to withhold bad news. This is because, upon no disclosure, the investors believe that the entrepreneur of a diversified firm is more likely to allocate capital to a positive-NPV project than the stand-alone firm. Thus, less disclosure alone may not be an indicative of too much agency conflict; organizational structure plays an important role in mitigating the information asymmetry problem.

The result also suggests that diversified firms choose not to implement every positive-NPV project if the expected NPV of each project is moderate. Evidence suggests that large organizations often choose not to undertake positive-NPV projects (e.g., Berger et al., 1999). The analysis suggests that this can happen if each project has a moderate level of NPV. The following proposition derives the condition under which the entrepreneur can engage in internal capital allocation.

**Proposition 2.** Let $\gamma^*(R, K) \equiv \sqrt{\frac{3(R-2K)}{3(R-2K)+2K^3}}$.

(i) $\gamma^*(R, K) < \gamma^{**}(R, K)$ for $R > 2K$ and $\gamma^*(R, K) = \gamma^{**}(R, K)$ for $R = 2K$.

(ii) $p^{**} \in \left[p_{BE}, \frac{1}{2}\right]$ exists if $\gamma \geq \gamma^*(R, K)$ and $\gamma \leq \gamma^{**}(R, K)$.

(iii) $p^{**}(\gamma)$ decreases in $\gamma$.

**No Internal Capital Allocation.** Proposition 2 shows that, if the entrepreneur is highly likely to be informed of signal $s_i = p_i$ (i.e., $\gamma > \gamma^{**}(R, K)$), there does not exist $p^{**}$ such that $V_{ND} > 0$. Intuitively, if the entrepreneur is highly likely to be informed of $s_i = p_i$, the investors perceive $d_1 = d_2 = ND$ as the entrepreneur hiding low $s_i = p_i$ rather than $s_i = \emptyset$. This makes the expected NPV of one project at the two-division firm negative. As a result, no capital is raised, making internal capital allocation upon no disclosure unviable. Without internal capital allocation across divisions, the two-division firm is technically the same as the two stand-alone firms. If the entrepreneur is less likely to be informed (i.e., $\gamma < \gamma^*(R, K)$), the entrepreneur can always raise capital $K$ for each project, rendering internal capital allocation irrelevant.
**Internal Capital Allocation.** Proposition 2 shows that, if the entrepreneur learns each signal $s_i$ with probability $\gamma$ such that $\gamma \geq \gamma^*(R, K)$ and $\gamma \leq \gamma^{**}(R, K)$, there exists $p^{**}$ such that $V_{ND} \geq 0$. Thus, the entrepreneur can raise capital for one project upon $d_1 = d_2 = ND$. This is because of (i) the diversification effect and (ii) internal capital allocation across the two projects.

Both effects can be seen as follows. Upon $d_1 = d_2 = ND$, the project with the highest expected success probability is the project with $s_i = \emptyset$ for two reasons. First, $E[p_i|s_i = \emptyset] = 1/2$. Second, the entrepreneur discloses any signal $s_i \geq p^{**}$, where $p^{**} \leq 1/2$. Recall that the entrepreneur with two independent projects observe $s_i = \emptyset$ with probability $(1 - \gamma)^2 + 2\gamma(1 - \gamma) = 1 - \gamma^2 > 1 - \gamma$.

This shows that the entrepreneur with two independent projects is more likely to observe signal $s_i = \emptyset$ than the entrepreneur with a single project. Thus, with the diversification effect, the investors become more confident that the entrepreneur with two projects has at least one positive-NPV project with $s_i = \emptyset$. Moreover, from lemma 3, the investors know that the entrepreneur allocates capital to the most profitable project, and, the project with $s_i = \emptyset$ is the most profitable in expectation upon $d_1 = d_2 = ND$. Thus, the combination of the effects of diversification and internal capital allocation makes the conditional success probability upon $d_1 = d_2 = ND$ and $k = 1$, $E[p|d_1 = d_2 = ND, k = 1]$, higher, where

$$E[p|d_1 = d_2 = ND, k = 1] = \left[\frac{1 - \gamma^2}{1 - \gamma^2 + \gamma^2 B(p^{**})} + \frac{\gamma^2 B(p^{**})}{1 - \gamma^2 + \gamma^2 B(p^{**})} A(p^{**})\right].$$

$B(p^{**})$ is the probability that $s_1 = p_1$ and $s_2 = p_2$ are withheld given $p^{**}$, and $A(p^{**})$ is the corresponding conditional expected success probability. $p^{**}$ is determined so that the equation below is satisfied:

$$V_{ND}(p^{**}) = \left[\frac{1 - \gamma^2}{1 - \gamma^2 + \gamma^2 B(p^{**})} + \frac{\gamma^2 B(p^{**})}{1 - \gamma^2 + \gamma^2 B(p^{**})} A(p^{**})\right] R - K = p^{**}R - K > 0,$$

**Date 1 – Information.** I calculate the ex-ante value of the two-division firm and show that it is higher than the ex-ante value of a portfolio of two separate stand-alone firms for $\gamma \in [\gamma^*(R, K), \gamma^{**}(R, K)]$. Proposition 3 summarizes the results.
**Proposition 3.** Suppose that projects held by firms are symmetric and independent.

(i) The ex-ante value of a two-division firm is higher than the ex-ante value of two stand-alone firms if \( \gamma^*(R, K) \leq \gamma \leq \gamma^{**}(R, K) \) holds.

(ii) The ex-ante value of a two-division firm and the ex-ante value of two stand-alone firms are the same if either \( \gamma < \gamma^*(R, K) \) or \( \gamma > \gamma^{**}(R, K) \).

Part (i) shows that the two-division firm performs better than the two stand-alone firms in terms of ex-ante value, if firms are *moderately* likely to learn success probabilities. This can potentially explain why we observe diversified firms in the first place. As shown in the analysis, the two-division firm withholds more bad news and sometimes foregoes a positive-NPV project. However, with the help of the diversification effect and internal capital allocation, the two-division firm can raise capital for one project upon \( d_1 = d_2 = ND \), thereby mitigating the under-investment problem. Part (ii) shows that if firms are either highly likely or unlikely to be informed of success probabilities, the two-division firms share the same ex-ante value as the two stand-alone firms.

![Figure 6: Ex-ante value of stand-alone and two-division firms](image)

**Empirical Implications of Proposition 3.** Suppose that entrepreneurs can choose to organize two projects under the same roof (i.e., a two-division firm) or two separate stand-alone firms. In addition, suppose that entrepreneurs are evenly distributed with respect to likelihood \( \gamma_i \) of observing \( s_i = p \) from \( \gamma_i = 0.4 \) to \( \gamma_i = 1 \). Figure 6 suggests that (i) entrepreneurs with \( \gamma_i \in (0.4, 0.8) \) choose a two-division structure and (ii) entrepreneurs with \( \gamma_i > 0.8 \) find no reason to
choose a two-division structure, and I assume that entrepreneurs choose stand-alone firms (e.g., if there are small costs of having diversified structure, as in Gertner et al., 1994). If we take the average of the ex-ante value of a two-division firm and the average of the ex-ante value of two stand-alone firms, Figure 6 suggests that the average ex-ante value of a two-division firm is lower than the average ex-ante value of two stand-alone firms. This can potentially explain why diversified firms appear to be traded at a discount compared to a portfolio of stand-alone firms.

