ACCOUNTING versus PRUDENTIAL REGULATION*

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Abstract

We develop a framework to study how accounting measurement and prudential regulation interact to affect a bank’s incentives to originate credit. Our main result is that the accounting measurement system and bank leverage are policy tools that should be used in tandem, generating more value than systems that rely either on accounting regulation or on prudential regulation. An important application of our analysis is on the current debate on the appropriate loan loss provisioning model. We show that while banks engage in excessive risk-taking under an incurred loss model, an expected loss model can lead to excessive liquidations. More interestingly, we show that as credit conditions in the economy improve, the optimal measurement system moves towards an expected loss model. Conversely, as credit conditions deteriorate, the optimal measurement regime tilts more towards an incurred loss model.

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1. Introduction

Prudential regulations typically rely on inputs from accounting numbers but, so far, banking and accounting regulators have failed to operationalize their joint objectives. The mission of the Board of Governors of the Federal Reserve is to “foster the stability, integrity, and efficiency of the nation’s monetary, financial, and payment systems so as to promote optimal macroeconomic performance” (Government Performance and Results Act Annual Performance Report, 2011) while the Financial Accounting Standard Board states its objective as “providing financial information about the reporting entity that is useful to existing and potential investors” (FASB Concepts Statement No.8). Interestingly, the FASB notes that concerns for stability of the banking system may be in conflict with its objective of providing useful information to outsiders (FASB Concepts Statement No.8, 2011, OB 2 and BC 1.19-1.23).

At the core of this problem, we believe, is the lack of a framework that explains the role that accounting measurement plays for banks that are subject to prudential regulation. In this paper, we offer a parsimonious theory that investigates the economic tradeoffs that tie decision-useful information in the accounting systems (accounting regulation) to a concern for prudential regulation. More precisely, we study the interactions between the two forms of regulations in order to understand whether one can dominate the other and how both should jointly respond to changes in the banking sector. This is, of course, a small subset of the vast set of questions faced by accounting and prudential regulators, but it offers some simple insights to a yet-obscure problem with implications as to the role of accounting numbers in bank regulation.

We explore this question in the context of a simple real effects model of a bank, in which the emphasis is on a capital requirement that constrains the ability of the bank to originate risky loans and a commitment by the regulator to a measurement system that can be used to control ex-post incentives to liquidate the bank’s loan portfolio. We
show that the regulator will choose to liquidate risky loans more than would be desirable ex-post in order to provide ex-ante incentives to choose safe loans. However, doing so is not credible absent a suitable reporting system. Under a prudential regulation setting, i.e., when the reporting system is such that a state of the world that would cause a loss is perfectly revealed, the regulator would optimally continue banks too often from an ex-ante perspective. As a result, under prudential regulation, prudential regulators require constraints on leverage that are too strict (to solve the risk-shifting problem), thereby constraining the bank’s ability to lend. Increasing ex-post liquidation via a measurement system that rely on an expectation of the state of the world-resulting in excessive liquidations of some loans in states where such losses would not incur-slackens the bank’s incentive-compatibility constraint and allows for higher bank leverage. Hence, our main result is that the accounting measurement system and bank leverage are policy tools that should be used in tandem, generating more surplus for banks than systems that rely either on accounting regulation or on prudential regulation, even though each type of regulation might individually provide incentives on its own.

An important application of our model is in terms of the recent debate about the move of accounting standards from an incurred loss recognition model to an expected loss model for loan portfolios. We interpret the incurred loss model as a reporting system where the state of the world is known and the bank is liquidated only if a loss is incurred, while we interpret an expected loss model as the case where liquidation might occur even though there is only partial information about the state of the world. Our results uncover an interesting tradeoff for regulators. While banks engage in excessive risk-taking under an incurred loss model which, in turn, implies that the regulator must constrain the leverage of the bank, an expected loss model can lead to excessive liquidations. More interestingly, we show that as credit conditions in the economy improve as captured by a higher proportion of safe loans and/or higher liquidations values for risky loans, the optimal measurement system resembles an expected loss regime. Conversely, as credit conditions deteriorate,
Literature review. From a theoretical standpoint, there is an extensive literature that shows how agency frictions place bounds on the size of firms, see Holmstrom and Tirole (1997) for a model with a strategic intermediary given incentives to monitor a loan. We borrow heavily from these ideas in that a capital requirement on the bank bounds the size of its loan portfolio. To our knowledge, this literature does not focus on the size of firms and the design of the information system which we view as the novel elements of our model. Nevertheless, the general question of the optimal design of an information system in response to agency problems has a long history in accounting, with contributions along two paths: first, in Arya, Glover and Sivaramakrishnan (1997), and an ongoing follow-up literature, the design of the information system can address commitment problems in dynamic contracting settings; second, the real effects literature finds many environments in which price pressure will distort investment decisions (Kanodia and Sapra 2016). Our model borrows from both types of approaches, in that we examine an optimal contract with only partial commitment but nest within this problem an investment decision. In summary, the theoretical side of our contribution is to bring together the theory of the size of banks with that of information system design in order to study their interactions.

There is a burgeoning strand of banking and debt-related literature in accounting, which takes specific institutional elements of these problems into a measurement question. Corona, Nan and Zhang (2013) and Corona, Nan and Zhang (2014) are closer in spirit to our question. However, their focus is different in that their variable of interest is the degree of bank competition which we do not explicitly model. The first study shows that fair-value may be strategically adopted in order to commit some banks to exit ex-post and reducing competition. The second study examines the channel through which more bank transparency can induce more risk-taking. A primary difference with our question is that they take the capital requirement as exogenous. Within this area, many other studies
investigate the role of accounting information given pre-existing capital requirements, see, e.g., Cifuentes, Ferrucci and Shin (2005), Allen and Carletti (2008), Heaton, Lucas and McDonald (2010), and Bleck and Gao (2016). In these models, capital requirements are socially undesirable so they are not well-suited to examine some of the questions we examine here.

The impact of accounting standards on financial institutions’ behaviors has received a great attention among academia since the 2007-08 financial crisis (Acharya and Ryan 2016, Barth and Landsman 2010, Bushman and Landsman 2010). Dewatripont and Tirole (1994) show that historical cost accounting may reduce the ability of prudential regulators to discipline banks. Some empirical studies provide evidence that financial reporting indeed affects banks’ risk-taking incentives. For instance, Chircop and Novotny-Farkas (2016) suggest that extending the use of fair values for regulatory purposes reduces ex-ante risk-taking. This is consistent with Ellul, Jotikasthira, Lundblad and Wang (2015), who find that the use of fair values in statutory accounting reduces ex-ante risk-taking incentives in insurance firms. In the same vein, Bushman and Williams (2012) find that forward-looking provisions reflecting timely recognition of expected future loan losses is associated with enhanced risk-taking discipline. We argue in consistent with those studies that a well-designed accounting system may help a regulator to control banks’ risk-taking incentives.

Lastly, the closest related paper to ours is Li (2017), who analyzes risk-taking incentives in banks in presence of capital regulation under different accounting regimes. She shows that the accounting regime that maximizes the social welfare is determined by a tradeoff between the social cost of capital regulation and the efficiency of the banks project discovery efforts. In our paper, there is no exogenous cost of capital regulation and the bank is not focused on short-term earnings. We solve for the optimal accounting system given a tradeoff between banks’ ex-ante risk-taking incentives and ex-post inefficient liquidations.
2. The model

The timeline has four event dates, indexed by \( t = 0, 1, 2, 3 \) and features a regulator, a bank, and passive insured depositors. We present the timeline of the model in Figure 1.

At date \( t = 0 \), the bank invests an exogenous amount of equity \( E \). The regulator chooses a leverage multiplier for the bank, which we model as a maximum size of the loan portfolio \( A \in [E, A_{\max}] \), where \( A_{\max} \) is chosen to be sufficiently large. Given size \( A \), the bank will borrow \( A - E \) from depositors, so that we shall think of \( A/E \) as permissible leverage. Deposits are perfectly insured for reasons outside of the model (e.g., bank runs, risk-aversion by depositors). Thus, the pricing of deposits does not incorporate the default risk of the bank. We normalize the interest rate of deposit, i.e. the risk-free rate, to zero.

The regulator commits to a reporting system (aka, a persuasion mechanism) that maps a variable \( s \), to an interim signal which will determine whether the bank is liquidated at date \( t = 2 \). The state \( s \) has a distribution \( F(.) \) and a density \( f(.) \), with full support on \([0, 1]\).

At date \( t = 1 \), the bank makes a binary risk choice \( e \in \{0, 1\} \). Conditional on low risk, \( e = 1 \), the loan portfolio has a probability \( q \in (0, 1) \) to be safe and a probability \( 1 - q \) to be risky. Conditional on high risk, \( e = 0 \), the probability that the loan portfolio is safe is equal to zero. Safe loans return a payoff \( \alpha \) regardless of the state \( s \) of the world. Risky loans return \( \beta > \alpha \) with probability \( s \) and 0 with probability \( 1 - s \) if they are continued.

At date \( t = 2 \), the regulator optimally liquidates banks if the expected total payoff from liquidation is greater than the expected total payoff from continuation. If the safe loan is liquidated, we assume that the payoff from liquidation is \( \alpha \). However, if the risky

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1. The deposit insurance is an inherent feature of the banking sector. Diamond and Dybvig (1983) model the bank’s role as a liquidity provider and rationalize the deposit insurance as a tool to prevent bank runs. In the US, deposits are insured by the Federal Deposit Insurance Corporation (FDIC).

2. In this paper, we focus our analysis on the conflict of interests between stockholders and depositors of the bank. Therefore, we assume that the bank’s manager acts in the best interest of the shareholders.

3. From linearity, the incentive-compatibility condition is unchanged if we assume, more generally, that the probability of safe loans is reduced by \( \Delta q \in (0, q) \).
loan is liquidated, for simplicity, we assume that the equity holders in liquidated banks with risky loans do not recover terminal dividends. This assumption is consistent with most bank liquidations observed in practice (Granja, Matvos and Seru 2017). It can be micro-founded if banks cannot efficiently liquidate loans and, instead, requires action by a regulator or a better-capitalized intermediary. Formally, we assume that only the regulator can restructure a risky loan and recover, possibly over time, a payoff $L \in (0, 1)$.

The decision to liquidate is made based on the information produced by the reporting system. Without loss of generality, we set the number of signals equal to the number of induced actions (Kamenica and Gentzkow 2011) and use a binary signal structure $m \in \{0, 1\}$, where $m = 1$ induces liquidation and $m = 0$ induces continuation. With a slight abuse of language, we define a reporting system as a probability $m(s) \equiv \mathbb{E}(m|s) \in [0, 1]$ that state $s$ triggers a liquidation signal.\(^5\) Because liquidating must be optimal when conditional on receiving $m = 1$, we assume that

$$\mathbb{E}(s \mid m = 1) \leq \frac{L}{\beta} \leq \mathbb{E}(s \mid m = 0). \quad (2.1)$$

At date $t = 3$, the loan payoffs are realized. The payoff $\pi$ of the bank’s loan is $\pi = \alpha$ if the loan is safe, $\pi = L$ if the loan is risky and liquidated, or $\pi = \beta$ with probability $s$ or $\pi = 0$ with probability $1 - s$ if the loan is risky and continued. The regulator compensates depositors if the bank fails, $\pi < A - E$, which we assume is financed via a frictionless

\(^4\)Note that, in our model, the risky loan changes payoffs in each state but not their probability. Specifically, we interpret states as an aggregate state of the economy which makes risky loans more likely to default, as in Furlong and Keeley (1989) or Hellmann, Murdock and Stiglitz (2000), and, for our model of conflict of interest between bank and regulator, emphasizes the fact that banks do not internalize payoffs in the low state when they do not repay depositors. This also differs from other models, such as Holmstrom and Tirole (1997), Goex and Wagenhofer (2009) or Bertomeu and Cheynel (2015), where the state is idiosyncratic to the firm and, therefore, productive effort (by an entrepreneur) affects the probability of each outcome.

\(^5\)In practice, prudential regulators cannot liquidate a bank arbitrarily upon receiving some negative news. Regulators may liquidate a bank that is violating the regulatory leverage constraint. Hence, one way of interpreting our accounting system is that upon receiving bad news, the regulator forces a bank to write-down the book value of the loan (via provisioning for loan losses) which, in turn, implies that the bank will violate the regulatory leverage constraint. We discuss the provisioning interpretation of our results in section 4.
ex-ante tax. The social surplus from the loans is measured as $\Sigma = \mathbb{E}(\pi - 1)A$ where $\mathbb{E}$ denotes the expectations operator.

