Inside and outside information*

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Abstract

We analyze a model of communication where uninformed parties receive verifiable inside information, which is disclosed strategically by self-interested parties, as well as outside information such as reviews, ratings or policy reports. For a range of parameters, the classic “unraveling” spiral works in reverse, and information becomes fragile: Second-order changes in the distribution of outside information can trigger first-order reductions in inside disclosures. This yields new insights into policy questions such as the optimal transparency of financial systems. We also show that the importance of outside information hinges on the shape of insiders’ payoff functions, which leads to testable predictions for corporate disclosures, bilateral trade and political contests.

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1 Introduction

Many transactions rely on information disclosed by self-interested insiders. An active literature studies the economic implications of verifiable inside information (e.g. Milgrom, 2008; Hagenbach et al., 2014; Hart et al., 2017). However, uninformed agents also tend to have access to outside information, such as online product reviews or announcements by policy-makers. In this paper, we argue that insiders’ incentive to disclose information depends critically on the availability and quality of outside information. One of our main results is that outside information leads to fragility: Small changes in the distribution of outside signals can discontinuously destroy incentives for inside disclosures.

We study a standard model of communication: An informed agent called Sender privately observes his type, and wants to persuade an uninformed agent called Receiver that this type is high. He can disclose his type verifiably at a cost. Receiver also observes outside information, which Sender cannot control. To understand the fragility of information, it is useful to recall that verifiable disclosures exhibit a form of strategic complementarity. If the best types of Sender are expected to disclose their type, Receiver rationally assumes that no news is bad news, which generates strong incentives for other types of Sender to also disclose. In models without outside information, this complementarity leads to unraveling: All but the worst types of Sender disclose in equilibrium, as long as disclosure costs are small enough (Grossman and Hart, 1980; Milgrom, 1981).

In our model, by contrast, outside information opens the door to reverse unraveling, because the strategic complementarity now works in favor of opacity. If outside information is sufficiently precise, the highest types of Sender expect an accurate and favorable outside signal, and rationally stay quiet to save on the costs of disclosure. Receiver now expect silence from the best, and no news is ambiguous news. Silence then becomes more attractive for the second-best types of Sender. If they too stay

\footnote{Lewis (2011) demonstrates the empirical relevance of disclosure costs in online auctions. Leuz and Wysocki (2016) survey a large body of research documenting that disclosures are costly, both for technological reasons and because of concerns about releasing proprietary data to competitors. Our main results continue to hold in the case where disclosure costs are small, in the sense that they would not matter in a model without outside information.}

\footnote{Motivated by the above examples, we focus on disclosures that are pre-emptive to an extent: When Sender decides whether to disclose, he cannot perfectly predict the realization of outside information. We discuss the foundations of this assumption in Section 3.}
quiet, no news becomes better news still, the third-best types are tempted to stay quiet, and so forth. As a result, a small improvement in the quality of outside signals can take us from full disclosure to an opaque equilibrium where no inside disclosures are made at all. This discontinuity is a stark case of a more general feature of our model, namely that better outside information tends to crowd out inside information. Indeed, we show that for any equilibrium where Sender discloses with positive probability, more informative outside signals (in the sense of Blackwell, 1953) can leave Receiver worse informed overall (also in the Blackwell sense).

We complement this analysis by showing how the impact of outside information depends not only on its quality, but also on the shape of Sender’s payoffs. If Sender’s payoffs are sufficiently concave as a function of his perceived type, then the marginal benefit of being perceived as the best is relatively low. Thus, the best types of Sender are happy to wait for outside information, and the reverse unraveling loop gains traction. The resulting equilibrium is either fully opaque, or features non-monotonic strategies with disclosures made only by mediocre Senders. If payoffs are sufficiently convex, on the other hand, we obtain monotone equilibria where only the best types disclose, as in games without outside information.

Our findings are relevant for contemporary economic policy. A much-debated question is whether the government should release more information to the public. For example, the recent financial crisis has triggered calls for more transparency about the health of financial institutions (e.g. Goldstein and Sapra, 2014).\textsuperscript{3} Existing work warns that better public information is not always good, since it can reduce the informativeness of the price system (Vives, 1997; Amador and Weill, 2010; Kohlhas, 2015) or facilitate wasteful coordination (Morris and Shin, 2002; Angeletos and Pavan, 2007). These effects arise in economies where private information is dispersed among many agents. Our results point to a different informational externality: Better public signals can crowd out the disclosure of inside information. This externality is particularly powerful due to the fragility of information, and arises in situations

\textsuperscript{3}Other related policy issues are whether to disclose national statistics in a timely manner (Amador and Weill, 2010), and the question of central bank transparency (Woodford, 2006). In these examples, the policy maker reveals outside signals about aggregate variables, while our model features signals about individual types. We do not formally extend the model to feature distinctions between individual and aggregate shocks. However, we conjecture that for large firms – with low idiosyncratic risk – performance correlates strongly with the overall state of the economy. Then, large firms’ incentive to disclose information is weakened when the government is expected to release information about aggregates, and the effects we highlight are likely to remain important.
where private information is concentrated in the hands of strategic insiders.

In an application to financial panics, in the spirit of Morris and Shin (2000) and Bouvard et al. (2015), we demonstrate that this externality is important for the optimal choice of transparency in macro-prudential policy. We consider the choice of a policy-maker who can determine the precision of outside information about banks’ health, for example, by publishing the results of stress tests in more or less detail. At the same time, banks can disclose inside information about their quality. Interestingly, it can be welfare-enhancing for strong banks to stay quiet in a crisis. This is an instance of the Hirshleifer (1971) effect: Opacity by strong banks enhances welfare by generating implicit insurance for weak banks, who would otherwise face a bank run. An optimal policy in this context must exploit the informational externality by crowding out disclosures from the strongest banks. This is implemented by releasing outside signals that meet a minimum standard of transparency. These results are new to the growing literature on stress testing and optimal financial transparency (Bouvard et al., 2015; Goldstein and Leitner, 2015; Orlov et al., 2017).

We further note that there are many other applications where opacity reduces welfare. To highlight but one example, we study a model of political communication during election campaigns, again examining the welfare impact of more informative public signals (e.g. more news coverage). In this case, crowding out disclosures by candidates is detrimental to welfare, because it introduces noise into voters’ decisions. Therefore, the effect of better public information on welfare is ambiguous. This application also yields a new testable prediction: Candidates make more effort to disclose their quality in close elections. Candidates who expect a landslide victory, by contrast, are content to stay quiet and wait for favorable outside signals.

Our results on convex and concave payoffs deliver further empirical predictions. In an application to corporate disclosure, we show that high-quality firms are most likely to disclose when they are financed by equity (a convex claim on returns), but less likely to disclose when financed by debt (a concave claim). The existing literature emphasizes managers’ desires to keep stock prices high (Verrecchia, 1983; Acharya et al., 2011) and to enhance market liquidity (Diamond and Verrecchia, 1991). Our model implies, in addition to these factors, capital structure and executive compensation play a key role in determining disclosure strategies.

The remainder of this Section summarizes the related literature. In Section 2, we lay out the key intuitions in the context of a simple example. In Section 3 we present
a general Sender-Receiver model, and the core results on equilibrium disclosures are in Section 4. Section 5 contains applications, and Section 6 concludes. All proofs not given in the main text can be found in the Appendix.

Related literature

Our work contributes to the literature on verifiable communication. Grossman and Hart (1980), Grossman (1981), Milgrom (1981) and Milgrom and Roberts (1986) point out the “unraveling” of information in various settings. Hagenbach et al. (2014) derive conditions for full disclosure in a wider class of games with pre-play certifiable communication. Kartik (2009) shows that disclosures only partially separate Senders when lying about one’s type is possible but costly. Our focus is instead on situations where little or no information is disclosed by insiders. Our results complement a literature that points out the failure of unraveling when disclosure costs are substantial (Jovanovic, 1982; Verrecchia, 1983), or when it is uncertain whether Sender has any information to disclose (Dye, 1985; Shin, 1994, 2003). Mathios (2000) and Jin and Leslie (2003) provide empirical evidence of incomplete disclosure.

We make two contributions in this context. First, we point out that information is fragile. In existing work based on disclosure costs and higher-order uncertainty, strong types of Sender still have the strongest incentive to disclose, so that strategic complementarities work in favor of disclosures by other types. In our model, the same strategic complementarities – working in reverse – favor opacity and lead to discontinuities in equilibrium play. Second, we establish outside information as a first-order determinant of incentives to communicate. In this regard, our work complements that of Acharya et al. (2011), who study corporate disclosures in a dynamic game and show that the anticipation of (outside) public revelations can lead to clustered announcements of bad (inside) news.

Outside information has also been studied in the context of signaling games. Fel-tovich et al. (2002) and Daley and Green (2014) study signaling games with outside information and two or three types of Sender. As in the first step of our reverse unraveling mechanism, the highest-quality Senders have weaker incentives to acquire signals if their quality is likely to be revealed. Luca and Smith (2015) and Bederson et al. (2016) present supporting empirical evidence. We focus instead on verifiable
disclosure, which is a special but much more tractable case of signaling. This focus allows us to derive new insights on the fragility of information, the informativeness of equilibria, and the importance of the shape of payoffs, in a setting with many types of Sender.

2 Inside and outside information: Binary actions

An informed player called Sender offers an indivisible good for sale at an exogenously determined price \( p \), and an uninformed player called Receiver decides whether to buy it. Receiver’s action is binary – buy the good or don’t buy. The value of the good to Receiver is its quality \( \theta \in [\underline{\theta}, \bar{\theta}] \), drawn from a commonly known distribution with smooth density \( f(\theta) \). Under full information, Receiver would like to buy the good if and only if \( \theta \geq p \).

We consider the following game:

1. Sender privately observes \( \theta \), and can send a message \( m \in \{\theta, \emptyset\} \). The message \( m = \theta \) amounts to verifiably disclosing \( \theta \), since it is not feasible for other quality types, but reduces Sender’s utility by \( c > 0 \). The null message \( m = \emptyset \) amounts to staying quiet. It is costless but reveals no verifiable information.

2. Receiver observes \( m \), as well as an outside signal \( s = \theta + k\epsilon \), where \( k \) is a parameter measuring noise, and the error \( \epsilon \in [-1, 1] \) is drawn from a distribution with smooth density \( g(\epsilon) \) and cumulative distribution \( G(\epsilon) \).

3. Receiver decides whether or not to buy the good. If she buys, her payoff is \( \theta - p \) and Sender’s payoff is \( p - c \times 1_{m=\theta} \). If she does not buy, her payoff is zero, and Sender’s payoff is \( -c \times 1_{m=\theta} \).

We assume that \( E[\theta] < p \), so that the average good is not worth buying without further information, and that \( p > c \), so that profits from a sale are sufficient to recoup the costs of disclosure. We further assume that higher realizations of \( s \) are good news about \( \theta \) in the sense of the Monotone Likelihood Ratio Property; this is the case if and only if the density \( g(\epsilon) \) of noise is log-concave.

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4One can think of verifiable disclosure as a limiting case of signaling, where the signal “my type is X” is infinitely costly to acquire for Senders whose type is not X. Working in this limiting case reduces the number of beliefs that need to be specified off the equilibrium path.
In this model, bad types of Sender with $\theta < p$ have a dominant strategy to stay quiet: Disclosing $\theta < p$ never leads to a sale, but incurs the cost of disclosure $c$. Thus we focus on the behavior of good types $\theta \geq p$. The most informative situation we can hope for is where all good types disclose, and we will call this a transparent equilibrium.

We begin by noting that a transparent equilibrium exists if and only if outside information is sufficiently noisy. To verify this, note that Receiver rationally forms skeptical beliefs when transparency is expected. In a transparent equilibrium, no news is bad news. When Sender stays quiet, it is rational to assume that $\theta < p$ unless the outside signal proves otherwise, and Receiver buys the good only if $s \geq p + k$, that is, if outside information reveals beyond doubt that $\theta \geq p$.

In response, it is rational for Sender to disclose if $\theta \geq p$ and $\mathcal{M}(\theta) \geq c$, where

$$\mathcal{M}(\theta) = p \times Pr[s < p + k|\theta] = p \times G\left(1 - \frac{\theta - p}{k}\right).$$

Intuitively, the function $\mathcal{M}(\theta)$ measures the maximal punishment that Sender can suffer by staying quiet: It is the value $p$ of making a sale, times the probability of drawing an outside signal $s < p + k$ (and thereby missing out on a sale). Sender prefers to prove his type when the maximal punishment exceeds the cost $c$ of disclosure, and therefore, a transparent equilibrium exists if and only if $\mathcal{M}(\theta) \geq c$ for all $\theta \geq p$. It is clear from (1) that the maximal punishment is decreasing in $\theta$, because high types face a smaller probability of drawing a low outside signal. Thus a transparent equilibrium exists if and only if $\mathcal{M}(\tilde{\theta}) \geq c$, or equivalently, if the noise $k$ is above a critical level:

$$k \geq \frac{\tilde{\theta} - p}{1 - G^{-1}\left(\frac{c}{p}\right)} \equiv \bar{k}.$$

When $k < \bar{k}$, by contrast, a set of the best types $\theta \in (\theta_0, \tilde{\theta}]$ have a dominant strategy to stay quiet in equilibrium, where $\theta_0$ is the critical type for whom the maximal punishment $\mathcal{M}(\theta) = c$. Figure 1 illustrates this in panel (a).

Crucially, the maximal punishment is no longer credible: When the best Senders

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5We impose a natural restriction on out-of-equilibrium beliefs here: If Receiver observes a signal $s \notin [\theta - k, \theta + k]$, which could not have come from type $\theta$, then she attaches zero probability to this type. Without loss of generality, we assume that Receiver buys the good if she is indifferent.
$\theta \in (\theta_0, \bar{\theta}]$ stay quiet, no news is ambiguous news, and Receiver interprets silence ($m = \emptyset$) more favorably. The critical outside signal that guarantees a sale in the absence of inside disclosure falls to $s_0 < p + k$, solving the equation

$$E[\theta|s_0, \theta \notin [p, \theta_0]) = p.$$  

Staying quiet now becomes more attractive, since it offers the opportunity to pool with the best. A wider set of high quality Senders $\theta \in (\theta_1, \theta_0]$ now have an (iterated) dominant strategy to stay quiet – see panel (b) in Figure 1. This strategic complementarity continues to amplify silence: In response to types above $\theta_1$ staying quiet the critical signal falls further, additional types $\theta \in (\theta_2, \theta_1]$ prefer to stay quiet, silence becomes better news still, and so forth. Letting $\theta_n$ be the highest type who still discloses at the $n^{th}$ iteration, we can see that no type above $\bar{\theta} = \lim_{n \to \infty} \theta_n$ discloses in any equilibrium.

