The Risk of Risk-Sharing:
Diversification and Boom-Bust Cycles

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Abstract

I model a shock whereby financial intermediaries can better diversify borrowers' idiosyncratic risks. This generates a fragile sectoral boom: intermediaries reallocate funds to the shocked sector, raising sectoral investment, but they simultaneously increase leverage, exposing the economy to a crisis. This cycle is amplified if the diversification-shocked sector is higher-risk or faces looser credit standards. Among other financial shocks – relaxed borrowing constraints, lower capital requirements, higher risk tolerance, lower uncertainty, and foreign safe-asset demand – none generate both sectoral reallocation and financial leveraging. I apply the model quantitatively to the recent housing cycle. Feeding in a novel mortgage diversification index, the model generates the household credit increase measured in data and a broad bust. Without mortgage diversification, neither occurs.

JEL Codes: D14, G11, G12.

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1 Introduction

I argue boom-bust cycles are the price to pay for a more efficient financial sector. In a general equilibrium model with financial frictions, I study a diversification shock, which allows financiers to better diversify idiosyncratic risks and improves financial efficiency. In the short run, a diversification shock leads to lower risk premia and elevated investment: the boom. A series of low risk premia depletes financial sector capital, leading to higher long-run financial leverage that destabilizes the economy: the bust.

If financiers’ diversification technology disproportionately improves in one sector, that sector will experience an outsized boom, through a funding reallocation. In summary, reallocation and leverage are the key model mechanisms. Reallocation helps explain why many booms are sector-specific. Leverage helps explain why a sectoral boom may produce a broader bust. The financial sector intermediates funds everywhere, so financier fragility translates into macroeconomic fragility.

Leverage may be “excessive” for many reasons, among which I choose one appropriate for the model. I incorporate a fire-sale mechanism in which undercapitalized financiers liquidate their loan portfolios to less qualified investors. If a fire sale occurs, the real economy suffers. I demonstrate the conditions under which improvements to diversification increase the likelihood of a crisis, and when they do not. Even in a setting with occasionally-binding constraints and fire sales, the model has a highly tractable solution with which I can analyze the effects of diversification.

The recent US housing boom, followed by the Great Recession, is a good example of such a diversification-induced cycle. Through looser financial regulations and a growing market for securitized products, financial sector diversification improved in the 1990s and 2000s. For example, deregulations that allowed banks to operate across state borders improved loan portfolio diversification. Securitized loan portfolios were meant to be immune to idiosyncratic shocks, which represents an extreme version of portfolio diversification, to the extent these securities were held within the financial sector.\footnote{Financial sector should be interpreted broadly here. Acharya, Schnabl and Suarez (2013) show that ABCP conduits were sponsored by banks, who were thus the ultimate bearers of its risk; He, Khang and Krishnamurthy (2010) study the asset-backed security holdings of hedge funds, commercial banks, and investment banks, who all participated; Chernenko, Hanson and Sunderam (2016) study mutual fund holdings of securitized products.}

At the same time, the financial sector accumulated leverage, possibly in response to its improved efficiency. And because the deregulations and the securitization boom were disproportionately directed toward household finance, household credit climbed relative to non-financial private credit, representing a reallocation.\footnote{The claim that deregulations and securitization were geared primarily towards household finance is supported by the following facts. First, the results of Rice and Strahan (2010) and Favara and Imbs (2015) together provide causal evidence that bank branching deregulations in the late 1990s and early 2000s disproportionately affected mortgage finance, relative to firm finance. Second, securitization of mortgages grew much faster in this period than securitization of commercial loans. For example, the ratio of outstanding mortgage securities (primarily MBS) to corporate securities (primarily corporate bonds and CLOs) was approximately 1 in 1990 and reached 1.5 in 2006, see Appendix D.1.}

Figure 1 summarizes these facts.

In the model, the reallocation and leverage mechanisms generate other stylized features of the housing boom. The diversification shock lowers financier discount rates, so mortgage spreads fall during the boom.\footnote{See Justiniano, Primiceri and Tambalotti (2017) for evidence that mortgage spreads did fall during the boom.} Lower discount rates increase valuations, so the house price-rent ratio rises.
In a quantitative application, I apply the model to the recent US housing cycle and show that diversification-fueled booms can be powerful. In doing this, I also make an empirical contribution. I construct an index of idiosyncratic banking risk that I use in calibrating the model. This index has two advantages relative to the current literature. First, it is a comprehensive measure of deregulation, financial innovations, mergers, and fundamental risks which tend to have similar effects but cannot easily be compared. Second, it has interpretable risk-based units, which is relevant for calibrating economic models. This index, which corresponds closely to the diversification shock in the model, rose steadily from 1990 to 2006. Feeding this index into the model, I match the household credit share depicted in figure 1. The calibrated model also generates a substantial bust that is completely absent without increased in diversification technology.

Reallocation and leverage, driven by financial innovations and changes in regulation, seem to be more general features of credit booms, and not isolated to the recent US housing cycle. As Brunnermeier and Schnabel (2015) write in their narrative account of many bubbles,

The overwhelming share of bubbles was accompanied by a lending boom, which appears to be an almost universal feature of asset price bubbles. This expansion of credit was frequently related to financial innovation...Often, the bursting of bubbles leads to a redirection of capital flows, spurring new asset price booms in other regions...Finally, bubbles often occur during phases of financial deregulation.

Diversification-type shocks may have explanatory power elsewhere. For instance, in the 1880s, a
market developed for securitization of farm mortgages, coinciding with a lending boom and preceding a market collapse. In the 1920s, a market developed for securitization of commercial real estate loans, coinciding with a construction boom and preceding the Great Depression.\footnote{See Simkovic (2013) and Goetzmann and Newman (2010) for a discussion of these episodes.}

But not all financial shocks generate the same types of cycles, even qualitatively. I examine five other financial shocks in the model – a loan-to-value shock, a capital requirement shock, a risk-tolerance shock, an uncertainty shock, and a foreign savings shock. Among these, none generate both sectoral reallocation and financier leveraging. To the extent these phenomena are pervasive credit cycle characteristics, this provides further support for the relevance of the diversification shock. I stress that this analysis is conditional on my framework and does not reject the fruitfulness of these shocks in other modeling environments. But the forces I uncover when analyzing these financial shocks are forces I expect to be present in other model economies as well.

The first shock relaxes credit standards and allows borrowers to take higher loan-to-value (LTV) ratios. This \textit{LTV shock} generates a reallocation but no financier leverage. In the model, higher LTV creates a risk-transfer between undiversified insiders and diversified financiers, who are compensated for their increased risk-taking with higher lending spreads. Financier profitability rises with spreads, pushing down leverage and bust probabilities over time. This suggests a leverage-induced bust must be generated by a credit supply shock, rather than a credit demand shock.

The second shock reduces equity-retention frictions for financial intermediaries, as might a reduction in capital requirements. This \textit{capital requirement shock} reduces financiers' cost-of-capital, but it also applies to the entire intermediary balance sheet. As a result, there is no endogenous reallocation between sectors. This suggests that a boom which disproportionately affects one asset class, as in the 2000s housing boom, requires an asset-class-specific shock.

The third shock decreases risk aversions of investors, which may also stand in for exogenous changes to investor optimism. A \textit{financier risk-tolerance shock} resembles a capital requirement shock: it affects financier decisions in all markets, so no reallocation occurs. A \textit{borrower risk-tolerance shock} tends to raise lending spreads, since borrowers are willing to pay more for their asset purchases, which then buffers intermediary balance sheets. If all agents become more risk tolerant, there is no reason why financiers would disproportionately react, so financial fragility remains unchanged.

The fourth shock reduces idiosyncratic volatility in one of the sectors. This \textit{uncertainty shock} generates a sector-specific boom, as it is a directed shock. But lower uncertainty benefits both borrowers and financiers, so the effect on lending spreads and financial leverage are ambiguous. This suggests that a leverage-fueled boom-bust cycle requires a shock that disproportionately affects financial intermediaries' risk-reward tradeoff.

The fifth shock serves to increase demand in the riskless bond market, similar to a global savings glut. This \textit{foreign savings shock} generates a lower riskless interest rate, the rate at which financiers borrow. But like the capital requirement shock, lower interest rates affect all sectors symmetrically, so no reallocation occurs.
Related Literature

My paper studies a macroeconomy in which financial intermediation plays an important role allocating funds. In that economy, I analyze the effects of various financial shocks, with emphasis on diversification. In my quantitative analysis, I apply the framework to the recent US housing boom. The paper thus contributes to three broad literatures: (1) the literature on the effects of financial intermediation on the macroeconomy; (2) the literature on diversification and other financial shocks; (3) the literature on the recent housing cycle.

By focusing on the financial sector, my framework shares many features with the “financial accelerator” literature on macroeconomic dynamics with financial frictions. Net worth of borrowers/producers/financiers acts as a buffer to fundamental economic shocks in these models, building off of insights by Bernanke and Gertler (1989) and Kiyotaki and Moore (1997). Using continuous-time methods, He and Krishnamurthy (2013) and Brunnermeier and Sannikov (2014) extend these ideas to study crisis dynamics and other nonlinearities.

There are three differences between the structure of my model and the structure common to this literature. First, the key role of my financial intermediaries are to diversify idiosyncratic risks. Most papers in this literature study financial intermediaries who are more productive investors but in fact less diversified than other agents. Second, I include two sectors, in order to study financiers’ reallocation between them. Third, I introduce financial shocks, which substantively alters the model dynamics. In most financial accelerator papers, the role of the financial sector is to amplify more-or-less standard fundamental shocks.

Net-worth-centric financial accelerator models generate a bust only after a long sequence of negative fundamental shocks, because intermediaries are well-capitalized following a boom. In contrast, by combining financial shocks with a risk-averse financial sector, my model features an endogenous bust. After a diversification shock, banks increase risk-taking, competing away lending profits, thus running down balance sheet buffers and increasing the likelihood of a downturn.