Conditional on low risk, a liquidating bank receives an expected payoff

$$U_l = q(\alpha A - A + E)$$

and a continuing bank receives an expected payoff

$$U_c(s) = q(\alpha A - A + E) + (1 - q)s(\beta A - A + E).$$

The regulator chooses two policies, a leverage multiplier $A$ and an optimal reporting mechanism, to maximize the total surplus from the loans. The optimal reporting mechanism is a control problem over the probability distribution of the liquidation signal $m$. The regulator therefore maximizes

$$\Sigma = (q\alpha + (1 - q)\mathbb{E}(mL + (1 - m)s\beta) - 1)A$$

subject to the following incentive-compatibility condition that induces the low risk loan:\(^6\)

$$\mathbb{E} \left[ \alpha A - A + E - s(1 - m)(\beta A - A + E) \right] \geq 0. \quad (2.3)$$

**Definition 1** An efficient policy $(A^*, m^*(\cdot))$ maximizes $\Sigma$ subject to (2.1) and (2.3).

For the entire analysis of the model, we impose the following assumptions.

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\(^6\)One potential solution to this risk-shifting problem would be to prohibit loans whose interest rate is too high, which we do not allow in our model since we assume that loan characteristics are not contractible. This is a strong assumption and, empirically, several institutional structures verify characteristics of loans (e.g., qualified loans must satisfy certain borrower requirements). However, not all loan characteristics are easily observable by regulators and issuing a set of acceptable interest rates conditional on each type of loan would require a degree of regulatory control that is far beyond current institutions.
The regulator chooses a leverage multiplier $A/E$ and a reporting system $m(s)$. The bank invests $A \geq E$. The bank makes a risk choice $e \in \{0, 1\}$ and gets a safe or a risky loan.

The state of the world $s$ is realized, the reporting system sends $m(s)$ and the regulator takes the liquidation decision. Payoffs are realized: $\alpha$ for a safe loan, $\beta$ with prob. $s$ and 0 with prob. $1 - s$ for a risky loan.

Figure 1: Model Timeline

\[ A_0 = \begin{cases} \frac{\alpha A_{\text{max}} - A_{\text{max}} + E}{\beta A_{\text{max}} - A_{\text{max}} + E} & \frac{1 - q\alpha}{(1 - q)\beta} - F(L/\beta)L/\beta) \\ < \int_{L/\beta}^1 sf(s)ds < \min(\alpha/\beta, 1/\beta - F(L/\beta)L/\beta)) \end{cases} \] (2.4)

A0 rules out degenerate cases for which the analysis of our model becomes straightforward. The first part of the left-hand side of the inequality implies that the bank cannot achieve its first-best surplus by lending to the maximal extent $A_{\text{max}}$ and implementing the ex-post surplus-maximizing liquidation policy. The second part requires the low risk portfolio to have positive value because otherwise, the regulator would always induce a bank of size zero.

Similarly, the first part of the right-hand side of the inequality rules out parameter values in which the agency problem is so severe that the bank would lend only its own equity. The second part of the right-hand side of the inequality guarantees that the risky loan is value-destroying and therefore rules out a solution in which the regulator prescribes risky loans with maximal size.
3. Analysis

3.1. Prudential regulation benchmark

Before solving the general model in which the regulator chooses both the optimal reporting system and prudential regulation, we first examine a benchmark setting in which the regulator controls the leverage of the bank but takes as given a reporting system that perfectly reveals the state. More precisely, we assume that the regulator perfectly observes the state $s$ and liquidates the bank whenever $\beta s < L$. We refer to this setting as a (pure) prudential regulation benchmark because it ignores the optimal design of the reporting system.

From A0, it follows that the regulator cannot induce a low risk portfolio if the bank has zero equity and therefore we assume for this benchmark setting that the bank has non-zero equity $E$ to commit per unit of loan. Consistent with the existing literature on financial intermediation (e.g., Holmstrom and Tirole (1997) or Biais, Mariotti, Plantin and Rochet (2007)), we show that the regulator must impose a leverage $A^{*}/E$ that is bounded by the incentive problem.

**Proposition 1** Under prudential regulation, the optimal leverage is given by

\[
\frac{A^{*}}{E} = \frac{1 - \int_{\frac{s}{E}}^{1} sf(s)ds}{1 - \int_{\frac{s}{E}}^{1} sf(s)ds - (\alpha - \beta \int_{\frac{s}{E}}^{1} sf(s)ds)} > 1.
\] (3.1)

Equation (3.1) illustrates the tradeoff between continuing valuable risky loans and increasing bank leverage. Specifically, if we denote the surplus from continuing risky loans as $S_c = \int_{\frac{s}{E}}^{1} sf(s)ds$ then,

\[
\frac{\partial A^{*}/E}{\partial S_c} = -\frac{\beta - \alpha}{(1 - S_c - (\alpha - \beta S_c))^2} < 0,
\]

implying that each unit of additional surplus from continuing a risky loan decreases the
size of the loan portfolio that the bank can manage. Further, the right-hand side of (3.1) is greater than one (from A0), implying that there is always a suitable loan portfolio size such that low risk can be induced.

**Corollary 1** The optimal leverage \( A^*/E \) increases in the payoff \( \alpha \) of the safe loan, in the liquidation value \( L \) of the risky loan, and decreases in the payoff \( \beta \) of the risky loan.

The only tool available to the regulator under prudential regulation is the capital requirement. Hence, as the payoff of the risky loan in the good state increases, the risk-shifting problem becomes more severe and the regulator reduces bank leverage. Conversely, as the payoff of the safe loan or the liquidation payoff increase, the risk-taking problem becomes less severe and the regulator increase the bank’s leverage.

Substituting the optimal leverage from equation (3.1) in equation (2.2) yields the following expression for the surplus of the bank as a function of the characteristics of its loan portfolio:

\[
\Sigma = \frac{(1 - \int_{\frac{1}{2}}^{1} s f(s) ds)(q \alpha + (1 - q)(L F(\frac{L}{\beta}) + \beta \int_{\frac{1}{2}}^{1} s f(s) ds) - 1)}{1 - \int_{\frac{1}{2}}^{1} s f(s) ds - (\alpha - \beta \int_{\frac{1}{2}}^{1} s f(s) ds)} E. \tag{3.2}
\]

As expected, the surplus is increasing in the liquidation payoff \( L \). The surplus also increases both as the likelihood \( q \) of the safe loan increases and as the profitability \( \alpha \) of the safe loan increases. Closer inspection of equation (3.2), however, reveals that the impact of \( \beta \) on the bank’s surplus is ambiguous due to two opposing effects of \( \beta \) on the bank’s surplus. First, an increase in \( \beta \) makes the risky loan—which reduces expected surplus—more attractive to banks and, as shown in Corollary 1, causes a reduction in the equilibrium leverage. A low risk portfolio increases surplus so the reduction in size is socially costly. Second, an increase in \( \beta \) makes the risky loan more attractive because it implies a first-order stochastic dominance shift in their payoff structure. Therefore, even though the risky loan reduces social surplus under our maintained assumption A0, the adverse impact
of the risky loan on the bank’s surplus becomes more muted as $\beta$ increases. The latter effect increases the total surplus generated by banks.

3.2. Accounting regulation benchmark

We now contrast the (pure) prudential setting discussed above to a polar opposite in which the regulator relies only on the design of a reporting system to discipline banks but does not control the bank’s leverage. In essence, we are interested in a version of the model in which the regulator does not impose any upper bound on the size of the bank’s assets so that the bank’s leverage will be infinite. Accordingly, we set $E = 0$ since the bank equity per unit of loan is nearly zero and refer to this as (pure) accounting regulation because we entirely forfeit prudential tools to control banks.

Because the incentive-compatibility constraint is proportional to $A$, it can be rewritten as either $A = 0$ or

\[ E\left[\alpha - 1 - s(1 - m)(\beta - 1)\right] \geq 0, \quad (3.3) \]

and so that regulator maximizes $\Sigma$ subject to (2.1) and (3.3).

**Proposition 2** Let $\kappa$ be given by $\int_0^1 sf'(s)ds = (\alpha - 1)/(\beta - 1)$. Under accounting regulation,

(i) if $L \geq \beta \int_0^\kappa sf'(s)ds/F(\kappa)$, the optimal policy is such that $A^* = A_{\text{max}}$ and the reporting system issues a liquidation signal $m(s) = 1$ if and only if $s < \kappa$, where $\kappa > L/\beta$;

(ii) otherwise, inducing low risk is infeasible.

The above result implies that if the liquidation value of the risky loan is high enough (case (i)), the optimal accounting regulation can discipline the bank to choose the low risk portfolio without any additional equity. However, the liquidation policy is inefficient as the bank is liquidated over some states of the world with $s \in [L/\beta, \kappa]$. Therefore, in the
absence of any prudential regulation, the need to provide ex-ante disciplining incentives is in conflict with the ex-post efficient liquidation constraint. Indeed, when the agency problem becomes sufficiently severe (case (ii)), the regulator is unable to commit to liquidate the loan portfolio sufficiently often to solve the risk-taking problem and the bank cannot induce the low risk portfolio.

**Corollary 2** Whenever $A^* > 0$, the optimal threshold $\kappa$ decreases in the payoff $\alpha$ of the safe loan and increases in the payoff $\beta$ of the risky loan.

Another important implication of this benchmark (which we shall generally demonstrate throughout our analysis) is given in Corollary 2. Absent any agency problem, the total surplus is increasing in $\beta$ because this raises the payoff from the risky loan for any liquidation policy. However, when $\beta$ is too large, it becomes infeasible to induce the low risk portfolio and, therefore, more profitable risky loans reduce the loan portfolio to zero.

This argument applies when switching from case (i) to case (ii) of Proposition 2 but we can also apply a similar logic within case (i) of the Proposition to show how an increase in $\beta$ may reduce total surplus via its effect on the liquidation choice. Rewriting the total surplus after substituting (3.3),

$$\Sigma = A_{\max}(q\alpha + (1 - q)(\mathcal{L}F(\kappa) + \beta\frac{\alpha - 1}{\beta - 1}) - 1).$$

Taking a total derivative with respect to $\beta$, the total surplus $\Sigma$ is increasing in $\beta$ if and only if

$$\frac{\alpha - 1}{(\beta - 1)^2}(\frac{\mathcal{L}}{\kappa} - 1) > 0. \quad (3.4)$$

The sign of the left-hand side of (3.4) has the sign of $\mathcal{L}/\kappa - 1$ and depends only on $\beta$ and $\alpha$ via their effects on $\kappa$. We have shown that $\kappa$ is increasing in $\beta$ in Corollary 2 and, further, this term is positive when $\kappa$ is close to $\mathcal{L}/\beta$. Hence, the total surplus is either increasing or inverse U-shaped in $\beta$, with its global maximum at the value of $\beta$ that induces $\kappa = \mathcal{L}$. 
Reinjecting this in the definition of $\kappa$, the socially preferred payoff for risky loans is given by

$$1 = \frac{\alpha - 1}{\int_{L}^{1} sf(s)ds}.$$  \hfill (3.5)

In other words, our framework implies that the regulator would have preferences about which type of risky loans the banks should be allowed to engage in. In particular, settings with higher safe loan payoffs and liquidation values are conducive to lending to more risky loan. Vice-versa, environments in which risky loans become more profitable, without any concurrent change in safe loans, will typically be detrimental.

As in Plantin, Sapra and Shin (2008), we next conduct a comparison of two modes of regulation taken in isolation and demonstrate that prudential regulation can do worse than accounting regulation. From a practical perspective, this result contrasts with the predominant model of bank regulation which primarily relies on controlling prudential tools. This is also, of course, an important step to motivating a theory where both pieces are jointly optimized. In our model, comparing prudential versus accounting regulation amounts to comparing total surplus when controlling only the reporting system (pure accounting regulation) versus requiring bank equity and controlling the amount of leverage (pure prudential regulation).

**Corollary 3** Accounting regulation with zero equity is preferred to prudential regulation if and only if $\mathcal{L} \geq \beta \int_{0}^{\infty} sf(s)ds/F(\kappa) + o(1/A_{max}).$

The main insight from Corollary 3 is that accounting regulation may be too rigid to tackle sufficiently important agency frictions but, if the risk-taking problem is not too severe, then accounting regulation is preferred to prudential regulation. Because the regulator cannot commit to shut down banks if they have positive surplus, there are limits as to what liquidations pure accounting regulation can induce and these can become insufficient to solve the agency problem. In this case, banks must be required to hold additional skin in the game in the form of an equity contribution and an upper bound on
the size of their loan portfolio. Equivalently, the regulator imposes capital requirements.

To summarize, accounting regulation become more efficient as the liquidation values of risky loans improve relative to the maximal payoffs of such loans.