Due to this “reverse unraveling” loop, we experience a discontinuous drop in disclosures as soon as the noise $k$ falls below $\bar{k}$: A discrete set $\theta \in (\bar{\theta}, \bar{\theta}]$ of good types stays quiet in equilibrium even when $k = \bar{k} - \epsilon$. Establishing this discontinuity in a rigorous way involves showing that the limit $\bar{\theta}$ is bounded away from the best type $\bar{\theta}$ whenever $k < \bar{k}$. We give a formal proof of this property, and fully characterize the equilibria of the binary action case, in Appendix B.

Panel (a) of Figure 2 illustrates how to compute all equilibria of this game. We show in the Appendix that in any equilibrium, Sender discloses whenever his type is in an interval $\theta \in (p, \theta').$ The best response function $B(\theta')$ in the Figure is defined as the highest type $\theta$ of Sender who prefers to disclose when Receiver expects disclosures from $\theta \in (p, \theta')$. The best response is increasing in $\theta'$ due to strategic complementarities: When $\theta'$ increases, no news becomes worse news, and more good types of Sender prefer to disclose. The equilibria are the intersections of $B(\theta')$ with the 45-degree line. Because of strategic complementarities, there can be multiple equilibria. Improvements in the precision of signals tend to reduce incentives for high types to disclose, and consequently shift $B(\theta')$ down. Panel (b) further illustrates the fragility of information around $k = \bar{k}$. On a technical level, the discontinuity arises because the best response becomes infinitely steep near the top of the type distribution.

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*I formally write $\theta' = p$ for a fully opaque equilibrium without any disclosure.*
(a) **Deviations from full disclosure.** The blue (solid) curve is the density of outside signals drawn by the best type of Sender $\bar{\theta}$, scaled by the price $p$. The blue shaded area is the maximal punishment $M(\bar{\theta})$, defined in Equation (1). For $\bar{\theta}$, the maximal punishment is smaller than the cost $c$ of disclosure. The red (dashed) curve is the density of signals for the critical type $\theta_0$ for whom $M(\theta_0) = c$. All types above $\theta_0$ have a dominant strategy to stay quiet.

(b) **Reverse unraveling.** When it is common knowledge that types $\theta > \theta_0$ stay quiet, the critical signal outside signal that ensures a sale falls to $s_0$. The probability that type $\theta_0$ draws a signal below $s_0$ (the red shaded area), scaled by the price, is less than the cost $c$ of disclosure. The thick black curve is the scaled density of signals for the new critical type $\theta_1 < \theta_0$ who is indifferent between disclosure and staying quiet.

**Figure 1:** Reverse unraveling
(a) **Best responses and equilibria.** The blue (upper) curve shows the best response when outside signals are imprecise; the unique equilibrium is transparency at point A. The black (middle) curve is drawn for intermediate signal precision; there are multiple equilibria at A, B and C due to strategic complements. The red (lower) curve is drawn for low signal precision; the only equilibrium in this case is full opacity at D.

(b) **Discontinuity around full disclosure.** The blue (upper) curve shows the best response when \( k = \bar{k} \); while there are multiple equilibria, the transparent equilibrium at point A exists. The red (lower) curve shows the best response after a marginal increase in precision to \( k = k - \epsilon \); since the best response function is infinitely steep near the top type \( \bar{\theta} \), a small shift of order \( \epsilon \) leads to a discrete shift in equilibrium play. The most informative equilibrium is now at point B.

**Figure 2:** *Equilibrium in the binary action case.*
This simple case illustrates the main mechanism in this paper: Outside information has a strong tendency to crowd out inside disclosures. Moreover, this effect is amplified by strategic complementarities, to the point where second-order improvements in outside information trigger first-order drops in inside disclosures.

It is worth noting what is special about the binary case: As long as Sender is perceived to be above the threshold $\theta = p$, there is no marginal benefit of being seen as a better type. This diminishing marginal benefit plays a crucial role, since it weakens incentives to disclose among the best types. In the more general case below, we extend this intuition and relate equilibrium disclosures to the shape of payoff functions.

We have also relied on the assumption that signals do not have full support: Signals $s > p + k$ cannot possibly come from types whose good is not worth buying. We believe that deviations from full support are empirically realistic, since very favorable news tend to rule out very bad outcomes and vice versa. However, some of our general results in the next Section go through even when we allow outside signals to have full support, and we will state explicitly when it is necessary to allow for deviations from it. Moreover, in the Online Appendix, we consider the binary action case when outside signals have full support. In this case, transparent equilibria always exist, but we demonstrate that they are unstable in a natural sense if the precision of outside information is above a certain threshold. Thus, our focus on non-transparent equilibria remains valid even with full support.

3 A general Sender-Receiver game

We study a game between a Sender (he), who has the opportunity to disclose verifiable inside information, and a Receiver (she), who decides on an action based on these disclosures and outside information. Receiver needs to choose an action $a \in A \subset \mathbb{R}$. Payoffs depend on this action and on the state of the world $\theta \in \Theta = \{\theta_1, ..., \theta_N\} \subset \mathbb{R}$, where $\theta_N > \theta_{N-1} > ... > \theta_1$.

We focus on the case of “pure persuasion,” where Sender’s payoff $v(a)$ depends

\footnote{Casual empiricism suggests that most small businesses and developing countries have no chance of obtaining an AAA credit rating from the major agencies; a worker who has been fired from his previous job for stealing would not expect a glowing reference with positive probability, and an innocent defendant in a trial might rationally attach zero probability to having her fingerprints found on the murder weapon.}
only on the action taken and is strictly increasing in \(a\). Receiver’s payoff \(u(a, \theta)\) is log-supermodular in \(a\) and \(\theta\), so that she optimally chooses higher actions when optimistic about \(\theta\).\(^8\) We assume that, when Receiver knows \(\theta\) with certainty, she has a unique best response denoted \(a^\star(\theta) = \text{arg}\max_{a \in A} u(a, \theta)\).

Sender privately observes the state \(\theta\), which we refer to as his type, and sends a message \(m \in \{\theta, \emptyset\}\). The message \(m = \theta\) is available only to type \(\theta\), and therefore amounts to full and verifiable disclosure of \(\theta\), but it reduces Sender’s utility by \(c > 0\). The null message \(m = \emptyset\), which we refer to as staying quiet, is costless but reveals no verifiable information.\(^9\) In addition, Receiver privately observes an outside signal \(s\) drawn from a finite set \(S \subset \mathbb{R}\).

We write \(\mu_0(\theta)\) for the prior distribution of \(\theta\) and \(\pi(s|\theta)\) for the conditional distribution of \(s\) given \(\theta\), which are common knowledge. We have \(\sum_{\theta \in \Theta} \mu_0(\theta) = 1\) and \(\sum_{s \in S} \pi(s|\theta) = 1\) for all \(\theta\). We assume that \(\mu_0(\theta) > 0\) for all \(\theta\).

The timing is as follows: First, Sender observes \(\theta\) and chooses \(m\). Second, Receiver observes inside information \(m\) and outside information \(s\) before choosing an action \(a\).\(^10\) We consider Perfect Bayesian Equilibria: Sender and Receiver choose messages and actions to maximize expected payoffs, and the Receiver’s posterior beliefs about \(\theta\) are derived using Bayes’ law on the equilibrium path. Off the equilibrium path, we require that Receiver places zero probability on type \(\theta'\) if she observes a signal such that \(\pi(s|\theta') = 0\).

### Pre-emptive disclosures

In our model, Sender commits to make a disclosure \(m\) before he knows the realization \(s\) of outside information. This is important for our results: One of our key intuitions is that the best types have weaker incentives to disclose if they anticipate a favorable realization of \(s\). In an alternative model where verifiable messages can be sent between the realization of \(s\) and Receiver’s action \(a\), the best types would have stronger incentives to disclose...
incentives to make such a disclosure. We focus on the case of pre-emptive disclosures because we believe that it captures frictions that arise in most of the applications that motivate our paper.

In many applications, Receivers react very quickly to outside news, and irreparable damage to Sender’s prospects may be done if he waits until after this event to prove his quality. For example, financial markets respond quickly to bad news or credit downgrades and managers may lose their job or reputation before they have a chance to respond. The release of bad news on the eve of an election may damage a candidate’s chances regardless of her subsequent communication.

This friction is especially relevant in situations where verifiable information takes time to prepare and circulate. In financial markets, reports need to be prepared and externally audited in advance of their release. In product markets, there can be considerable delays in circulating inside information, for example via advertising campaigns, to reach a dispersed audience of potential customers. More generally, if economic agents have limited capacity for processing information as in Sims (2003), Receiver may be unable to (or rationally choose not to) process further communications by Sender once the outside signal $s$ has resolved a significant portion of the uncertainty.

In other applications, an alternative friction leads to a formally equivalent game: In the case of job applications, the outside information, such as past employers’ reference letters, remains private information for the Receiver (employer) until the hiring decision has been made, so that the Sender (applicant) must decide on a communication strategy, such as the disclosures he makes on his C.V., before he knows the realization of outside information.

11A potential variation on our model is a setting where the verifiable report $m = \theta$ takes time to prepare, but where Sender can prepare it in advance and decide whether to release it once $s$ has been observed. In this environment, Sender has stronger incentives to prepare the report than in our model, because he retains the option to keep it to himself in case $s$ turns out to be better news than the truth. However, similar arguments to our main results are likely to go through: The best types of Sender have a relatively weak incentive to prepare a verifiable report, because they anticipate that the outside signal $s$ will be good enough to secure a favorable action. Therefore, the effects we emphasize will continue to arise.
Preliminaries and notation

We write $V(\theta) = v(a^*(\theta))$ for Sender’s payoff when he is taken to be type $\theta$ for certain. When Sender stays quiet and outside information is $s$, we write

$$\alpha(s) \in \arg\max E_{\mu}[u(a, \theta)|s, m = \emptyset]$$

for Receiver’s (potentially random) best response. This action depends on equilibrium beliefs $\mu$ about $\theta$. We further define the net payoff from a verifiable disclosure as

$$\mathcal{N}(\theta) \equiv V(\theta) - E[v(\alpha(s))|\theta],$$

so that Sender prefers to disclose if $\mathcal{N}(\theta) \geq c$.

As pointed out by Milgrom and Roberts (1986) and others, it is often useful to consider “skeptical” beliefs, where Receiver assumes that Sender is of the worst type $\bar{\theta}(s) = \min\{\theta|\pi(s|\theta) > 0\}$ consistent with her outside information $s$. We define the maximal punishment that Sender can suffer by staying quiet as the difference between the payoff he obtains under full disclosure, and the payoff he obtains by staying quiet and facing a skeptical Receiver:

$$\mathcal{M}(\theta) = V(\theta) - E[V(\bar{\theta}(s))|\theta].$$

The maximal punishment is an upper bound on the net payoff from disclosure; so that $\mathcal{N}(\theta) \leq \mathcal{M}(\theta)$ for all feasible equilibrium beliefs. Indeed, type $\theta$ has a dominant strategy to stay quiet if and only if $\mathcal{M}(\theta) < c$. Note in particular that $\mathcal{M}(\theta_1) = 0$, so that the worst type $\theta_1$ always has a dominant strategy to stay quiet.

We call an equilibrium monotone increasing if the probability of disclosure $Pr[m = \theta|\theta]$ is increasing in the type $\theta$, and strictly increasing for some pair of types. We call an equilibrium opaque if nobody discloses and $Pr[m = \theta|\theta] = 0$. Finally, we call an equilibrium non-monotone if $Pr[m = \theta|\theta]$ is strictly increasing for some pair of types and strictly decreasing for another. The fact that the worst type $\theta_1$ has a dominant strategy to stay quiet guarantees that these are the only possibilities.

A focal point in the literature on inside information is the transparent equilibrium: Sender discloses unless he is the worst type $\theta_1$, and Receiver responds to silence by adopting skeptical beliefs. In our setting a transparent equilibrium exists if and only
if
\[
c \leq \min_{\theta > \theta_1} M(\theta) \equiv c_0,
\]
where \( M(\theta) \) denotes type \( \theta \)'s maximal punishment.\(^{12}\)

To motivate our analysis, it is useful to consider equilibria in the absence of outside information. In this case, disclosure strategies remain monotone increasing, and thus qualitatively similar to a transparent equilibrium, even when (4) does not hold. Indeed, Receiver now responds to silence with a fixed optimal action \( \alpha \). Thus Sender’s payoff from staying quiet is independent of \( \theta \) and given by \( E[v(\alpha)] \).\(^{13}\) The net payoff \( N(\theta) = V(\theta) - E[v(\alpha)] \) is now increasing in \( \theta \). Since all types are valued equally if quiet, high types have the most to gain from disclosure. It is immediate that all equilibria without outside information are monotone increasing, unless costs \( c \) are prohibitively high. Our focus is on situations where the presence of outside information breaks the monotone relationship between Sender’s type and his disclosure strategy.

### 4 Inside and outside information in equilibrium

#### 4.1 Fragility of information

An important feature of the “reverse unraveling” mechanism is amplification: Due to strategic complementarities, second-order changes in the environment can trigger first-order responses in Sender’s equilibrium communication strategy.

To analyze discontinuities, we conduct a comparative statics exercise: We consider how equilibrium disclosure changes as we vary the distribution of outside information from pure noise to full information in a continuous fashion. Formally, we write the conditional distribution \( \pi(s|\theta;t) \) of signals given types as a function of a parameter \( t \in [0,1] \). The stochastic matrix \( \Pi(t) = (\pi(s|\theta;t))_{s,\theta} \) then defines the \( t \)-dependent structure of outside information. We say that \( \Pi(t) \) is pure noise if \( \pi(s|\theta;t) \) is inde-

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\(^{12}\)The argument behind (4) is simple. Positing a transparent equilibrium, we need to check that no type \( \theta > \theta_1 \) wishes to deviate and stay quiet, which is the case if the net payoff from disclosure \( N(\theta) \geq c \). With skeptical beliefs, the net payoff equals the maximal punishment \( N(\theta) = M(\theta) \), so that (4) is sufficient to rule out profitable deviations. It is easy to check that (4) is also necessary: If maximal punishments cannot entice Sender to disclose, he prefers to stay quiet regardless of Receiver’s beliefs.

\(^{13}\)Receiver may randomize \( \alpha \), but if he does, then this randomization must be conditionally independent of \( \theta \) in the standard definition of Perfect Bayesian Equilibrium.
dependent of $\theta$, and that $\Pi(t)$ is fully revealing if for each $s$ there is only one $\theta$ such that $\pi(s|\theta; t) > 0$.