This endogenous offsetting of lower risk by higher risk-taking is related to the “Peltzman effect” in automobile safety (Peltzman (1975)) or the “volatility paradox” in macro-finance (Brunnermeier and Sannikov (2014)). Demsetz and Strahan (1997) document this effect empirically for the case of larger bank holding companies, whose better diversification is offset by increased risk-taking. This offsetting is captured by the welfare analysis of my model. Although better risk-sharing improves

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5An important exception is Di Tella (2017), which shows that removing ad-hoc contracting frictions severs all amplification of fundamental TFP shocks. In that paper, uncertainty shocks are proposed as a driver of the financial accelerator. Some models not based on net worth are exceptions to this. For example, Boissay, Collard and Smets (2016), a model based on information asymmetries and adverse selection, generates a finance-centric boom-bust cycle. The model of Gorton and Ordoñez (2016) generates cycles based on the interaction between real productivity and financiers’ incentives to accept collateral. The model of Geanakoplos (2010) generates a leverage cycle based on heterogeneous beliefs.

6Also related: Wolf Wagner develops a series of theoretical models to illustrate downsides of financial diversification in Wagner (2008), Wagner (2010), and Wagner (2011). For example, the closest to my paper is Wagner (2008), in which the banking sector features a risk-taking externality. Individual banks do not take into account that a high-risk, low-liquidity portfolio choices increase other banks’ probability of inefficiently liquidating their portfolios. In my model, the analogous externality is a fire-sale externality: when financiers are undercapitalized, they begin selling assets to the market, which puts further downward pressure on asset prices and induces asset sales by other financiers. Better diversification improves risk-reward trade-offs, thereby worsening the externalities in both papers.
welfare by eliminating some idiosyncratic risks, increased risk-taking scales the importance of those risks in net worth volatility, reducing welfare.

Technically, my approach to modeling diversification is related to the model in Gârleanu, Panageas and Yu (2015). That paper uses a Brownian bridge on a circle of locations to model correlated shocks. Investors allocate funds along arcs of the circle, which prevents full diversification of idiosyncratic risks. I use the theory of Gaussian processes to develop a new stochastic process, which I call a Brownian cylinder, that maintains cross-sectional correlations on a circle but accommodates an infinite-horizon, continuous-time setting. This apparatus could be useful in other settings where continuous time methods have proved fruitful (e.g., optimal stopping problems, occasionally-binding portfolio constraints, heterogeneous-agent macro models).

Motivating the quantitative application of this model is an empirical literature arguing that credit supply increases drove the recent US housing boom. Favara and Imbs (2015) study the effect of credit supply on house prices, using bank branching deregulations of the late 1990s and early 2000s as a credit supply instrument. The deregulations plausibly allowed some banks to achieve better-diversified loan portfolios.8 There are many other papers in this literature.9 My paper argues that better mortgage diversification is an important credit supply shock driving the boom and bust.

A quantitative modeling literature examines the plausibility of various shocks in the housing boom and bust. For example, Justiniano, Primiceri and Tambalotti (2015a) find that a relaxation of lending constraints (credit supply shock) can generate a house price boom, whereas a relaxation of borrowing constraints (credit demand shock) cannot, because of their opposing effects on real interest rates.10 In contrast, my model features risk premia, so an LTV increase generates a housing boom, as financiers are better diversified than borrowers. The impact of an LTV increase in my model is higher mortgage spreads, counter to evidence in Justiniano, Primiceri and Tambalotti (2017) that non-conforming mortgage spreads declined by 2% during the 2000s. Diversification improvements generate a mortgage spread decline. Additionally, my focus on financial leverage provides discipline on the type of credit supply shock models should feature.

Finally, most of the extant literature incorporating financial shocks generates a bust only after applying a negative financial shock.11 My model differs in that a positive diversification shock endogenously generates higher likelihood of a bust, through increased financial fragility.

8The deregulations were created by the Riegle-Neal Interstate Banking and Branching Efficiency Act of 1994 (IBBEA). These deregulations led to asset-side diversification, deposit-side diversification, and bank competition.

9Adelino, Schoar and Severino (2012) use a regression discontinuity design to show that (conforming) mortgage securitization reduces lending rates and raises house prices. Mian and Sufi (2018) provide causal evidence that the abrupt increase in private-label MBS activity dramatically increased house prices and led to the bust. See also Mian and Sufi (2009), Mian and Sufi (2011), and Di Maggio and Kermani (2017). This credit supply view is not uncontroversial. For example, Haughwout, Lee, Tracy and Van der Klauuw (2011), Chincu and Mayer (2015), and Albanesi, De Giorgi and Nosal (2017) point to housing investors (who mortgage more than one property) as a key driver of the 2000s boom-bust cycle. Adelino, Schoar and Severino (2016) argue that mortgage credit increased proportionally to all income groups. These findings are at odds with the traditional rationing-based credit supply view.

10Also see Kiyotaki, Michaeldides and Nikolov (2011) and Justiniano, Primiceri and Tambalotti (2015b). Favilukis, Ludvigson and Van Nieuwerburgh (2017) and Greenwald (2016) show that relaxed borrowing constraints can generate a large boom, if we consider a careful calibration of a model inclusive of aggregate risk.

11For example, Guerrieri and Lorenzoni (2017) provides a thorough analysis of the effects of borrowing constraints tightening. Tightening constraints can generate a large bust and slow recovery, through a large deleveraging episode.
2 Two-Sector Model: Reallocation and Leverage

The model of this section is meant to introduce the primary mechanisms of my framework: reallocation and leverage. I introduce two sectors that produce with their own capital stocks. I will show that an increase in diversification of one sector’s risks leads to reallocation towards that sector and simultaneously to increased overall risk-taking by financial intermediaries.

2.1 Setup

Time is continuous \( t \geq 0 \). The model features two groups of agents: insiders and outside financiers. Insiders are additionally split into two groups, depending on which of two productive sectors they inhabit, \( A \) or \( B \). These insiders invest in capital, and consume goods from both sectors. To finance their capital purchases, insiders issue outside securities and put up some of their own net worth. These outside securities are held by financial intermediaries, which are operated by financiers. To finance their investment activities, financiers use their own net worth as well as risk-free debt. To start, financiers cannot directly manage productive capital. Agents in each group are indexed by \( i \in [0, 1] \), which will represent an agent’s location, to be described below. Figure 2 summarizes the model, with the flow of funds between insiders and financiers.

![Figure 2: Typical Flow of Funds. Arrows show the direction of investment.](image)

Preferences

All agents are infinitely-lived and have logarithmic utility over a Cobb-Douglas aggregate of the two sectors’ consumption goods, i.e., \( c := a^{1-\beta}b^\beta \). Mathematically,

\[
U_t := E_t \left[ \int_t^\infty pe^{-\rho(s-t)} \log(c_s)ds \right], \quad \rho > 0.
\]

(1)

Cobb-Douglas implies the two consumption goods have expenditure shares of \( 1-\beta, \beta \). I assume the composite good \( c \) is the numeraire. Let the relative prices of \( a \) and \( b \) be \( p_A \) and \( p_B \).
Locations and Idiosyncratic Risk

Agents are arranged on a circle, which has locations indexed by \( i \in [0, 1] \). Locations will be special because they feature different idiosyncratic shocks.

These shocks directly hit the evolution of productive capital. Mathematically, capital held by an insider at location \( i \) evolves dynamically as

\[
dk_{i,t}^A = k_{i,t}^A[\iota_{i,t}^A dt + \sigma_A \cdot dZ_t + \hat{\sigma}_A dW_{i,t}^A], \\
dk_{i,t}^B = k_{i,t}^B[\iota_{i,t}^B dt + \sigma_B \cdot dZ_t + \hat{\sigma}_B dW_{i,t}^B].
\]

In (2)-(3), \( \iota^A, \iota^B \) are the desired investment rates, \( Z := (Z^A, Z^B) \) is a standard Brownian motion (aggregate shock), and \( W^A, W^B \) are idiosyncratic shocks (more on these stochastic processes below). For simplicity, I assume \( \sigma_A \cdot \sigma_B = 0 \), i.e., orthogonal aggregate shocks. These “capital-quality shocks” are a simple way to capture permanent productivity or depreciation shocks, without introducing additional state variables.

For two reasons, I assume no investment adjustment costs, as in Cox, Ingersoll and Ross (1985). First, my focus is on incomplete financial markets rather than investment frictions. A minimal number of frictions affords maximum theoretical clarity, as my results on boom-bust cycles must be attributed to the remaining frictions. Second, zero adjustment costs allows me to obtain analytical solutions to the equilibrium of this economy.

I assume the idiosyncratic shocks \( W_{i,t}^A \) and \( W_{i,t}^B \) are independent copies of a stochastic process with the following properties.

**Assumption 1 (Shock Structure).** Assume the following for \( W := \{W_{i,t} : i \in [0, 1], t \geq 0\} \).

(i) At each location \( i \in [0, 1] \), \( W_{i,t} \) is a standard Brownian motion, independent of \( Z_t \).

(ii) For any two locations \( i, j \in [0, 1] \), the shock correlation is

\[
\text{corr}(dW_{i,t}, dW_{j,t}) = 1 - 6\text{dist}(i, j)(1 - \text{dist}(i, j)),
\]

where \( \text{dist}(i, j) := \min(|i - j|, 1 - |i - j|) \) is a distance metric on the circle of circumference 1.

(iii) \( W_{i,t} \) is continuous in \((i, t)\) almost-surely, under the Euclidean distance metric on the cylinder \( \text{dist}((i, s), (j, t)) := [t - s]^2 + \text{dist}(i, j)^2]^{1/2} \).

Given part (i) of Assumption 1, \( dW_{i,t} \) is iid over time, for fixed location \( i \). Part (ii) of Assumption 1 means that the shock correlations between locations decrease with their distance from one another.\(^{13} \)

\(^{12}\)In contrast, models like Brunnermeier and Sannikov (2014) rely on capital adjustment costs to generate real effects of financial frictions. Indeed, in their model, the probability of capital misallocation vanishes as adjustment costs shrink to zero. Intuitively, the most-productive users can avoid selling capital at a discount if they can disinvest costlessly.