### 3.3. Joint prudential and accounting regulation

We next derive the optimal accounting standard when the regulator jointly optimizes the reporting system and the capital requirement. Without loss of generality, we decompose this joint choice as a choice of $A$ and, for a given $A$, a choice of the measurement $m(\cdot)$, hereafter, the subproblem. As we will demonstrate, the bank will always choose to lend to the maximal extent allowed by the regulator, so we shall equivalently refer to $A$ as allowed bank leverage or the size of the bank.

We can simplify the analysis by noting that the subproblem is equivalent to maximizing a Lagrangian objective function that is pointwise linear in $m(\cdot)$, that is,

$$
(P) \quad \max_{m(\cdot) \in [0,1]} \int ((q\alpha + (1 - q)(m(s)\mathcal{L} + (1 - m(s))s\beta)) - 1)A f(s) ds
$$

s.t.

$$
\frac{\mathcal{L}}{\beta} \int m(s)f(s) ds - \int sm(s)f(s) ds \geq 0 \quad (3.6)
$$

$$
\int s(1 - m(s))f(s) ds - \frac{\mathcal{L}}{\beta} \int (1 - m(s))f(s) ds \geq 0 \quad (3.7)
$$

$$
\alpha A - A + E - (\beta A - A + E) \int s(1 - m(s))f(s) ds \geq 0. \quad (3.8)
$$

where (3.6) and (3.7) are the left-hand and right-hand side of (2.1), respectively, and (3.8) is the incentive-compatibility condition.\(^7\) We show next that the liquidation policy must take the form of a threshold above which the bank is continued.

\(^7\)Note that this is a relaxed version of the subproblem as we allow for a choice of $m(s)$ on the unit interval. However, the relaxed subproblem coincides with the original subproblem to the extent that, as a result of linearity, the solution must be an extreme value of the set of feasible policies.
Lemma 1 A solution to (P) must be such that \( m(s) = 1 \) if and only if \( s < \tau \), where \( \tau \geq \mathcal{L}/\beta \).

Lemma 1 contains two key parts, which we discuss separately. The fact that only low states must be liquidated may seem surprising given that, in the incentive-compatibility constraint, liquidating the loan conditional on higher states increases the left-hand side of (3.8) the most. However, from the objective function, higher states also improve the payoffs of the bank if the loan is continued and this latter effect dominates the former effect.

To see why, we offer below a heuristic proof which carries the intuition better than the formal proof. Assume that the incentive-compatibility condition binds (as will turn out to be the case) so that

\[
\int s(1 - m(s))f(s)ds = \frac{\alpha A - A + E}{\beta A - A + E}.
\]

(3.9)

This condition means that a sufficiently large fraction of the surplus from continuation must be forfeited in order to solve the risk-taking problem. Intuitively, continuation gives the agent the ability to achieve the \( \beta s \) payoff and thus makes it more difficult to induce the low risk choice. Note that this condition does not say whether high or low states should be liquidated and, indeed, high states contribute more to this constraint because they are multiplied by \( s \) (and affect a manager choosing high risk the most). But, reinjecting in the objective function and regrouping terms, implies

\[
\Sigma = A(q\alpha + (1 - q)\frac{\alpha A - A + E}{\beta A - A + E} - 1) + A(1 - q)\mathbb{E}(m)\mathcal{L} \quad \text{(3.10)}
\]

where \( \text{cst}(A) \) is a function of \( A \) that does not contain \( m(.) \). Hence, once the incentive-compatibility binds, the objective function is increasing in the probability of liquidation.

The policy that maximizes this probability while meeting incentive-compatibility is therefore to shut down firms conditional on low states. To further explain this feature of our
model, note that the second term in (3.10) is the expected payoff from the risky loan, with the benefit being determined entirely from the incentive-compatibility. In other words, the entire continuation surplus cannot be increased beyond what is permissible by the risk-taking agency problem.

The second part of Lemma 1 demonstrates that the liquidation threshold is always weakly greater than the first-best threshold $\tau_{fb} = \mathcal{L}/\beta$. Put differently, the equilibrium may feature excess liquidations for incentive purposes. Liquidations, in this model, serve a dual objective, namely, to shut down ex-post inefficient risky projects and discipline ex-ante risk-taking.

The first objective is best solved by setting a threshold at $\tau_{fb} = \mathcal{L}/\beta$, such that states below $\tau_{fb}$ induce a liquidation. This constraint is very similar to the ex-post efficiency constraint in (2.1) except that, in the constraint, it is evaluated against the perceived state revealed by the reporting system. If one were to focus only on the first objective, the reporting system $m(s) = 1$ if and only if $s < \tau_{fb}$ would be chosen.

The second objective requires to use liquidations to elicit a low risk portfolio. In turn, inspecting (2.3), the incentive-compatibility condition becomes easier to meet when $\mathbb{E}(sm)$ increases, that is, liquidating higher states of the world helps discipline risk-taking. Whenever risk-taking incentives bind, the implied distortion must take the form of more liquidations than would be demanded by the first objective.

**Lemma 2** The efficient continuation threshold $\tau$ satisfies

$$\int_{\tau}^{1} s f(s) ds = \frac{\alpha A - A + E}{\beta A - A + E}.$$  

(3.11)

Lemma 2 demonstrates that the incentive-compatibility condition must bind. The intuition for this observation in our model is straightforward. The first-best policy - lending the maximal amount of loans with no distortion to liquidation - is infeasible (from A0). Of interest, we can rewrite (3.11) as a statement about the leverage of a bank, by
dividing both sides of the right-hand side of (3.11) by $E$ and solving for bank leverage:

$$
\frac{A}{E} = \frac{1 - \int_1^1 s f(s)ds}{1 - \int_1^1 s f(s)ds - (\alpha - \beta \int_1^1 s f(s)ds)}.
$$

(3.12)

This characterization illustrates how, holding a liquidation threshold fixed, more leverage is possible if the payoff from safe assets increases or the payoff from risky assets decreases.

To further characterize the optimal policies chosen by the regulator, we write the optimal continuation threshold $\tau(A)$ in terms of $A$ from (3.11). Applying the implicit function theorem,

$$
\tau'(A) = \frac{(\beta - \alpha)E}{\tau(A)f(\tau(A))(\beta A - A + E)^2} > 0,
$$

which is intuitive because the risk-taking problem is more severe when the ratio of loans to equity increases and thus requires a greater fraction of liquidation.

We have for now left aside the ex-post constraint $\mathbb{E}(s|m(s) = 1) \leq \mathcal{L}/\beta$, which is required for the bank to be credibly liquidated by the regulator. Yet, bringing back this constraint yields another crucial implication of our analysis. The more $A$ is increased, the more difficult it becomes to meet the incentive-compatibility condition: to meet this condition, $\tau(A)$ must increase to raise $\mathbb{E}(s \mid m(s) = 1)$. But, if $A$ is set too high, $\tau(A)$ will no longer satisfy $\mathbb{E}(s \mid s \leq \tau(A)) \leq \mathcal{L}/\beta$. This implies an upper bound $\tau$ on the liquidation threshold, defined by $\mathbb{E}(s \mid s \leq \tau) = \mathcal{L}/\beta$, and an implied upper bound on the size of the bank $A$, where $\tau = \tau(\mathcal{A})$.

Reinjecting the optimal leverage (3.12) into the social surplus (2.2), this problem can be rephrased as choosing the optimal liquidation threshold to maximize

$$
\Sigma(\tau) \equiv \frac{(q \alpha + (1 - q)(\mathcal{L}F(\tau) + \beta \int_\tau^1 s f(s)ds - 1)(1 - \int_\tau^1 s f(s)ds)}{1 - \int_\tau^1 s f(s)ds - (\alpha - \beta \int_\tau^1 s f(s)ds)E}
$$

(3.14)

with a necessary condition for an interior solution given by $\Sigma'(\tau) = 0$. 

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We show next that the regulator would always choose to distort the ex-post liquidation threshold, i.e., choose $\tau > \tau_{fb} = L/\beta$. As a corollary, the optimal reporting system induces a greater bank size and higher efficiency than a reporting system in which $s$ is perfectly revealed. We prove this statement in the next proposition.

**Proposition 3** The efficient continuation threshold $\tau(A^*)$ is set at a level strictly higher than the first-best continuation threshold $\tau_{fb} = L/\beta$.

Recall that the liquidation is also governed by an ex-post constraint which bars policies in which the regulator would liquidate a bank with greater value if it were continued. The higher the liquidation threshold, the more this constraint becomes difficult to satisfy. Specifically, the ex-post constraint is satisfied if and only if

$$\int_0^\tau sf(s)ds = \frac{L}{\beta} F(\tau).$$

(3.15)

For convenience, we assume here that $\mathbb{E}(s)\beta > L$, which implies that $\tau < 1$. This has little bearing on our main insights as long as, if this condition is not satisfied, the regulator can set the liquidation threshold at the maximal level, i.e. $\tau = 1$.

To characterize the optimal threshold, note that $\Sigma'(\tau) < 0$ is a sufficient condition for the existence of an interior solution $\tau(A^*) < \tau$ since, then, the regulator would increase welfare by reducing the liquidation threshold. In the next proposition, we show that this condition is necessary and sufficient, and can be expressed in terms of a statement about the fraction of safe loans.

**Proposition 4** $\tau(A^*) = \tau$ if and only if

$$q > \bar{q} = \frac{\nu}{(\alpha - 1)(\beta - \alpha)\tau + \nu} \in (0, 1),$$

(3.16)
where

\[ \nu = (1 - \beta \mathbb{E}(s))(\beta - \alpha)\tau - (L - \tau \beta)(1 + \frac{L F(\tau)}{\beta} - \mathbb{E}(s))(1 - \alpha + (\beta - 1)(\mathbb{E}(s) - \frac{L}{\beta} F(\tau))). \]

The intuition for Proposition 4 is given in two steps, starting with a comparative static in \( q \) in this paragraph and followed by the rationale for \( \tau(A^*) = \tau \). Although the probability of a safe loan \( q \) did not play an important role in any of the benchmark settings, we show here that it is a key determinant of how the regulator chooses the optimal reporting system. Specifically, when \( q \) is large, there is a greater net benefit from increasing leverage. In the prudential benchmark, this leverage was fully determined by the incentive condition, so that there was little the regulator could do to increase it further. Here, on the other hand, the regulator can increase the inefficient liquidation of risky loans. Whether this is desirable depends on the trade-off with the cost of excessive liquidation. But, to pin down this trade-off, recall that the fraction of risky loans is lower so the cost of relying on inefficient liquidations is reduced. Putting both regulatory channels together, we conclude that the regulator will (weakly) increase the threshold \( \tau(A^*) \) in response to an increase in the fraction of safe loans \( q \).

Continuing on this logic, this implies that expectations about failing banks increase as a function of \( q \), and it becomes increasingly difficult for the regulator to credibly shut down banks. When \( q \) becomes greater than \( \bar{q} \), the required liquidation threshold to implement the ideal leverage would be above \( \tau \). At this point, the ex-post liquidation constraint \( \mathbb{E}(s \mid s \leq \tau(A^*)) \leq L/\beta \) becomes binding and the regulator implements the maximal credible threshold \( \tau \). Put differently, when the loans become sufficiently safe, the regulator implements the maximal credible level of liquidation (and would have been better-off with a policy of commitment to liquidation and even higher leverage). Naturally, when this point \( q \geq \bar{q} \) is reached, the credible level of liquidation no longer depends on \( q \) since safe loans neither gain nor lose from liquidating the bank. We summarize these comparative
Corollary 4 \( A^*/E \) and \( \tau(A^*) \) are increasing in \( q \), strictly if and only if \( q \leq \bar{q} \).

A different logic is at play for an increase in the liquidation value \( L \) but the intuition remains entirely transparent. As for \( q \), a greater liquidation payoff increases the total value earned by increasing leverage, leading to a greater desirability of higher leverage (provided it remains incentive-compatible). It also reduces the cost of liquidating risky loans for a given state. Therefore, the comparative static is similar to that of \( q \) and an increase in \( L \) leads to higher leverage and more liquidation for any given state. In addition, because higher \( L \) increases the credibility of a regulatory intervention (via its effect on \( L/\beta \) in the ex-post liquidation constraint), this comparative static holds even when the maximal threshold \( \tau \) is attained. We state it below.

Corollary 5 \( A^*/E \) and \( \tau(A^*) \) are increasing in \( L \).

An increase in \( \alpha \) increases the attractiveness of safe loans to banks and reduces the severity of agency problems while increasing the surplus generated by banks. Hence, we argue and find that the leverage increases as a function of \( \alpha \). If \( q \) is high, however, because the maximal liquidation threshold is attained and the credibility of liquidations does not depend on \( \alpha \), it will no longer affect the liquidation threshold.

Corollary 6 \( A^*/E \) and \( \tau(A^*) \) are increasing in \( \alpha \), strictly for \( A^*/E \) or if \( q < \bar{q} \).