**Proposition 1.** For any payoffs $\{u, v\}$ and any prior $\mu_0$, assume that $c$ is sufficiently small to ensure that there is a transparent equilibrium when public signals are pure noise. Then there exists a continuous path of signal structures $\Pi(t)$ for $t \in [0, 1]$ with the following properties:

- $\Pi(0)$ is pure noise, while $\Pi(1)$ is fully revealing, and
- There exists a critical point $t^* \in (0, 1)$ such that, when Receiver observes the signal induced by $\Pi(t)$, there is a transparent equilibrium for $t \leq t^*$, while full opacity is the unique equilibrium for $t > t^*$.

Proposition 1 states that, regardless of payoffs and priors, we can find a way to gradually increase the informativeness of outside signals from pure noise to full revelation, such that at some intermediate point, Sender’s incentives to disclose in any equilibrium are discontinuously destroyed.

This is an existence result, and discontinuities do not necessarily arise along every sequence of signals. However, we have significant degrees of freedom when choosing the path of signal structures $\Pi(t)$. In particular, we show in the Online Appendix that the results continue to go through on an appropriately defined open set of signal structures.

The following sketch of the proof, illustrated further in Figure 3, reveals why “reverse unraveling” drives the result. We find a path of signal structures $\Pi(t)$ which runs from pure noise to full revelation and leads to a discontinuity at $t^*$. Consider the maximal punishment $M(\theta; t)$ for staying quiet. Maximal punishments are tougher when signals are noisy. For $t < t^*$, we construct noisy signals such that the maximal punishment is tough and $M(\theta; t) > c$ for all $\theta > \theta_1$. In this region, Equation (4) holds, so that there is a transparent equilibrium.

Let $t^*$ be the first point where the best type $\theta_N$ has a weakly dominant strategy to stay quiet with $M(\theta_N; t^*) = c$. At $t^*$, our path of signals reaches a point such that lower types $\theta \in (\theta_1, \theta_N)$ are at most $\delta$ utility units away from having such a dominant strategy, so that

$$c + \delta > M(\theta, t^*) > c.$$

(5)
Figure 3: **Discontinuity around full disclosure.** The informativeness of outside signals increases from pure noise to full revelation as $t$ increases. The black (lower solid) curve is the maximal punishment perceived by the best type $\theta_N$. At $t^*$, this maximal punishment crosses $c$. For $t = t^* + \epsilon$, type $\theta_N$ strictly prefers to stay quiet. The blue (upper solid) curve is the maximal punishment perceived by a worse type $\theta$ who shares an outside signal with $\theta_N$. When type $\theta_N$ is known to stay quiet, the maximal punishment is no longer credible and type $\theta$’s net payoff $N(\theta; t)$ from disclosure – the red (dashed) line – jumps down, giving $\theta$ a strict incentive to stay quiet too.
Moving from \( t^* \) to \( t^* + \epsilon \), we improve the informativeness of signals further so that the best type’s maximal punishment falls just below \( c \). The best type \( \theta_N \) now has a strictly dominant strategy to stay quiet. Nonetheless, Equation (5) implies that for small \( \epsilon \), lower types \( \theta < \theta_N \) still prefer to disclose for at this point, as long as they expect a maximal punishment for silence.

The crucial step in the proof is that at \( t^* + \epsilon \), the maximal punishment is no longer a credible threat. It is now common knowledge that type \( \theta_N \) does not disclose, since doing so is strictly dominated. For any outside signal that can be drawn by \( \theta_N \), Receiver’s posterior beliefs are now more optimistic than the skeptical beliefs that underpin the maximal punishment. For any type \( \theta \) who shares an outside signal with \( \theta_N \), the net payoff from disclosure is now strictly lower than the maximal punishment: \( N(\theta; t) < M(\theta; t) \). If \( \delta \) is low enough, that is if type \( \theta \)’s incentives to disclose are not too strong at point \( t^* \), this improvement in beliefs will be enough to entice type \( \theta \) to also stay quiet. Thus a set of types \( \Theta^{(0)} \), who share signals with the best type, decide to copy the best type when he ceases to make disclosures.\(^{14}\)

This process of “reverse unraveling” continues, as it did in the example of Section 2. Once types \( \Theta^{(0)} \) decide to stay quiet, Senders who share signals with \( \Theta^{(0)} \) no longer consider the maximal punishment to be credible, the set \( \Theta^{(1)} \) of types who share signals with \( \Theta^{(0)} \) stay quiet in response, and so forth. If the possible signals form a connected network,\(^{15}\) then this process eventually encompasses all types so that \( \Theta^{(n)} = \Theta \) for some \( n \). At this point, the only strategy profile surviving iterated deletion of strictly dominated strategies is full opacity, and this must be the unique equilibrium of the game.

We note a caveat to Proposition 1: The sequence of signals that leads to a discontinuity does not have full support, that is, the set of possible signals is sometimes different for different types. With full support, the maximal punishment is the same for all types of Sender, since Receiver’s most pessimistic beliefs place probability one on the worst type \( \theta_1 \) for all possible realizations of the outside signal \( s \). If full support is enforced, we cannot construct a threshold \( t^* \) where the best type’s maximal

\(^{14}\)It is not crucial for this step that the best type \( \theta_N \) is the first to have a dominant strategy to stay quiet. However, the reduction in net payoffs from disclosure is strongest, and the bounds on \( \delta \) are least stringent, if it is indeed \( \theta_N \).

\(^{15}\)By a connected network, we mean that for any two signals \( s \) and \( s' \), there exists a path of types \( \{\theta^{(1)}, \ldots, \theta^{(M)}\} \) so that \( \pi(s|\theta^{(1)}) > 0 \), \( \pi(s'|\theta^{(M)}) > 0 \), and types \( \theta^{(i)} \) and \( \theta^{(i+1)} \) share a signal everywhere along the path.
punishment equals the cost $c$ of disclosure while other types’ maximal punishment exceed it as in Figure 3. However, as we have argued in Section 2, deviations from full support are empirically reasonable in many settings.

### 4.2 Better outside information can reduce total information

In addition to establishing the fragility of information, we consider how access to better outside information affects the quality of the information that Receiver observes in equilibrium. We use the Blackwell (1953) order to rank information structures. A general signal $\tau' \in T'$ is said to be more informative about $\theta$ than another signal $\tau \in T$ if $\tau$ is a “garbled” version of $\tau'$: Nature first draws the clean signal $\tau'$ and then randomly converts it to the garbled signal $\tau$, so that we can write

$$
Pr[\tau|\theta] = \sum_{\tau' \in T'} Pr[\tau'|\theta]g(\tau'|\tau')
$$

for some conditional distribution $g(\tau'|\tau')$. Blackwell’s theorem shows that this notion of informativeness is equivalent to requiring that every Bayesian decision-maker weakly prefers to observe realizations of $\tau'$ instead of $\tau$.

In the context of our model, outside information $s'$ is more informative than $s$ in the sense of Blackwell if the conditional distribution of $s$ can be written as a garbling of $s'$ in this way. We can also rank equilibrium outcomes by informativeness. Take an equilibrium $\mathcal{E}$ where Sender discloses with probability $Pr[m|\theta]$ as a function of his type. On the equilibrium path, Receiver observes the signal $\tau = \{s, m\}$, which contains both outside and inside information and has conditional distribution $Pr[\tau|\theta] = \pi(s|\theta) \times Pr[m|\theta]$. We consider situations where outside information changes to $s'$ and Sender changes his equilibrium disclosure strategy. The resulting new equilibrium $\mathcal{E}'$ induces an appropriately defined signal $\tau' = \{s', m'\}$. Receiver is less informed in the new equilibrium in the sense of Blackwell if we can write $\tau'$ as a garbling of $\tau$ according to Equation (6).

We show that more informative signals can always leave Receiver worse informed in equilibrium. Proposition 1 motivates this idea. The path $\Pi(t)$ in the proposition represents successive improvements in the informativeness of outside information. For $t \leq t^*$, there is a transparent equilibrium where Receiver perfectly learns $\theta$, while for $t > t^*$, there is only an opaque equilibrium in which Receiver has imperfect...
information about $\theta$. When we cross the threshold $t^*$, it is clear that outside signals become more informative, but equilibrium information deteriorates. This is a special case because we are comparing a fully revealing equilibrium with a fully opaque one. We can generalize this insight. Better outside signals can always decrease informativeness as long as there is some disclosure in the initial equilibrium:

**Proposition 2.** Suppose that, when outside information is $s$, there is an equilibrium $E$ in which Sender makes a disclosure $m = \theta$ with strictly positive probability. Then there exists an outside signal $s'$ such that

- $s'$ is more informative than $s$ in the sense of Blackwell, and
- In the game where outside information is $s'$, there is an equilibrium $E'$ in which Receiver is less informed than in $E$ in the sense of Blackwell.

Proposition 2 follows from the interaction between outside signals and insiders’ incentives to disclose. When outside signals become more informative, they crowd out incentives for voluntary disclosures by Sender. Since Sender is better informed than Receiver, this crowding-out can unambiguously reduce information sharing in the economy.

A simple example, which we illustrate in Figure 4, reveals the logic of the proof. Suppose that outside information is $s$ and that Sender strictly prefers to disclose in equilibrium when his type is $\theta_i$. We can define a more Blackwell-informative outside signal $s'$ as a compound lottery: With probability $z_i$, type $\theta_i$ draws a “special signal” $s' = \vartheta_i$ that reveals type $\theta_i$ perfectly. With probability $1 - z_i$ he draws a realization of the original signal $s$. It is easy to see that $s'$ is more informative than $s$ in the sense of Blackwell, as illustrated in Panel (a) of Figure 4. If we take $z_i$ sufficiently close to one, then type $\theta_i$ expects to get close to the full disclosure payoff, and stays quiet in order to save on the costs of disclosure. Moreover, Receiver’s beliefs and the incentives for other types $\theta_j$ change only marginally when type $\theta_i$ changes his strategy in this way. Assuming that all other types $\theta_j$ have a strict preference for their equilibrium strategy to begin with, none of them have an incentive to change their behavior in response. We can therefore find a new equilibrium in which type

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16Note that $s'$ is drawn from the augmented signal space $S' = S \cup \{\vartheta_i\}$. The Blackwell order does not require two signals to come from the same support.

17Before the change, type $\theta_i$ discloses with probability one; afterwards, he is revealed with probability $z_i$ close to one. Thus, adaptations to Receiver’s Bayesian beliefs are on the order of $1 - z_i$. 

20
θᵢ switches from full disclosure to staying quiet, but no other type of Sender changes his strategy. In this new equilibrium, Receiver no longer learns the identity of type θᵢ perfectly as long as zᵢ < 1, while the information she obtains about other types θⱼ is unchanged. Thus, Receiver is less informed in a Blackwell sense, as illustrated in panel (b) of Figure 4. In the Appendix, we deal with the complications that arise when equilibria are not strict, such as when Sender plays a mixed strategy.

In this example, the new signal s’ does not have full support (only type θᵢ can send the new “special signal” s’ = ϑᵢ), but we show in the Online Appendix that this is not necessary. The argument is robust to small deviations in the signal structure so that if the original signal s has full support, we can construct a full-support signal s” close to s’ which satisfies the claim of Proposition 2.

4.3 Disclosures and the shape of payoffs

So far, we have shown that specific distributions of outside information have a strong tendency to crowd out inside disclosures. Key to these effects was the fact that incentives to disclose are weakened by precise outside information, particularly for strong
types of Sender. Before moving on to applications, we conduct a complementary exercise: For a given distribution of outside information, we study whether strong types of Sender have an incentive to stay quiet in equilibrium, thus giving rise to the effects we have emphasized. As we noted above, the best types of Sender always have the strongest incentive to disclose in the absence of outside information. In the presence of outside information, we show that the shape of Senders’ payoff plays a crucial role.

We begin by focusing on a class of models for which we can directly relate disclosure strategies to payoff functions. Then, we use iterated deletion of dominated strategies – generalizing the analysis of the binary case in Section 2 – to derive a characterization of equilibrium disclosures in the general model.

**Concavity and convexity in a model with “virtual types”**

Consider the common special case where Receiver’s optimal action takes the form

$$\arg \max E[\mu(u(a, \theta))] = E[\mu(X(\theta))],$$

for some increasing function $X(\theta)$, and where Sender’s utility is simply $v(a) = a$. Such preferences are natural in standard seller-buyer interactions where $a$ denotes the willingness-to-pay of a buyer (or indeed of a mass of buyers in a competitive market) for an indivisible item that gives her utility $X(\theta)$. This case permits a useful re-interpretation of payoffs in terms of “virtual types”. If Sender stays quiet and has true type $\theta = \theta_i$, his expected payoff is

$$E[\alpha(s)|\theta_i] = \sum_{s \in S} \pi(s|\theta_i) E[X(\theta)|s, m = \emptyset] = \sum_{j=1}^N q_{ij} X(\theta_j),$$

where $q_{ij} = E[Pr[\theta_j|s, m = \emptyset]|\theta_i]$ is the expected probability mass that Receiver places on type $\theta_j$. Payoffs in the absence of disclosure are therefore equivalent to a game in which Sender draws a “virtual type” $\theta_j$ according to the conditional distribution $q_{ij}$. Note that $q_{ij}$ is indeed a probability distribution since $\sum_j q_{ij} = 1$ for all $i$. We write $Q_{ij} = \sum_{k \leq j} q_{ik}$ for the cumulative distribution of virtual types.

We assume that outside signals satisfy the strict Monotone Likelihood Ratio Property (MLRP; defined as in Milgrom, 1981): For $\theta' > \theta$ and $s' > s$, we impose that

$$\pi(s'|\theta') \pi(s|\theta) > \pi(s'|\theta) \pi(s|\theta').$$
We further assume that neighboring types share signals: For each $i$, there is an $s$ such that $\pi(s|\theta_i) > 0$ and $\pi(s|\theta_{i-1}) > 0$ (clearly, any signal distribution with full support satisfies this restriction).