\(^{13}\)This presumption on the shock correlation owes to Gârleanu et al. (2015). Using the Brownian bridge on a “circle”, they construct discrete-time idiosyncratic shocks that are cross-sectionally correlated but contain zero aggregate risk. In doing so, they find that the dividend correlation is exactly \( 1 - 6\text{dist}(i, j)(1 - \text{dist}(i, j)) \). The proof of Lemma 2.1 would apply for any appropriate correlation function \( \nu(i, j) \) that depends only on \( \text{dist}(i, j) \) (i.e., stationary correlation function).
Nearby locations have nearly perfect shock correlation. Two locations which are “far away” from
one another (e.g., \( i = 1/4 \) and \( j = 3/4 \)) will have a large negative correlation. A key question is
whether any such stochastic process exists.

**Lemma 2.1.** A stochastic process \( W := \{W_{i,t} : i \in [0, 1], t \geq 0\} \) exists which satisfies Assumption 1.

The preceding lemma, proved in Appendix B, establishes existence of \( W \). The key step is proving
\( W \) can be constructed as a Gaussian process with the appropriate covariance function, which needs
to be symmetric and positive semi-definite. Because \( W \) evolves on a circle over time, which looks
like a cylinder, I will call it the *Brownian cylinder*.

With the properties in Assumption 1, we can establish some distributional properties of the Browni-
nian cylinder, in particular that it contributes no aggregate risk. These are stated below in Lemma
2.2. The proof of Lemma 2.2, as well as all other results concerning the stochastic process \( W \), are in
Appendix B.

**Lemma 2.2.** Under Assumption 1, there is no aggregate risk, i.e., \( \int_{0}^{1} (dW_{i,t})di = 0 \) almost-surely. More
generally, the local variance of a unit investment divided amongst the shocks along an arc of length \( \Delta \)
is equal to \( (1 - \Delta)^2 \), i.e., \(^{14}\)

\[
\text{Var}_i \left( \int_{i}^{i+\Delta} \Delta^{-1} dW_{j,t}dj \right) = (1 - \Delta)^2 dt.
\]

Consequently, the process \( W_{i,t}^\Delta := (1 - \Delta)^{-1} \Delta^{-1} \int_{i}^{i+\Delta} W_{j,t}dj \) is a standard Brownian motion.

Given Lemma 2.2, the shock \( dW_{i,t} \) is correlated across locations but washes out in the aggregate,
the sense in which it is idiosyncratic. The surprising part of this result is that we only needed to
specify the covariance structure of the shocks, and this property alone allows us to pin down the
integral of all the shocks.

Figure 3 plots one simulation of the Brownian cylinder for \( t \in [0, 1] \) at 500 evenly spaced locations
\( i \). See Appendix B for details on how to simulate \( W \). Each cross-section of the cylinder represents
the circle of locations. To represent the shocks, the cylinder is shaded according to the size of
\( W_{i,t}/\sqrt{t} \).

**Asset Markets**

I assume sectoral capital is homogeneous and can be traded frictionlessly across locations, which
implies the location-invariant unit prices \( q_{A,t} \) and \( q_{B,t} \). With zero investment adjustment costs, we
will have \( q_{A,t} \equiv q_{B,t} \equiv 1 \) in equilibrium.

There is also a zero-net-supply futures market for trading claims directly on aggregate risk. Investing
one unit of net worth in this claim earns the excess return \( \pi_t dt + dZ_t \), where \( \pi_t \) is the
market price of risk associated with the \( dZ_t \) shock. These futures contracts are continuously settled.
Finally, there is a zero-net-supply riskless bond market that returns \( r_t dt \). All agents can access both
the futures and riskless bond markets frictionlessly.

\(^{14}\)I always take the notational convention that “\( i + \Delta \)” represents \( i + \Delta - |i + \Delta| \) when indexing a position on the circle.
Risk of Risk-Sharing

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Return-on-Capital

A firm is just a collection of capital, which produces according to an “AK” technology. The representative insiders at location $i$ produce $G_A k_{i,t}^A$ and $G_B k_{i,t}^B$. As a result of this and the absence of adjustment costs, the return-on-capital is given by

$$dR_{i,t}^A = p_{A,t} G_A dt + \sigma_A \cdot dZ_t + \tilde{\sigma}_A dW_{i,t}^A$$

$$dR_{i,t}^B = p_{B,t} G_B dt + \sigma_B \cdot dZ_t + \tilde{\sigma}_B dW_{i,t}^B.$$  \(4\)

$$5\)

Thus, capital returns have a location-invariant distribution and time-invariant risk.

Insider Problem

Because of the symmetry of the two sectors and their insiders, I describe the problem of an insider in generic sector $z \in \{A, B\}$. On the asset side, insiders hold capital that returns (4)-(5). They are also marginal in the risk-free debt market, at the interest rate $r_t$. On the liability side, insiders can borrow against their capital from financial intermediaries, by signing a contract promising the return of

$$d\tilde{R}_{i,t}^z := (r_t + s_{i,t}^z) dt + (dR_{i,t}^z - \mathbb{E}_t[dR_{i,t}^z]), \quad z \in \{A, B\}.$$  \(10\)

This is a way for insiders to shed some of the idiosyncratic risk associated with production. The “spread” charged by financial intermediaries is given by $s_{i,t}^z$.

I assume that insiders borrow a fixed fraction $\phi_z$ of the value of their enterprise from intermediaries in the form of outside equity. With the fixed fraction, insiders pay $\phi_z k_{i,t}^z d\tilde{R}_{i,t}^z$ to financiers. In Appendix C.1, such a risk-sharing arrangement is micro-founded from a standard moral hazard problem.\(^{15}\) I explore alternative risk-sharing arrangements resembling debt in Appendix C.4.

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\(^{15}\)Under that interpretation, the restriction that insiders keep $1 - \phi_z$ fraction of capital risk on their balance sheets is called a “skin-in-the-game” constraint. Given sufficient skin-in-the-game, the exact composition of outside contracts is irrelevant. Indeed, once moral hazard problems are resolved between insiders and outsiders, the outside securities issued
Combining the assumptions above, insider net worth $n^z_{i,t}$ evolves as

$$
 dn^z_{i,t} = (n^z_{i,t} r_t - c^z_{i,t}) dt + k^z_{i,t} (dR^z_{i,t} - r_t dt) - \phi^z_{i,t} (d\hat{R}^z_{i,t} - r_t dt)
$$

$$
 + n^z_{i,t} \theta^z_{i,t} \cdot (\pi_t dt + dZ_t), \quad z \in \{A, B\}. 
$$

Given the ability to frictionlessly trade aggregate risk but not the idiosyncratic risk of capital, one can think of the differential $k^z_{i,t} \mathbb{E}_t [dR^z_{i,t} - r_t dt - \phi^z_{i,t} (d\hat{R}^z_{i,t} - r_t dt)]$ as a compensation for idiosyncratic risk. Mathematically, households solve

$$
 \max_{n^z_{i,t}, c^z_{i,t}, k^z_{i,t}, \theta^z_{i,t}} U^z_{i,t}, \quad z \in \{A, B\},
$$

subject to (6), $n^z_{i,t} \geq 0$, $k^z_{i,t} \geq 0$, where $U^z_{i,t}$ is given by the logarithmic utility function (1).

**Financier Problem**

Financiers manage intermediaries, which serve a diversification and safe-asset-creation role. On the asset side, intermediaries hold a partially diversified portfolio of equity in each of the two sectors. On the liabilities side, intermediaries are financed by riskless debt and the financier’s internal equity.

I model diversification as follows. Financiers are tied to locations/intermediaries, just as insiders are. An intermediary located at $i \in [0, 1]$ invests in a portfolio of insiders’ securities located “nearby” by insiders are indeterminate due to Modigliani-Miller holding on these securities. In particular, there are no taxes, costs of default, incomplete financial markets, or any other frictions that would violate MM, after agency problems are resolved. Therefore, the equity-like contract is without loss of generality under this interpretation.

For example, we may think of changing the degree of risk in the outside contract by setting

$$
 d\hat{R}^z_{i,t} := (r_t + s^z_{i,t}) dt + \zeta (dR^z_{i,t} - \mathbb{E}_t [dR^z_{i,t}]),
$$

for $\zeta \leq 1$. The parameter $\zeta$ might capture the fact that insiders empirically borrow in debt which is less risky than the underlying asset. If $\zeta = 1$, the contract is equity. As $\zeta \to 0$, the contract approaches riskless debt. However, the parameter $\zeta$ is irrelevant in the following sense. One can verify that $\phi^z_{i,t} \zeta$ enter all formulas multiplicatively, and so enters all equilibrium expressions multiplicatively. What matters is that $\phi^z_{i,t}$ of risk is sold off to outsiders.
in the sense that they lie in a connected interval adjacent to location \( i \). Define \( \Delta_z \in [0, 1] \) to be the length of this interval for insiders in sector \( z \in \{A, B\} \). Insiders financed by intermediary \( i \) are those with \( j \in [i, i + \Delta_z] \mod [0, 1] \). Here, the \( \Delta_z \) are exogenously fixed numbers, not choices by financiers. I explore endogenous \( \Delta_z \) choices in Appendix C.3. This partial but imperfect diversification arc on the circle may be visualized in figure 4.