The analysis suggests that one source of a value discount for diversified firms may come from their relative inability to learn the profitability of their investment projects. Still, under the same conditions (e.g., likelihood $\gamma_i$ of observing $s_i = p_i$), the ex-ante value of a diversified firm is higher than the ex-ante value of the two stand-alone firms.

The following corollary shows how cash flows $R$ upon success affects the value of internal capital allocation, given a fixed $\gamma$.

**Corollary 2.** Let $\gamma \in (0,1)$ be given such that $\gamma^*(R^*, K) = \gamma^{**}(R^{**}, K) = \gamma$ for $R^*, R^{**} > 0$.

1. $R^* > R^{**} > 0$;
2. The ex-ante value of the two-division firm and the ex-ante value of the two stand-alone firms are the same if cash flows $R$ are either lower than $R^{**}$ or higher than $R^*$.
3. The ex-ante value of the two-division firm is higher than the ex-ante value of the two stand-alone firms if $R \in (R^{**}, R^*)$.

![Figure 7: The graph of $\gamma^{**}(R, K)$ and $\gamma^*(R, K)$ with $K = 1$](image)
Let $\gamma \in (0,1)$ be given. With low cash flows $R$, the firm is in region A in Figure 7, and we have $\gamma > \gamma^*(R^{**}, K)$. Part (ii) of proposition 3 indicates that the ex-ante value of the two-division firm is the same as the ex-ante value of the two stand-alone firms. Intuitively, with low cash flows $R$, the investors become skeptical about the expected profitability of the two-division firm, leading to no-investment upon $d_1 = d_2 = ND$ and rendering internal capital allocation unviable. As cash flows $R$ increases, $\gamma$ lies in $(\gamma^*(R, K), \gamma^{**}(R, K))$. That is, the firm is in region B in Figure 7, and part (i) of proposition 3 indicates that the ex-ante value of the two-division firm is higher than the ex-ante value of the two stand-alone firms. With high $R$, the firm is in region C of Figure 7, and the entrepreneur always raises capital for two projects, and both organizational structures deliver the same ex-ante firm value.

**Empirical Implications of Corollary 2.** If cash flows $R$ can be interpreted as indicative of productivity and $R$ is bounded above by a reasonable number, my analysis implies that diversified firms can to be more productive than stand-alone firms. With higher productivity, the diversified firms can easily attract capital from the investors upon no disclosure, and these firms can enjoy internal capital allocation. This can potentially explain that diversified firms are, on average, more productive (i.e., Schoar, 2002).

V. Conclusion

In this paper, I jointly consider both firms’ disclosure and their internal capital allocation decisions. I show that diversified firms withhold more information than stand-alone firms. Despite less disclosure, diversified firms may enjoy higher firm values because of internal capital allocation across divisions. This result can potentially explain why firms choose to be diversified even if the diversified structure may lead to the withholding of more information.

This paper also investigates conditions under which the value of a diversified firm is higher than the value of a comparable set of stand-alone firms. What determines the value of the firm is the likelihood of being informed of the profitability of each divisional project. With higher likelihood of being informed, firms disclose abundant information to investors. With more
information to investors, capital is efficiently allocated to profitable projects by external markets, leading to an increase in firm value. As a result, the benefit of running an internal capital market is reduced. However, with moderate likelihood of being informed, firms do not disclose abundant information to investors, and diversified structure leads to even less disclosure. Thus, the external investors cannot make efficient investment decisions, and firm value suffers as a result. However, the relative benefit of running an internal capital market increases, which induces firms to be diversified. Thus, the different likelihood of being informed of divisional profitability may explain heterogeneous firm values across different organizational structures.

The paper has two additional results. First, I show that diversified firms may choose not to implement every positive-NPV project if the expected NPV of each project is moderate. The result also implies that the disclosure strategies of two independent projects can become *interdependent* due to internal capital allocation. Second, firms choose to be diversified if their projects generate a moderate level of cash flows upon success. This result can be used to think about why diversified firms can be more productive, if cash flows upon success can be interpreted as indicative of productivity.

Overall, the analysis suggests that disclosure environment plays an important role in determining the boundary of the firm. As Coase (1990) argues, “the theory of the accounting system is part of the theory of the firm.”
VI. Appendix

Proof of Lemma 1

(i) If \( s \geq p_{BE} \), the investors can break even and \( k = 1 \). The entrepreneur’s expected payoff upon signal \( s \geq p_{BE} \) becomes \( s(R - R_e) = sR - K \), and \( E[p(R - R_e)|s \geq p_{BE}] = \frac{p_{BE} + 1}{2}R - K \). If \( s < p_{BE} \), the investors cannot break even and choose \( k = 0 \). If \( s = \emptyset \), \( E[p|s = \emptyset] = \frac{1}{2} > p_{BE} \). Thus, the investors choose \( k = 1 \). The entrepreneur’s expected payoff becomes \( E[p(R - R_e)|s = \emptyset] = \frac{1}{2}R - K \). At date 1, every player observes signal \( s = p \) with probability \( \gamma \) and \( s = \emptyset \) with probability \( 1 - \gamma \). Then, the ex-ante value of the project is calculated as follows:

\[
\gamma \Pr(s \geq p_{BE}) \left( \frac{p_{BE} + 1}{2}R - K \right) + (1 - \gamma) \left( \frac{1}{2}R - K \right) = \frac{1}{2}R - K + \frac{\gamma K^2}{2R},
\]

proving part (i).

(ii) Suppose that the entrepreneur of the stand-alone firm can credibly disclose both \( s = p \) and \( s = \emptyset \). Then, I show that the following equilibrium exists:

a) The investors’ belief about \( p \) is as follows: Given \( d = D \), \( E[p|s] = s \) if \( s \neq \emptyset \) and \( E[p|s = \emptyset] = \frac{1}{2} \). Given \( d = ND \), \( E[p|s \leq p_{BE}] = \frac{p_{BE}}{2} \).

b) The investors choose \( k = 1 \) upon \( s \geq p_{BE} \) or \( s = \emptyset \). The investors choose \( k = 0 \) upon \( d = ND \).

c) The entrepreneur chooses \( d = ND \) if \( s \leq p_{BE} \) and \( d = D \) if either \( s > p_{BE} \) or \( s = \emptyset \).