We write $\Delta X_i = X(\theta_{i+1}) - X(\theta_i)$ for the increment in Receiver’s action if she learns that Sender’s type increases from $\theta_i$ to the next-best type $\theta_{i+1}$. Integrating by parts, we can re-write the net payoff from disclosure in Equation (3) as:

$$\mathcal{N}(\theta_i) = \sum_{j=1}^{i-1} \Delta X_j Q_{ij} - \sum_{j=i}^{N-1} \Delta X_j (1 - Q_{ij}).$$

Equation (7) expresses the net payoff from disclosure as the sum of two components. The first term is the downside risk that Sender takes by staying quiet; with probability $Q_{ij}$ his virtual type is below $\theta_j$ for $j < i$, and the associated incremental loss is $\Delta X_j$. The second term is the upside risk; with probability $1 - Q_{ij}$, Sender’s virtual type is above $\theta_j$ for $j > i$, and the associated incremental gain is again $\Delta X_j$. If the downside exceeds the upside by more than $c$, Sender prefers to disclose. Virtual types are useful because they inherit the ordering of signals: In the Appendix, we show that type $\theta_{i+1}$ draws better virtual types – in the sense of first-order stochastic dominance – than type $\theta_i$. Thus, the probability weights $Q_{ij}$ on downside risk decrease as Sender’s true type $i$ improves, while the weights $1 - Q_{ij}$ on upside risk increase.

We define measures of the concavity and convexity of payoffs:

$$\text{Concavity} \equiv \min_i \frac{\Delta X_i}{\Delta X_{i+1}}$$

$$\text{Convexity} \equiv \min_i \frac{\Delta X_{i+1}}{\Delta X_i}$$

When Concavity $> 1$, payoffs are concave in the sense that the marginal value of being perceived as a better type diminishes as Sender’s type improves. Similarly, when Convexity $> 1$, the marginal value of being perceived as a better type increases as Sender’s type improves, and payoffs are convex. We can relate these parameters to disclosure strategies in equilibrium:

**Proposition 3.** If the Concavity of payoffs is sufficiently large, then for any level of disclosure costs $c > c_0$, all equilibria are non-monotone or fully opaque. Conversely, if Convexity is sufficiently large, then there are no non-monotone equilibria.

In other words, when payoffs are concave enough, the strongest types of Sender
stay quiet in any equilibrium, as soon as the level of disclosure cost \( c \) rises above the threshold \( c_0 \) that rules out a transparent equilibrium.\(^{18}\) Conversely, when payoffs are convex enough, the strongest types of Sender always disclose in equilibrium (unless costs are prohibitive so that nobody wants to disclose). The result uses our characterization (7) of the net benefit of disclosure: When payoffs are sufficiently concave, incentives to disclose come mainly from the downside increments \( \Delta X_1, \ldots, \Delta X_{i-1} \). Since the probability weights on these increments fall as Sender’s true type improves, strong types have weak incentives to disclose. Then, the logic of reverse unraveling leads to a non-monotone or fully opaque equilibrium, as in the binary example of Section 2.

When payoffs are sufficiently convex, by contrast, we can rule out non-monotone equilibria as follows: Let \( \theta_n \) be the highest quiet type in a non-monotone equilibrium, and \( \theta_d < \theta_n \) a disclosing type below him. We can show that type \( \theta_n \) has strictly stronger incentives to disclose than \( \theta_d \) because of the large, convex, utility he gains by raising his virtual type to \( \theta_n \). The formal proof constructs uniform bounds on Concavity and Convexity that ensure these properties for all possible (pure or mixed) strategy profiles; the bounds depend on the prior signal distribution, but not on equilibrium play. A simple example clarifies the logic of Proposition 3.

**Example.** Consider the “virtual type” case with three types \( \theta \in \{\theta_1, \theta_2, \theta_3\} \), five outside signals \( s \in \{s_0, \ldots, s_4\} \), and a uniform prior \( \mu_0(\theta_i) = 1/3 \). Each type draws the outside signal to the left of his type with probability \( \pi(s_i-1|\theta_i) = p \), that to the right with probability \( \pi(s_{i+1}|\theta_i) = r \), and the signal matching his type with the remaining probability \( \pi(s_i|\theta_i) = q = 1 - p - r \).

We have Concavity \( = \frac{\Delta X_1}{\Delta X_2} \). The maximal punishment for silence is \( M(\theta_2) = \Delta X_1(p+q) \) for the middle type and \( M(\theta_3) = \Delta X_1p + \Delta X_2(p+q) \) for the top type. We have \( M(\theta_3) < M(\theta_2) \) when Concavity \( > 1 + \frac{p}{q} \), that is, when the concavity of payoffs is large relative to the likelihood ratio of left-tail outside signals to intermediate ones. Under this condition, the high type \( \theta_3 \) has the strongest incentives to deviate from a transparent equilibrium, and by Equation (4), such an equilibrium exists if and only if \( c \leq M(\theta_3) \equiv c_0 \). Whenever \( c > c_0 \), the top type must therefore stay quiet, and

\(^{18}\)Note, however, that this threshold \( c_0 \) is larger than it would be in the absence of outside information, because the maximal punishment \( M(\theta) \) is smaller when there are outside signals. Hence, we can focus on the case where \( c > c_0 \), but still assume that transparency would be an equilibrium if there were no outside signals.
since the bottom type also has a dominant strategy to stay quiet, resulting equilibria must be non-monotonic of fully opaque, in line with the first part of Proposition 3.

Moreover, we have Convexity = \( \frac{1}{\text{Concavity}} = \frac{\Delta X_2}{\Delta X_1} \). Consider a non-monotone equilibrium in pure strategies, where only the middle type \( \theta_2 \) discloses. In this equilibrium, Receiver is certain that \( \theta = \theta_1 \) when the outside signal is \( s \leq s_1 \), and equally certain that \( \theta = \theta_3 \) when \( s \geq s_3 \). When \( s = s_2 \), she places probability \( \frac{r}{p+r} \) on type \( \theta_1 \) and complementary probability \( \frac{p}{p+r} \) on type \( \theta_3 \). The implied distribution of virtual types has \( Q_{21} = p + q \frac{r}{p+r} = Q_{22} \) and \( Q_{31} = p \frac{r}{p+r} = Q_{32} \). For optimality, the middle type must prefer to disclose and the top type must prefer to stay quiet: \( \mathcal{N}(\theta_2) \geq c \geq \mathcal{N}(\theta_3) \). Thus a non-monotone equilibrium exists for some \( c \) if and only if \( \mathcal{N}(\theta_2) \geq \mathcal{N}(\theta_3) \).

Substituting into (7) and rearranging, this is equivalent to Convexity \( \leq \frac{\lambda}{1-\lambda} \), where \( \lambda = Q_{21} - Q_{31} \) is the downside risk perceived by the top type relative to the middle type. Conversely, when Convexity \( > \frac{\lambda}{1-\lambda} \), there is no non-monotone equilibrium.

The example further helps us to understand disclosures when parameters fall between the bounds we have established in Proposition 3, that is, when payoffs are neither very convex nor very concave. It turns out that the skewness of outside information becomes critical in this case. For instance, when payoffs are linear (Concavity = 1), non-monotone equilibria exist in the example if and only if \( \lambda = p(1 - \frac{r}{p+r}) + q \frac{r}{p+r} \geq \frac{1}{2} \). With a symmetric outside signal distribution \( (p = r) \) this is impossible unless signals are perfectly revealing. When outside signals are precise with \( q > \frac{1}{2} \), we have a non-monotone equilibrium if and only if outside information is right-skewed, with \( \frac{r}{p} \) sufficiently large. Intuitively, a right skew increases the advantage of top types over mediocre types, since the outside signals drawn by mediocre types are now interpreted chiefly as having come from low types. As a result, payoffs must now be strictly convex to rule out non-monotone disclosures.

Iterated deletion of dominated, non-monotone strategies

We return to our general model and characterize equilibrium disclosures in terms of the maximal punishment \( \mathcal{M}(\theta) \). We first define a procedure for iterated deletion of strictly dominated, non-monotone disclosures (DNMD): Let \( \Theta_n \) be the set of types who stay quiet after \( n \) iterations of the procedure. At stage \( n \) of the procedure, we first delete from Sender’s strategy any disclosure by a type \( \theta_i \) for whom staying quiet is strictly dominant, assuming that Receiver expects all previously deleted types
\( \theta \in \Theta_{n-1} \) to stay quiet. Then, we identify the highest of these types, say \( \theta_{i^n} \), and then delete any disclosure by \( \theta \leq \theta_{i^n} \) to enforce a monotone strategy profile up \( \theta_{i^n} \). At the end of stage \( n \), the set of types who stay quiet in DNMD strategies is a set of the worst types \( \Theta_n = \{\theta_1, \ldots, \theta_{i^n}\} \). Since \( \Theta_n \) always constitutes an interval of consecutive types \( \theta_1, \ldots, \theta_{i^n} \), and \( \Theta_{n-1} \subseteq \Theta_n \), the set clearly converges to some connected interval \( \Theta^* = \{\theta_1, \theta_2, \ldots, \theta_{i^*}\} \) no later than stage \( N \) of the procedure. Moreover, since the worst type has a strictly dominant strategy to stay quiet, \( \Theta^* \) is non-empty. We derive a simple test for evaluating whether monotone or non-monotone equilibria of a particular disclosure game exist:

**Proposition 4.** If no type \( \theta \in \Theta^* \) prefers to disclose when Receiver expects the cutoff strategy \( Pr[m = \theta|\theta] = 1(\theta \in \Theta^*) \), then a monotone increasing equilibrium exists. Conversely, if some type \( \theta \in \Theta^* \) prefers to disclose when Receiver expects the fully opaque strategy \( Pr[m = \theta|\theta] \equiv 1 \), then all equilibria are non-monotone.

Proposition 4 gives a simple test for identifying whether monotone equilibria survive the introduction of outside information, which will allow us to generalize the intuition about concave payoffs from Proposition 3. The test involves checking whether types in \( \Theta^* \) prefer to deviate to disclosure under two sets of Receiver’s beliefs. For the first part, assume that Receiver expects the cutoff disclosure strategy where \( m = \emptyset \) if and only if \( \theta \in \Theta^* \), and forms beliefs that are consistent with this strategy. If no type of Sender in \( \Theta^* \) wishes to disclose in response, Proposition 4 tells us that this (monotone) profile is an equilibrium. Intuitively, this follows because types outside \( \Theta^* \) do not have an iterated dominant strategy to stay quiet. Thus, there must be some profile at which they would be willing to disclose. Indeed, the stipulated cutoff strategy, where only a set of the worst types \( \theta \in \Theta^* \) stay quiet, makes disclosure most attractive. For the converse, we show that any conjectured monotone disclosure strategy makes it less attractive for any types to disclose than the opaque strategy. Thus, if some \( \theta \in \Theta^* \) wishes to disclose under opacity, there cannot be a monotone equilibrium.

For an example of how we can detect non-monotone equilibria in the general game, consider the case where the best type \( \theta_N \) has a dominant strategy to stay quiet (the equivalent of Equation (2) in Section 2):

\[
M(\theta_N) < c \iff \sum \pi(s|\theta_N) [V(\theta_N) - V(\theta(s))] < c. \tag{8}
\]
This condition arises naturally if outside signals are sufficiently precise relative to the 
"steepness" of Sender’s payoffs. In particular, if the probability $\pi(s|\theta_N)$ that the best 
type draws a signal in common with a much worse type is low, or if the difference in 
payoffs $V(\theta_N) - V(\theta_1(s))$ between type $\theta_N$ and types with whom he shares signals is 
small, then the best type will prefer to stay quiet in any equilibrium. In this case, 
our iterations lead to $\Theta^* = \Theta$. Thus, we know that all equilibria are strictly non-
monotone as long as any type has an incentive to deviate from full opacity. This occurs 
if payoffs are steeper, or signals are less precise, for some mediocre types $\theta < \theta_N$.\footnote{For example, recall the simple model in Section 2, where there is a discontinuous jump in payoffs around the cutoff type $\theta = p$.} 
Hence, concave payoffs, which are flat for high types and steep for intermediate types, 
generate non-monotonic disclosures and give rise to “reverse unraveling” effects.

Condition (8) becomes trivial if signals have full support. However, we can still 
use Proposition 4 to establish non-monotonicity of equilibrium by considering later 
rounds of iterated deletion. Since, with full support, bad types have the lowest 
maximal punishment, the set of types who have a dominant strategy to stay quiet 
consists of the worst types $\{\theta_1, \theta_2, \ldots, \theta_n\}$ for some $n$. Receiver’s response to silence, 
in any equilibrium, can be no worse than to assume that $\theta \in \Theta_1$ for sure, and then 
to interpret outside information $s$ on this basis. If signals are sufficiently precise, the 
best type is sure that Receiver will place most of the probability mass on $\theta_n$. The 
net payoff from disclosure is then close to $V(\theta_N) - V(\theta_n)$. Therefore if the costs of 
disclosure $c > V(\theta_N) - V(\theta_n)$, we can rule out disclosures from the best type for 
sufficiently precise signals. Again, non-monotonicity is closely related to the shape of 
payoffs: If they are flat near the top of the type distribution, then $V(\theta_N) - V(\theta_n)$ is 
small relative to the net payoff of types $\theta < \theta_N$.

5 Applications

5.1 Financial panics

We apply our theory to study the reaction of financial institutions when they face 
a panic among investors. We are ultimately interested in a policy question: How 
much public (outside) information should a policy-maker such as the Fed release into 
a potentially panicked banking system?
Consider a bank (Sender) interacting with a group of short-term investors (Receivers) and a policy-maker (the Fed). Investors can run on the bank and force immediate liquidation of the its assets, in which case the assets yield one dollar. Alternatively, they can roll over their funds, in which case the bank’s assets are held to maturity and yield $1 + \theta$, where $\theta \in [0, \tilde{\theta}]$ is the bank’s privately observed quality. The bank can send investors a message $m = \theta$, which verifiably reveals its quality and costs $c \times \theta$, or stay quiet with $m = \emptyset$. In addition, investors observe an outside public signal $s = \theta + k\epsilon$, defined as in Section 2.

As for preferences, the bank chooses its communications $m$ to maximize the joint utility of its investors ($1$ if there is a run, $1 + \theta$ otherwise), net of disclosure costs. We stipulate that investors choose to run on the bank if and only if

$$E_{\mu}[\theta|m, s] < p$$

where $p > 0$ is a parameter capturing the illiquidity of the bank’s assets, and $\mu$ denotes their (endogenous) beliefs about the joint distribution of $\{\theta, m, s\}$. We assume that $E[\theta] < p$, so that there would be a bank run without any further information.