For simplicity, I assume intermediaries finance all insiders their investment arc symmetrically. In other words, the intermediary in location \( i \) supplies \( \lambda^z_{i,t} \Delta^{-1} n^F_{i,t} \) of funds to sector-\( z \) insiders in each location it lends to, rather than allowing \( \lambda^z_{i,t} \) to also vary by destination.\(^{16}\)

Putting everything together, the financier’s net worth evolves dynamically as follows:

\[
dn^F_{i,t} = \left( n^F_{i,t} r_t - c^F_{i,t} \right) dt + \lambda^A_{i,t} \Delta^{-1} \int_i^{i+\Delta_A} (d\tilde{R}^A_{j,t} - r_t dt) dj + \lambda^B_{i,t} \Delta^{-1} \int_i^{i+\Delta_B} (d\tilde{R}^B_{j,t} - r_t dt) dj \\
+ n^F_{i,t} \theta^F_{i,t} \cdot (\sigma_t dt + dZ_t).
\]  

Financiers solve

\[
\max n^F_{i,t}, c^F_{i,t}, \lambda^A_{i,t}, \lambda^B_{i,t}, \theta^F_{i,t} \mathcal{U}^F_{i,t}
\]

subject to (8), \( n^F_{i,t} \geq 0, \lambda^A_{i,t} \geq 0, \lambda^B_{i,t} \geq 0 \), where \( \mathcal{U}^F_{i,t} \) is given by the logarithmic utility function (1).

**Free Mobility**

At this point, I make an important technical assumption that keep the equilibrium construction tractable. Specifically, I assume a free-mobility condition between locations, which allows us to study a symmetric equilibrium.

**Assumption 2 (Free Mobility).** *Insiders and financiers are freely mobile among locations \( i \).*

Under Assumption 2, idiosyncratic shocks will wash out in aggregate, but the expectation that they will hit matters for individual behavior. A similar free-mobility assumption has been used across the idiosyncratic “islands” of Gertler and Kiyotaki (2011). For details on an equilibrium of a similar model without Assumption 2, see Khorrami (2018). In that setting, a symmetric equilibrium is not possible. Indeed, the entire distribution of net worth across locations becomes a state variable, and prices are location-dependent.\(^{16}\)

\(^{16}\)Relaxing this does not change the results significantly. With the maintained symmetry assumptions, \( \{\lambda_{i \to j,t}\} \) could be chosen in two stages (if spreads \( s_{i,t} \) are independent of location \( j \)). First, leverage \( \lambda_{i,t} := \Delta^{-1} \int_i^{i+\Delta} \lambda_{i \to j,t} dj \) could be chosen to trade off return and risk, as in the main text. Second, risky share allocations \( \lambda_{i \to j,t}/\lambda_{i,t} \) could be chosen to minimize the portfolio variance of a unit investment. One can verify that the resulting portfolio is exactly a symmetrically funded portfolio.
2.2 Equilibrium

Definition 1. An equilibrium consists of price and allocation processes, adapted to the aggregate and idiosyncratic shocks \( \{(Z^A_t, Z^B_t, W^A_{i,t}, W^B_{i,t}) : i \in [0,1], t \geq 0 \} \), such that all agents solve their optimization problems and all markets clear. Prices consist of the interest rate \( r_t \), aggregate risk price \( \pi_t \), spreads \( s^A_{i,t}, s^B_{i,t} \), and goods prices \( p^A_{i,t}, p^B_{i,t} \). Allocations consist of capital and equity holdings \( (k^A_{i,t}, k^B_{i,t}, \lambda^A_{i,t}, \lambda^B_{i,t}) \), consumption choices \( (a^A_{i,t}, a^B_{i,t}, c^F_{i,t}) \), and aggregate risk hedging choices \( (\theta^A_{i,t}, \theta^B_{i,t}, \theta^F_{i,t}) \). A symmetric equilibrium is an equilibrium in which all objects are independent of \( i \) for each \( t \). The market clearing conditions at every point in time are as follows.

- **Goods markets:**
  \[
  \int_0^1 G^A k^A_{i,t} \, di = \int_0^1 [a^A_{i,t} + c^B_{i,t} + c^F_{i,t}] \, di \\
  \int_0^1 G^B k^B_{i,t} \, di = \int_0^1 [b^A_{i,t} + b^B_{i,t} + b^F_{i,t}] \, di \\
  \int_0^1 (G^A k^A_{i,t})^{1-\beta} (G^B k^B_{i,t})^{\beta} \, di = \int_0^1 [c^A_{i,t} + c^B_{i,t} + c^F_{i,t}] \, di + \int_0^1 (\theta^A_{i,t} k^A_{i,t} + \theta^B_{i,t} k^B_{i,t}) \, di
  \]

- **Funding markets:**
  \[
  \int_{i-\Delta z}^i \Delta z^{-1} \lambda^z_{j,t} n^F_{j,t} \, dj = \phi_z k^z_{i,t}, \quad \forall i \in [0,1], \quad z \in \{A, B\}
  \]

- **Aggregate risk market:**
  \[
  \int_0^1 [\theta^A_{i,t} n^A_{i,t} + \theta^B_{i,t} n^B_{i,t} + \theta^F_{i,t} n^F_{i,t}] \, di = 0.
  \]

- **Bond market:**
  \[
  \int_0^1 [n^A_{i,t} + n^B_{i,t} + n^F_{i,t}] \, di = \int_0^1 [k^A_{i,t} + k^B_{i,t}] \, di.
  \]

First, we have a lemma which shows that, under additional restrictions, any equilibrium will be “location-invariant” in a certain sense.

**Lemma 2.3 (Location-Invariance).** Let Assumptions 1 and 2 hold. If an equilibrium is such that

\[
\frac{n^A_{i,t}}{n^A_{i,t} + n^B_{i,t}} \quad \text{and} \quad \frac{n^F_{i,t}}{n^A_{i,t} + n^B_{i,t} + n^F_{i,t}}
\]

are independent of \( i \), then that equilibrium must be location-invariant in the sense that \( k^A_{i,t} / k^B_{i,t}, k^A_{i,t} / n^A_{i,t}, k^B_{i,t} / n^B_{i,t}, \lambda^A_{i,t}, \lambda^B_{i,t}, s^A_{i,t}, \) and \( s^B_{i,t} \) are independent of \( i \). Furthermore, a symmetric equilibrium is feasible.

Among such location-invariant equilibria, I analyze the special one in which locations are exactly identical in their net worths, which is feasible (and weakly optimal) under free-mobility. Studying
this equilibrium allows me to avoid keeping track of the full distribution of wealth among locations, which would otherwise be necessary to know the evolution of aggregates like wealth.

For construction of the symmetric equilibrium, define aggregate capital \( K_t := \int_0^1 k_{i,t}^A + k_{i,t}^B \) and the capital distribution \( \kappa_t := K_t^{-1} \int_0^1 k_{i,t}^A di \). Define the wealth shares

\[
\alpha_t := \frac{N_{A,t}}{N_{A,t} + N_{B,t}} \quad \text{and} \quad \eta_t := \frac{N_{F,t}}{N_{F,t} + N_{H,t}},
\]

where \( N_{A,t} := \int_0^1 n_{i,t}^A di \), \( N_{B,t} := \int_0^1 n_{i,t}^B di \), and \( N_{F,t} := \int_0^1 n_{i,t}^F di \) are aggregate net worths. The only state variables in a symmetric equilibrium will be \((\alpha_t, \eta_t, K_t)\). Therefore, in what follows, I drop location \( i \) subscripts from all variables, whenever the meaning is clear. All stationary variables will be solely functions of \((\alpha_t, \eta_t)\), while growing variables will form a stochastic trend around \( K_t \). The state dynamics are

\[
d\alpha_t = \mu^\alpha_t dt + \sigma^\alpha_t dZ_t \\
d\eta_t = \mu^n_t dt + \sigma^n_t dZ_t \\
dK_t = K_t[t_t dt + \sigma_t dZ_t],
\]

where the aggregate investment rate is given by \( \iota_t \) and the aggregate diffusion vector is given by \( \sigma_t := \kappa_t \sigma_A + (1 - \kappa_t) \sigma_B \). The following proposition characterizes the equilibrium up to the solution of a single nonlinear equation.

**Proposition 2.4 (Two-Sector Equilibrium).** Let Assumptions 1 and 2 hold. Then, there exists a unique symmetric equilibrium with state variables \((\alpha, \eta)\). The equilibrium is non-stochastic in the sense that \( \sigma^\alpha \equiv \sigma^n \equiv 0 \). The state drifts are

\[
\mu^\alpha_t = \alpha_t(1 - \alpha_t)[\bar{\pi}^2_{A,t} - \pi^2_{B,t}] \\
\mu^n_t = \eta_t(1 - \eta_t)[\bar{\pi}^2_{F \rightarrow A,t} + \bar{\pi}^2_{F \rightarrow B,t} - \alpha_t \bar{\pi}^2_{A,t} - (1 - \alpha_t) \bar{\pi}^2_{B,t}],
\]

where

\[
\bar{\pi}_{A,t} := \frac{\kappa_t(1 - \phi_A) \hat{\sigma}_A}{\alpha_t(1 - \eta_t)} \quad \text{and} \quad \bar{\pi}_{B,t} := \frac{(1 - \kappa_t)(1 - \phi_B) \hat{\sigma}_B}{(1 - \alpha_t)(1 - \eta_t)}
\]

\[
\bar{\pi}_{F \rightarrow A,t} := \frac{\kappa_t \phi_A (1 - \Delta_A) \hat{\sigma}_A}{\eta_t} \quad \text{and} \quad \bar{\pi}_{F \rightarrow B,t} := \frac{(1 - \kappa_t) \phi_B (1 - \Delta_B) \hat{\sigma}_B}{\eta_t}
\]

are shadow idiosyncratic risk prices. The aggregate risk price vector is \( \pi = \kappa \sigma_A + (1 - \kappa) \sigma_B \). The capital distribution \( \kappa_t = \kappa(\alpha_t, \eta_t) \) is the unique solution to the following nonlinear equation:

\[
\rho \left[ \frac{1 - \beta}{\kappa} - \frac{\beta}{1 - \kappa} \right] - \left( \kappa \| \sigma_A \|^2 - (1 - \kappa) \| \sigma_B \|^2 \right) = \left[ (1 - \phi_A) \hat{\pi}_A + \phi_A (1 - \Delta_A) \bar{\pi}_{F \rightarrow A} \right] \hat{\sigma}_A - \left[ (1 - \phi_B) \hat{\pi}_B + \phi_B (1 - \Delta_B) \bar{\pi}_{F \rightarrow B} \right] \hat{\sigma}_B.
\]
Finally, the growth rate \( i \) and interest rate \( r \) are given by

\[
\begin{align*}
i &= (G_A \kappa)^{1-\beta} (G_B (1 - \kappa))^\beta - \rho \\
r &= \rho + i - \|\pi\|^2 - (1 - \eta)[\alpha \hat{\pi}_A^2 + (1 - \alpha) \hat{\pi}_B^2] - \eta[\hat{\pi}_{F\rightarrow A}^2 + \hat{\pi}_{F\rightarrow B}^2].
\end{align*}
\]