First, given investors’ belief, we have \( E[p|d = ND] = \frac{p_{BE}}{2} \leq p_{BE} \) upon \( d = ND \). Then, the investors choose \( k = 0 \) if they observe either \( d = ND \) or \( s \leq p_{BE} \). The investors choose \( k = 1 \) if they observe \( s > p_{BE} \) or \( s = \emptyset \). Given the investors’ strategy, the entrepreneur with \( s > p_{BE} \) chooses \( d = D \) to enjoy higher valuation \( s > p_{BE} \geq E[p|d = ND] \). The entrepreneur with \( s \leq p_{BE} \) is indifferent between \( d = D \) and \( d = ND \) because the investors cannot break even and choose \( k = 0 \), regardless of \( d \in \{D, ND\} \). Given the indifference between \( d = D \) and \( d = ND \), \( d = ND \) is chosen by the assumption that the entrepreneur chooses \( d =
ND if she expects not to raise capital from the investors. Thus, the investors’ belief and the entrepreneur’s strategy are consistent.

I show that the entrepreneur’s disclosure strategy is unique. Let a compact set \( A \) denote the equilibrium nondisclosure set such that the entrepreneur chooses \( d = ND \) if \( s \in A \) and \( d = D \) if \( s \notin A \). Then, I show that \( A = [0, p_{BE}] \). Suppose that there exists \( s^* = \sup A \) such that \( s^* > p_{BE} \) and \( [0, p_{BE}] \subset A \). Then, the investors believe that either \( s \in [0, p_{BE}] \) or \( s \notin A \setminus [0, p_{BE}] \).

Thus, \( E[p|d = ND] < s^* \). Then, the entrepreneur has an incentive to disclose \( s^* \), contradicting that \( s^* \in A \).

Since any positive-NPV projects (either \( s = \emptyset \) or signal \( s \geq p_{BE} \)) are implemented and any negative-NPV project (i.e., \( s < p_{BE} \)) is not implemented, the first-best investment efficiency is achieved as in part (i).

**Lemma A1.** Let \( p^*(\gamma) \) and \( \theta(\gamma, p) \) be defined as follows:

\[
p^*(\gamma) \equiv \frac{\sqrt{1-\gamma} - (1-\gamma)}{\gamma} \text{ and } \theta(\gamma, p) \equiv \frac{1-\gamma}{1-\gamma+\gamma p} \times \frac{1}{2} + \frac{\gamma p}{1-\gamma+\gamma p} \times \frac{p}{2}.
\]

Then, \( p^*(\gamma) \) is the unique solution to \( \theta(\gamma, p) = p \) for \( p \in [0,1] \), and \( p^*(\gamma) \) decreases in \( \gamma \).

**Proof of Lemma A1**

Let \( \gamma, p \in [0,1] \) be given. Note that \( \theta(\gamma, 0) = \frac{1}{2} > 0 \) and \( \theta(\gamma, 1) = \frac{1}{2} < 1 \). The solution of

\[
\theta(\gamma, p) = p \text{ is uniquely found as } p = \frac{\sqrt{1-\gamma} - (1-\gamma)}{\gamma} = p^*(\gamma), \text{ after discarding the negative root.}
\]

Thus, we have \( \theta(\gamma, p) > p \) for \( p < p^*(\gamma) \) and \( \theta(\gamma, p) < p \) for \( p > p^*(\gamma) \).

We have \( \frac{\partial}{\partial \gamma} p^*(\gamma) = \frac{2\sqrt{1-\gamma} - \gamma - 2}{2\sqrt{1-\gamma^2}} \). Note that \( 2\sqrt{1-\gamma} + \gamma - 2 < 0 \) since \( 2\sqrt{1-\gamma} < 2 - \gamma \iff 4(1-\gamma) < (2-\gamma)^2 = 4 - 4\gamma + \gamma^2 \iff 0 < \gamma^2 \), which is always true. Thus, \( \frac{\partial}{\partial \gamma} p^*(\gamma) < 0 \).

**End of Proof of Lemma A1**
Proof of Lemma 2

(i) Suppose that \( p^*(\gamma) > p_{BE} \) and the investors’ belief about success probability \( p \) is as follows. Given \( d = ND \), \( E[p|s \leq p^*(\gamma)] = \theta(\gamma, p^*(\gamma)) \); given \( d = D \), \( E[p|d = D] = p \). I show that the investors choose \( k = 1 \). Given \( d = ND \), we have \( E[p|d = ND] = \theta(\gamma, p^*(\gamma)) = p^*(\gamma) \) by Lemma A1. Since \( p^*(\gamma) > p_{BE} \), the investors choose \( k = 1 \). Given \( d = D \), \( E[p|d = D] = s \geq p^*(\gamma) > p_{BE} \). Thus, the investors choose \( k = 1 \).

If \( s \leq p^*(\gamma) = E[p|ND] \), the entrepreneur chooses \( d = ND \). The entrepreneur chooses \( d = D \) for \( s \geq E[p|ND] \) because \( sR - K \geq E[p|ND]R - K \). Thus, the investors’ belief is consistent with the strategies of the entrepreneur. In addition, the entrepreneur’s disclosure strategy is unique since \( p^*(\gamma) \) is uniquely defined by part (i).

(ii) Suppose that \( p^*(\gamma) < p_{BE} \). Suppose that the investors’ belief about success probability \( p \) is as follows: Upon \( d = ND \), \( E[p|d = ND] < p_{BE} \); \( E[p|d = D, s] = s \). Then, the investors choose \( k = 1 \) if \( s > p_{BE} \). If \( s \leq p_{BE} \) is disclosed, the investors choose \( k = 0 \). If signal \( s \) is withheld, \( E[p|d = ND] < p_{BE} \) and the investors choose \( k = 0 \).

Given the investors’ belief, \( d = D \) is chosen for \( s > p_{BE} \geq E[p|d = ND] \). If \( s \in A \) or \( s \notin A \) and \( s < p_{BE} \), the entrepreneur is indifferent between \( d = ND \) and \( d = D \) since both actions result in \( k = 0 \). So, the entrepreneur chooses \( d = ND \) by the assumption that the entrepreneur chooses \( d = ND \) if she expects not to raise capital from the investors. Thus, the entrepreneur’s disclosure strategy is consistent with the investors’ belief about success probability \( p \).