In the Online Appendix, we develop an exact micro-foundation for this condition from a global game of bank runs, following Morris and Shin (2000) and Bouvard et al. (2015). The economic intuition is simple: We model a global game where many investors decide simultaneously whether to withdraw their funds or roll over (as in Diamond and Dybvig, 1983). Pessimistic beliefs about $\theta$ lead to coordination on a “panic” equilibrium where everybody withdraws because they expect withdrawals

\[\text{Practitioners commonly think of the costs of disclosure for financial institutions as proprietary, such as the costs of revealing one’s investment portfolio to competitors. These costs are likely increasing in portfolio quality $\theta$. Assuming that the costs are proportional to $\theta$ is convenient but not crucial. Since the benefit of avoiding a bank run is $\theta$, the cost-benefit ratio is now constant at $c$, allowing us to solve the disclosure game in exact analogy to Section 2. Results with a general cost function $c(\theta)$ (nesting the fixed cost case) are available on request.}\]

\[\text{\text{\footnotesize{20}}Practitioners commonly think of the costs of disclosure for financial institutions as proprietary, such as the costs of revealing one’s investment portfolio to competitors. These costs are likely increasing in portfolio quality $\theta$. Assuming that the costs are proportional to $\theta$ is convenient but not crucial. Since the benefit of avoiding a bank run is $\theta$, the cost-benefit ratio is now constant at $c$, allowing us to solve the disclosure game in exact analogy to Section 2. Results with a general cost function $c(\theta)$ (nesting the fixed cost case) are available on request.}\]
from others.\textsuperscript{22,23}

We now turn to our policy question. Suppose that the Fed can choose the noise parameter $k \geq 0$ \textit{ex ante} and commit to this choice.\textsuperscript{24} In general, a subset $\mathcal{D}(k) \subset [p, \bar{\theta}]$ of banks disclose in equilibrium (banks with $\theta < p$ have a dominant strategy to stay quiet). In response, investors run on the bank if and only if (9) holds, or equivalently $s < s^\star$, where the critical signal $s^\star$ solves $E[\theta|s^\star, \theta \notin \mathcal{D}] = p$. Welfare is then:

$$W(k) = 1 + (1 - c) \int_{\theta \in \mathcal{D}(k)} \theta dF(\theta) + \int_{\theta \notin \mathcal{D}(k)} Pr[s \geq s^\star|\theta; k] \theta dF(\theta)$$

Intuitively, if the bank discloses ($\theta \in \mathcal{D}(k)$), it avoids a run for certain. Thus welfare is enhanced by $\theta - c\theta$; the value of avoiding the run less the cost of disclosure. The first integral in (10) measures these gains. If the bank stays quiet ($\theta \notin \mathcal{D}(k)$), a run is avoided, and welfare $\theta$ gained, as long the outside signal $s \geq s^\star$. This gain in welfare is the second integral. We note that welfare also coincides with the aggregate utility of all banks, since the banks are acting to maximize the total value of assets.

The mechanics of equilibrium disclosure are as in the model of Section 2, since investors’ collective action is binary (run or don’t run). As before, we refer to a situation where all strong banks $\theta > p$ disclose as a \textit{transparent} equilibrium. Whenever there are multiple equilibria for a given choice of $k$ (as in Figure 2 (a), for example), we assume that the welfare-maximizing $\mathcal{D}(k)$ is selected. We demonstrate that it is always optimal for the Fed to release a minimal amount of information:

\textbf{Proposition 5.} There is a $k$ such that transparency is the unique equilibrium if and only if $k > k^\star$. Any optimal policy has $k \leq k^\star$, and welfare increases discontinuously when $k$ crosses $k^\star$ from above.

\textsuperscript{22}More precisely, the Online Appendix lets bank returns $r = 1 + \theta + \eta$, and endows each investor with a private signal $\eta^i = \eta + \zeta^i$. When investor $i$ is pessimistic, he expects other investors $j \neq i$ to be pessimistic too, and therefore to start liquidating their investment. Since liquidation is wasteful, this further reduces the value of the bank, making it rational for $i$ to also withdraw. We take the usual limit as both $\eta$ and $\zeta^i$ collapse to zero.

\textsuperscript{23}The role of disclosure in global games is non-trivial when public signals interact with private ones (e.g. Angeletos et al., 2006). We sidestep these effects by assuming that inside information $m$ and outside information $s$ are about $\theta$, while the private signals are about another, independent factor $\eta$. In any case, the presence of a global game is not crucial for our analysis. Our arguments apply in any economy where pessimistic beliefs about a fundamental variable $\theta$ cause (re)financing constraints to bind.

\textsuperscript{24}The case where reducing the noise $k$ is costly leads to similar results: The optimal policy depends on the shape of the cost function, but it remains true that the policy-maker will optimally choose less noise when there are inside disclosures, in line with Proposition 5 below.

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When outside information is noisy with $k > k$, all strong banks $\theta \geq p$ disclose, and all weak banks $\theta < p$ suffer a run. When $k < k$, by contrast, there are equilibria in which some strong banks stay quiet. By revealed preference, all strong banks are weakly better off in these equilibria than they are under transparency. Moreover, the silence of strong strictly improves the utility of the weak, who can now avoid a run with positive probability (if they obtain a sufficiently impressive $s$). This is an instance of the Hirshleifer (1971) effect: Opacity provides implicit insurance. Overall, nobody is worse off and weak banks strictly gain, so that setting $k < k$ unambiguously improves welfare. The discontinuity around $k = k$ arises due to reverse unraveling: Once noise is reduced to $k$, the strongest banks stay quiet, others join in, and a discrete mass of banks must stay quiet in equilibrium. Thus, there is a discrete improvement in the implicit insurance provided to weak banks, and therefore in welfare.

This result is relevant in the context of a growing literature on stress test design (Bouvard et al., 2015; Goldstein and Leitner, 2015; Orlov et al., 2017). Existing work focuses on the benchmark case where banks cannot make inside disclosures (i.e. $D(k) \equiv \emptyset$). A common result is that public signals should be noisy when investors’ prior beliefs are not too pessimistic.\footnote{In general, finding the best $k$ is not a concave problem and its properties are very sensitive to the distribution of bank types. The main trade-off is between giving investors sufficient information to avoid a run, and introducing noise in order to implicit insurance.}

Proposition 5 leads us to a concrete but different policy recommendation: Policymakers should ensure a minimum level of transparency regardless of prior beliefs. This is because, in our setting, reducing the noise in public information enhances implicit insurance because it serves to crowd out efforts that strong banks make to separate themselves from the crowd. Because of the fragility of information, the welfare benefits of following this policy are first-order.

### 5.2 Political contests

Consider a political contest with two candidates $j \in \{A, B\}$ and a continuum of voters $i \in [0, 1]$. This is an extension of our game with two competing Senders and a population of Receivers. Candidate $j$ derives utility 1 if he is elected by majority vote, and zero otherwise. Voter $i$ derives utility $u = b^i + \theta$ if $B$ is elected, and $-u$ if $A$ is elected. The parameter $b^i$ measures individual preferences for $B$, and $\theta$ measures $B$’s inherent quality. We represent the distribution of $b^i$ in the population
by a smooth cumulative probability function $\beta(b)$. We assume that the distribution of $b^i$ is symmetric and normalize preferences such that the median voter is neutral with $\int b^i d\beta = 0$. We assume for simplicity that $\theta \sim N(0, \sigma^2_{\theta})$, so that the candidates are equally desirable ex ante.

Both candidates observe $\theta$ and can send a public message $m_j \in \{\emptyset, \theta\}$, where the verifiable disclosure $m_j = \theta$ leads to a utility cost $c \in (0, \frac{1}{2})$. Voters observe $\theta$ and an outside signal $s = \theta + \epsilon$, where $\epsilon \sim N(0, \sigma^2_{\epsilon})$, and then cast their votes to maximize expected utility. We focus on equilibria where each voter follows her individual preference. Thus, $B$ wins if $E[\theta|m, s] > 0$. The number of votes cast for $A$ is $\beta(-E[\theta|m, s])$.

If neither candidate makes a disclosure, then there is generally a critical outside signal $s_0$ so that $B$ wins if and only if $s > s_0$. Candidate $B$ then prefers to disclose if $\theta \geq 0$ and

$$Pr[s \leq s_0|\theta] = \Phi\left(\frac{s_0 - \theta}{\sigma_{\epsilon}}\right) \leq c.$$ 

In analogy to Section 2, this condition is most likely to hold for mediocre $B$-candidates, because the probability of drawing an unimpressive signal becomes small for sufficiently high $\theta$. The same incentives apply for $A$-candidates, who prefer to disclose if $\theta$ is negative but not too negative. We must therefore look for a threshold equilibrium where disclosures are made if neither candidate knows that he has a large advantage. In equilibrium, $B$ discloses if $\theta \in [0, \theta_B]$ and $A$ discloses if $\theta \in [\theta_A, 0)$. The thresholds $\theta_j$ and the critical signal $s_0$ solve the system of equations:

$$1 - \Phi\left(\frac{\theta_A - s_0}{\sigma_{\epsilon}}\right) = c = \Phi\left(\frac{s_0 - \theta_B}{\sigma_{\epsilon}}\right),$$

$$E[\theta|s_0, \theta \notin [\theta_A, \theta_B]] = 0.$$ 

The first equation, along with the symmetry of normal distributions, gives $\theta_A = -\theta_B$. Then, using symmetry again, the second equation implies that the critical signal $s_0 = 0$. We can now find a unique threshold equilibrium. Disclosures are made if and only if

$$|\theta| \leq \theta^* = -\sigma_{\epsilon}\Phi^{-1}(c).$$

(11)

We now establish that welfare in this model, as measured by the aggregate welfare of voters, increases whenever voters have more information about $\theta$. Denote voters’
information in this model by $\mathcal{I} = \{m_A, m_B, s\}$. Then aggregate voter utility is

$$W(\mathcal{I}) = \begin{cases} 
\int (b_i + E[\theta|\mathcal{I}])d\beta(b_i) & \text{if candidate B wins} \\
-\int (b_i + E[\theta|\mathcal{I}])d\beta(b_i) & \text{if candidate A wins} 
\end{cases}$$

$$= |E[\theta|\mathcal{I}]|$$

Therefore, welfare is a convex function of voters’ expectation of $\theta$. It follows from Rothschild and Stiglitz (1970) that any Blackwell-improvement in voters’ information $\mathcal{I}$ must strictly increase welfare. This has an immediate implication for the optimal amount of outside information in political contests: It is optimal to either release no information at all ($\sigma_\epsilon \to \infty$), in which case all candidates make disclosures according to (11), or to eliminate all noise in outside signals ($\sigma_\epsilon = 0$), in which case all candidates stay quiet. In either case, voters end up perfectly informed, which is the first-best outcome. For intermediate levels $\sigma_\epsilon \in (0, \infty)$, welfare is lower, and the effect of increasing noise on aggregate utility is ambiguous.

Moreover, we find an intriguing testable prediction. We observe an inside disclosure only if the advantage $|\theta|$ of either candidate is small. In this state of the world, the candidate who has an advantage makes an inside disclosure and wins the election by a small margin. Landslide victories, in which the number of votes cast in favor of A is less than $\beta(-\theta^*)$ or greater than $\beta(\theta^*)$, are associated with no inside information being revealed.

### 5.3 Corporate disclosures

We now model corporate disclosures by a firm wishing to raise funds from investors. We consider two modes of finance: issuing bonds and issuing shares. Since bonds give investors a concave claim and shares give a convex one, Proposition 3 suggests that incentives to disclose will differ between the two scenarios.

Consider a firm whose profits are $\theta \sim U[0, 1]$. The firm wishes to maximize the amount of money it raises by selling a given security to risk-neutral financial investors. As usual, the firm privately observes $\theta$ and can verifiably disclose it ($m = \theta$) at a cost $c$. Investors subsequently observe an outside signal $s = \theta + k\epsilon$, where $\epsilon \sim U[-1, 1]$. For simplicity, we assume that the noise parameter $k > 1/2$, so that any two types have some signals in common.
If the firm sells shares, then the payoff to buyers of shares is the convex claim
\[ \max\{\theta - d, 0\} \], where \( d \) denotes the face value of any existing debt. The firm’s payoff
is the market price of shares, which is given by investors’ willingness to pay given
inside and outside information:
\[
p(m, s) = E[\max\{\theta - d, 0\}|m, s].
\]
Low-quality firms with \( \theta \leq d \) have a dominant strategy to stay quiet, since disclosing
\( \theta \) would yield \( p = 0 \). For firms with \( \theta > d \), the net payoff from disclosure is the
expected gain in share prices
\[
N(\theta) = (\theta - d) - \frac{1}{2k} \int_{\theta-k}^{\theta+k} p(\theta, s)ds.
\]
This net payoff is strictly increasing in \( \theta \): The first term (the payoff from full disclo-
sure) increases with \( \theta \) at rate 1. The second term increases at rate
\[
\frac{1}{2k} [p(\theta + k) - p(\theta - k)]
\]
since increasing \( \theta \) shifts probability mass from low signals around \( \theta - k \) to high signals
around \( \theta + k \). Under the assumption that signals are not too precise \((k > 1/2)\), this
rate is always less than one. Since the net payoff from disclosure is increasing in \( \theta \),
all equilibria must have a cutoff property, where only firms with high quality \( \theta \geq \theta^* \)
make disclosures, with \( \theta^* > d \).

If the firm sells bonds, by contrast, the payoff to buyers is the concave claim
\( \min\{d, \theta\} \). The firm’s payoff is the market price of bonds
\[
q(m, s) = E[\min\{d, \theta\}|m, s].
\]
The net payoff from disclosure is now
\[
N(\theta) = \min\{d, \theta\} - \frac{1}{2k} \int_{\theta-k}^{\theta+k} q(\theta, s)ds.
\]
It is easy to see that incentives to disclose are strongest at the kink of the payoff
function where $\theta = d$. For lower-quality firms with $\theta < d$, the first term increases at rate 1, while the second term increases at rate $\frac{1}{2k} [q(0, \theta + k) - q(0, \theta - k)] < 1$. For high-quality firms with $\theta > d$, the first term is fixed, while the second term is still increasing. Therefore, the net payoff from disclosure has a peak at $\theta = d$. It follows that all equilibria must have interval strategies, where only firms with intermediate quality $\theta \in (\theta_L^*, \theta_H^*)$ make disclosures, with $\theta_L^* \leq d \leq \theta_H^*$.

The empirical predictions of this model are that disclosures come mainly from high-quality firms if they are selling shares, and mainly from intermediate-quality firms if they are selling bonds. Moreover, since firm quality and outside information are positively correlated, we predict that disclosures come mainly from firms with favorable subsequent realizations of outside information (e.g. optimistic analyst opinions) if selling shares, and mainly from firms with intermediate signals (e.g. mediocre credit ratings) if selling bonds. These predictions need to be qualified by allowing for sample selection: We have assumed that the security sold by the firm is exogenously determined and independent of its quality $\theta$. In the classic “pecking order” theory of Myers and Majluf (1984), debt is selected by high-quality firms as a signal. Daley et al. (2017) study a related model to ours, where debt issuance serves as an (inside) signal of quality in a model with (outside) credit ratings and two possible realizations of firm quality. Our model complements this literature by showing that – conditional on selecting debt, or in situations where debt is unambiguously more attractive (e.g. due to tax advantages) – disclosures tend to be made by firms of intermediate quality.

