Title: Online Linear Programming: Dual Convergence, New Algorithms, and Regret Bounds

Job talk abstract:

In this talk, we discuss the problem of online linear programming (LP) in which the objective function and constraints are observed sequentially and not known a priori. Specifically, we consider a random input model where the columns of the constraint matrix along with the corresponding coefficients in the objective function are generated i.i.d. from an unknown distribution and revealed sequentially over time. First, we establish convergence properties on the dual optimal solution to a large-scale LP problem – this answers an open question on this topic and justifies the wide application of dual-based algorithms to the online LP problems. Next, we derive upper and lower bounds for the online LP problem. The bounds relate the regret of an online LP algorithm to several key quantities, including the estimation error of the dual optimal solution, the remaining constraints (resources/inventories) at the end of the horizon, and a stopping time associated with the constraint consumption process. Based on the preceding results, we develop a new nonstationary learning algorithm which improves the previous algorithm performances by taking into account the past input data as well as and decisions/actions already made. We derive an O(log n log logn) regret bound for this new algorithm, against the O(sqrt n) bound for typical dual price based learning algorithms. Finally, inspired by the findings on the online LP problem, we present a fast algorithm for approximately solving a class of large-scale binary integer LPs. The algorithm is free of matrix multiplication and requires only one single pass over the inputs of the integer LP.