I show that the entrepreneur’s disclosure strategy is unique. Let a compact set \( A \) denote the equilibrium nondisclosure set such that the entrepreneur chooses \( d = ND \) if \( s \in A \) and \( d = D \) if \( s \notin A \). Then, I show that \( A = [0, p_{BE}] \). Suppose that there exists \( s^* = \sup A \) such that \( s^* > p_{BE} \) and \( [0, p_{BE}] \subset A \). Then, the investors believe that either \( s \in [0, p_{BE}] \) or \( s \in A\setminus[0, p_{BE}] \). Thus, \( E[p|d = ND] < s^* \). Then, the entrepreneur has an incentive to disclose \( s^* \), contradicting that \( s^* \in A \).
Proof of Proposition 1

(i) Suppose that \( \gamma \leq \frac{R(R-2K)}{(R-K)^2} \). By rearranging, we have \( p^*(\gamma) \geq p_{BE} \). This implies that nondisclosure of signal \( s \) does not lead to credit rationing by the investors due to part (ii) of lemma 2. Then, the ex-ante value of the project becomes

\[
\gamma(1 - p^*(\gamma)) \left( \frac{1 + p^*(\gamma)}{2} R - K \right) + (1 - \gamma(1 - p^*(\gamma))) \left( \theta(\gamma, p^*(\gamma)) \frac{R}{2} - K \right) = \frac{1}{2} R - K.
\]

(ii) Suppose that \( \gamma > \gamma^* = \frac{R(R-2K)}{(R-K)^2} \). By rearranging, we have \( p^*(\gamma) < p_{BE} \). Thus, part (iii) of lemma 2 shows that, with nondisclosure of signal \( s < p_{BE} \), the entrepreneur faces credit rationing by the investors. Then, with \( p_{BE} = K/R \), the date-1 expected value of the project becomes

\[
\gamma \left\{ (1 - p_{BE}) \left( \frac{1 + p_{BE}}{2} R - K \right) + p_{BE} \times 0 \right\} = \gamma \left( \frac{R - K}{2} \right)^2,
\]

which is increasing in \( \gamma \). Note that, with \( \gamma = \frac{R(R-2K)}{(R-K)^2} \), we have \( \gamma \frac{(R-K)^2}{2R} = \frac{1}{2} R - K \). That is, \( \gamma \frac{(R-K)^2}{2R} \) is bounded below by \( \frac{1}{2} R - K \) at \( \gamma = \frac{R(R-2K)}{(R-K)^2} \).

(iii) \( \frac{\partial}{\partial R} \left( \frac{R(R-2K)}{(R-K)^2} \right) = \frac{2R^2}{(R-K)^3} > 0 \) and \( \frac{\partial}{\partial K} \left( \frac{R(R-2K)}{(R-K)^2} \right) = -\frac{2KR}{(R-K)^3} < 0 \).

(iv) Note that \( p_{BE} X = K \). Then, \( p_{BE} = K/R \). Then, \( \gamma(1 - p_{BE}) \left( \frac{1 + p_{BE}}{2} R - K \right) - \left( \frac{1}{2} R - K \right) \)

increases in \( \gamma \). In addition, we have

\[
\frac{\partial}{\partial R} \left[ \gamma(1 - p_{BE}) \left( \frac{1 + p_{BE}}{2} R - K \right) - \left( \frac{1}{2} R - K \right) \right] = -\frac{K^2 \gamma + (1 - \gamma) R^2}{2R^2} < 0
\]

and

\[
\frac{\partial}{\partial K} \left[ \gamma(1 - p_{BE}) \left( \frac{1 + p_{BE}}{2} R - K \right) - \left( \frac{1}{2} R - K \right) \right] = 1 - \gamma(1 - p_{BE}) > 0,
\]

proving part (iv).
Proof of Lemma 3

(i) If \( k = 1 \), the entrepreneur allocates capital \( K \) to one project. Then, she chooses either \( l_1 = 1 \) and \( l_2 = 0 \) or \( l_1 = 0 \) and \( l_2 = 1 \). The entrepreneur’s payoff of \( l_i = 1 \) and \( l_j = 0 \) for \( i \neq j \) is 
\[
E[p_i|s_i](R - R_f), \quad \text{where} \quad R_f \in [0, R] \quad \text{is repayment to the investors upon cash flows} \quad R.
\]
Thus, she chooses \( l_i = 1 \) and \( l_j = 0 \) if \( E[p_i|s_i] > E[p_j|s_j] \). \( E[p_i|s_i] > E[p_j|s_j] \) happens in two cases.

First, with \( s_i > s_j \), we have \( E[p_i|s_i] = s_i > s_j = E[p_j|s_j] \). Second, if \( s_j < \frac{1}{2} \) and \( s_i = \emptyset \), \( E[p_i|s_i = \emptyset] = \frac{1}{2} > s_j = E[p_j|s_j] \). If the entrepreneur is indifferent between the two projects, I assume \( l_i = 1 \) with probability \( 1/2 \). This happens if \( s_i = s_j \).

(ii) The implementation of the project \( i \) yields \( E[p_i|s_i](R - R_f) \), which is greater than zero payoff upon no implementation. Thus, if \( k = 2 \), \( l_1 = l_2 = 1 \) is chosen. If \( k = 0 \), \( l_1 = l_2 = 0 \) is chosen due to the lack of capital.

Proof of Lemma 4

Let \( ND^p \) be the set of signal \( s_i \) that is withheld by the entrepreneur who has observed \( s_i = p_i \) and \( s_j = \emptyset \). Let \( ND^f \) be the set of signal \( (s_1, s_2) \) such that, if \( (s_1, s_2) \in ND^f \), the entrepreneur who has observed \( s_1 = p_1 \) and \( s_2 = p_2 \) withholds both signals.

I first derive the investors’ optimal investment strategy given their belief about the success probability. Then, I derive the optimal disclosure strategy of the entrepreneur. Lastly, I show that the investors’ belief is consistent with the entrepreneur’s disclosure strategy.

Let \( p^{**} \in [p_{BE}, 1/2] \) be given, and suppose that the investors’ belief about success probabilities \( p_1 \) and \( p_2 \) is as follows:

\[ a) \quad \text{With} \quad s_i = p_i \text{ and } s_j = \emptyset \text{ for } i \neq j, \text{ the entrepreneur chooses } d_i = ND \text{ for } s_i = p_i \leq p^{**}, \quad d_i = D \text{ for } s_i = p_i > p^{**} \text{ and } d_j = ND. \]

\[ b) \quad \text{With } s_1 = p_1 \text{ and } s_2 = p_2, \text{ the entrepreneur picks } d_1 = d_2 = ND \text{ for } s_i \leq p^{**}, i \in \{1,2\} \text{ and } s_1 + s_2 \leq p^{**} + p_{BE}. \text{ The entrepreneur chooses } d_i = D \text{ and } d_j = ND \text{ for } s_i \geq p^{**} \text{ and } s_j < p_{BE}. \text{ The entrepreneur chooses } d_1 = d_2 = D \text{ for } s_i \geq p^{**}, i \in \{1,2\} \text{ and } s_1, s_2 > p^{**} + p_{BE}. \]
Then, given investors’ belief, I show $k = 2$ is chosen upon $d_1 = d_2 = D$ and $k = 1$ otherwise.

a) The investors choose $k = 2$ if $s_1, s_2 \geq p_{BE}$ are disclosed.

b) If $s_i > p^*$ is disclosed and $s_j$ is withheld, the investors choose $k = 1$ for the two reasons. First, we have $s_i > p^* \geq p_{BE}$, and the investors can break even. Second, I show that $E[p_j | s_i, d_j = ND] < p_{BE}$. The investors believe that, if the entrepreneur is informed, $s_j < p_{BE}$. Then, we have

$$E[p_j | s_i, d_j = ND] = \theta(y, p_{BE}) < p_{BE},$$

due to (i) $p^*(y) < p_{BE}$, and (ii) $\theta(y, p_{BE}) < p_{BE}$ for $p_{BE} > p^*(y)$ from part (i) of lemma 2.