(15) (16)

I should make a few preliminary comments on the equilibrium in Proposition 2.4. The expected excess return on each capital stock can be decomposed into the aggregate risk premium, plus idiosyncratic risk premia earned by insiders and financiers. These idiosyncratic risk premia are non-trivial due to imperfect diversification by both insiders (who must hold cratic risk premia earned by insiders and financiers. These idiosyncratic risk premia are non-trivial due to imperfect diversification by both insiders (who must hold
discount rates, rather than just insiders’. Indeed,

\[
p_z \Delta_z - r - \sigma_z \cdot \pi = \begin{cases} (1 - \phi_z) \eta_z \hat{\pi}_z & \text{insiders’ idio risk premium} \\ \phi_z (1 - \Delta_z) \eta_z \hat{\pi}_{F\rightarrow z,t} & \text{financiers’ idio risk premium} \end{cases}, \quad z \in \{A, B\}.
\]

(17)

For example, \( \sigma_z \cdot \pi \) is the aggregate risk premium in sector \( z \), as the product of the quantity of risk (loading on \( dZ_t \)) and the aggregate risk price \( \pi \). Similarly, the latter two terms represent idiosyncratic risk premia: \( (1 - \phi_z) \eta_z \) and \( \phi_z (1 - \Delta_z) \eta_z \) represent the quantity of idiosyncratic risk held by insiders and financiers, respectively. In short, each capital stock is priced by a weighted-average of insider and financier discount rates, rather than just insiders’.

In addition, the non-stochastic nature of the economy is due to the combination of identical risk preferences and complete markets over aggregate risk. In particular, agents may frictionlessly pick their level of exposure to \( dZ_t \), given the existence of a hedging securities market (i.e., \( \theta_{i,t}^A, \theta_{i,t}^B, \) and \( \theta_{i,t}^F \) are unconstrained). I will relax this below.

Finally, since the state variables are deterministic, a reasonable conjecture is that the system eventually reaches a “steady-state” as \( t \to \infty \). This is the subject of the following proposition.

Proposition 2.5 (Steady-State). In the equilibrium of Proposition 2.4, wealth shares \( \alpha_t \to \alpha_\infty \) and \( \eta_t \to \eta_\infty \) asymptotically, where

\[
\alpha_\infty := \kappa_\infty (1 - \phi_A) \hat{\sigma}_A /
\kappa_\infty (1 - \phi_A) \hat{\sigma}_A + (1 - \kappa_\infty) (1 - \phi_B) \hat{\sigma}_B
\]

(18)

\[
\eta_\infty := \sqrt{(\kappa_\infty \phi_A (1 - \Delta_A) \hat{\sigma}_A)^2 + ((1 - \kappa_\infty) \phi_B (1 - \Delta_B) \hat{\sigma}_B)^2 /
\sqrt{(\kappa_\infty \phi_A (1 - \Delta_A) \hat{\sigma}_A)^2 + ((1 - \kappa_\infty) \phi_B (1 - \Delta_B) \hat{\sigma}_B)^2 + \kappa_\infty (1 - \phi_A) \hat{\sigma}_A + (1 - \kappa_\infty) (1 - \phi_B) \hat{\sigma}_B
\}

(19)

The capital distribution \( \kappa_t \to \kappa_\infty \), where \( \kappa_\infty \) is the unique member of

\[
\rho \left[ \frac{1 - \beta}{\kappa_\infty} - \frac{\beta}{1 - \kappa_\infty} \right] - \kappa_\infty \|\sigma_A\|^2 - (1 - \kappa_\infty) \|\sigma_B\|^2
\]

\[
= \left[ (1 - \phi_A) \hat{\pi}_{A,\infty} + \phi_A (1 - \Delta_A) \hat{\pi}_{F\rightarrow A,\infty} \right] \hat{\sigma}_A - \left[ (1 - \phi_B) \hat{\pi}_{B,\infty} + \phi_B (1 - \Delta_B) \hat{\pi}_{F\rightarrow B,\infty} \right] \hat{\sigma}_B.
\]

The equilibrium from Propositions 2.4 and 2.5 can be conveyed graphically. The left panel of
figure 5 plots the supply and demand in sector A’s lending market, with the idiosyncratic risk price $\hat{\pi}_{F \rightarrow A}$ against the lending portfolio $\lambda^A$.

\[ \lambda^A = \frac{\hat{\pi}_{F \rightarrow A}}{(1 - \Delta_A)\bar{\sigma}_A} \]

\[ \chi^A = \frac{\phi_A(\hat{\pi}_{F \rightarrow A})}{\eta} \]

\[ \mu^\eta = \eta(1 - \eta)[\sigma^2_{F \rightarrow A} + \sigma^2_{F \rightarrow B} - \kappa \sigma^2_A - (1 - \kappa)\sigma^2_B] \]

\[ \mu^\eta = 0 \]

Figure 5: Steady-state equilibrium.

The increasing line is loan supply: financiers’ optimal portfolio $\lambda^A$ is simply a mean-variance portfolio trading off idiosyncratic risk compensation, $\hat{\pi}_{F \rightarrow A}$, against the idiosyncratic volatility of the portfolio, $(1 - \Delta_A)\bar{\sigma}_A$. This investment can be chosen using only idiosyncratic risk considerations because of the frictionless market for trading aggregate risk. The other curve plots loan market clearing and can be thought of as loan demand. This demand curve is downward-sloping because the capital share $\kappa$ is declining in financiers’ required idiosyncratic risk price $\hat{\pi}_{F \rightarrow A}$. This can be seen in equation (14), which converges to (20) in steady-state.

The right panel shows the dynamics of $\eta$. The drift $\mu^\eta$ is typically strictly decreasing in $\eta$, since $\hat{\pi}_{F \rightarrow A}$ and $\hat{\pi}_{F \rightarrow B}$ are decreasing in $\eta$, while $\hat{\pi}_A$ and $\hat{\pi}_B$ are increasing in $\eta$. This is also why the economy converges to the steady-state, defined by $\mu^\eta = 0$, from any starting point.

2.3 Long-Run Effects of Better Diversification

In this section, I illustrate the reallocation and leverage effects discussed in the introduction. Suppose diversification improves in sector $A$, i.e., $\Delta_A \uparrow$. Figure 6 illustrates the adjustment of the economy to the new steady-state.

First, better diversification increases the loan supply because it improves financiers’ risk-reward trade-off. Greater competition for taking this risk reduces risk compensation $\hat{\pi}_{F \rightarrow A}$. This effect is captured by the outward rotation of the supply curve (left panel), which results in a shift from the diamond to the hollow circle. Note that lower risk compensation reduces financier profitability, so the drift $\mu^\eta$ shifts downwards (right panel).

Second, lower financier profitability reduces long-run $\eta$, meaning financiers must take more loan risk per unit of their net worth in equilibrium. They are happy to end up with a lower wealth share, since a lower quantity of idiosyncratic risk necessitates a lower precautionary savings buffer. This effect is captured by the outward shift in the demand curve, which results in a shift from the hollow
circle to the solid circle.\textsuperscript{17}

The result of these two forces are the \textit{reallocoation} and \textit{leverage} effects. Sectoral reallocation occurs for a simple reason: better diversification lowers the idiosyncratic risk premia in sector $A$, which raises incentives for capital to flow there. At the same time, lower financier idiosyncratic risk premia translates into lower lending spreads and thus lower lender profitability. In the long run, this reduces financiers’ relative wealth, so financiers must accumulate leverage to continue their scale of operations. Indeed, financier leverage (assets/equity) is

$$\text{leverage} := \lambda^A + \lambda^B = \frac{\phi_A \kappa + \phi_B (1 - \kappa)}{\eta},$$

so declines in $\eta$ through better diversification tend to increase leverage. The following result, a corollary of the steady-state equilibrium equations, formalizes the preceding discussion.

\textbf{Proposition 2.6 (Reallocation and Leverage). In the steady-state equilibrium of Proposition 2.5, the following comparative statics hold:}

\begin{enumerate}
\item[(i)] If $\Delta_B = 1$, then
\[ \frac{d\kappa_\infty}{d\Delta} > 0 \quad \text{and} \quad \frac{d\eta_\infty}{d\Delta} < 0. \]
\item[(ii)] If $\phi_A = \phi_B$, $\hat{\sigma}_A = \hat{\sigma}_B$, $\|\sigma_A\| = \|\sigma_B\|$, $\beta = 1 - \beta$, and $\Delta_A = \Delta_B := \Delta$, then
\[ \frac{d\kappa_\infty}{d\Delta} = 0 \quad \text{and} \quad \frac{d\eta_\infty}{d\Delta} < 0. \]
\item[(iii)] If $\phi_A = \phi_B = 1$, then
\[ \frac{d\kappa_\infty}{d\Delta} > 0 \quad \text{and} \quad \frac{d\eta_\infty}{d\Delta} = 0. \]
\end{enumerate}

\textsuperscript{17}Note that there is a small outward shift in the supply on impact because equation (14) shows that $\kappa$ is increasing in $\Delta_A$, independently of $\hat{\pi}_{F \rightarrow A}$. One can show that this is second order relative to the shift in the supply curve, i.e., $\hat{\pi}_{F \rightarrow A}$ still falls on impact, holding $\eta$ fixed.
Part (i) of Proposition 2.6 demonstrates a case in which both reallocation and leverage effects are in play. The assumption $\Delta_B = 1$ is made to derive unambiguous comparative statics in this case. Parts (ii) and (iii) demonstrate necessary conditions for the reallocation and leverage effects. Part (ii) shows that some asymmetric diversification increase is necessary for the reallocation channel. With completely symmetric sectors, sectoral allocations are immune to a broad increase in diversification. Part (iii) shows that some segmented markets (through borrowing limits) are necessary for the leverage channel. Without segmented markets between insiders and financiers, the insiders of both sectors have zero long-run wealth, meaning there can be no financial sector leverage.\(^{18}\)

### 2.4 Applications of Diversification Shocks

In the introduction, I have already discussed the plausibility of a diversification-fueled boom in US housing markets during the 1990s-2000s. Under this interpretation, sector $A$ of the model economy would correspond to housing, while sector $B$ would correspond to productive capital. As shown in figure 1, this episode featured a large sectoral credit reallocation from corporate credit to household credit, a rise of financial intermediary leverage, and a wave of deregulations and securitizations. Section 5 quantifies the effect of increased mortgage diversification on a calibrated model economy.