6 Conclusion

In this paper, we study the determinants of a privately informed Sender’s incentives to produce and disclose verifiable evidence to a Receiver, who also has access to outside sources of information. We argue that the interaction between competing information sources matters for bilateral trade under asymmetric information, corporate disclosures, and political contests, as well as for the optimal amount of transparency in financial systems.

The presence of outside information can starkly alter insiders’ incentives to dis-
close. In contrast to the classic literature on verifiable disclosure, our Senders may succumb to reverse unraveling, where “high quality” experts have an incentive to withhold information, generating an amplification mechanism towards even greater opacity. This amplification effect can have large negative consequences for equilibrium disclosures and informativeness. In particular, improvements in outside information can induce discontinuous drops in inside disclosures, and it is always possible for such an improvement to leave Receiver strictly worse informed in equilibrium. Further, we show how the tendency for reverse unraveling equilibria depends naturally on the shape of the Sender’s payoff function.

Our work also has potential implications for the literature on mechanism design with verifiable evidence (Green and Laffont, 1986; Bull and Watson, 2004; Kartik and Tercieux, 2012; Ben-Porath et al., 2016). These papers study the constraints that players’ ability to disclose verifiable evidence places on the implementability of social choice functions. In our model, the availability of outside information crucially impacts on incentives to disclose evidence. Therefore, optimal mechanisms are likely to take a different shape when the designer can also choose to release outside information.

Finally, our results suggest a difficulty for the problem of selling information, which is the topic of recent work by Hörner and Skrzypacz (2016) and Bergemann et al. (2016), among others. Holding inside disclosures constant, uninformed parties would be willing to pay for additional outside information in our models. However, we have seen that outside information tends to crowd out inside disclosures, and a Receiver who purchases outside information may obtain less information overall in equilibrium. A full exploration of this tension is beyond the scope of our paper, but the role of information markets for overall informativeness and real market efficiency remains an important topic for future research.
References


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A Proofs

Throughout the Appendix, we write $\sigma(\theta) = \text{Pr}[m = \theta | \theta]$ for (potentially mixed) disclosure strategies, and $\sigma = \{\sigma(\theta_1), ..., \sigma(\theta_n)\}$ for strategy profiles. We write Receiver’s beliefs interchangeably as functions of realizations of $\theta$, as in $\mu(\theta_i)$, or with superscripts, as in $\mu^i$.

Proposition 1

Proof. We construct a simple path of signals $\Pi(t)$ that satisfies the claim of the Proposition. Let $p_i : [0, 1] \rightarrow [0, 1]$ be a $C^2$, strictly increasing function with $p_i(0) = 0$, $p_i(1) = 1$ and whose derivative is equicontinuous, for $i = 1, \ldots, N$. Iteratively define the following class of outside signals: let $\hat{\Pi}(t)$ be an $N \times N$ matrix whose elements are

$$
\hat{\pi}(s | \theta; t) = \begin{cases} 
(1 - p_i(t)) \hat{\pi}(s | \theta_{i-1}; t), & s < i \\
p_i(t), & s = i \\
0, & s > i.
\end{cases}
$$

$\hat{\Pi}(t)$ satisfies MLRP for all $t$. We show first that

$$
\mathcal{M}(\theta_i; t) = (1 - p_k(t)) \mathcal{M}(\theta_{i-1}; t) + p_k(t) (V(\theta_i) - V(\theta_k))
$$

is decreasing in $t$, with $\mathcal{M}(\theta_i; 0) > c, \mathcal{M}(\theta_i; 1) = 0, \forall i > 1$.

We argue inductively: If $\mathcal{M}(\theta_{i-1}; t)$ is increasing in $t$ then clearly so too is $\mathcal{M}(\theta_i; t)$, since $p_k(t)$ is increasing in $t$ and $V(\theta_i) - V(\theta_k)$ is strictly decreasing in $s$. Observing that $\mathcal{M}(\theta_2; t) = (1 - p_2(t)) (V(\theta_2) - V(\theta_1))$ is decreasing establishes monotonicity. Thus for each $\theta$, there is a unique $t^*_i$ at which $\mathcal{M}(\theta_i; t) = c$. Moreover, we can find a $\Pi(t)$ such that $t^*_i = t^*; \forall i, j$. To do this, we iteratively adjust $\hat{\Pi}(t)$; suppose a matrix $\Pi'_k(t)$ induces $\mathcal{M}(\theta_i; t_k') = \mathcal{M}(\theta_j; t_k')$, $\forall i, j \leq k$. Construct $\Pi'_{k+1}(t)$ as follows: if $t^*_{k+1} > t^*_k$, replace row $k$ of $\Pi'_k(t)$ with the functions

$$
\left(\pi' \left( s | \theta_k; \frac{t}{t_k} \right) \right)_{s=1}^n.
$$
When in any equilibrium the net payoff to disclosure for type
we show there exists a
Proposition 2
for any
Otherwise, replace each row \( i < k \) with \((\pi'(s | \theta_i; \hat{t}) t)_{s=1}^{n}\). Applying this process to
then for all \( t > t^* \), full opacity is the unique equilibrium of the
time. Finally, we show there exists a \( \delta > 0 \) such that if at \( t^* \), \( c \leq \mathcal{M}(\theta_i; t^*) \leq c+\delta \), with \( \mathcal{M}(\theta_k; t^*) = c \) for at least some \( k \), then for all \( t > t^* \), full opacity is the unique equilibrium of the
disclosure game. At any \( t > t^* \), we have \( \mathcal{M}(\theta_k; t) < c \). Thus, for any equilibrium
strategy profile, \( \theta_k \)'s net payoff from disclosure is
\[
\sum_{s=1}^{k-1} \pi(s | \theta_k; t) (V(\theta_k) - v(\alpha(s))) \leq \mathcal{M}(\theta_k; t) < c
\]
since for any log-supermodular \( u \) and increasing \( v \), \( v(\alpha(s)) \geq V(\theta_s) \). Thus, in any equilibrium \( m(\theta_k) = \emptyset \), \( \forall t > t^* \). Given any signal \( s < k \), define the vector of beliefs
\( \mu_s = (Pr[\theta|s, \emptyset])_{\theta \in \Theta} \), and define \( \nu(\mu_s) \) as Sender’s utility when Receiver has these beliefs. We can bound \( \mu_s \geq \kappa_s^11_k + (1 - \kappa_s^1)1_s \), where \( \kappa_s^1 > 0 \) satisfies
\[
\kappa_s^t = \min_{\{\sigma|\sigma(\theta_k) = 0\}} Pr[\theta_k | s, m = \emptyset] = \frac{Pr[s, m = \emptyset | \theta_k] \mu_0(\theta_k)}{Pr[s, m = \emptyset]} \geq \frac{\pi(s | \theta_k) \mu_0(\theta_k)}{Pr[s]} > 0
\]
for any \( t < 1 \), which follows since \( Pr[m = \emptyset | \theta_k] = 1 \) in equilibrium and \( Pr[s, m = \emptyset] \leq Pr(s) \). Since \( \alpha(s) \) is strictly increasing in the LR order, \( \nu(\mu_s) > \nu(1_s) = V(\theta_s) \). Define \( \delta = \min_{s}[1 - \pi(s | \theta_i; t)][\nu(\mu^*_s) - \nu(1_s)] \), where \( \mu^*_s = \kappa^*_s 1_k + (1 - \kappa^*_s) 1_s \).
Then in any equilibrium the net payoff to disclosure for type \( \theta_i \) satisfies
\[
\mathcal{N}(\theta_i; t) \leq \mathcal{M}(\theta_i; t) - \sum_{s < k} \pi(s | \theta_i; t) [\nu(\mu^*_s) - \nu(1_s)] \leq \mathcal{M}(\theta_i; t) - \delta
\]
When \( \mathcal{M}(\theta_i; t) \leq c+\delta \), \( \forall i \), all types strictly prefer to set \( m(\theta_i) = \emptyset \) in any equilibrium.
\[
\square
\]
Proposition 2

Proof. Fix \((S, \Pi)\) and a corresponding equilibrium strategy profile \( \sigma^* \), actions \( \{\alpha(s)\}_{s \in S} \) and Receiver posterior beliefs \( (\mu_s)_{s \in S} \). Suppose further that \( \sigma(\theta_i) > 0 \) for some \( \theta_i \).

\[\text{It is simple to reparameterize } \Pi(t) \text{ to ensure that } p_i(t) < 1 \text{ for } t < 1.\]
Partition $\Theta$ as follows: $\theta \in Q \iff \mathcal{N}^* (\theta) < c$, $\theta \in D$ otherwise. Now consider the following modified signal structure, $(S \cup \Theta, \Pi')$ which satisfies

$$\pi' (s \mid \theta_i) = \begin{cases} 
\pi (s \mid \theta_i), & \theta_i \in Q, s \in S \\
0, & \theta_i \in Q, s \in \Theta \\
(1 - z_i) \pi (s \mid \theta_i), & \theta_i \in D, s \in S \\
z_i, & \theta_i \in D, s = \theta_i \in \Theta
\end{cases}. $$

where $z_i \leq \sigma (\theta_i)$. For each $\theta_i$, $\exists \exists z_i < \sigma (\theta_i)$ such that, for all $\exists z_i < z_i$ and $\theta_i \in D$:

$$\sum_{s \in S \cup \Theta} \pi' (s \mid \theta_i) (V (\theta_i) - v (\alpha (s))) = (1 - z_i) \sum_{s \in S \cup \Theta} \pi (s \mid \theta_i) (V (\theta_i) - v (\alpha (s))) < c. $$

Fix $\exists z_i \leq z_i \leq \sigma (\theta_i)$. Let $\sigma' (\theta) = 0$. Given strategy profile $\zeta'$, and outside signals $(S \cup \Theta, \Pi')$, Receiver’s posterior beliefs $\hat{\mu}^N_{i=1}$ given $s \in S$ can be written

$$\hat{\mu}_s = \frac{\mu_0 (\theta_i) (1 - z_i) \pi (s \mid \theta_i)}{\mu_0 (\theta_j) (1 - z_j) \pi (s \mid \theta_j)}.$$

As $z_i \to \sigma (\theta_i)$, $\mu_s \to \mu_s$. Given finiteness of $\Theta, S$, for any $\varepsilon > 0$ there exists bounds $(\exists i)_{i=1}^N$ such that $|\hat{\mu}_s - \mu_s| < \varepsilon$ whenever $\exists z_i < z_i < \sigma (\theta_i)$, $\forall i$. If $\alpha (s)$ is continuous in $\mu$ (which holds because Receiver has a unique best response), then given strict preference for nondisclosure of all types under action profile $\{\alpha (s)\}_{s=1}^N$, and outside signals $(S \cup \Theta, \Pi')$, we can therefore find a $\varepsilon > 0$ such that opacity is an equilibrium of this game.

Finally, the opaque equilibrium with outside signals $(S \cup \Theta, \Pi')$ is a Blackwell garbling of equilibrium information structure with $\sigma^*$ and $(S, \Pi)$. To see this, note that one can construct the equilibrium signal Receiver observes under the former equilibrium by the garbling the Sender’s disclosures in the latter equilibrium as follows: given message $m = \emptyset$ and signal $s$, use the ‘truthful’ garbling $\Pr (s \mid s, m = \emptyset) = 1$; given message $m = \theta$, garble to signal $s \in S \cup \Theta$ with probabilities $\Pr (s = \theta \mid m = \theta) = \frac{z_i}{\sigma^* (\theta_i)}$ for $s = \theta$, $\Pr (s \mid m = \theta) = \left(1 - \frac{z_i}{\sigma^* (\theta_i)} \right) \pi (s \mid \theta_i)$ for $s \in S$. \hfill \Box
Proposition 3

Proof. We write Concavity $= \chi$ and Convexity $= \xi$. We split the proof into two parts. First, we show that sufficiently concave payoffs imply that all equilibria are non-monotone or opaque. Second, we show that sufficiently convex payoffs imply that there are no non-monotone equilibria.

Part 1: Concave payoffs

Let $\Sigma_m \subset [0,1]^N$ be the space of monotone increasing disclosure strategies. For any $\sigma \in \Sigma_m$, define $d(\sigma) = \min\{i|\sigma_i > 0\}$ as the lowest disclosing type, and $q(\sigma) = d(\sigma) - 1$ the highest type who stays quiet with probability 1. We first derive a bound on the weights that these two types attach to being perceived as type $q(\sigma) - 1$ or worse, assuming that this type exists (i.e., that $q(\sigma) > 1$). For all $j < q(\sigma)$, Receiver’s beliefs if Sender stays quiet are interior with $\Pr[\theta \leq \theta_j | \emptyset] \in (0,1)$. For any pair of signals $(s', s)$, where $s' > s$ and at least one of them is drawn by type $j$ or worse with positive probability, the strict MLRP of signals implies strict first-order stochastic dominance (see Milgrom 1981, Theorem 1):

$$\Pr[\theta \leq \theta_j | \emptyset, s'] < \Pr[\theta \leq \theta_j | \emptyset, s] \text{ for all } s' > s.$$ 

The cumulative distribution of virtual types $Q^\sigma_{ij} = E_s[\Pr[\theta \leq \theta_j | s, \emptyset] | \theta_i]$ is therefore the expectation of an decreasing function of $s$, where the superscript $\sigma$ is introduced to highlight the dependence of Receiver’s beliefs on equilibrium play. Using the MLRP again, we find that $Q^\sigma_{ij}$ is strictly decreasing in $i$ for $j < q(\sigma)$. Thus for all feasible $\sigma$,

$$Q^\sigma_{d(\sigma), q(\sigma) - 1} - Q^\sigma_{q(\sigma), q(\sigma) - 1} < 0,$$

Define

$$Q_m = \max_{\{\sigma \in \Sigma_m | q(\sigma) > 1\}} Q^\sigma_{d(\sigma), q(\sigma) - 1} - Q^\sigma_{q(\sigma), q(\sigma) - 1}.$$ 

It is easy to see that the constraint set is compact, so that the maximum is achieved and satisfies $Q_m < 0$. Note that we can repeat the stochastic dominance argument above for $j \geq q(\sigma)$: In this case we may have $Q^\sigma_{ij} = 0$ for a range of $i$ (for example, if only types below $j$ stay quiet with positive probability), but a parallel argument establishes that $Q^\sigma_{ij}$ is non-increasing in $i$. 