c) Suppose that $d_1 = d_2 = ND$. Let $B(p^*)$ denote the probability of $d_1 = d_2 = ND$ upon $\sigma_1 = p_1$ and $s_2 = p_2$. The investors believe that the entrepreneur has observed $s_1 = s_2 = \emptyset$ with probability $\frac{(1-\gamma)^2}{(1-\gamma)^2 + 2\gamma(1-\gamma) + \gamma^2 B(p^*)}$ and the conditional expected success probability for either project is $1/2$. Then, internal capital allocation leads to investment in the project with $p_i = \emptyset$. The investors believe that, with probability $\frac{2\gamma(1-\gamma)}{(1-\gamma)^2 + 2\gamma(1-\gamma) + \gamma^2 B(p^*)}$, the entrepreneur has observed $s_i \leq p^* \leq 1/2$ and $s_j = \emptyset$, and the entrepreneur allocates capital $K$ to project $j$ with $s_j = \emptyset$ due to part (i) of lemma 3. Then, the conditional expected success probability is also $1/2$. The investors believe that, with probability $\frac{\gamma^2 B(p^*)}{(1-\gamma)^2 + 2\gamma(1-\gamma) + \gamma^2 B(p^*)}$, the entrepreneur has observed $s_1 = p_1$ and $s_2 = p_2$, and the entrepreneur allocates capital $K$ to the project with $s_i \geq s_j$ due to part (i) of lemma 3. Given the investors’ belief, we have

$$B(p^*) = (p_{BE})^2 + 2 \times p_{BE}(p^* - p_{BE}) + \frac{(p^* - p_{BE})^2}{2} = \frac{(p^*)^2 + 2p^*p_{BE} - (p_{BE})^2}{2}$$

Let $A(p^*)$ denote the expected success probability upon $d_1 = d_2 = ND$ and $s_1 = p_1$ and $s_2 = p_2$. $A(p^*)$, along with $V_{ND}$, will be derived in proposition 2. For now, assume that $A(p^*)$ is given so that $V_{ND} > 0$. Then, the expected value of the project upon $d_1 = d_2 = ND$ and $k = 1$, $V_{ND}(p^*)$, is

$$V_{ND}(p^*) = \left[ \frac{1 - \gamma^2}{1 - \gamma^2 + \gamma^2 B(p^*)} + \frac{\gamma^2 B(p^*)}{1 - \gamma^2 + \gamma^2 B(p^*)} A(p^*) \right] R - K.$$
By assumption, \( p^* \) satisfies \( V_{ND}(p^*) > 0 \). Thus, the investors indeed choose \( k = 1 \). The conditions under which \( V_{ND}(p^*) > 0 \) holds is shown in proposition 2.

I derive the entrepreneur’s optimal disclosure strategy given the investors’ belief and show that the investors’ belief is consistent with the entrepreneur’s disclosure strategy.

a) Suppose that the entrepreneur has observed \( s_i = p_i \) and \( s_j = \emptyset \). Then, \( d_j = ND \). If \( s_i < p_{BE} \), the entrepreneur chooses \( d_i = ND \) to raise capital \( K \) given that the investors invest capital \( K \) upon no disclosure of both signals. With \( d_i = D \), the entrepreneur cannot raise capital.

Suppose that the entrepreneur has observed \( s_i \geq p_{BE} \). If \( s_i \geq p_{BE} \) is disclosed and \( d_j = ND \), the investors choose \( k = 1 \) and the expected payoff becomes
\[
  s_i R - K.
\]
Since \( s_i R - K \) is increasing in \( s_i \), the entrepreneur who has observed \( s_i \) withholds the signal if and only if it is less than or equal to a cutoff value, which I denote \( p^* \). That is, \( ND^p = [0, p^*] \). Since \( V_{ND}(p^*) \) is the expected value of the diversified firm upon \( d_1 = d_2 = ND \) and \( k = 1 \), \( p^* \) satisfies
\[
  p^* R - K = V_{ND}(p^*).
\]
Thus, the investors’ belief about disclosure threshold \( p^* \) is consistent with the entrepreneur’s disclosure strategy.

b) Suppose that the entrepreneur has observed \( s_1 = p_1 \) and \( s_2 = p_2 \). Then, given that \( V_{ND}(p^*) \) is the equilibrium project value given no disclosure and \( k = 1 \), \((s_1, s_2)\) \( \in ND^f \) must satisfy the following incentive compatibility conditions:
\[
  s_i R - K \leq V_{ND}(p^*) = p^* R - K, i \in \{1, 2\}
\]
\[
  (s_1 + s_2) R - 2K \leq V_{ND} = p^* R - K \Rightarrow s_1 + s_2 \leq p^* + p_{BE}
\]
Thus, \( ND^f = \{(s_1, s_2)|s_1 \leq p^*, s_2 \leq p^*, and s_1 + s_2 \leq p^* + p_{BE}\} \). The entrepreneur has an incentive to disclose \( s_i \) and withhold \( s_j \) if \( s_i \geq p^* \) and \( s_j < p_{BE} \). The entrepreneur enjoys \( s_i R - K \geq V_{ND} \) with disclosure of \( s_i \) and \( V_{ND} \) with the withholding of \( s_i \). The entrepreneur has no incentive to disclose \( s_j \) since the disclosure of \( s_j < p_{BE} \) leads to zero payoff.
Note that $p^* \geq p_{BE}$ for the equilibrium to exist. Recall that $p^*R - K = V_{ND}(p^*)$. This implies that $p^*$ needs to be greater than $p_{BE}$ so that $V_{ND} \geq 0$. If $p^* < p_{BE}$, $V_{ND} < 0$ and the investors chooses $k = 0$ upon no-disclosure of either both signals or one signal.

**Proof of Corollary 1**

I consider three cases: (i) $s_1 = s_2 = \emptyset$, (ii) $s_i = p_i$ and $s_j = \emptyset$ for $i \neq j$, and (iii) $s_1 = p_1$ and $s_2 = p_2$. In each case, I show that the entrepreneur of the two-division firm withholds more information than the entrepreneurs of the two stand-alone firms.

First, upon $s_1 = s_2 = \emptyset$, the entrepreneur of the two-division firm and the entrepreneurs of stand-alone firm 1 and 2 remain silent.

Second, upon $s_i = p_i$ and $s_j = \emptyset$, the entrepreneur of the two-division firm withholds $s_i < p^*$ from part (i) of Lemma 4. The entrepreneur of stand-alone firm $i$ withholds $s_i < p_{BE}$ from part (iii) of Lemma 2. Since $p^* > p_{BE}$, the entrepreneur of the two-division firm withholds more information.

Last, upon $s_1 = p_1$ and $s_2 = p_2$, the entrepreneur of stand-alone firm 1 and 2 withhold $s_i < p_{BE}$ from part (iii) of lemma 2. Part (ii) of Lemma 4 implies that the entrepreneur of the two-division firm withholds at least as much as $s_i \leq p_{BE}$.