Mortgage securitization occurred at least two other times in US history.\(^{19}\) In the 1880s, a market developed for securitization of farm mortgages, coinciding with a lending boom and preceding a market collapse. In the 1920s, a market developed for securitization of commercial real estate loans, coinciding with a construction boom and preceding the Great Depression. My model could apply similarly to these two episodes, insofar as securitization waves generate better risk-sharing for lenders in these markets.

The mechanism in my model also has implications for the general nature of sectoral capital allocations. As Bansal, Ward and Yaron (2017) point out, sectoral wealth shares tend to be negatively correlated with sectoral risk premia. This stylized fact is at odds with standard two-sector models with only fundamental shocks: larger sectors must covary more with aggregate consumption, requiring higher risk premia, e.g., Cochrane, Longstaff and Santa-Clara (2007). My model can replicate the stylized fact, if diversification shocks are important: better diversification lowers sectoral risk premia but still raises sectoral capital because the return variance falls more.\(^{20}\)

The model can also speak to credit allocation within a sector that has heterogeneity in borrower quality. This may be relevant for the recent US housing boom, because of the emphasis on lower-quality borrowers. Empirically, diversification of lower-quality loans plausibly increased more than diversification of higher-quality loans in the 2000s.\(^{21}\)

---

\(^{18}\)An alternative, that preserves segmented markets but allows insiders unlimited borrowing, is to assume financiers’ subjective discount rate is greater than insiders’, i.e., $\rho_F > \rho_A = \rho_B := \rho$, so that insiders have wealth in the long run. Under this assumption, one can show that $\kappa_\infty$ is increasing in $\Delta_A$ and $\eta_\infty$ is decreasing in $\Delta_A$, even if $\phi_A = \phi_B = 1$.

\(^{19}\)See Simkovic (2013) and Goetzmann and Newman (2010) for discussion of the following episodes.

\(^{20}\)A similar argument holds for other financial shocks, like volatility shocks, that improve the risk-reward trade-off.

\(^{21}\)Securitization of non-conforming loans (private-label MBS containing subprime, alt-A, and jumbo loans) increased dramatically, even relative to conforming loans. See Appendix D.1.
Theoretically, suppose sector $A$ has a greater amount of idiosyncratic risk ($\hat{\sigma}_A > \hat{\sigma}_B$) and borrows a greater fraction of their asset purchases ($\phi_A > \phi_B$). Then, a diversification boom in sector $A$ produces larger reallocation and leverage effects than a diversification boom in sector $B$. The following proposition formalizes this statement. A priori, this is nontrivial because sectors with greater idiosyncratic risk and greater borrowing must pay higher idiosyncratic risk premia.  

**Proposition 2.7** (High-risk borrowers). In the steady-state equilibrium of Proposition 2.5, the following comparative statics hold:

(Idiosyncratic Risk) \[ \frac{\partial^2 \kappa_{\infty}}{\partial \Delta_A \partial \hat{\sigma}_A} > 0 \quad \text{and} \quad \frac{\partial^2 \eta_{\infty}}{\partial \Delta_A \partial \hat{\sigma}_A} < 0. \]

(Outside Funding) \[ \frac{\partial^2 \kappa_{\infty}}{\partial \Delta_A \partial \phi_A} > 0 \quad \text{and} \quad \frac{\partial^2 \eta_{\infty}}{\partial \Delta_A \partial \phi_A} < 0. \]

Thus, the reallocation and leverage effects of a diversification improvement are amplified under greater sectoral idiosyncratic risk and greater sectoral borrowing.

Proposition 2.7 explains why improved diversification of lower-quality borrowers’ idiosyncratic risks might lead to a large cycle, and it helps reconcile the timing of the 2000s housing boom with the timing of 2000s private-label MBS boom, rather than an earlier increase in diversification of conforming mortgages. See Mian and Sufi (2018) for evidence that the private-label MBS boom caused a large housing boom.

Finally, one could interpret sector $A$ as domestic producers and sector $B$ as foreign producers, with funds intermediated by a single global financial sector. The equilibrium “exchange rate” in this economy is given by

\[ \frac{p_{A,t}}{p_{B,t}} = \frac{1 - \beta \mu_B}{\beta \mu_A} \frac{1 - \kappa_t}{\kappa_t} . \]

The presence of $\kappa_t$ implies exchange rates are determined by global capital flows, unlike a frictionless complete-markets economy. As $\kappa_t$ is influenced by financial variables like $\Delta_A, \Delta_B$ and intermediary wealth $\eta_t$, global financial shocks affect exchange-rate dynamics, similar to the intermediary-centric theoretical analysis of Gabaix and Maggiori (2015).

A diversification boom in one country can thus have spillovers to the global economy, through leverage increases in the global financial system. Such leverage increases do not create any inefficiencies in the current model, but they will after extending the model to incorporate a particular fire-sale externality, which I do in Section 3.

---

22One microfoundation for why lower-quality insiders might have higher issuance is to introduce asymmetric information about insider types. In a standard signalling equilibrium, higher-quality types must retain a greater share of risk in order to separate themselves from low-quality types. When interpreting $\phi_z$ as a borrowing constraint, it is not clear why lower-quality borrowers would have looser borrowing constraints. But when interpreting $\phi_z$ as a reduced-form for borrowing demand, it becomes reasonable to assume lower-quality types (who are typically poorer) will have higher $\phi_z$.

23The Cobb-Douglas consumption aggregator is less appropriate for a setting in which the two sectors represent different quality tiers within the same asset class. That said, it is straightforward to make the sectoral goods perfect substitutes, and all of the results of this section still go through.
2.5 Dynamic Response to a Diversification Shock

Due to the tractability offered by logarithmic utility and frictionless physical investment, computing impulse response functions (IRFs) is not problematic. An important feature is that there is no “impact response” in this model.

**Lemma 2.8.** There is no state variable impact response to an unanticipated shock to $\Delta_A$ or $\Delta_B$ at time $t$, i.e., $(\alpha_t, \eta_t) = (\alpha_{t-}, \eta_{t-})$. The capital share responds on impact, $\kappa_t \neq \kappa_{t-}$.

The key intuition for Lemma 2.8 is that portfolio holdings are pre-determined before a shock, so wealth can only jump if unit prices jump. But frictionless investment implies capital prices are always equal to one; in particular, they cannot jump. In fact, it is the jump in $\kappa_t$, possible due to frictionless investment, which preserves continuity of $(\alpha_t, \eta_t)$.

In figure 7, I illustrate the time-path of a one-time unanticipated shock from $\Delta_A$. The shock occurs at time $t = 0$ and the system has approximately reached steady-state at that time. The left panel shows the time-path for $\Delta_A$. The blue curves in the middle and right panels illustrate the responses of $(\kappa_t, \eta_t)$ when the two sectors are parameterized identically. There is a jump in the capital share $\kappa_t$ of sector $A$ – the reallocation effect. The fast adjustment of $\kappa_t$ is possible because of the absence of investment adjustment costs.\(^{24}\)

![Figure 7: IRFs to a one-time shock from $\Delta_A = 0$ to $\Delta_A = 1$ at time $t = 0$. In this example, $\|\sigma_A\| = \|\sigma_B\| = 0.02$, $G_A = G_B = 0.1$, and $\beta = 0.5$. The blue curves set $\Delta_B = 0.5$, $\tilde{\sigma}_A = \tilde{\sigma}_B = 0.20$, and $\phi_A = \phi_B = 0.50$, and the red curves set $\Delta_B = 1$, $\tilde{\sigma}_A = 0.40$, and $\phi_A = 0.90$.](image)

Meanwhile, there is a slow decline in the financier wealth share $\eta_t$ – the leverage effect. The slow adjustment of $\eta_t$ occurs because improved diversification hurts financial sector profitability, thus retained earnings, which takes time to accumulate.

The red curves illustrate responses when the economy is parameterized to reflect differences between a low- and high-quality sector. Sector $A$ has greater idiosyncratic risk ($\tilde{\sigma}_A > \tilde{\sigma}_B$) and greater borrowing needs ($\phi_A > \phi_B$), while sector $B$ is already fully-diversified at time 0 ($\Delta_B = 1$). As Proposition 2.7 suggests, this setting features amplified responses.

\(^{24}\)Gradual increases in $\Delta_A$ lead to gradual reallocation, even without adjustment costs. See Appendix A.5.
3 Diversification-Induced Financial Crises

The model of Section 2 is meant to illustrate the reallocation and leverage mechanisms in a simple way. The key shortcoming of that model is the absence of any financial fragility: even though diversification reduces the financier wealth share, this does not translate into macroeconomic fluctuations. Below, I modify the model to allow for stochastic dynamics and the possibility of financial crises.

3.1 Stochastic Dynamics

To introduce stochastic fluctuations into the economy, I make the following two assumptions.

First, I constrain insiders’ ability to hedge aggregate risk. I continue to assume financiers may hedge aggregate risk. In reality, insiders of firms may be prevented from market trading due to incentive problems, whereas financiers are more likely to be marginal in these markets. Mathematically, I constrain $\theta_A^{i,t} \equiv \theta_B^{i,t} \equiv 0$. This generates stochastic fluctuations, because aggregate risk cannot be shared perfectly among agents.