Finally, the effect of the risky payoff \( \beta \) on leverage is ambiguous because it has two opposite effects on the liquidation threshold. On the one hand, a higher \( \beta \) makes it more costly to liquidate a firm for any level of \( s \) and this tends to reduce the liquidation threshold. On the other hand, a higher \( \beta \) also makes the risky loan portfolio more attractive which requires more liquidation to meet the incentive constraint.

As it turns out, the effect of \( \beta \) depends on whether the ex-post liquidation constraint \( \tau \) is reached (equivalently, whether the fraction of safe loans is high enough). If \( \tau(A^*) = \tau \),...
the constraining factor is to make the liquidation of failing banks credible, so that higher \( \beta \), because it makes it more tempting for the regulator to continue, leads to a reduction in the threshold and a lower leverage. If \( q < \bar{q} \), we have the only case where the optimal leverage and the liquidation may enter in different directions, as a result of the two trade-offs discussed earlier. We elaborate more on this in the next corollary.

**Corollary 7** If \( q \geq \bar{q} \), \( A^*/E \) and \( \tau^* \) are decreasing in \( \beta \). Otherwise, for any parameter values such that \( \partial \tau^*/\partial \beta < 0 \) or \( \partial (A^*/E)/\partial \beta > 0 \), \( \tau^* \) and \( A^*/E \) vary in the same direction as a function of \( \beta \).

The main new claim is contained in the second part of Corollary 7 which reveals that, for certain settings, the regulatory tools vary in the same direction as for the other comparative statics.\(^8\) In particular, if the regulator raises leverage in response to higher \( \beta \), a more severe agency problem is faced for the ex-post liquidation and liquidations must increase. Vice-versa, if the regulator reduces liquidations, then the more severe agency problem must be solved with lower leverage.

Table 1 wraps up with the comparative statics of the general model, with the main observation being that, for all variables but the payoff of the risky loan, accounting and prudential regulations move in tandem. When using only accounting regulation (third row) or only prudential regulation (fourth row), the regulations do not depend on the fraction of good projects \( q \) and, in the case of accounting regulation, the continuation threshold does not depend on the liquidation value. The regulations respond to more profitable risky loans by either reducing leverage or by increasing liquidations, and more profitable safe loans reduce the equilibrium level of inefficiency by either increasing bank leverage or reducing inefficient liquidations.

\(^8\)This claim is, unfortunately, based on endogenous objects but, conceptually, it aims at establishing when we should observe the two regulatory variables co-moving and helps clarify some intuitions.
Joint regulation with $q > \bar{q}$  

Joint regulation with $q \leq \bar{q}$  

Accounting only benchmark  

Prudential only benchmark  

<table>
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<tr>
<th></th>
<th>$\tau(A')$</th>
<th>$q$</th>
<th>$\mathcal{L}$</th>
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<td>with $q &gt; \bar{q}$</td>
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<td>benchmark</td>
<td>$A' / E$</td>
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Table 1: Comparative statics

4. An Application to loan loss provisioning

A practical application of our model is to inform the debate on the optimal provisioning model for loan losses that has been received a lot of attention since the 2007-08 financial crisis. Namely, the incurred loss approach has been heavily criticized for reinforcing procyclical effects of bank regulation by delaying the recognition of loan losses until such losses have occurred. Accounting standard setters are currently requiring a switch to an expected loss approach where loan losses would be recognized even though such losses have not occurred yet but might occur in the future based on the bank’s information about expected loan defaults. Stated differently, the threshold for recognizing losses would be lower under an expected loss approach than the incurred loss approach.

In our model, we can approach this policy debate from a mechanism design perspective and view the incurred loss approach as a reporting system where the state $s$ is known and the bank is liquidated only if a loss is incurred conditional on $s$, while the expected loss approach refers to reporting systems that induce the regulator to form expectations and, occasionally, liquidate loans that could have been valuable had the state been known. For expositional purposes, we refer to an expected loss model as the extreme case with the maximum probability of liquidation, that is, when $\tau^* = \overline{\tau}$, while an incurred loss model refers to $\tau^* = L / \beta$ - other regimes will be referred to as mixed reporting models which bear closer to an incurred loss model when $\tau^*$ is close to $L / \beta$. Below, we compare first the
surplus for the two provisioning regimes in the accounting regulation benchmark.

**Corollary 8** Under accounting regulation, an expected loss model is always weakly preferred to an incurred loss model, strictly if

$$q > \frac{1 - \mathcal{LF}(\bar{\tau}) - \beta \int_{s}^{1} sf(s)ds}{\alpha - \mathcal{LF}(\bar{\tau}) - \beta \int_{s}^{1} sf(s)ds}, \quad (4.1)$$

where $\bar{\tau}$ is such that $E(s \mid s \leq \bar{\tau}) = \mathcal{L}/\beta$.

Corollary 8 is a key step to lay out the economic tradeoff between the two provisioning regimes in our model. Under an incurred loss model, incentives tend to be too low: in fact, so much so that absent bank equity, the low risk loan portfolio cannot be induced and the bank should not operate. Under an expected loss model, incentives can be given to choose the low risk portfolio under the same conditions as (ii) in Proposition 2, but the additional inefficient liquidations of the risky loan cause the value of the portfolio to decrease. When $q$ is low, as noted in (4.1), excessive reliance on expected loss can also make it undesirable to lend.

We can draw the following implications from our general model. First, an incurred loss model is never the optimal reporting system because it implies excessively low bank leverage (that is, prudential regulations that are too strict). Put differently, under an incurred loss model, banks engage in excessive risk-taking which, in turn, implies that the regulator must constrain the size of the bank to compensate for this effect. But expected loss may be problematic as well, as a reporting system that maximizes the total amount of liquidations can lead to excessive liquidations, in particular when the proportion of risky loans is high enough and such liquidations could be socially costly.

Second, the comparative statics on the liquidation threshold tell us to what extent the reporting system moves from an incurred to an expected loss model. As we show in our joint regulation, higher capital requirements typically come together with reporting systems that are more tilted towards expected loss; in particular, if the safe loans or the
liquidation payoffs are higher, or the likelihood of safe loans is higher, the regulator will tend to readjust toward an expected loss model, increasing bank leverage in the process. We view these settings as situations when the economy as a whole features more favorable conditions so, according to our model, an expected loss model is more suitable to expansionary credit periods. The following corollary, which is a direct consequence of Proposition 4 and Corollaries 4, 5 and 6, sums up this discussion.

**Corollary 9** The optimal measurement regime is as follows.

- If \( q > \bar{q} \), an expected loss model is optimal;
- otherwise, if \( q \leq \bar{q} \), the optimal measurement regime is mixed. As \( q, \alpha \) and \( \mathcal{L} \) increase, the optimal regime moves towards an incurred loss model.

Lastly, let us slightly modify the framework to consider this policy debate in settings where characteristics of the information environment do not lend themselves to implementing the ideal reporting system. So far, we have assumed that there are no frictions in how information can be controlled ex-ante. However, some practical cases may feature constrained choices of mechanisms where certain information may arrive over time. To set ideas, consider first a setting in which information about the state over an horizon \( t \in [0, 1] \) where state \( s \geq t \) becomes public information after state \( t \). Naturally, in this context, we could not implement certain reporting systems that were allowed in our baseline model. However, we can think about the reporting system choice as an intervention point in the timeline, with a date \( t_0 \) (committed ex-ante) where the regulator would decide to liquidate. In other words, the regulator commits to intervention once the information has a certain precision.

Inspecting this problem further, we know that by setting \( t_0 = \tau^* \), the regulator would observe all states with \( s \geq t_0 \) (and would not liquidate as we have shown that \( \mathbb{E}(s \mid s \geq \tau^*) \geq \mathcal{L}/\beta \)) and, for the remaining banks, observe the state \( s < t_0 \) which would yield the
liquidation policy of the baseline model. So, we can think about the optimal mechanism as commitment to intervene at a particular level of knowledge about the state. An incurred loss model, with \( \tau^* = \mathcal{L}/\beta \) will correspond to intervention when it is certain that the loan is losing value while an expected loss model will correspond to intervention when the loan starts losing value in expectation.

It is of course also possible that the arrival of information may not allow for a solution that implements the “full-control” mechanism considered earlier and, unfortunately, there are too many processes of arrival of information to consider all cases. Instead, we analyze the opposite version of the previous example to make this point clear and assume that it is now the low states \( s < t \) that are revealed as time progresses.\(^9\) Intervention occurs at pre-committed time \( t_0 \). Setting \( t_0 = \tau^* \) will not work here because it would imply that, at \( t_0 \), the regulator would know the state for all projects below \( t_0 \) and thus would liquidate if and only if \( s \leq \mathcal{L}/\beta \) (incurred loss model) and not liquidate in the region \( (\mathcal{L}/\beta, \tau^*) \) or the region above \( \tau^* \) since we know that \( \mathbb{E}(s \mid s \geq \tau^*) \geq \mathcal{L}/\beta \). So what is the best measurement given this constrained mechanism problem. Indeed, as it should be intuitively apparent, an incurred loss model becomes the only viable option.

**Corollary 10** Suppose that states \( s \leq t \) become observable as \( t \in [0,1] \) increases, and the regulator commits to intervene at \( t_0 \). Then, the stopping time is at \( t_0 = \mathcal{L}/\beta \) and implements an incurred loss model.

The intuition for this is only useful to set up the additional considerations that come from an exogenous arrival of information and which reduce the scope for a mechanism. When low states are revealed purely sequentially over time, then an expected loss model is not possible since all incurred loss states will be fully revealed before we get to the point where we can induce additional liquidations. Of course, more generally, we can extend this

\(^9\)Keep in mind that these refers to states of the world, not characteristics of individual loans so it would not be evident that this process would be more plausible than the earlier one as this is different from one individual loan failing a payment (further, payments occur at the end in our model).
argument to any deterministic revelation of states $s$ which is more complex and refers to both good and bad states, noting that arrival of information about good states will tend to bias the analysis toward adoption of an expected loss model while bad states will tend to bias the analysis toward an incurred loss model due to feasibility issues.

5. Extensions

5.1. Costly liquidation of safe loans

We have assumed until this point that safe loans can be carried to maturity even if the bank is liquidated, so that they pay off the same amount $\alpha$ regardless of the liquidation policy. We explore below the consequences of a polar opposite to this assumption, by supposing that the safe loan payoff changes to $L$ when liquidated. In other words, safe loan suffer from liquidations in all states of the world.

In this setting, the first-best ex-post liquidation threshold $\tau_{fb}$ is now such that $q\alpha + (1 - q)\tau_{fb}\beta = L$, i.e. $\tau_{fb} = (L - q\alpha)/(1 - q)\beta$. Let us assume that $q\alpha < L$ in order to have an interior first-best ex-post liquidation threshold. This threshold is lower than in our baseline model because the regulator incurs an additional social loss when liquidating safe loans, especially when safe loans are likely. We can rewrite the ex post efficient continuation/liquidation constraints as

$$E(s \mid m = 1) \leq \tau_{fb} = \frac{L - q\alpha}{(1 - q)\beta} \leq E(s \mid m = 0). \tag{5.1}$$

The total surplus is now given by

$$\Sigma = \left( E(mL + (1 - m)(q\alpha + (1 - q)s\beta)) - 1 \right) A \tag{5.2}$$
and the incentive-compatibility condition that makes low risk the preferred choice is

\[ E \left[ (1 - m)(\alpha A - A + E - s(\beta A - A + E)) \right] \geq 0. \] (5.3)

As in the baseline model, the regulator can increase the liquidation \((m)\) or decrease the bank leverage \((A)\), in order to elicit a low risk portfolio choice. In this setting, as in our baseline model, the optimal reporting system is a threshold \(\tau^*\) above which loan portfolios are continued. To illustrate this intuition formally, let us write the Lagrangian of the baseline model, indicating by multiplier \(\mu_0\) the incentive constraint (5.3) (we omit the ex-post constraints for expositional purpose but they can be easily reincorporated). Differentiating with respect to the probability \(m(s)\) of liquidating conditional on state \(s\),

\[ \frac{\partial L}{\partial m(s)} = s(\mu_0(\alpha A - A + E) - (1 - q)\beta) + \mathcal{L} - q\alpha - \mu_0(\alpha A - A + E). \]

Noting that this function is linear in \(s\), we know that \(m(s) = 1\) if and only if \(s \leq \tau\), where \(\tau\) is a threshold in \([0,1]\). Hence, the incentive-compatibility constraint can be rewritten as

\[ \int_{\tau}^{1} (\alpha A - A + E - s(\beta A - A + E))f(s)ds \geq 0. \]