Therefore, the entrepreneur of the two-division firm withholds more signals than the entrepreneurs of the two stand-alone firms.

**Proof of Proposition 2**

(i) I show that $\gamma^*(R, K) - \gamma^*(R, K) = \sqrt{\frac{3(R-2K)}{3(R-2K) + \frac{2K^3}{R^2}}} - \frac{R(R-2K)}{(R-K)^2} > 0$ for $R > 2K$.

(a) The unique solution to $\gamma^*(R, K) = \gamma^*(R, K)$, given $K$, is $R = 2K$. That is, $\gamma^*(R, K) = \gamma^*(R, K)$ only when $R = 2K$. Recall that $\frac{1}{2} R - K > 0$. Thus, the minimum possible $R$ is $R = 2K$. Then, $\gamma^*(R, K) = \gamma^*(R, K)$ when $R$ is at the lowest possible value, which is $R = 2K$. 

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(b) Both $\gamma^*(R, K)$ and $\gamma^{**}(R, K)$ increase in $R$ for $R \geq 2K$: 
$$\frac{\partial}{\partial R} \frac{R(R-2K)}{(R-K)^2} = \frac{2K^2}{(R-K)^2} > 0$$
and 
$$\frac{\partial}{\partial R} \sqrt{\frac{3(R-2K)}{3(R-2K)+\frac{2K^3}{R^2}}} > 0.$$ 

(c) 
$$\frac{\partial}{\partial R} \frac{R(R-2K)}{(R-K)^2} = \frac{2K^2}{(R-K)^2} = \frac{2}{K} \text{ at } R = 2K, \text{ and } \frac{\partial}{\partial R} \sqrt{\frac{3(R-2K)}{3(R-2K)+\frac{2K^3}{R^2}}} \text{ at } R = 2K \text{ is infinity.}$$

(c) implies that the slope of $\gamma^{**}(R, K)$ is higher than the slope of $\gamma^*(R, K)$ at $R = 2K$. Thus, there exists $\epsilon > 0$ such that $\gamma^{**}(R + \epsilon, K) > \gamma^*(R + \epsilon, K)$ for $R = 2K$.

(a) and (b) together imply that either $\gamma^*(R, K) - \gamma^{**}(R, K) > 0$ or $\gamma^*(R, K) - \gamma^{**}(R, K) < 0$ for $R > 2K$, and $\gamma^*(R, K) = \gamma^{**}(R, K)$ for $R = 2K$.

Thus, (a), (b), and (c) together imply that $\gamma^{**}(R, K) > \gamma^*(R, K)$ for $R > 2K$ and $\gamma^*(R, K) = \gamma^{**}(R, K)$ for $R = 2K$.

(ii) I first calculate the expected value of the project upon no $d_1 = d_2 = ND$ and $k = 1$, that is, $V_n(p^{**})$. Recall $B(p^{**})$, the probability of $d_1 = d_2 = ND$ upon $s_1 = p_1$ and $s_2 = p_2$, 
$$B(p^{**}) = \frac{(p^{**})^2 + 2p^{**}p_{BE} - (p_{BE})^2}{2}.$$ 

There are three cases to consider.

a. With probability $(p_{BE})^2 / B(p^{**})$, $s_1, s_2 \leq p_{BE}$. Then, the entrepreneur allocates capital $K$ to the project with the highest expected cash flows. That is, 
$$E[\max(p_1, p_2) | s_1, s_2 \leq p_{BE}] = \frac{2}{3} p_{BE}. $$

b. With probability $2 \times \frac{p_{BE}(p^{**} - p_{BE})}{B(p^{**})}$, $s_i \leq p_{BE}$ and $s_j \in (p_{BE}, p^{**}]$. Thus, $l_i = 0$, $l_j = 1$ and 
$$E \left[ p_j | s_j \in (p_{BE}, p^{**}] \right] = \frac{p_{BE} + p^{**}}{2}. $$
c. With probability $\frac{(p^* - p_{BE})^2}{2} / B(p^*)$, $s_1, s_2 \geq p_{BE}$ such that $s_1 + s_2 \leq p_{BE} + p^*$. With probability $1/2$, $s_1, s_2 \in [p_{BE}, \frac{p_{BE} + p^*}{2}]$ and the entrepreneur picks the project with the highest expected cash flows. That is,

$$E \left[ \max\{s_1, s_2\} \right]_{s_1, s_2 \in \left[ p_{BE}, \frac{p_{BE} + p^*}{2} \right]} = \frac{p_{BE} + p^*}{2} + \frac{1}{3}p_{BE} = \frac{1}{3}[2p_{BE} + p^*].$$

With probability $1/4$, $s_1 \in \left[ \frac{p_{BE} + p^*}{2}, p^* \right]$ and $s_2 \in \left[ p_{BE}, \frac{p_{BE} + p^*}{2} \right]$ and the entrepreneur chooses $I_1 = 1$ and $I_2 = 0$. The same calculation is performed for $s_2 \in \left[ \frac{p_{BE} + p^*}{2}, p^* \right]$ and $s_1 \in \left[ p_{BE}, \frac{p_{BE} + p^*}{2} \right]$. Then, we have

$$2 \times E \left[ p_i \left| s_i \in \left[ \frac{p_{BE} + p^*}{2}, p^* \right] \right\} \right] = \frac{p_{BE} + 3p^*}{2}.$$

Then, the expected cash flows upon $s_1, s_2 \geq p_{BE}$ such that $s_1 + s_2 \leq p_{BE} + p^*$ is

$$\frac{1}{2} \left[ \frac{2p_{BE} + p^*}{3} + \frac{p_{BE} + 3p^*}{2} \right] = \frac{7p_{BE} + 11p^*}{12}.$$

Then, the expected success probability $A(p^*)$ upon $s_1 = p_1$ and $s_2 = p_2$ is

$$A(p^*) = \frac{1}{B(p^*)} \left[ \frac{(p_{BE})^2}{3} p_{BE} + 2p_{BE}(p^* - p_{BE}) \frac{p_{BE} + p^*}{2} + \frac{(p^* - p_{BE})^2 7p_{BE} + 11p^*}{12} \right].$$

Then, we have

$$V_{ND}(p^*) = \left[ \frac{1 - \gamma^2}{1 - \gamma^2 + \gamma^2 B(p^*)} \frac{1}{2} + \frac{\gamma^2 B(p^*)}{1 - \gamma^2 + \gamma^2 B(p^*)} A(p^*) \right] R - K.$$  

The equilibrium threshold $p^*$ satisfies $V_{ND}(p^*) = p^* R - K$. By rearranging, we have

$$\frac{1 - \gamma^2}{1 - \gamma^2 + \gamma^2 B(p^*)} \frac{1}{2} + \frac{\gamma^2 B(p^*)}{1 - \gamma^2 + \gamma^2 B(p^*)} A(p^*) = p^*. \quad (AA.1)$$