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Next, suppose that \( \sigma \in \Sigma_m \) is a monotone increasing strategy played in equilibrium, and assume that \( c > c_0 \). By the definition of \( c_0 \), there is a type \( \theta_j = \arg \min_{i \geq 2} M(\theta_i) \) who has a dominant strategy to stay quiet, so that \( \sigma_j = 0 \). By monotonicity, we have \( \sigma_i = 0 \) for all \( i \leq j \), and it follows that the highest quiet type \( q(\sigma) > 1 \). Optimality requires that this type prefers to stay quiet and the lowest discloser \( d(\sigma) \) prefers to disclose. We obtain \( N(\theta_{d(\sigma)}) \geq c \geq N(\theta_{q(\sigma)}) \), implying

\[
0 \leq N(\theta_{d(\sigma)}) - N(\theta_{d(\sigma) - 1}) = \Delta X_{d(\sigma) - 1} + \sum_{i=1}^{N-1} \Delta X_i (Q^\sigma_{d(\sigma),i} - Q^\sigma_{d(\sigma) - 1,i}) \leq \Delta X_{d(\sigma) - 1} + \Delta X_{d(\sigma) - 2} (Q^\sigma_{d(\sigma),i} - Q^\sigma_{d(\sigma) - 1,i}) \leq \Delta X_{d(\sigma) - 1} + \Delta X_{d(\sigma) - 2} Q_m.
\]

where the second inequality follows by first-order stochastic dominance, and the third imposes the bound derived above. Dividing by \( \Delta X_{d(\sigma)} \) and using \( Q_m < 0 \), we obtain

\[
\frac{\Delta X_{d(\sigma) - 2}}{\Delta X_{d(\sigma) - 1}} \leq \frac{1}{|Q_m|}.
\]

This further implies that the concavity parameter \( \chi \leq \frac{1}{|Q_m|} \). We have now shown that the existence of a monotone increasing equilibrium with \( c > c_0 \) implies that \( \chi \) is bounded above. By contrapositive, if \( \chi \) is sufficiently large, then there is no monotone equilibrium for the range of disclosure costs \( c > c_0 \), as required.

**Part 2: Convex payoffs**

Let \( \Sigma_{nm} \subset [0, 1]^N \) be the space of non-monotones strategy profiles. For \( \sigma \in \Sigma_{nm} \), we can define \( q(\sigma) = \max\{i|\sigma_i < 1\} \) as the highest type who stays quiet with positive probability, and \( d(\sigma) = \max\{i < q(\sigma)|\sigma_i > 0\} \) as the highest discloser below \( q(\sigma) \). Since type \( \theta_1 \) has a dominant strategy, we have \( d(\sigma) > 1 \) and \( q(\sigma) > 2 \). We first derive a bound on the weights that these two types attach to being perceived as type \( q(\sigma) - 1 \) or worse. Let

\[
Q_{nm} = \inf_{\sigma \in \Sigma_{nm}} \{Q^\sigma_{q(\sigma),q(\sigma) - 1} - Q^\sigma_{d(\sigma),q(\sigma) - 1}\}.
\]


Since the cumulative probabilities $Q^{\sigma}_{ij} \in [0, 1]$, we have $Q_{nm} \geq -1$. We show that this inequality is strict. Suppose, for a contradiction, that $Q_{nm} = -1$. Then for every $\epsilon$ we can find strategies $\sigma \in \Sigma_{nm}$ such that $Q^{\sigma}_{q(\sigma), q(\sigma)} - 1 - Q^{\sigma}_{d(\sigma), q(\sigma)} < -1 + \epsilon$. This implies two requirements: $Q^{\sigma}_{q(\sigma), q(\sigma)} < \epsilon$ and $Q^{\sigma}_{d(\sigma), q(\sigma)} > 1 - \epsilon$, that is, type $q(\sigma)$ almost never draws a virtual type worse than himself, while type $d(\sigma)$ almost never draws a better virtual type than $q(\sigma) - 1$. Since neighboring types share signals, we can find a realization $s = s'$ that is drawn with positive probability by both $q(\sigma)$ and $q(\sigma) - 1$. Our first requirement implies that Receiver’s posterior belief, after observing $m = \emptyset$ and $s = s'$, satisfies $Pr[\theta \leq \theta_{q(\sigma)} - 1|\emptyset, s'] \leq \delta(\epsilon)$, where $\delta(\epsilon) \to 0$ as $\epsilon \to 0$. For small enough $\epsilon$, this is only possible if type $q(\sigma) - 1$ discloses with positive probability (otherwise Receiver would place a discrete probability mass on this type when she observes $m = \emptyset$). Therefore, we know that the highest discloser below $q(\sigma)$ is his neighbor: $d(\sigma) = q(\sigma) - 1$. Our second requirement now implies that Receiver’s posterior belief satisfies $Pr[\theta > \theta_{q(\sigma)} - 1|\emptyset, s'] \leq \hat{\delta}(\epsilon)$, where $\hat{\delta}(\epsilon) \to 0$ as $\epsilon \to 0$. We can write

$$1 = Pr[\theta \leq \theta_{q(\sigma)} - 1|\emptyset, s'] + Pr[\theta > \theta_{q(\sigma)} - 1|\emptyset, s'] \leq \delta(\epsilon) + \hat{\delta}(\epsilon),$$

and taking limits as $\epsilon \to 0$, we get a contradiction. Therefore, $Q_{nm} > -1$.

Next, suppose that $\sigma \in \Sigma_{nm}$ is a non-monotone strategy played in equilibrium. Optimality requires that $N(\theta_{d(\sigma)}) \geq c \geq N(\theta_{q(\sigma)})$, implying

$$0 \geq N(\theta_{q(\sigma)}) - N(\theta_{d(\sigma)}) = \sum_{i=d(\sigma)}^{N-1} \Delta X_i + \sum_{i=1}^{N-1} \Delta X_i(Q^i_{q(\sigma)} - Q^i_{d(\sigma)}).$$

Note that $Q_{ij} = 1$ for all $i$ and $j \geq q(\sigma)$, since Receiver attaches probability $Pr[\theta_j|\emptyset] = 0$ to types $j > q(\sigma)$ when Sender stays quiet. Moreover, a parallel argument to Part 1 of this proof establishes that the distribution of virtual types given $\theta = q(\sigma)$ first-order stochastically dominates that given the lower type $\theta = d(\sigma)$, so that $Q_{q(\sigma)}^i - Q_{d(\sigma)}^i \leq \Delta X_i$. Since neighboring types share signals, we can find strategies $\sigma \in \Sigma_{nm}$ such that $Q_{nm} \geq -1$. We show that this inequality is strict. Suppose, for a contradiction, that $Q_{nm} = -1$. Then for every $\epsilon$ we can find strategies $\sigma \in \Sigma_{nm}$ such that $Q^{\sigma}_{q(\sigma), q(\sigma)} - 1 - Q^{\sigma}_{d(\sigma), q(\sigma)} < -1 + \epsilon$. This implies two requirements: $Q^{\sigma}_{q(\sigma), q(\sigma)} < \epsilon$ and $Q^{\sigma}_{d(\sigma), q(\sigma)} > 1 - \epsilon$, that is, type $q(\sigma)$ almost never draws a virtual type worse than himself, while type $d(\sigma)$ almost never draws a better virtual type than $q(\sigma) - 1$. Since neighboring types share signals, we can find a realization $s = s'$ that is drawn with positive probability by both $q(\sigma)$ and $q(\sigma) - 1$. Our first requirement implies that Receiver’s posterior belief, after observing $m = \emptyset$ and $s = s'$, satisfies $Pr[\theta \leq \theta_{q(\sigma)} - 1|\emptyset, s'] \leq \delta(\epsilon)$, where $\delta(\epsilon) \to 0$ as $\epsilon \to 0$. For small enough $\epsilon$, this is only possible if type $q(\sigma) - 1$ discloses with positive probability (otherwise Receiver would place a discrete probability mass on this type when she observes $m = \emptyset$). Therefore, we know that the highest discloser below $q(\sigma)$ is his neighbor: $d(\sigma) = q(\sigma) - 1$. Our second requirement now implies that Receiver’s posterior belief satisfies $Pr[\theta > \theta_{q(\sigma)} - 1|\emptyset, s'] \leq \hat{\delta}(\epsilon)$, where $\hat{\delta}(\epsilon) \to 0$ as $\epsilon \to 0$. We can write

$$1 = Pr[\theta \leq \theta_{q(\sigma)} - 1|\emptyset, s'] + Pr[\theta > \theta_{q(\sigma)} - 1|\emptyset, s'] \leq \delta(\epsilon) + \hat{\delta}(\epsilon),$$

and taking limits as $\epsilon \to 0$, we get a contradiction. Therefore, $Q_{nm} > -1$.

Next, suppose that $\sigma \in \Sigma_{nm}$ is a non-monotone strategy played in equilibrium. Optimality requires that $N(\theta_{d(\sigma)}) \geq c \geq N(\theta_{q(\sigma)})$, implying

$$0 \geq N(\theta_{q(\sigma)}) - N(\theta_{d(\sigma)}) = \sum_{i=d(\sigma)}^{N-1} \Delta X_i + \sum_{i=1}^{N-1} \Delta X_i(Q^i_{q(\sigma)} - Q^i_{d(\sigma)}).$$

Note that $Q_{ij} = 1$ for all $i$ and $j \geq q(\sigma)$, since Receiver attaches probability $Pr[\theta_j|\emptyset] = 0$ to types $j > q(\sigma)$ when Sender stays quiet. Moreover, a parallel argument to Part 1 of this proof establishes that the distribution of virtual types given $\theta = q(\sigma)$ first-order stochastically dominates that given the lower type $\theta = d(\sigma)$, so that $Q_{q(\sigma)}^i - Q_{d(\sigma)}^i \leq \Delta X_i$.
0. Dividing the above inequality by $\Delta X_{q(\sigma)-1}$ and combining these observations,

\[
0 \geq 1 + Q_{q(\sigma),q(\sigma)-1}^\sigma - Q_{d(\sigma),q(\sigma)-1}^\sigma + \sum_{i=1}^{q(\sigma)-1} \left( \frac{\Delta X_i}{\Delta X_{q(\sigma)-1}} \right) (1_{i \geq d(\sigma)} + Q_{q(\sigma),i}^\sigma - Q_{d(\sigma),i}^\sigma)
\]

\[
\geq 1 + Q_{q(\sigma),q(\sigma)-1}^\sigma - Q_{d(\sigma),q(\sigma)-1}^\sigma + \sum_{i=1}^{q(\sigma)-2} \xi^{-[q(\sigma)-1-i]} (1_{i \geq d(\sigma)} + Q_{q(\sigma),i}^\sigma - Q_{d(\sigma),i}^\sigma)
\]

\[
\geq (1 + Q_{nm}) - \sup_{\sigma \in \Sigma_{nm}} \sum_{i=1}^{q(\sigma)-2} \xi^{-[q(\sigma)-1-i]},
\]

where the last line follow noting that $1_{i \geq d(\sigma)} + Q_{q(\sigma),i}^\sigma - Q_{d(\sigma),i}^\sigma \geq -1$ and then taking the infimum. We know that the first term $1 + Q_{nm} > 0$. Thus the second term must be smaller than $- (1 + Q_{nm})$. However, it is easy to see that the limit of this term as $\xi \to \infty$ is zero, so that the above series of inequalities gives us $\xi \leq \xi_0$ for some finite $\xi_0$. We have now shown that the existence of a non-monotone equilibrium implies an upper bound on $\xi$. By contrapositive, if $\xi$ is sufficiently large, then there is no non-monotone equilibrium, as required.

\[\square\]

**Proposition 4**

*Proof.* For the first claim, suppose that all types $\theta \in \Theta^*$ prefer to stay quiet when Receiver’s beliefs are consistent with the cutoff strategy $\sigma^*(\theta) = 1_{\theta \in \Theta^*}$. To verify that $\sigma^*$ is an equilibrium, we show that all types $\theta \notin \Theta^*$ prefer to disclose. By definition, type $\theta \notin \Theta^*$ does not have a dominant strategy to stay quiet, so there exists a strategy profile $\tilde{\sigma}$ with corresponding beliefs for Receiver such that $\theta$ prefers to disclose. We can write Receiver’s posteriors under $\tilde{\sigma}$ as

\[
Pr_{\tilde{\sigma}}[\theta_k|\emptyset] = w Pr_{\tilde{\sigma}}[\theta_k|\emptyset, \theta \in \Theta^*] + (1 - w) Pr_{\tilde{\sigma}}[\theta_k|\emptyset, \theta \notin \Theta^*],
\]

where $w = Pr_{\tilde{\sigma}}[\theta \in \Theta^*]$. These beliefs MLRP-dominate any possible posteriors under $\sigma^*$, since the latter attach no weight to $\theta \notin \Theta^*$ on the equilibrium path. Thus, Receiver’s payoff from staying quiet is weakly higher under $\tilde{\sigma}$ than under $\sigma^*$, and it follows that disclosure must also be a best response to $\sigma^*$ for $\theta \notin \Theta^*$, as required.

For the second claim, suppose that some type $\theta_l \in \Theta^*$ prefers to disclose when Receiver’s beliefs are consistent with the opaque strategy $\sigma^o(\theta) \equiv 0$. Suppose for a contradiction that there is a monotone increasing equilibrium $\sigma$. Receiver’s posterior
likelihood ratio of states $\theta_i < \theta_j$, given $m = \emptyset$ and some $s \in S$, satisfy

\[ \frac{\mu_0(\theta_j)(1 - \sigma(\theta_j))\pi(s|\theta_j)}{\mu_0(\theta_i)(1 - \sigma(\theta_i))\pi(s|\theta_i)} \leq \frac{\mu_0(\theta_j)\pi(s|\theta_j)}{\mu_0(\theta_i)\pi(s|\theta_i)} \]  \tag{12}

where the inequality follows since $\sigma(\theta_i) \leq \sigma(\theta_j)$ for any monotone strategy. But the right-hand side of (12) is Receiver’s likelihood ratio for $\theta_i < \theta_j$, given $s$, under the opaque strategy $\sigma^o$. Therefore, if Sender does not disclose, Receiver takes weakly higher actions under $\sigma^o$ than under the candidate equilibrium strategy $\sigma$. By assumption, some type $\theta \in \Theta^*$ strictly prefers to disclose under $\sigma^o$, and so this type must also prefer to disclose under $\sigma$. Also, it follows from the definition of DNMD that type $\theta^* = \max \Theta^*$ stays quiet in any monotone equilibrium, and type $\theta_1$ has a dominant strategy to stay quiet. Thus, we know that $\theta_1$ and $\theta^*$ stay quiet, while $\theta \in (\theta_1, \theta^*)$ discloses, contradicting monotonicity. \hfill \Box

**Proposition 5**

The following are true by the same argument as in Appendix B:

- All equilibria feature interval strategies: $\sigma(\theta) = 1_{\theta \in (p, \theta')}\) for almost all $\theta$ in equilibrium for some cutoff $\theta'$, and investors run on the bank if $s < s^*(\theta')$ for an appropriately defined critical signal $s^*(\theta')$. We have $s^*(\theta') \leq p + k$, with equality only if $\theta' = \bar{\theta}$.