The left-hand side is negative if \(\tau \geq (\alpha A - A + E)/(\beta A - A + E)\), which implies that \(\tau^* < (\alpha A^* - A^* + E)/(\beta A^* - A^* + E)\). Moreover, the left-hand side is decreasing in \(\tau\) for \(\tau < (\alpha A - A + E)/(\beta A - A + E)\). Further, the objective function is increasing in \(\tau\) when \(\tau \in (0, \tau_{fb})\), hence, it must be that, \(\tau^* \leq \tau_{fb}\). In contrast with our baseline model, the optimal liquidation policy is to induce excessive continuations. Indeed, for low realizations of \(s\), the bank is strictly better-off with a safe rather than a risky loan in case of continuation, whereas the bank is indifferent between a safe and a risky loan in case of liquidation. As a result, excessive continuations are a way for the regulator to provide incentives to the bank to choose a low risk portfolio.
The incentive-compatibility constraint binds in equilibrium because, otherwise, the regulator set $\tau^* = \tau_{fb}$ and $A = A_{\text{max}}$, a case that we rule out as in the baseline model.\textsuperscript{10} Thus, the optimal leverage is such that

$$ A = \frac{1 - F(\tau) - \int_{\tau}^{1} s f(s) ds}{\int_{\tau}^{1} s f(s) ds (\beta - 1) - (\alpha - 1)(1 - F(\tau))}.$$ \hfill (5.4)

Reinjecting, the total surplus is given by

$$\Sigma = A(F(\tau) \mathcal{L} + q(1 - F(\tau)) \alpha + (1 - q) \beta \int_{\tau}^{1} s f(s) ds - 1)$$

$$= E \frac{(1 - F(\tau) - \int_{\tau}^{1} s f(s) ds)(F(\tau) \mathcal{L} + q(1 - F(\tau)) \alpha + (1 - q) \beta \int_{\tau}^{1} s f(s) ds - 1)}{\int_{\tau}^{1} s f(s) ds (\beta - 1) - (\alpha - 1)(1 - F(\tau))}.$$  

Taking the first-order condition with respect to $\tau$ yields

$$H(\tau) = (F(\tau) \mathcal{L} + q(1 - F(\tau)) \alpha + (1 - q) \beta \int_{\tau}^{1} s f(s) ds - 1)(\beta - \alpha)(1 - F(\tau)) - \int_{\tau}^{1} s f(s) ds$$

$$+ (1 - F(\tau) - \int_{\tau}^{1} s f(s) ds)(\mathcal{L} - q \alpha - (1 - q) \tau \beta)((\beta - 1) \int_{\tau}^{1} s f(s) ds - (\alpha - 1)(1 - F(\tau))) = 0.$$ \hfill (5.5)

Evaluating the function $H$ at the first-best threshold yields

$$H(\tau_{fb}) = (F(\tau_{fb}) \mathcal{L} + q(1 - F(\tau_{fb})) \alpha + (1 - q) \beta \int_{\tau_{fb}}^{1} s f(s) ds - 1)(\beta - \alpha) \int_{\tau_{fb}}^{1} (\tau_{fb} - s) f(s) ds.$$  

The first term of this product is the social surplus of a low-risk loan portfolio using the first-best liquidation policy and this surplus is assumed to be positive.\textsuperscript{11} The second term is obviously negative. Hence, we can conclude that $H(\tau_{fb}) < 0$. Thus, the ex-post

\textsuperscript{10}Specifically, we assume that $\alpha A_{\text{max}} - A_{\text{max}} + E - (\beta A_{\text{max}} - A_{\text{max}} + E) \int_{\tau_{fb}}^{1} s f(s) ds < 0$.

\textsuperscript{11}This is equivalent to assume that $F(\tau_{fb}) \mathcal{L} + q(1 - F(\tau_{fb})) \alpha + (1 - q) \beta \int_{\tau_{fb}}^{1} s f(s) ds > 1$.  

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first-best threshold is always a local maximum of the total surplus $\Sigma$. This contrasts with our main model in which the ex post first-best threshold is never a local maximum. Note, nevertheless, that, because the social surplus need not be concave, there can be cases in which the optimal loss measurement is set below $\tau_{fb}$, i.e. $\tau^* < \tau_{fb}$.

In summary, when the liquidation value of the safe loan is equal to the liquidation value of the risky loan, the optimal accounting policy never induces excessive liquidations and the ex-post first-best liquidation threshold may be the optimal threshold. This contrasts with the results of our baseline model.

5.2. Residual bank equity with risky loans

Next, we analyze below another important variation of our baseline model, in which the bank can liquidate the risky loan without transferring it to the regulator. In this environment, when forced to liquidate, the bank obtains $L$ per loan instead of having to transfer all of its equity to the regulator. One interpretation is that the bank can resell both risky and safe loans at the same market price $L$. For high enough leverage $A$ (to be derived later on), $A(qA + \alpha - (1 - q)L) A - (A - E) < 0$, the bank will still have zero equity left in case of liquidation, so this case is only relevant for environments where banks start with high levels of equity or the agency friction is very high. Perhaps, for example, this model fits more closely the problem of certain financial intermediaries (e.g., guaranteed funds, specialized banks) which rely more on their own equity capital than on outside depositors. For simplicity, we assume that the bank learns perfectly the realization of the state of the world $s$, but the bank cannot credibly reveal $s$ to the regulator.

The first-best ex-post liquidation threshold, $\tau_{fb} = L/\beta$, is the same as in our baseline model. Given that a bank never defaults, a liquidating bank receives an expected payoff

$$U_l = q(\alpha A - A + E) + (1 - q)(\alpha A - A + E).$$  (5.6)
The total surplus is the same than in our baseline model. The regulator cannot force the bank to keep a loan if the expected payoff for the bank in case of liquidation is higher than the expected payoff in case of continuation, i.e., for all $s$ such that $m(s) = 0$, we have

\[ q(\alpha A - A + E) + (1 - q)(\mathcal{L}A - A + E) \leq q(\alpha A - A + E) + (1 - q)s(\beta A - A + E), \quad (5.7) \]

which is equivalent to $\mathcal{L}A - A + E \leq s(\beta A - A + E)$. Further, the incentive-compatibility condition that makes low risk the preferred choice is

\[ \mathbb{E}\left[ \alpha A - A + E - m(\mathcal{L}A - A + E) - s(1 - m)(\beta A - A + E) \right] \geq 0. \quad (5.8) \]

As in our baseline model, the regulator can decrease the bank leverage ($A$), in order to elicit a low risk portfolio choice. The effect of liquidations on the incentives of the bank to choose the low risk portfolio becomes ambiguous in this setting. On the one hand, for high values of $s$ (such that $s(\beta A - A + E) > \mathcal{L}A - A + E$), liquidating the loan decreases the payoff to the bank and increases the incentives. On the other hand, for low values of $s$ (such that $s(\beta A - A + E) < \mathcal{L}A - A + E$), the expected payoff of the bank increases in case of liquidation and the regulator would be willing to continue. However, in those states, the bank voluntarily liquidates the loan portfolio. As a result, the optimal liquidation policy cannot induce excessive continuations and, as in our baseline model, induces excessive liquidations.

Formally, we can write the Lagrangian of the baseline model, indicating by multiplier $\mu_0$ the incentive constraint (5.8) and omitting the ex-post constraints. Differentiating this Lagrangian with respect to the probability $m(s)$ of liquidating conditional on state $s$,

\[
\frac{\partial L}{\partial m(s)} = s(\mu_0(\alpha A - A + E) - (1 - q)\beta) + (1 - q)\mathcal{L} - \mu_0(\mathcal{L}A - A + E).
\]

Noting that this function is linear in $s$, we know there exists $\tau$ in $[0, 1]$ such that
\( m(s) = 1 \) for \( s < \tau \), and \( m(s) = 0 \) for \( s > \tau \). Then, we can rewrite the incentive-compatibility constraint as

\[
\alpha A - A + E - (\mathcal{L}A - A + E)F(\tau) - (\beta A - A + E) \int_{\tau}^{1} sf(s)ds \geq 0. \tag{5.9}
\]

The left-hand side increases in \( \tau \) for \( \tau > (\mathcal{L}A - A + E)/(\beta A - A + E) \). Moreover, we know that condition (5.7) requires that \( \tau^* \geq (\mathcal{L}A - A + E)/(\beta A - A + E) \) and \( \tau_{fb} = \mathcal{L}/\beta \geq (\mathcal{L}A - A + E)/(\beta A - A + E) \). Hence, the optimal liquidation policy induces excessive liquidations, i.e. \( \tau^* \geq \tau_{fb} \). As previously, we rule out the solution with \( \tau_{fb} \) and \( A_{\max} \), which implies that the incentive compatibility constraint is binding in equilibrium.\(^\text{12}\) The optimal leverage is such that

\[
\frac{A}{E} = \frac{1 - F(\tau) - \int_{\tau}^{1} sf(s)ds}{\int_{\tau}^{1} sf(s)ds(\beta - 1) - F(\tau)(1 - \mathcal{L}) - (\alpha - 1)}. \tag{5.10}
\]

After deriving the optimal leverage, the condition that makes sure that the bank with a risky loan does not default in case of liquidation is given by \( A^*/E \leq \mathcal{L} + 1 \). This condition is satisfied if we assume that the parameters \( (\alpha, \beta, \mathcal{L}, q) \) are such that

\[
\frac{1 - F(\tau) - \int_{\tau}^{1} sf(s)ds}{\int_{\tau}^{1} sf(s)ds(\beta - 1) - F(\tau)(1 - \mathcal{L}) - (\alpha - 1)} \leq \mathcal{L} + 1. \tag{5.11}
\]

To summarize, when the bank does not default in case of liquidation and the bank can take the liquidation decision, the optimal accounting policy also induces excessive continuations.

\(^{12}\)We assume that \( \alpha A_{\max} - A_{\max} + E - (\mathcal{L}A_{\max} - A_{\max} + E)F(\tau_{fb}) - (\beta A_{\max} - A_{\max} + E) \int_{\tau_{fb}}^{1} sf(s)ds < 0. \)
6. Conclusion and perspectives on integrated accounting policy

Economists worry about the efficient allocation of resources. Unfortunately, transparency need not always serve this objective and the FASB’s position on increasing access will lead to recurrent conflicts between accounting and economic policy. This paper, as part of many others in this stream of literature, offers an alternative perspective on accounting as integrated to economic policy sharing common objectives, and where dissemination information is a means-to-an-end. To do this, accounting standard setters need to be equipped to think about economic consequences and place these consequences in their primary goal.

We have approached this research problem in the context of a setting that is currently affected by both accounting and prudential regulators. The current institutional arrangement is odd, as prudential regulators use accounting information as input, but then transform some elements of these accounting numbers or entirely ignore information that the FASB views as important. Occasionally, various bodies have noted that the actions of accounting standard setters have gone against efforts to stabilize credit markets by other bodies. The position, stated repeatedly, that accountants should give public access to as much information as possible while letting other regulatory bodies deal with consequences using other levers is difficult to justify. In fact, we show here that accounting choices can only be partially addressed using a capital requirement, often leading to prudential choices that are too strict. Put differently, if accounting choices yield outcomes that are detrimental to investors, other policy tools are only partial substitutes.

More work is needed to address various difficult problems that may rise when using economic objectives. First, we do not know yet how accounting regulators would effectively implement provisioning rules that are time-varying, thus creating informational levers that activate or deactivate as conditions in the capital market change. A threshold in a simple
model offers a high-level perspective but does not speak much of its implementation at a micro-level in terms of the measurements of particular transactions. Should regulators control impairment ceilings, in terms of varying percentages of loss of value causing an impairment? Should accounting numbers be indexed on distance from a capital ratio if banks access different profiles of risky loans?