Second, I introduce a fourth category of agent, which I call “distressed investors”, who may also extend financing to insiders, but are less qualified to do so. In particular, for each unit of financing, distressed investors must pay a pecuniary cost $\chi$ out of their returns. Such costs may be a reduced-form for search costs, information-acquisition costs, etc. I also assume distressed investors are worse at diversifying idiosyncratic risk than financiers, with arc-lengths $\Delta_A \leq \Delta_A$ and $\Delta_B \leq \Delta_B$. To ensure these agents have non-negligible relative wealth in the long-run, I assume now that financiers have a higher discount rate, $\rho_F > \rho$, than distressed investors and insiders. Distressed investors are otherwise identical to financiers.

Competition among insiders ensures that distressed investors must also charge spreads $s_A$ and $s_B$. Consequently, their return-on-investment is given by

$$\Delta_A^{-1} \int_{i-}^{i+\Delta_A} (d\tilde{R}_{j,t}^A - \chi dt) dj + \Delta_B^{-1} \int_{i-}^{i+\Delta_B} (d\tilde{R}_{j,t}^B - \chi dt) dj.$$

Their portfolio choices, alongside financiers’, are given by

$$\lambda_D^z = \frac{(s_z - \sigma_z \cdot \pi - \chi)^{+}}{(1 - \Delta_z)^2 \hat{\sigma}_z^2} \quad \text{and} \quad \lambda_F^z = \frac{s_z - \sigma_z \cdot \pi}{(1 - \Delta_z)^2 \hat{\sigma}_z^2}, \quad z \in \{A, B\}.$$

Distress occurs when spreads rise beyond distressed investors’ participation cost $\chi$. Since the participation cost is modeled as a pecuniary cost, any equilibrium distress leads to inefficiency.

Now, equilibrium requires that we keep track of distressed investors’ aggregate net worth $N_{D,t}$. In symmetric equilibrium, the wealth distribution is now characterized by three state variables:

$$\alpha := \frac{N_A}{N_A + N_B}, \quad \eta := \frac{N_F + N_D}{K}, \quad \text{and} \quad x := \frac{N_F}{N_F + N_D}.$$ 

The following proposition characterizes when distressed investors will enter the market.
Proposition 3.1 (Distressed Investors). Let Assumptions 1 and 2 hold. Then, there exists a unique symmetric equilibrium with state variables \((\alpha, \eta, x)\). Distressed investors lend to sector \(z \in \{A, B\}\) if and only if \(\eta_t \leq \eta^*_t, \) where

\[
\eta^*_A_t := \chi^{-1}x_t^{-1}\kappa(1 - \Delta_A)^2\bar{\sigma}^2_A \tag{21}
\]
\[
\eta^*_B_t := \chi^{-1}x_t^{-1}(1 - \kappa_t)\phi(1 - \Delta_B)^2\bar{\sigma}^2_B. \tag{22}
\]

Proposition 3.1 illustrates the possibility of misallocation in this economy. Although financiers may freely hedge aggregate risk, they still hold idiosyncratic risk. If their wealth is low relative to the amount of idiosyncratic risk they must take on, distressed investors have an incentive to enter the market and begin lending. These incentives are summarized by the thresholds \((\eta^*_A, \eta^*_B)\).

In stationary equilibrium, there is a positive probability of misallocation. This occurs because the wealth share \(\eta_t\) now evolves stochastically. In particular, the fact that insiders in sectors \(A, B\) do not hedge aggregate risk implies that insiders and financiers do not share this aggregate risk perfectly. When distressed investors are out of the market, i.e., \(\lambda^A_D = \lambda^B_D = 0\), the equilibrium diffusion vector of \(\eta\) is given by

\[
\sigma^\eta = (\phi_A - \eta)\kappa\sigma_A + (\phi_B - \eta)(1 - \kappa)\sigma_B,
\]

which is generically non-zero. Consider the case when \(\eta_t < \min(\phi_A, \phi_B)\). Then, negative sectoral shocks, \(dZ^A < 0\) or \(dZ^B < 0\), translate into lower \(\eta_t\). As negative shocks accumulate, \(\eta_t\) eventually crosses \(\eta^*_A\) or \(\eta^*_B\), leading to distressed investing. This possibility arises because of a pecuniary externality: individual financiers do not internalize the downward pressure their risk-taking puts on lending spreads and thus \(\eta_t\).

That said, when \(\Delta_A = \Delta_B = 1\), distressed investors never take positive positions, as formulas (21)-(22) show. Under perfect diversification, financiers can perfectly hedge all the risks on their loan portfolio, so their lending and leverage decisions are again completely decoupled from the risks they must bear. There is no reason for a less efficient lender to enter if they can finance a more efficient lender to do the same. Better diversification nullifies the externality in the limit.

As diversification improves marginally, a similar logic holds true. Solving for the full equilibrium, I compute the stationary average of distressed investment \(\lambda^A_D + \lambda^B_D\), as well as the probability this portfolio is strictly positive. The left panel of figure 8 shows the result for different values of \(\Delta_A\). Distressed investment vanishes as financiers’ sector \(A\) diversification improves. As the degree of financial market completeness rises, the chances of inefficient outcomes decline.

This occurs even though financiers are taking more leverage in response to better diversification (right panel). Intuitively, higher leverage is an efficient outcome with better diversification, because financiers face lower portfolio-level risk and thus have lower precautionary savings demand. This model is not well-designed to generate financial fragility out of diversification improvements.

\[25\text{In the language of Dávila and Korinek (2017), a “distributive externality” arises.}\]
Figure 8: Distressed investor leverage (left panel) and financier leverage (right panel). In this example, \( \| \sigma_A \| = \| \sigma_B \| = 0.04, \sigma_A = \sigma_B = 0.20, \phi_A = \phi_B = 0.50, G_A = G_B = 0.1, \beta = 0.5, \Delta_B = \Delta_A = 0.99, \rho = 0.01, \rho_F = 0.08, \) and \( \chi = 0.05. \) Note that \( \Delta_A = \Delta_A \) varies on the x-axis.

### 3.2 Leverage Constraints

The possibility of an inefficient leverage-induced financial crisis re-emerges if financiers face a leverage constraint: \(^{26}\)

\[
\lambda^A_{F,t} + \lambda^B_{F,t} \leq \bar{\lambda}. \tag{23}
\]

In the previous section, with no leverage constraint and with complete markets for hedging aggregate shocks, financier net worth is only useful for defraying the costs of untradeable idiosyncratic risk. As diversification improves, financier net worth is less needed in this role. Thus, financial leverage can rise as the probability of inefficient distressed investment falls.

In this new setting, financier wealth is additionally desirable in order to avoid constraint (23). Upon hitting the constraint, financiers will be forced to de-lever, independent of their risk exposures and degree of hedging activities. De-leveraging automatically results in inefficient participation by distressed investors. Thus, inefficiencies are magnified under the leverage constraint.

**Proposition 3.2** (Leverage Constraint and Distress). *Let Assumptions 1 and 2 hold, and augment financiers’ problem with constraint (23). In a symmetric equilibrium, the following hold:*

(i) *Generically, \( \lambda^A_{F,t} + \lambda^B_{F,t} = \bar{\lambda} \) implies \( \lambda^A_{D,t} + \lambda^B_{D,t} > 0. \)*

(ii) *Suppose \( \chi \geq \bar{\lambda} \max \{ (1 - \Delta_A)^2 \hat{\sigma}^2_A, (1 - \Delta_B)^2 \hat{\sigma}^2_B \}. \) Then, \( \lambda^A_{F,t} + \lambda^B_{F,t} < \bar{\lambda} \) implies \( \lambda^A_{D,t} + \lambda^B_{D,t} = 0. \)*

Part (i) of Proposition 3.2 says that, under almost all parameter constellations, distressed investors lend whenever (23) binds. This is the sense in which leverage constraints can introduce additional times of distressed investing. Part (ii) says that, under certain parameterizations, (23) must bind whenever distressed investors lend. This is the sense in which distress is unlikely without the presence of leverage constraints. For example, under \( \Delta_A = \Delta_B = 1, \) this economy experiences distress if and only if leverage constraints bind. Then, by taking \( \bar{\lambda} = +\infty, \) we revert to an economy

\(^{26}\)Constraints like (23) may arise due to incentive problems – e.g., Hart and Moore (1994), Kiyotaki and Moore (1997), and Gertler and Kiyotaki (2011) – or due to financial system regulation.
in which leverage constraints never bind and so distress never occurs, as in Proposition 3.1. But by taking \( \lambda \) low enough, we can ensure some equilibrium distress.

To construct equilibrium, note that the leverage constraint modifies portfolio choices by introducing an auxiliary variable (Lagrange multiplier) that I denote \( \zeta_t \). Thus,

\[
\lambda_D^\pi = \frac{(s_z - \sigma_z \cdot \pi - \lambda)^+}{(1 - \Delta_z)^2 \sigma_z^2} \quad \text{and} \quad \lambda_F^\pi = \frac{(s_z - \sigma_z \cdot \pi - \zeta)^+}{(1 - \Delta_z)^2 \sigma_z^2}, \quad z \in \{A, B\}. \tag{24}
\]

The standard complementary slackness condition determines when leverage constraints bind:

\[
0 = \min \{ \zeta, \lambda - \lambda^A_F - \lambda^B_F \}. \tag{25}
\]

These portfolio choices are simple because the constraints (leverage and shorting) are homogeneous in wealth, and because all agents have log utility. See Cvitanić and Karatzas (1992). The presence of \( \zeta \) helps us understand that a binding leverage constraint works similarly to a rise in intermediary funding costs. Indeed, as (24) suggests, the equilibrium with a leverage constraint is identical to an unconstrained economy in which financiers perceive a funding cost of \( r + \zeta \) rather than \( r \).