- Define the best response $B(\theta'; k)$ as the largest $\theta$ satisfying $G\left(\frac{s^*(\theta') - \theta}{k}\right) \geq c$ (if this is impossible, let $B(\theta') = p$ ). The cutoff $\theta'$ induces an equilibrium, given that the Fed chooses $k$, if and only if $B(\theta', k) = \theta'$,

- If $B(\theta'; k)$ is interior in a neighborhood of $\bar{\theta}$, it is infinitely steep with

\[ \lim_{\theta' \to \bar{\theta}} \frac{\partial}{\partial \theta'} B(\theta', k) = \infty \]

Now define $\hat{\theta}(k) = \inf \{\theta | B(\theta', k) = \theta'\}$ as the largest disclosing type in the least informative equilibrium given $k$. Let $k = \inf \{k | \hat{\theta}(k) = \bar{\theta}\}$ be the lower bound on $k$ such that transparency ($\theta' = \bar{\theta}$) is the unique equilibrium.

Note first that $k < \infty$: Suppose instead that $k = \infty$ and there is an equilibrium with $\theta' < \bar{\theta}$. Then, investors’ beliefs when the bank stays quiet are $E_\mu[\theta|m, s] = \ldots$
\[ E[\theta | \theta \notin (p, \theta')] \leq E[\theta] < p. \] Thus, quiet banks with \( \theta > p \) face a run with probability 1, and prefer to deviate to disclosure.

We next show that \( \hat{\theta}(k) < \bar{\theta} \). Suppose instead that \( \hat{\theta}(k) = \bar{\theta} \). Then in a neighborhood of \( \bar{\theta} \), \( B(\theta', k) \) must be interior; otherwise, moving to \( k = k - \epsilon \) would preserve transparency as the unique equilibrium. Since \( \lim_{\theta \to \bar{\theta}} \frac{\partial}{\partial \theta} B(\theta', k) = \infty \) whenever interior, we have \( B(\theta', k) < \theta' \) in a neighborhood of \( \bar{\theta} \). Then either \( B(\theta', k) = \theta' \) for all \( \theta' \), in which case \( B(p, k) = p \), or there exists a crossing \( B(\theta', k) = \theta' \) for \( \theta' \in (p, \bar{\theta}) \). In both cases, it is clear that \( \hat{\theta}(k) < \bar{\theta} \), a contradiction.

In any equilibrium with cutoff \( \theta' \), welfare is
\[
\tilde{W}(\theta') = 1 + (1 - c) \int_{\theta \in (p, \theta')} \theta dF(\theta) + \int_{\theta \notin (p, \theta')} \theta \left( 1 - G \left( \frac{s^*(\theta') - \theta}{k} \right) \right) dF(\theta)
\]
\[
= W + \int_{\theta > \theta'} \theta \left[ c - G \left( \frac{s^*(\theta') - \theta}{k} \right) \right] dF(\theta) + \int_{\theta < p} \theta \left[ 1 - G \left( \frac{s^*(\theta') - \theta}{k} \right) \right] dF(\theta)
\]
where \( W \equiv 1 + (1 - c) \int_{\theta > p} \theta dF(\theta) \) measures welfare in a transparent equilibrium. When the Fed chooses \( k = k \), welfare is at least \( \tilde{W}(\hat{\theta}(k)) \). When it chooses \( k > k \), welfare is \( W \). Thus for any \( k > k \),
\[
W(k) - W(k) \geq \int_{\theta > \hat{\theta}(k)} \theta \left[ c - G \left( \frac{s^*(\hat{\theta}(k)) - \theta}{k} \right) \right] dF(\theta)
\]
\[
+ \int_{\theta < p} \theta \left[ 1 - G \left( \frac{s^*(\hat{\theta}(k)) - \theta}{k} \right) \right] dF(\theta)
\]
For all disclosing types \( \theta > \theta' \), \( c \geq G \left( \frac{s^* - \theta}{k} \right) \) by revealed preference, so the first integral is non-negative. The second integral is strictly positive because \( s^*(\hat{\theta}(k)) < p + k \). Since neither integral depends on \( k \), we get \( W(k) - W(k) = \delta > 0 \) for all \( k > k \). The proposition follows: It is never optimal to choose \( k > k \), and the welfare gain from crossing \( k \) is discontinuous since \( \lim_{k \downarrow k} W(k) = W(k) - \delta \neq W(k) \).
B Derivations for Section 2

We write $\sigma(\theta) = Pr[m = \theta | \theta]$ for (potentially mixed) disclosure strategies. We first show that Receiver’s response, if Sender does not disclose, is to buy if and only if the outside signal is good enough.

**Lemma 1.** In any equilibrium, there exists a critical signal $s^*$ such that Receiver buys with probability one if $m = \emptyset$ and $s > s^*$, and buys with probability zero if $m = \emptyset$ and $s < s^*$.

**Proof.** Take any equilibrium with disclosure strategy $\sigma$. Consider Receiver’s response in the event $\{m = \emptyset, s\}$. If $s > p + k$, she strictly prefers to buy. If $s < p + k$, we can define the intermediate prior $F_{\emptyset}(\theta') = Pr[\theta \leq \theta' | m = \emptyset]$ which denotes the distribution of $\theta$ given the null message alone, and which is non-degenerate since it must place positive density on all $\theta \in (s - k, p)$ that have a dominant strategy to send $m = \emptyset$. Since $m$ and $s$ are independent conditional on $\theta$, Receiver’s posterior beliefs given $\{m = \emptyset, s\}$ are formed by updating $F_{\emptyset}$ given the outside signal $s$:

$$F_{\emptyset,s}(\theta') = Pr[\theta \leq \theta' | m = \emptyset, s]$$

$$= \frac{\int_{\emptyset}^{\theta'} g \left( \frac{s-\theta}{k} \right) dF_{\emptyset}(\theta)}{\int_{\emptyset}^{\theta} g \left( \frac{s-\theta}{k} \right) dF_{\emptyset}(\theta)}$$

Strict log-concavity of $g(.)$ implies that signals have the strict Monotone Likelihood Ratio Property (MLRP) with respect to $\theta$. Proposition 1 in Milgrom (1981) then shows that the expected value $E[\theta|m = \emptyset, s] = \int \theta dF_{\emptyset,s}(\theta)$ is strictly increasing in $s$. Moreover, note that this expectation is strictly less than $p$ for all signals $s < p - k$. We now have two possible cases: First, if $\lim_{s \uparrow p+k} E[\theta|m = \emptyset, s] > p$, then there exists a unique $s^*(\sigma)$ such that Receiver strictly prefers to buy if $s > s^*(\sigma)$ and strictly prefers not to buy if $s < s^*(\sigma)$. Second, if $\lim_{s \uparrow p+k} E[\theta|m = \emptyset, s] \leq p$, then Receiver strictly prefers not to buy for all $s < p + k$, and we may set $s^*(\sigma) = p + k$.

We now derive the equilibrium characterization stated in Section 2:
Proposition 6. In any equilibrium, one of the following is true:

1. Transparency: Sender fully discloses with $\sigma(\theta) = 1$ for all $\theta > p$ and stays quiet with $\sigma(\theta) = 0$ for all $\theta < p$;

2. Opacity: Sender never discloses with $\sigma(\theta) = 0$ for almost all $\theta$;

3. Interval disclosure: There is a threshold $\theta^*$ such that Sender fully discloses with $\sigma(\theta) = 1$ for all $\theta \in (p, \theta^*)$ and stays quiet with $\sigma(\theta) = 0$ for all $\theta < p$ and $\theta > \theta^*$.

A transparent (case 1) equilibrium exists if and only if $k \geq \bar{k}$.

Proof. Take any equilibrium with disclosure strategy $\sigma$. Types $\theta < p$ have a dominant strategy to stay quiet, so we focus on the strategies of types $\theta > p$, who strictly prefer to disclose if $\mathcal{N}(\theta) \equiv pG\left(\frac{s^* - \theta}{k}\right) - c > 0$, where $s^*$ is the cutoff signal defined in Lemma 1. Note that the net payoff $\mathcal{N}(\theta)$ from disclosure is non-increasing, and strictly decreasing whenever $G\left(\frac{s^* - \theta}{k}\right) \in (0, 1)$. In particular, $c < p$ implies that $\mathcal{N}(\theta)$ is strictly decreasing in the neighborhood of any point solving $\mathcal{N}(\theta) = 0$. We then have three possible cases. First, if $\mathcal{N}(\bar{\theta}) > 0$, then $\mathcal{N}(\theta) > 0$ for all $\theta > p$, so that all types above $\theta$ must disclose, which corresponds to case 1. Second, if $\mathcal{N}(p) \leq 0$, then $\mathcal{N}(\theta) < 0$ for all $\theta \in (p, \bar{\theta})$, so that nobody discloses, which corresponds to case 2. Third, if $\mathcal{N}(p) \geq 0 \geq \mathcal{N}(\bar{\theta})$, then there is unique root $\theta^* \in (p, \bar{\theta})$ such that $\mathcal{N}(\theta) > 0$ for $\theta < \theta^*$ and $\mathcal{N}(\theta) < 0$ for $\theta > \theta^*$. Therefore types $\theta \in (p, \theta^*)$ disclose, and types $\theta \in (\theta^*, \bar{\theta}]$ stay quiet, which corresponds to case 3.

For the second part of the proposition, suppose that $k < \bar{k}$, so that $pG\left(\frac{p+k-\bar{\theta}}{k}\right) - c < 0$. Noting that $s^* \leq p + k$ in any equilibrium, we have $\mathcal{N}(\bar{\theta}) < 0$, and therefore case 1 is impossible. Conversely, suppose that a case 1 equilibrium exists. Then $\mathcal{N}(\bar{\theta}) \geq 0$, and the critical signal $s^* = p + k$ (all lower signals are interpreted by Bayesian Receivers as coming from types $\theta < p$ with probability 1). It follows that $pG\left(\frac{p+k-\bar{\theta}}{k}\right) - c \geq 0$, implying $k \geq \bar{k}$. 

Lastly, we show that the highest disclosing type is bounded away from $\bar{\theta}$ when transparency is not an equilibrium, which motivates our analysis of discontinuity:

Proposition 7. There is a type $\tilde{\theta} < \bar{\theta}$ such that, whenever $k < \bar{k}$, we have $\sigma(\theta) = 0$ for all $\theta \geq \tilde{\theta}$ in any equilibrium.
Proof. Suppose \( k < \bar{k} \). Then all equilibria fall into case 2 or 3 in Proposition 6, that is, Senders disclose with probability 1 if \( \theta \in (p, \theta^*) \) for some threshold \( \theta^* \) (in case 2, where there is no disclosure, we formally write \( \theta^* = p \)). We must show that \( \theta^* \) is bounded away from \( \bar{\theta} \). For the remainder of the proof we use the best response function defined in the text. More precisely, define \( s^*(\theta^*) \) as the smallest public signal \( s \) satisfying \( E[\theta|\theta \notin (p, \theta^*), s] \geq p \). Define \( B(\theta^*) \) as the highest type satisfying \( pG\left(\frac{s^*(\theta^*) - \theta}{k}\right) - c \geq 0 \) (if this is impossible, let \( B(\theta^*) = p \)). It is easy to see that a cutoff \( \theta^* \) induces an equilibrium if and only if \( B(\theta^*) = \theta^* \).

We know that \( B(\theta) < \bar{\theta} \) since transparency is not an equilibrium. If \( B(\bar{\theta}) = p \) then \( \theta^* = p \) is the unique equilibrium and we are done. If \( \bar{\theta} > B(\theta^*) > p \) then by continuity, \( B(\theta) \) is interior in a neighborhood of \( \bar{\theta} \), in which case it solves

\[
B(\theta^*) = s^*(\theta^*) - G^{-1}\left(\frac{c}{p}\right),
\]

implying the first derivative

\[
B'(\theta^*) = \frac{ds^*(\theta^*)}{d\theta^*}.
\]

To complete the proof it is sufficient to show that \( \lim_{\theta^* \to \bar{\theta}} \frac{ds^*(\theta^*)}{d\theta^*} = \infty \). If this is the case, then the best response function is infinitely steep near \( \bar{\theta} \), and therefore there can be no fixed point solving (13) in a neighborhood of \( \bar{\theta} \). It is easy to see that \( \lim_{\theta^* \to \bar{\theta}} s^*(\theta^*) = p + k \), and Equation (2) implies that \( p + k < \bar{\theta} - k \). When \( \theta^* \) is close to \( \bar{\theta} \), the critical signal \( s^*(\theta^*) \) is interior and solves \( E[\theta|\theta \notin (p, \theta^*), s^*(\theta^*)] = p \). By Bayes’ law this is equivalent to

\[
\int_{[s^*(\theta^*) - k, p] \cup [\bar{\theta}, \theta^*]} (\theta - p)g\left(\frac{s^*(\theta^*) - \theta}{k}\right)f(\theta)d\theta = 0.
\]

Totally differentiating with respect to \( \theta^* \) we obtain

\[
\frac{ds^*(\theta^*)}{d\theta^*} = \frac{(\theta^* - p)g\left(\frac{s^*(\theta^*) - \theta^*}{k}\right)f(\theta^*)}{\int_{[s^*(\theta^*) - k, p] \cup [\bar{\theta}, \theta^*]} (\theta - p)g\left(\frac{s^*(\theta^*) - \theta}{k}\right)f(\theta)d\theta + (p + k - s^*(\theta^*))g(1)f(s^*(\theta^*) - k)}.
\]

As \( \theta^* \to \bar{\theta} \), the numerator converges to a finite positive number. The second term
in the denominator converges to zero since $s^*(\theta^*) \uparrow p + k$. The first term in the denominator converges to zero because $g'(.)$ is finite. Therefore $\frac{ds^*(\theta^*)}{d\theta^*} \uparrow \infty$, which completes the proof.