Note that at $p^* = 1/2$, the left-hand side of the equation in (AA.1) becomes

$$\frac{(85 - 18p_{BE} + 12(p_{BE})^2 + 8(p_{BE})^3)\gamma^2 - 96}{24((7 - 4p_{BE} + 4(p_{BE})^2)\gamma^2 - 8)},$$

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which is less than 1/2, since

\[
\frac{1}{2} \left( \frac{85 - 18p_{BE} + 12(p_{BE})^2 + 8(p_{BE})^3}{24((7 - 4p_{BE} + 4(p_{BE})^2)y^2 - 8)} \right) = \frac{(1 + 6p_{BE}(5 - 6p_{BE}) + 8(p_{BE})^3)y^2}{24(8 - (7 - 4p_{BE}(1 - p_{BE}))y^2)} > 0.
\]

At \( p^{**} = p_{BE} \), the left-hand side of the equation in (AA.1) becomes

\[
\frac{3 - (3 - 4(p_{BE})^3)y^2}{6 - 6(1 - (p_{BE})^2)y^2}.
\]

I identify the value of \( \gamma \) that makes the left-hand side of the equation in (AA.1) greater than \( p_{BE} \), and, with the help of the intermediate value theorem, this allows us to show the existence of the real solution to the equation in (AA.1).

Note that \( \frac{\partial}{\partial \gamma} \frac{3 - (3 - 4(p_{BE})^3)y^2}{6 - 6(1 - (p_{BE})^2)y^2} \) \( < 0 \) since \( p_{BE} \leq \frac{1}{2} \) and \( 4p_{BE} - 3 \leq 0 \). Then, \( \gamma \) needs to be low enough so that \( \frac{3 - (3 - 4(p_{BE})^3)y^2}{6 - 6(1 - (p_{BE})^2)y^2} > p_{BE} \). By solving \( \frac{3 - (3 - 4(p_{BE})^3)y^2}{6 - 6(1 - (p_{BE})^2)y^2} = p_{BE} \), we get

\[
\gamma^{**}(R, K) = \sqrt{\frac{3(R - 2K)}{3(R - 2K) + \frac{2K^3}{R^2}}}.
\]

Note that with \( p_{BE} = 0 \) or \( K = 0 \), we have \( \gamma^{**}(R, K) = 1 \). This implies that if \( p_{BE} \) is low, there exists probability \( \gamma < 1 \) that ensures the existence of solution \( p^{**} \) in (AA.1); with \( p_{BE} = 1/2 \) or \( R = 2K \), \( \gamma^{**}(R, K) = 0 \). This implies that, with \( p_{BE} \) close to 1/2, probability \( \gamma \) needs to be close to zero to ensure the existence of \( p^{**} \). Thus, \( p^{**} \in [p_{BE}, 1/2] \). Then, if \( \gamma < \gamma^{**}(R, K) \), we have

\[
\frac{(1 - \gamma)(1 + \gamma)}{(1 - \gamma)(1 + \gamma) + \gamma^2B(p_{BE})^2} + \frac{\gamma^2B(p_{BE})}{(1 - \gamma)(1 + \gamma) + \gamma^2B(p_{BE})}A(p_{BE}) > p_{BE}, \text{ and}
\]

\[
\frac{(1 - \gamma)(1 + \gamma)}{(1 - \gamma)(1 + \gamma) + \gamma^2B(\frac{1}{2})^2} + \frac{\gamma^2B(\frac{1}{2})}{(1 - \gamma)(1 + \gamma) + \gamma^2B(\frac{1}{2})}A(\frac{1}{2}) < \frac{1}{2}
\]

Thus, there exists \( p^{**} \in [p_{BE}, 1/2] \) that satisfies the equation in (AA.1). That is,

\[
\frac{(1 - \gamma)(1 + \gamma)}{(1 - \gamma)(1 + \gamma) + \gamma^2B(p^{**})^2} + \frac{\gamma^2B(p^{**})}{(1 - \gamma)(1 + \gamma) + \gamma^2B(p^{**})}A(p^{**}) = p^{**}.
\]
that satisfies the cubic equation in (AA.1) is the unique real solution such that

\[ p^* = \frac{22^{2/3} \left( (2 + 7(p_{BE})^2) \gamma^2 - 2 \right)}{A} + \frac{2^{1/3} A}{\gamma^2} - 5p_{BE}, \]

where

\[ A = \left( 3(1 + 10p_{BE})\gamma^4(1 - \gamma^2) - 74(p_{BE})^3\gamma^6 \right. \]

\[ + \gamma^3 \sqrt{16(2 - (2 + 7(p_{BE})^2)\gamma^2)^3 + \gamma^2(74(p_{BE})^3\gamma^2 - 3(1 - \gamma^2)(1 + 10p_{BE}))^2} \right)^{1/3}. \]

The other two solutions are complex numbers.

(iii) Using the implicit function theorem, we have

\[ p^{***}(\gamma) = \frac{2\gamma \left( 12(1 - 2p^{**}(\gamma)) + (p_{BE})^3 + 15p_{BE}(p^{**}(\gamma))^2 - 9(p_{BE})^2p^{**}(\gamma) + (p^{**}(\gamma))^3 \right)}{3 \left( -8(1 - \gamma^2) + 3(p_{BE})^2\gamma^2 - 10p_{BE}p^{**}(\gamma)\gamma^2 - \gamma^2(p^{**}(\gamma))^2 \right)} < 0, \]

since the denominator is negative due to \(3(p_{BE})^2\gamma^2 - 10p_{BE}p^{**}(\gamma)\gamma^2 < 3(p_{BE})^2\gamma^2 - 3p_{BE}p^{**}(\gamma)\gamma^2 = 3p_{BE}\gamma^2(p_{BE} - p^{**}(\gamma)) < 0; \) the numerator is positive due to \(12(1 - 2p^{**}(\gamma)) > 0 \) with \(p^{**}(\gamma) < 1/2\) and \(15p_{BE}(p^{**}(\gamma))^2 - 9(p_{BE})^2p^{**}(\gamma) > 9p_{BE}p^{**}(\gamma)(p^{**}(\gamma) - p_{BE}) > 0 \) with \(p^{**}(\gamma) > p_{BE}.\)
Proof of Proposition 3

(i) With probability \((1 - \gamma)^2\), the entrepreneur observes \(s_1 = s_2 = \emptyset\). Then, the entrepreneur chooses \(d_1 = d_2 = ND\), and the expected value of the project becomes \(V_{ND}\).