Second, we have still almost no research about other regulatory levers that interact with accounting policy. From a macroeconomic perspective, regulators control access to credit via interest rate policy, budgetary choices or tax policy, all of which are based on accounting information but do not seem coordinated with accounting. It seems an underlying belief in the accounting profession that such choices are purely politically-minded and would damage accounting regulation. There is confusion here as to economic definition of political as "what affects welfare in society" to the accountants’ definition as "what affects special interests at the expense of others" and we hope that, taking the first, the profession can see the benefit of justifying the political economy of such choices.
Appendix 1: proofs

Proof of Proposition 1: We can rewrite the (IC) constraint as

\[
\int_0^{L/\beta} (A\alpha - A + E)f(s)ds + \int_{L/\beta}^1 (A\alpha - A + E - s(A\beta - A + E))f(s)ds \geq 0. \tag{6.1}
\]

By contradiction, if the (IC) is not binding, the regulator will choose \(A = A_{\text{max}}\), large, which contradicts \(A_0\). Hence, binding the (IC), the optimal size \(A^*\) is given by equation (3.1) in text. Further \(A^*/E\) is greater than 1, implying that there is always a loan portfolio size such that the (IC) constraint is satisfied, i.e. low risk can be induced. □

Proof of Proposition 2: It is convenient to state the optimal reporting problem for each unit of loan as a program linear in the reporting policy where, with a slight abuse in notation, we write the control as a function \(m(s) \in [0, 1]\) indicating the probability that a firm is liquidated.

\[
(P) \quad \max_{m(s), A} A \int (q_\alpha + (1 - q)(m(s)L + (1 - m(s))s\beta - 1))f(s)ds \\
\text{s.t.} \quad \alpha - 1 - (\beta - 1) \int s(1 - m(s))f(s)ds \geq 0 \quad (\mu_0) \\
\quad \int (1 - m(s))sf(s)ds \geq \frac{L}{\beta} \int (1 - m(s))f(s)ds \quad (\mu_a) \\
\quad \frac{L}{\beta} \int m(s)f(s)ds \geq \int sm(s)f(s)ds \quad (\mu_b)
\]

Differentiating the lagrangian \(L\) in \(m(s)\), we obtain

\[
\frac{\partial L}{\partial m(s)} = (1 - q)(L - s\beta) + \mu_0 s(\beta - 1) + (\mu_a + \mu_b) \left( \frac{L}{\beta} - s \right)
\]

\[
= s(\mu_0(\beta - 1) - (1 - q)\beta - \mu_a - \mu_b) + (1 - q + \frac{\mu_a + \mu_b}{\beta})L.
\]

In turn, noting that this function is linear in \(s\) and positive at \(s = 0\), we know that
$m(s) = 1$ if and only if $s \leq \tau$, where $\tau$ is a threshold in $[0, 1]$.

**Case 1.** Suppose that $\mu_0 = 0$. Then, the solution $\mu_a = \mu_b = 0$ and $\tau_{ne} = \mathcal{L}/\beta$ maximizes the Lagrangian and satisfies the constraints associated to multipliers $\mu_a$ and $\mu_b$. Reinjecting in the incentive-compatibility condition,

$$\alpha - 1 - (\beta - 1) \int_{\tau}^{1} s f(s) ds \geq 0,$$

which contradicts A0.

**Case 2.** Suppose that $\mu_0 > 0$, in which case the complementary slackness condition implies $\alpha - 1 - (\beta - 1) \int_{\tau}^{1} s f(s) ds = 0$, which is equivalent to

$$\int_{\tau}^{1} s f(s) ds = \frac{\alpha - 1}{\beta - 1}. \quad (6.3)$$

Therefore, $\tau \in (\frac{\alpha}{\beta}, 1)$, where the lower bound is from A1.

Next, note that $\tau > \mathcal{L}/\beta$ implies that the constraint associated to $\mu_a$ is not binding. The constraint associated to $\mu_b$ requires the following inequality:

$$\mathcal{L} F(\tau) \geq \beta \int_{0}^{\tau} s f(s) ds,$$

which is equivalent to

$$\mathcal{L} \geq \frac{\beta \int_{0}^{\tau} s f(s) ds}{F(\tau)} \quad (6.5)$$

To conclude the proof, note that if (6.5) is not satisfied, then inducing low risk is not feasible and, therefore, the optimal choice is $A^* = 0$. Otherwise, the optimal choice is $A^* = A_{\max}$.

**Proof of Corollary 2:** Suppose that we are in the case with $A^* = A_{\max}$. The optimal threshold $\kappa$ is defined such that $\int_{\kappa}^{1} s f(s) ds = (\alpha - 1)/(\beta - 1)$. Therefore, as $\alpha$ increases, $\int_{\kappa}^{1} s f(s) ds$ increases, which implies that $\kappa$ decreases. Similarly, as $\beta$ increases, $\int_{\kappa}^{1} s f(s) ds$
decreases, which implies that $\kappa$ increases. \(\square\)

**Proof of Lemma 1:** We define the Lagrangian similarly with the lagrange multipliers $\mu_0$, $\mu_a$ and $\mu_b$, associated to constraints (3.8), (3.6) and (3.7) respectively. Differentiating with respect to $m(s)$,

$$\frac{\partial L}{\partial m(s)} = s(\mu_0(A\beta - A + E) - (1-q)\beta - \mu_a - \mu_b) + (1-q + \frac{\mu_a + \mu_b}{\beta})\mathcal{L}.$$ 

Noting that this function is linear in $s$ and positive at $s = 0$, we know that $m(s) = 1$ if and only if $s \leq \tau$, where $\tau$ is a threshold in $[0, 1]$.

**Case 1.** Suppose that $\mu_0 = 0$. Then, the solution $\mu_a = \mu_b = 0$ and $\tau = \mathcal{L}/\beta$ maximizes the Lagrangian and satisfies the constraints associated to multipliers $\mu_a$ and $\mu_b$. But, then, it is desirable to set $A = A_{\text{max}}$, which contradicts $A_0$.

**Case 2.** Suppose that $\mu_0 > 0$. Writing the incentive-compatibility explicitly after reinjecting $m(s) = 1$ if and only if $s < \tau$,

$$\Delta_{IC} = \alpha A - A + E - (\beta A - A + E) \int_{\tau}^1 s f(s) ds \geq 0 \quad (6.6)$$

The left-hand side is increasing in $\tau$ and the objective function is increasing in $\tau$ when $\tau \in (0, \mathcal{L}/\beta)$, hence, it must be that, if the (IC) is binding, $\tau \geq \mathcal{L}/\beta$. \(\square\)

**Proof of Lemma 2:** Suppose that the efficient continuation threshold is such that the incentive-compatibility constraint does not bind. Then, it is optimal for the regulator to set the threshold $\tau = \mathcal{L}/\beta$. This first-best ex post threshold satisfies the constraints (3.6) and (3.7).

The expected utility of the regulator is

$$(qa + (1-q)F(\frac{\mathcal{L}}{\beta})\mathcal{L} + (1-q) \int_{\xi}^1 s \beta f(s) ds - 1)A.$$ 

This last expression is positive from $A_0$. But then, it is desirable to set $A = A_{\text{max}}$. 37
This is a contradiction with A0. Hence, the incentive-compatibility condition binds in equilibrium. □

**Proof of Proposition 3:** Let us denote

\[
H(\tau) = \Sigma'(\tau)(1 - \int_\tau^1 sf(s)ds - (\alpha - \beta \int_\tau^1 sf(s)ds)^2/E
\]

(6.7)

Taking the first order condition of the optimization problem,

\[
H(\tau) = (q\alpha + (1 - q)(\mathcal{L}F(\tau) + \beta \int_\tau^1 sf(s)ds) - 1)(\beta - \alpha)\tau \\
+ (1 - q)(\mathcal{L} - \tau\beta)(1 - \int_\tau^1 sf(s)ds)(1 - \int_\tau^1 sf(s)ds - (\alpha - \beta \int_\tau^1 sf(s)ds)) = 0.
\]

(6.8)

Evaluating this expression at \(\tau = \mathcal{L}/\beta\),

\[
H\left(\frac{\mathcal{L}}{\beta}\right) = (q\alpha + (1 - q)(\mathcal{L}F(\frac{\mathcal{L}}{\beta}) + \beta \int_{\frac{\mathcal{L}}{\beta}}^1 sf(s)ds) - 1)(\beta - \alpha)\frac{\mathcal{L}}{\beta} > 0,
\]

so that the first-best ex post threshold \(\mathcal{L}/\beta\) is never the ex ante optimal choice for the regulator. Further, we know from Lemma 1 that \(\tau(A^*) \geq \mathcal{L}/\beta\) which implies that the liquidation threshold must be in the set \((\mathcal{L}/\beta, \tau(A))\). □

**Proof of Proposition 4:** Since the 'if' part is immediate, we prove here the 'only if' part. Differentiating \(H\) in \(\tau\),

\[
H'(\tau) = (q\alpha + (1 - q)(\mathcal{L}F(\tau) + \beta \int_\tau^1 sf(s)ds) - 1)(\beta - \alpha) \\
+ \tau(\beta - \alpha)(1-q)f(\tau)(\mathcal{L} - \tau\beta)(1 - \int_\tau^1 sf(s)ds)(1 - \int_\tau^1 sf(s)ds - (\alpha - \beta \int_\tau^1 sf(s)ds)) \\
+ (1 - q)(\mathcal{L} - \tau\beta)\left(\tau f(\tau)(1 - \int_\tau^1 sf(s)ds - (\alpha - \beta \int_\tau^1 sf(s)ds)) + f(\tau)(1 - \beta)(1 - \int_\tau^1 sf(s)ds)\right).
\]

(6.9)

To simplify the above equation, let us rewrite equation (6.8) at the optimal threshold
\[ \tau^* \equiv \tau(A^*) \]
\[ (q \alpha + (1 - q))(\mathcal{L}F(\tau^*) + \beta \int_{\tau^*}^1 s f(s) ds - 1)(\beta - \alpha)\tau^* \]
\[ = -(1 - q)(\mathcal{L} - \tau^* \beta)(1 - \int_{\tau^*}^1 s f(s) ds)(1 - \int_{\tau^*}^1 s f(s) ds - (\alpha - \beta \int_{\tau^*}^1 s f(s) ds)) \]  \hspace{1cm} (6.10)

Reinjecting this last equality into (6.9) evaluated at \( \tau^* \) yields

\[ H'(\tau^*) = \tau^*(\beta - \alpha)(1 - q) f(\tau^*) (\mathcal{L} - \tau^* \beta) \]
\[ - (1 - q) \frac{\mathcal{L}}{\tau^*} (1 - \int_{\tau^*}^1 s f(s) ds)(1 - \int_{\tau^*}^1 s f(s) ds - (\alpha - \beta \int_{\tau^*}^1 s f(s) ds)) \]
\[ + (1 - q)(\mathcal{L} - \tau^* \beta) \left( \tau^* f(\tau^*)(1 - \int_{\tau^*}^1 s f(s) ds - (\alpha - \beta \int_{\tau^*}^1 s f(s) ds)) + \tau^* f(\tau^*)(1 - \beta)(1 - \int_{\tau^*}^1 s f(s) ds) \right). \]

\[ = - (2\tau^* (1 - q) f(\tau^*) (\tau^* \beta - \mathcal{L}) + (1 - q) \frac{\mathcal{L}}{\tau^*} (1 - \int_{\tau^*}^1 s f(s) ds)) \]
\[ \times (\beta \int_{\tau^*}^1 s f(s) ds + 1 - \int_{\tau^*}^1 s f(s) ds - \alpha). \]  \hspace{1cm} (6.11)

It is immediate to verify that \( H_1 < 0 \). Next, \( H_2 > 0 \) can be written as

\[ \frac{\beta - 1}{\alpha - 1} > \frac{1}{\int_{\tau^*}^1 s f(s) ds}. \]  \hspace{1cm} (6.12)

This last inequality is an implication from the incentive-compatibility condition, since we know from the incentive-compatibility condition that

\[ \frac{1}{\int_{\tau^*}^1 s f(s) ds} = \frac{\beta A - A + E}{\alpha A - A + E} < \frac{\beta - 1}{\alpha - 1}. \]  \hspace{1cm} (6.13)
Therefore, we have shown that $\Sigma'(\tau^*) < 0$. Note that this holds at $\tau^*$ as well as at any root of $\Sigma'(.)$, implying that $\Sigma'(.)$ can cross zero at most once and from below so that $\Sigma$ has a single peak. Evaluating $\Sigma'(.)$ at $\bar{\tau}$, we know that the the peak is located below $\bar{\tau}$ if $\Sigma'(\bar{\tau}) \leq 0$ and above $\bar{\tau}$ if $\Sigma'(\bar{\tau}) > 0$.

We can further rewrite

$$H(\bar{\tau}) = (q \alpha + (1-q)(\mathcal{L}F(\bar{\tau}) + \beta \int_{\tau}^1 sf(s)ds - 1)(\beta - \alpha)\bar{\tau}$$

(6.14)

$$+ (1-q)(\mathcal{L} - \bar{\tau}\beta)(1 - \int_{\tau}^1 sf(s)ds)(1 - \int_{\tau}^1 sf(s)ds - (\alpha - \beta \int_{\tau}^1 sf(s)ds))$$

$$= (q \alpha + (1-q)\beta \mathbb{E}(s) - 1)(\beta - \alpha)\bar{\tau}$$

(6.15)

$$+ (1-q)(\mathcal{L} - \bar{\tau}\beta)(1 + \frac{\mathcal{L}F(\bar{\tau})}{\beta} - \mathbb{E}(s))(1 - \alpha + (\beta - 1)(\mathbb{E}(s) - \frac{\mathcal{L}}{\beta}F(\bar{\tau})))$$

(6.16)

where the last equality is obtained from the definition of $\bar{\tau}$. Thus, $\Sigma'(\bar{\tau}) > 0$ is equivalent to

$$q > \frac{(1-\beta \mathbb{E}(s))(\beta - \alpha)\bar{\tau} - (\mathcal{L} - \bar{\tau}\beta)(1 + \frac{\mathcal{L}F(\bar{\tau})}{\beta} - \mathbb{E}(s))(1 - \alpha + (\beta - 1)(\mathbb{E}(s) - \frac{\mathcal{L}}{\beta}F(\bar{\tau})))}{(\alpha - \beta \mathbb{E}(s))(\beta - \alpha)\bar{\tau} - (\mathcal{L} - \bar{\tau}\beta)(1 + \frac{\mathcal{L}F(\bar{\tau})}{\beta} - \mathbb{E}(s))(1 - \alpha + (\beta - 1)(\mathbb{E}(s) - \frac{\mathcal{L}}{\beta}F(\bar{\tau})))}$$

(6.17)

Therefore, we get condition (3.16) in text. □

**Proof of Corollaries 4, 5 and 6:**

Let us start by deriving the comparative statics in the case $q > \bar{q}$. The threshold $\bar{\tau}$ is defined by (3.15), which is equivalent to $\mathbb{E}(s \mid s \leq \bar{\tau}) = \mathcal{L}/\beta$. Obviously, $\bar{\tau}$ does not depend on $\alpha$. Further, as $\mathcal{L}$ increases, $\mathbb{E}(s \mid s \leq \bar{\tau})$ increases, which implies that $\bar{\tau}$ increases. Similarly, as $\beta$ increases, $\mathbb{E}(s \mid s \leq \bar{\tau})$ decreases, which implies that $\bar{\tau}$ decreases.

The optimal leverage is given by

$$\frac{A^*}{\mathbb{E}} = \frac{1 - \int_{\tau}^1 sf(s)ds}{1 - \int_{\tau}^1 sf(s)ds - (\alpha - \beta \int_{\tau}^1 sf(s)ds)}.$$  

(6.18)
Therefore,

\[
\frac{\partial (A^*_{\bar{E}})}{\partial \bar{\tau}} = \frac{\bar{\tau} f(\bar{\tau})(1 - \int_{\bar{\tau}}^{1} \sigma f(s)ds - (\alpha - \beta \int_{\bar{\tau}}^{1} \sigma f(s)ds)) + (1 - \int_{\bar{\tau}}^{1} \sigma f(s)ds)(\beta - 1)\bar{\tau} f(\bar{\tau})}{(1 - \int_{\bar{\tau}}^{1} \sigma f(s)ds - (\alpha - \beta \int_{\bar{\tau}}^{1} \sigma f(s)ds))^{2}} > 0.
\]

(6.19)

As a result,

\[
\frac{\partial (A^*_{\bar{E}})}{\partial \bar{L}} = \frac{\bar{\tau} \bar{\tau} f(\bar{\tau})(1 - \int_{\bar{\tau}}^{1} \sigma f(s)ds - (\alpha - \beta \int_{\bar{\tau}}^{1} \sigma f(s)ds)) + (1 - \int_{\bar{\tau}}^{1} \sigma f(s)ds)(\beta - 1)\bar{\tau} f(\bar{\tau})}{(1 - \int_{\bar{\tau}}^{1} \sigma f(s)ds - (\alpha - \beta \int_{\bar{\tau}}^{1} \sigma f(s)ds))^{2}} > 0
\]

(6.20)

and

\[
\frac{\partial (A^*_{\bar{E}})}{\partial \alpha} = \frac{1 - \int_{\bar{\tau}}^{1} \sigma f(s)ds}{(1 - \int_{\bar{\tau}}^{1} \sigma f(s)ds - (\alpha - \beta \int_{\bar{\tau}}^{1} \sigma f(s)ds))^{2}} > 0.
\]

(6.21)

Finally, \(\bar{\tau}\) decreases in \(\beta\). Hence,

\[
\frac{\partial (A^*_{\bar{E}})}{\partial \beta} = \frac{\bar{\tau} \bar{\tau} f(\bar{\tau})(1 - \int_{\bar{\tau}}^{1} \sigma f(s)ds - (\alpha - \beta \int_{\bar{\tau}}^{1} \sigma f(s)ds))}{(1 - \int_{\bar{\tau}}^{1} \sigma f(s)ds - (\alpha - \beta \int_{\bar{\tau}}^{1} \sigma f(s)ds))^{2}} + \frac{(1 - \int_{\bar{\tau}}^{1} \sigma f(s)ds)((\beta - 1)\bar{\tau} f(\bar{\tau}) \frac{\partial \bar{\tau}}{\partial \alpha} - \int_{\bar{\tau}}^{1} \sigma f(s)ds)}{(1 - \int_{\bar{\tau}}^{1} \sigma f(s)ds - (\alpha - \beta \int_{\bar{\tau}}^{1} \sigma f(s)ds))^{2}} < 0.
\]

(6.22)

which implies that \(A^*_{\bar{E}}\) decreases in \(\beta\).

Then, we can derive the comparative statics in the case \(q \leq \bar{q}\). Let \(H(\tau) = \Sigma'(\tau)\) be defined as the derivative of the social surplus in \(\tau\). We know from Proposition 4 that \(\tau^* \leq \bar{\tau}\) and, given that it is a local maximum of \(\Sigma\), \(H'(\tau^*) \leq 0\). We will assume here that it is regular maximum, \(H'(\tau^*) < 0\) so that the comparative statics are always well-defined.

It then follows that the comparative static of \(\tau^*\) in a variable \(X\) has the sign of \(\partial H/\partial X\),
which we conduct next:

\[
\frac{\partial H}{\partial \alpha} = q(\beta - \alpha)\tau - \tau(q\alpha + (1 - q)(\mathcal{L}F(\tau) + \beta \int^1_\tau sf(s)ds) - 1) \\
- (1 - q)(\mathcal{L} - \tau\beta)(1 - \int^1_\tau sf(s)ds);
\]

\[
\frac{\partial H}{\partial \beta} = (1 - q)\int^1_\tau sf(s)ds(\beta - \alpha)\tau + \tau(q\alpha + (1 - q)(\mathcal{L}F(\tau) + \beta \int^1_\tau sf(s)ds) - 1) \\
+ (1 - q)(\mathcal{L} - \tau\beta)(1 - \int^1_\tau sf(s)ds) \int^1_\tau sf(s)ds \\
- (1 - q)\tau(1 - \int^1_\tau sf(s)ds)(1 - \alpha + (\beta - 1) \int^1_\tau sf(s)ds);
\]

\[
\frac{\partial H}{\partial \mathcal{L}} = (1 - q)F(\tau)(\beta - \alpha)\tau \\
+ (1 - q)(1 - \int^1_\tau sf(s)ds)(1 - \alpha + (\beta - 1) \int^1_\tau sf(s)ds) > 0;
\]

\[
\frac{\partial H}{\partial q} = (\alpha - \mathcal{L}F(\tau) - \beta \int^1_\tau sf(s)ds)(\beta - \alpha)\tau \\
- (\mathcal{L} - \tau\beta)(1 - \int^1_\tau sf(s)ds)(1 - \alpha + (\beta - 1) \int^1_\tau sf(s)ds).
\]

We know that \( H(\tau(A^*)) = 0 \). Therefore

\[
- \tau^*(q\alpha + (1 - q)(\mathcal{L}F(\tau^*) + \beta \int^1_{\tau^*} sf(s)ds) - 1) \\
= \frac{1 - q}{\beta - \alpha}(\mathcal{L} - \tau^*\beta)(1 - \int^1_{\tau^*} sf(s)ds)(1 - \alpha + (\beta - 1) \int^1_{\tau^*} sf(s)ds), \quad (6.23)
\]

which implies that \( \frac{\partial H}{\partial q}_{\tau=\tau(A^*)} = (\alpha + \frac{1}{1 - q}(q\alpha - 1))(\beta - \alpha)\tau(A^*) > 0 \). Similarly, \( H(\tau(A^*)) = 0 \), is equivalent to

\[
- \tau^*(q\alpha + (1 - q)(\mathcal{L}F(\tau^*) + \beta \int^1_{\tau^*} sf(s)ds) - 1) \\
= \frac{1 - q}{\beta - \alpha}(\mathcal{L} - \tau^*\beta)(1 - \int^1_{\tau^*} sf(s)ds)(1 - \alpha + (\beta - 1) \int^1_{\tau^*} sf(s)ds), \quad (6.24)
\]

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which implies that

\[
\frac{\partial H}{\partial \alpha}(\tau(A^*)) = q(\beta - \alpha)\tau^* + (1 - q)(\tau^* \beta - \mathcal{L})(1 - \int_{\tau^*}^{1} s f(s) ds) (1 - \frac{1 - \alpha + (\beta - 1) \int_{\tau^*}^{1} s f(s) ds}{\beta - \alpha}) \\
= q(\beta - \alpha)\tau^* + \frac{1 - q}{\beta - \alpha}(\tau^* \beta - \mathcal{L})(1 - \int_{\tau^*}^{1} s f(s) ds) (\beta - 1 - (\beta - 1) \int_{\tau^*}^{1} s f(s) ds) > 0.
\]

(6.25)

We can conclude that the optimal threshold \( \tau^* \) increases in \( \alpha \), in \( q \) and in \( \mathcal{L} \).

From (3.12), we can also conclude that the optimal leverage moves in the same direction than the optimal threshold with respect to \( q \), \( \alpha \) and \( \mathcal{L} \). More precisely, we know that

\[
\frac{\partial (\frac{A^*}{E})}{\partial \mathcal{L}} = \frac{\partial \tau^* f(\tau^*)}{\partial \mathcal{L}} (1 - \int_{\tau^*}^{1} s f(s) ds - (\alpha - \beta \int_{\tau^*}^{1} s f(s) ds)) + (1 - \int_{\tau^*}^{1} s f(s) ds) (\beta - 1) \tau^* f(\tau^*) > 0,
\]

(6.26)

\[
\frac{\partial (\frac{A^*}{E})}{\partial q} = \frac{\partial \tau^* f(\tau^*)}{\partial q} (1 - \int_{\tau^*}^{1} s f(s) ds - (\alpha - \beta \int_{\tau^*}^{1} s f(s) ds)) + (1 - \int_{\tau^*}^{1} s f(s) ds) (\beta - 1) \tau^* f(\tau^*) > 0,
\]

(6.27)

and

\[
\frac{\partial (\frac{A^*}{E})}{\partial \alpha} = \frac{\partial \tau^* f(\tau^*)}{\partial \alpha} (1 - \int_{\tau^*}^{1} s f(s) ds - (\alpha - \beta \int_{\tau^*}^{1} s f(s) ds)) + (1 - \int_{\tau^*}^{1} s f(s) ds) ((\beta - 1) \tau^* f(\tau^*) \frac{\partial \tau^*}{\partial \alpha} + 1) > 0.
\]

(6.28)

Therefore, the optimal leverage \( \frac{A^*}{E} \) also increases in \( \alpha \), \( q \) and \( \mathcal{L} \).

The comparative statics with respect to \( \beta \) is slightly more complicated. Indeed, we
know that
\[
\frac{dH}{d\beta} = (1 - q) \int_\tau^1 sf(s)ds(\beta - \alpha)\tau + \tau(q\alpha + (1 - q)(L_F(\tau) + \beta \int_\tau^1 sf(s)ds) - 1) \\
- (1 - q)(\tau\beta - L)(1 - \int_\tau^1 sf(s)ds) \int_\tau^1 sf(s)ds \\
- (1 - q)\tau(1 - \int_\tau^1 sf(s)ds)(1 - \alpha + (\beta - 1) \int_\tau^1 sf(s)ds).
\] (6.29)

The first two terms are positive whereas the last two terms are negative. There is an ambiguity for the following reason. As \( \beta \) increases, the risk-shifting problem becomes more severe (first two terms) and the regulator should provide more incentives to the banker to choose the low risk portfolio by increasing the liquidation threshold. On the other hand, as \( \beta \) increases, the payoff in case of continuation is increasing (last two terms) and hence, the regulator is willing to reduce the liquidation threshold.\( \square \)

**Proof of Corollary 7:** Taking the derivative of (3.12) with respect to \( \beta \) yields

\[
\frac{\partial A^*/E}{\partial \beta} = \frac{\partial \pi^*/E(\tau^*)(1 - \int_\tau^1 sf(s)ds - (\alpha - \beta \int_\tau^1 sf(s)ds))}{(1 - \int_\tau^1 sf(s)ds - (\alpha - \beta \int_\tau^1 sf(s)ds))^2} \\
- \frac{\int_\tau^1 sf(s)ds(1 - (\beta - 1) \frac{\partial \pi^*/E}{\partial \beta}(\tau^*) + \int_\tau^1 sf(s)ds)}{(1 - \int_\tau^1 sf(s)ds - (\alpha - \beta \int_\tau^1 sf(s)ds))^2}.
\]

Hence, \( \frac{\partial \pi^*/E}{\partial \beta} < 0 \) implies that \( \frac{\partial A^*/E}{\partial \beta} < 0 \).