With probability \(2\gamma(1 - \gamma)\), the entrepreneur observes \(s_i = p_i\) and \(s_j = \emptyset\) for \(i \neq j\). Then, the entrepreneur chooses \(d_j = ND\), and the expected value of the projects becomes

\[
(1 - p^{**}(\gamma))(E[p_i|s_i \geq p^{**}(\gamma)]R - K) + p^{**}V_{ND} = (1 - p^{**})\left(\frac{1 + p^{**}}{2}X - K\right) + p^{**}V_{ND}.
\]

With probability \(\gamma^2\), the entrepreneur observes both signal \(s_1 = p_1\) and \(s_2 = p_2\). Then, the entrepreneur chooses \(d_1 = d_2 = D\) if \(s_1, s_2 \geq p^{**}(\gamma)\), and this is realized with probability \((1 - p^{**})^2\). Then, the expected value of the projects conditional on \(s_1, s_2 \geq p^{**}(\gamma)\) becomes

\[
\frac{1 + p^{**}(\gamma)}{2}R - K.
\]

With probability \(2 \times (1 - p^{**})p_{BE}\), \(s_i \in [p^{**}, 1]\) and \(s_j < p_{BE}\). Then, the entrepreneur chooses \(d_i = D\) and \(d_j = ND\) and the expected value of the project is \(\frac{1 + p^{**}}{2}R - K\). With probability \(2 \times (p^{**} - p_{BE})(1 - p^{**})\), \(s_i \in [1 - p^{**}, 1]\) and \(s_j \in [p_{BE}, p^{**}]\), and the entrepreneur chooses \(d_1 = d_2 = D\), and the expected value of the projects is

\[
\left(\frac{1 + p^{**}}{2} + \frac{p^{**} + p_{BE}}{2}\right)R - 2K.
\]

With probability \(\frac{(p^{**} - p_{BE})^2}{2}\), \(s_1, s_2 \leq p^{**}\) such that \(s_1 + s_2 \geq p_{BE} + p^{**}\). Then, the entrepreneur chooses \(d_1 = d_2 = D\), and the expected value of the projects is calculated as follows. We know that \(p_{BE} + p^{**} - s_1 \leq s_2 \leq p^{**}\) and \(p_{BE} \leq s_1 \leq p^{**}\). Then, \(E[s_2|s_1] = \frac{p_{BE} + 2p^{**} - s_1}{2}\). Then, \(E[E[s_2|s_1]] = E[E[s_2]] = \frac{p_{BE} + 2p^{**}}{2} - \frac{1}{2}E[s_1]\). By symmetry, \(E[s_1] = E[s_2]\). Thus, we have \(E[s_i] = \left(\frac{p_{BE} + 2p^{**}}{3}R - K\right)\). Since there are two implemented projects, we have

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\[ 2 \times \left( \frac{p_{BE} + 2p^{**}}{3} R - K \right). \]

With probability \((p^{**})^2 - \frac{(p^{**} - p_{BE})^2}{2}\), the entrepreneur chooses \(d_1 = d_2 = ND\), and the expected value of the project is \(V_{ND}\). Then, the expected value of the projects is calculated as follows:

\[
\gamma^2 \left[ 2(1 - p^{**})^2 \left( \frac{1 + p^{**}}{2} R - K \right) + 2p_{BE}(1 - p^{**}) \left( \frac{1 + p^{**}}{2} R - K \right) 
+ 2(1 - p^{**})(p^{**} - p_{BE}) \left( \frac{1 + p^{**}}{2} + \frac{p^{**} + p_{BE}}{2} \right) R - 2K \right] 
+ \frac{(p^{**} - p_{BE})^2}{2} 2 \times \left( \frac{p_{BE} + 2p^{**}}{3} R - K \right) 
+ \{ (p^{**})^2 - \frac{(p^{**} - p_{BE})^2}{2} \} V_{ND} \right] + 2\gamma(1 - \gamma) \left[ (1 - p^{**}) \left( \frac{1 + p^{**}}{2} - K \right) + p^{**} V_{ND} \right] + (1 - \gamma)^2 V_{ND}.
\]

By rearranging, we have

\[
2\gamma(1 - p^{**}) \left( \frac{1 + p^{**}}{2} R - K \right) 
+ \left[ \{ 1 - \gamma(1 - p^{**}) \}^2 + \gamma^2 (p^{**} - p_{BE})(1 - p^{**}) + \frac{\gamma^2(p^{**} - p_{BE})^2}{6} \right] V_{ND}.
\]

Subtracting the ex-ante value of the two separate stand-alone firms \(2\gamma(1 - p_{BE}) \left( \frac{1 + p_{BE}}{2} R - K \right)\) from the ex-ante value of the diversified firm, we have

\[
[1 - \gamma(1 - p_{BE})] (1 - \gamma(1 - p^{**})) V_{ND} + \gamma^2 (p^{**} - p_{BE})^2 \frac{V_{ND}}{6} \geq 0. \quad (A.A.3)
\]

Thus, the ex-ante value of the diversified firm is higher than the ex-ante value of the two stand-alone firms for \(\gamma \leq \sqrt{\frac{3(R-2K)}{3(R-2K) + \frac{2K^2}{R^2}}}\). Note that at \(\gamma = \sqrt{\frac{3(R-2K)}{3(R-2K) + \frac{2K^2}{R^2}}}\), we have \(p^{**} = p_{BE}\) and the ex-ante value of the diversified firm is the same as the two stand-alone firms:

\[
2\gamma(1 - p_{BE}) \left( \frac{1 + p_{BE}}{2} R - K \right).
\]

(ii) If \(\gamma < \gamma^*(R, K)\), part (ii) of proposition 1 implies that the entrepreneur can always raise capital for each project. Then, the ex-ante value of the two-division firm is \(2 \times \left( \frac{1}{2} R - K \right)\), as in
section 4.1. \( \gamma > \gamma^*(R, K) \), proposition 2 implies that there does not exist \( \gamma^* \in [p_{BE}^1, \frac{1}{2}] \) such that results in lemma 4 hold. Then, with \( \gamma > \gamma^*(R, K) \), part (iii) of proposition holds for each project in the two-division firm. Thus, and the ex-ante value of the two-division firm is the same as the ex-ante value of the two stand-alone firms.

Proof of Corollary 2

(i) Part (i) of Proposition 2 implies that \( \gamma^*(R, K) > \gamma^*(R, K) \) for \( R > 2K \). Also, note that \( \gamma^*(R, K) \) and \( \gamma^*(R, K) \) increase in \( R \), and \( \lim_{R \to \infty} \gamma^*(R, K) = \lim_{R \to \infty} \gamma^*(R, K) = 1 \) for \( K > 0 \). This implies that for \( R^* \) such that \( \gamma = \gamma^*(R, K) \), we can find \( R^* > R^* \) such that \( \gamma = \gamma^*(R^*, K) = \gamma^*(R^*, K) < 1 \).

(ii) For \( R \leq R^* \), we have \( \gamma \geq \gamma^*(R, K) \). For \( R \geq R^* \), we have \( \gamma \leq \gamma^*(R, K) \). Part (ii) of proposition 3 implies that the ex-ante value of the two-division firm is the same as the ex-ante value of the two stand-alone firms.

(iii) For \( R \in (R^+, R^*) \), Part (i) of proposition 3 implies that the ex-ante value of the two-division firm is higher than the ex-ante value of the two stand-alone firms.

References