Further, we know that the leverage is given by

\[
\int_\tau^1 sf(s)ds = \frac{\alpha A^* - A^* + E}{\beta A^* - A^* + E}.
\] (6.30)
Taking the derivative of the right hand side with respect to beta yields

\[
\frac{(\alpha - 1)\frac{\partial A^*}{\partial \beta} (\beta A^* - A^* + E) - (\alpha A^* - A^* + E)((\beta - 1)\frac{\partial A^*}{\partial \beta} + A^*)}{(\beta A^* - A^* + E)^2}
= \frac{E(\alpha - \beta)\frac{\partial A^*}{\partial \beta} - (\alpha A^* - A^* + E)A^*}{(\beta A^* - A^* + E)^2}
\]

(6.31)

Hence, \( \frac{\partial A^*}{\partial \beta} > 0 \) implies that the right-hand side of (6.30) decreases in \( \beta \), which in turn implies that \( \int_{s^*}^{1} sf(s)ds \) decreases in \( \beta \), i.e. \( \tau^* \) increases in \( \beta \). \( \square \)

**Proof of Corollary 8:** Under incurred loss, the loan portfolio is liquidated if and only if \( s < \mathcal{L}/\beta \). Reinjecting in the incentive-compatibility condition:

\[
\alpha - 1 - (\beta - 1) \int_{\mathcal{L}/\beta}^{1} sf(s)ds \geq 0,
\]

which contradicts the left-hand side inequality in A0. So (as noted in text) inducing the low risk portfolio is not feasible under incurred loss and the regulator would set bank size equal to zero.

Under expected loss, the incentive-compatibility constraint is given by

\[
\alpha - 1 - (\beta - 1) \int_{\tau^*}^{1} sf(s)ds \geq 0.
\]

Since the incentive-compatibility becomes easier to meet as the liquidation threshold increases, we know that this constraint is satisfied if and only if \( \kappa \) is below the expected loss threshold, that is, \( \mathcal{L} \geq \frac{\beta \int_{\tau^*}^{1} sf(s)ds}{\mathcal{F}(\kappa)} \), in which case \( A^* = A_{\text{max}} \) and the surplus is strictly greater than under incurred loss. \( \square \)
Appendix 2: uniform states

In this appendix, we develop the main results in the context of the uniform distribution where the main tradeoffs can sometimes be expressed in closed-form. Specifically, we assume that $s$ is uniformly distributed with support on $[0, 1]$. A point of interest of this uniform choice is that it can be viewed as a principle of maximal ignorance (absent any well-formed prior) or as a linear approximation for low levels of uncertainty as, for example, in Plantin, Sapra and Shin (2008). Our assumption that $\mathbb{E}(s) \beta > \mathcal{L}$ is equivalent to $2\mathcal{L}/\beta < 1$ and implies that the liquidation threshold $\bar{s}$ is interior.

Solving explicitly for $\kappa$ in Proposition 2,

$$\kappa = \sqrt{\frac{1 + \beta - 2\alpha}{\beta - 1}}. \quad (6.32)$$

With uniform states, the continuation threshold $\kappa$ is also the probability of liquidation. This probability is decreasing and concave in $\alpha$, which is intuitive because the value of liquidating loans becomes increasingly attractive when most loans are safe and there is very little opportunity cost from excess liquidation of risky loans. Further, it is increasing and convex in $\beta$ if and only if $3\alpha - 1 - 2\beta > 0$, which reflects a situation in which safe loans are sufficiently common that increasing the probability of liquidation (as $\beta$ increases) benefits incentives more than they reduce total surplus.

Specifically, total surplus is given by

$$\Sigma = A_{\max}(q\alpha + (1 - q)(\mathcal{L}\sqrt{\frac{1 + \beta - 2\alpha}{\beta - 1}} + \beta\frac{\alpha - 1}{\beta - 1}) - 1). \quad (6.33)$$

The surplus is increasing in $\beta$ if and only if $\mathcal{L} \geq \sqrt{\frac{1 + \beta - 2\alpha}{\beta - 1}}$. In particular, the payoff of the risky loan that maximizes total surplus can be derived explicitly as

$$\beta - 1 = \frac{2(\alpha - 1)}{1 - \mathcal{L}^2}, \quad (6.34)$$

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which represents the ideal loan risk profile for a bank with no equity.

Similarly, we derive total surplus under the maximum liquidation threshold $\tau = 2\mathcal{L}/\beta$. This implies a surplus equal to

$$\Sigma = A_{max}(q\alpha + (1 - q)(\mathcal{L}F(2\frac{\mathcal{L}}{\beta}) + \beta \int_{2\frac{\mathcal{L}}{\beta}}^{1} sds) - 1)$$

$$= A_{max}(q\alpha + (1 - q)\frac{\beta}{2} - 1)$$

and this measurement system yields positive surplus if and only if $q > \frac{1 - \beta/2}{\alpha - \beta/2}$.

Under the pure prudential regulation benchmark, equation (3.1) implies a bank leverage

$$\frac{A^*}{E} = \frac{\beta^2 + \mathcal{L}^2}{\beta^2(1 + \beta - 2\alpha) - \mathcal{L}^2(\beta - 1)}$$

which reveals how the bank becomes smaller in response to a more severe agency problem. Plugging this expression into the total surplus,

$$\Sigma = E\frac{\beta^2 + \mathcal{L}^2}{\beta^2(1 + \beta - 2\alpha) - \mathcal{L}^2(\beta - 1)}(q\alpha + (1 - q)(\mathcal{L}^2/\beta + \frac{\beta}{2}(1 - \frac{\mathcal{L}^2}{\beta^2})) - 1),$$

which is, as expected, increasing in the liquidation payoff since this helps sustain a greater bank size and higher payoffs conditional on liquidation. It is also increasing in fraction of sale loans $q$ because $\alpha > \mathcal{L}^2/\beta + \frac{\beta}{2}(1 - \frac{\mathcal{L}^2}{\beta^2})$ and ambiguous in $\beta$.

We solve next for the general case with an endogenous continuation threshold and an optimally set capital requirement.

**Proposition 5** Let

$$\bar{q} = \frac{1 + Q}{\alpha + Q}.$$  

where $Q > 0$ (see proof below). If $q > \bar{q}$, the reporting system issues a liquidation signal $m(s)$ if and only if $s < \tau(A^*) = 2\mathcal{L}/\beta$.  

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Lastly, in the specific case of uniform states, we show the ex-post liquidation constraint is always binding if the low risk portfolio has positive value whatever the liquidation policy.

**Corollary 11** If \( q\alpha + (1-q)L > 1 \), the continuation threshold is always set at the maximal level consistent with the ex-post liquidation constraint, i.e. \( \tau(A^*) = 2L/\beta \).

Under uniform states and if \( q\alpha + (1-q)L > 1 \), we show that the ex-post liquidation constraint is always binding (for any parameter value) so that the liquidation threshold is located at \( \tau = 2L/\beta \). Intuitively, because the mass of the distribution tends to be spread out to a greater degree than a purely unimodal distribution, the expected payoff in the liquidation region increases faster per each dollar liquidated than many distributions, and thus it becomes more difficult to meet the ex-post liquidation constraint. This also implies that, under close enough to uniform states, the choice of the reporting system depends only on the liquidation payoff and the payoff from risky loans, but not the probability of safe loans.

**Proof of Proposition 5:** With a uniform distribution, the incentive-compatibility condition is now given by

\[
\Delta_{IC} = \int \left( A\alpha - A + E - s(1 - m(s))(A\beta - A + E) \right) ds \geq 0. \tag{6.38}
\]

We define the Lagrangian similarly with associated multipliers \( \mu_0, \mu_a \) and \( \mu_b \). Differentiating with respect to \( m(s) \),

\[
\frac{\partial L}{\partial m(s)} = s(\mu_0(A\beta - A + E) - (1-q)\beta - \mu_a - \mu_b) + (1-q + \frac{\mu_a + \mu_b}{\beta})L.
\]

Noting that this function is linear in \( s \) and positive at \( s = 0 \), we know that \( m(s) = 1 \) if and only if \( s \leq \tau \), where \( \tau \) is a threshold in \([0, 1]\).

**Case 1.** Suppose that \( \mu_0 = 0 \). Then, the solution \( \mu_a = \mu_b = 0 \) and \( \tau = L/\beta \) maximizes the Lagrangian and satisfies the constraints associated to multipliers \( \mu_a \) and \( \mu_b \). But, then,
it is desirable to set \( A = A_{\text{max}} \), which contradicts \( A_0 \).

**Case 2.** Suppose that \( \mu_0 > 0 \). Writing the incentive-compatibility explicitly after reinjecting \( m(s) = 1 \) if and only if \( s < \tau \),

\[
\Delta_{IC} = \tau^2 (A(\beta - 1) + E) + E - A(\beta + 1 - 2\alpha) \geq 0 \tag{6.39}
\]

The left-hand side is increasing in \( \tau \) and the objective function is increasing in \( \tau \) when \( \tau \in (0, \mathcal{L}/\beta) \), hence, it must be that, if the (IC) is binding, \( \tau \geq \mathcal{L}/\beta \). In addition, the optimality of ex-post liquidations requires \( \mathcal{L} \geq \beta \tau/2 \), that is, \( \tau \leq 2\mathcal{L}/\beta \).

Solving for the optimal leverage,

\[
\frac{A}{E} = \frac{1 + \tau^2}{1 + \beta - 2\alpha - (\beta - 1)\tau^2} \tag{6.40}
\]

and reinjecting into the social surplus

\[
\Sigma = A \int (q\alpha + (1 - q)(m(s)\mathcal{L} + (1 - m(s))s\beta - 1))ds
\]

\[
= E(1 + \tau^2) \frac{(q\alpha + (1 - q)(\mathcal{L}\tau + \frac{1}{2}\beta(1 - \tau^2)) - 1)}{1 + \beta - 2\alpha - (\beta - 1)\tau^2}. \tag{6.41}
\]

Differentiating this expression with respect to \( \tau \),

\[
H(\tau) = k_0 \sum_{i=0}^{5} \nu_i \tau^i,
\]
where $k_0 > 0$ and

\begin{align*}
\nu_0 &= (\beta + 1 - 2\alpha)(1 - q)C > 0 \\
\nu_1 &= \beta(\beta + q - \beta q - 5) + 4\alpha(1 + \beta q) - 4\alpha^2 q \\
\nu_2 &= 2C(1 - q)(1 + 2\beta - 3\alpha) > 0 \\
\nu_3 &= -2\beta(1 - q)(1 + \beta - 2\alpha) < 0 \\
\nu_4 &= -(\beta - 1)(1 - q)C < 0 \\
\nu_5 &= \beta(1 - q)(\beta - 1) > 0.
\end{align*}

It is readily verified that this polynomial is positive at $\tau = C/\beta$:

\begin{align*}
H(\frac{C}{\beta}) &= 2(1 - q)(\beta - \alpha)\frac{C^3}{\beta^2} + 4(\beta - \alpha)(q\alpha - 1)\frac{C}{\beta} + 2(1 - q)(\beta - \alpha)C \\
&= 4(\beta - \alpha)\frac{C}{\beta}(q\alpha + (1 - q)(\frac{C}{\beta} + \beta \int_{\frac{1}{\beta}}^1 sds) - 1) > 0.
\end{align*}

Therefore, the solution implies excess liquidations, i.e., $\tau > C/\beta$, and full-information is not optimal.

Further, we have

\begin{align*}
H(2C/\beta) &= 16(1 - q)(\beta - 1)\frac{C^5}{\beta^4} + 8(1 - q)(\alpha - 1)\frac{C^3}{\beta^2} + 8\alpha \frac{C}{\beta}(1 + \beta q - \alpha q) \\
&\quad + C(2(q - 5) + (1 - q)(3\beta + 1 - 2\alpha)).
\end{align*}

The condition $q \geq \bar{q}$ is equivalent to $H(2C/\beta) > 0$. From our analysis of the model with a general distribution, the optimal solution is the corner solution $\tau(A^*) = \tilde{\tau} = 2C/\beta$.

Otherwise, if $q < \bar{q}$, the solution is interior. □

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Bibliography


Corona, Carlos, Lin Nan, and Gaoqing Zhang (2013) ‘Banks’ voluntary adoption of fair value accounting and interbank competition.’ *Available at SSRN 2298184*