**Proposition 3.3.** Let Assumptions 1 and 2 hold, and augment financiers’ problem with constraint (23). In a symmetric equilibrium, \((\kappa, \zeta)\) solve a nonlinear system given by (25) and

\[
\rho \left[ \frac{1 - \beta}{\kappa} - \frac{\beta}{1 - \kappa} \right] - \phi_A s_A + \phi_B s_B = (1 - \eta)^{-1} \left[ \frac{\kappa}{\alpha}(1 - \phi_A)^2(\|s_A\|^2 + \delta_A^2) - \frac{1 - \kappa}{1 - \alpha}(1 - \phi_B)^2(\|s_B\|^2 + \delta_B^2) \right],
\]

where \( \rho := x\eta \rho_F + (1 - x\eta) \rho \) is a wealth-weighted discount rate, \((s_A, s_B)\) are equilibrium spreads,

\[
s_A - \sigma_A \cdot \pi = x_A^\Delta \frac{\kappa \phi_A (1 - \Delta_A)^2 \delta_A^2}{x\eta} + x_A^\Delta \zeta + (1 - x_A^\Delta) \chi \tag{27}
\]

\[
s_B - \sigma_B \cdot \pi = x_B^\Delta \frac{(1 - \kappa) \phi_B (1 - \Delta_B)^2 \delta_B^2}{x\eta} + x_B^\Delta \zeta + (1 - x_B^\Delta) \chi \tag{28}
\]

\[
\pi = \eta^{-1} [\kappa \phi_A \sigma_A + (1 - \kappa) \phi_B \sigma_B] \text{ is the aggregate risk price, } (x_A^\Delta, x_B^\Delta) \text{ are diversification-weighted wealth distributions,}
\]

\[
x^\Delta_z := \frac{1 - x}{(1 - \Delta_z)^2} + \frac{1}{(1 - \Delta_z)^2} \quad \text{and} \quad x^\Delta_z := \frac{1 - x}{(1 - \Delta_z)^2} + \frac{1}{(1 - \Delta_z)^2} \cdot 1_{\{\eta^*_A < \eta(1 - \zeta / \chi)\}}, \quad z \in \{A, B\}, \tag{29}
\]

and \((\eta^*_A, \eta^*_B)\) are defined in (21)-(22). Define the idiosyncratic risk prices

\[
\hat{\pi}_A := \frac{\kappa(1 - \phi_A) \delta_A}{(1 - \eta) \alpha} \quad \text{and} \quad \hat{\pi}_A := \frac{(1 - \kappa)(1 - \phi_B) \delta_B}{(1 - \eta)(1 - \alpha)} \tag{30}
\]

\[
\hat{\pi}_{F \rightarrow z} := \frac{(s_z - \sigma_z \cdot \pi - \zeta)^+}{(1 - \Delta_z) \delta_z} \quad \text{and} \quad \hat{\pi}_{D \rightarrow z} := \frac{(s_z - \sigma_z \cdot \pi - \chi)^+}{(1 - \Delta_z) \delta_z} \tag{31}
\]
and the shadow aggregate risk prices $\pi_A := \hat{\pi}_A \sigma_A / \hat{\sigma}_A$ and $\pi_B := \hat{\pi}_B \sigma_B / \hat{\sigma}_B$. State variable dynamics are given by

$$
\mu^\alpha = \alpha(1 - \alpha)[\hat{\pi}^2_A - \hat{\pi}^2_B + \|\pi_A\|^2 - \|\pi_B\|^2] - (\alpha \pi_A + (1 - \alpha) \pi_B) \cdot \sigma^\alpha
$$

$$
\sigma^\alpha = \alpha(1 - \alpha)[\pi_A - \pi_B],
$$

$$
\mu^\eta = \eta(1 - \eta)[x(\rho - \rho_f) + \zeta \lambda + x(\hat{\pi}_{F\rightarrow A}^2 + \hat{\pi}_{F\rightarrow B}^2) + (1 - x)(\hat{\pi}_{D\rightarrow A}^2 + \hat{\pi}_{D\rightarrow B}^2) + \|\pi\|^2 - \alpha(\hat{\pi}_{A}^2 + \|\pi_A\|^2 - (1 - \alpha)(\hat{\pi}_{B}^2 + \|\pi_B\|^2)] - (\eta \pi + (1 - \eta)(\alpha \pi_A + (1 - \alpha) \pi_B)) \cdot \sigma^\eta
$$

$$
\sigma^\eta = \eta(1 - \eta)[\pi - \alpha \pi_A - (1 - \alpha) \pi_B],
$$

and

$$
\mu^x = x(1 - x)[\rho - \rho_f + \zeta \lambda + \hat{\pi}_{F\rightarrow A}^2 + \hat{\pi}_{F\rightarrow B}^2 - \hat{\pi}_{D\rightarrow A}^2 - \hat{\pi}_{D\rightarrow B}^2]
$$

$$
\sigma^x = 0.
$$

**Remark 1** (Occasionally-Binding Constraints). In equilibrium, the leverage constraint (23) will be “occasionally-binding”, holding with equality in a strict subset of the state space. Distressed investor participation also has this feature. To numerically determine the region where (23) binds, I solve the complementary slackness equation (25), jointly with (26), as a system of nonlinear equations in $(\kappa, \zeta)$, at each point in the state space $(\alpha, \eta, x)$. An important property of equations (25)-(26) is the following. The equations at state $(\alpha, \eta, x)$ are independent of the equations at another state $(\alpha', \eta', x')$, e.g., because these are algebraic equations rather than differential equations. So we may solve the equations “state-by-state” by providing a sparse Jacobian structure, which codes the independence of the discretized system of equations, to the numerical solver.

Figure 9 reproduces figure 8 after imposing a leverage constraint, $\bar{\lambda} = 10$. The presence of a leverage constraint induces distress. As $\Delta_A$ increases, financiers take more leverage, increasing the likelihood of a binding leverage constraint. As Proposition 3.2 suggests, the binding leverage constraint requires entry of distressed investors. This reverses the finding of Section 3.1, in which distress probability decreases in $\Delta_A$.

In economies in which distress is more likely, there is more variability in equilibrium spreads. This can be seen in figure 10. Indeed, as $\Delta_A$ increases, the typical level of lending spreads falls. But with small probability, spreads can rise dramatically. Binding leverage constraints require entry by distressed investors, who in turn require high lending spreads as compensation. Notice that improvements in $\Delta_A$ impact the variability of $s_B$, as well as $s_A$. When distress occurs, it tends to affect the entire economy, as financiers intermediate funds to both sectors.

Finally, note that distress induces real costs. Because the distressed investors incur a cost $\chi$ when
they lend, investment and consumption must be reduced in such times, by goods market clearing:

$$\iota + x_\eta \rho_F + (1 - x_\eta) \rho = \big(G_A \kappa\big)^{1 - \beta} \big(G_B (1 - \kappa)\big)^{\beta} - \chi_\eta (1 - x)(\lambda_A^A + \lambda_B^B).$$

(38)

### 4 Comparison to Other Shocks

Financial diversification shocks offer an answer to why booms are often sectoral (rereallocation) and why sectoral booms may produce broad busts (financial leverage). In this section, I study several other “financial shocks” in my model: an LTV shock, a capital requirement shock, a risk-tolerance shock, an uncertainty shock, and a foreign savings shock. The motivation to study these shocks is that the extant literature has linked them at some point to boom-bust cycles, most recently related to the 2000s US housing boom. I show that none, other than the diversification shock, can produce both a sectoral reallocation and intermediary leveraging in my model. The results of this analysis are summarized in Table 1.
4.1 LTV Shock

Another important financial shock is an increase in $\phi$, which reduces the idiosyncratic risk insiders must bear when investing in capital. Since $\phi$ is the fraction of assets that can be borrowed against by insiders, like a loan-to-value ratio, I refer to this shock as an LTV shock. This type of shock is widely studied in the quantitative modeling literature, with somewhat disparate results. Here, I study the implications of this shock in my model. We have the following result.

**Proposition 4.1** (LTV Shock). Suppose $\Delta_B = 1$. If $\Delta_A$ is sufficiently large, then

$$\frac{d\kappa}{d\phi_A} > 0 \quad \text{and} \quad \frac{d\eta}{d\phi_A} > 0.$$  

The key thing to note about $\phi$ is that it is a risk transfer between insiders and financiers. Since financiers are better-diversified than insiders, this is value-enhancing and generates sectoral reallocation. Mathematically, equation (17) shows that higher $\phi_A$ lowers sector $A$’s idiosyncratic risk premia, which are equal to

$$\text{idio rp}_A = \kappa \left[ \frac{(1 - \phi_A)^2}{\alpha(1 - \eta)} + \frac{\phi_A^2(1 - \Delta_A)^2}{\eta} \right] \hat{\sigma}_A^2.$$  

This quantity is decreasing in $\phi_A$ for a well-diversified sector.

That said, the risk transfer to financiers shifts idiosyncratic risk compensation from insiders to financiers. In response to the LTV shock, lending spreads increase, the sense in which this is a “credit demand shock”. Thus, an increase in $\phi_A$ unambiguously raises financier profitability and their long-run wealth share, which tends to decrease financier leverage. Any credit demand shock that raises spreads is likely to generate similar effects.

---

27 For example, Kiyotaki et al. (2011), Justiniano et al. (2015b), Favilukis et al. (2017), Kaplan et al. (2017).
4.2 Capital Requirement Shock

Another possible finance-centric explanation for boom-bust cycles is improved financier access to leverage. Perhaps financiers are equity-issuance constrained, as might arise from capital requirements or from more fundamental agency frictions. A relaxation in capital requirements increases financiers’ ability to leverage. To model this scenario, I allow financiers to partially issue equity against their assets, requiring them to keep \(1 - \phi_F\) fraction of skin-in-the-game, like the insiders of sectors \(A\) and \(B\). Shocks to the parameter \(\phi_F\) can be called capital requirement shocks. We have the following result.

**Proposition 4.2** (Capital Requirement Shock). Consider equilibrium with capital requirement \(1 - \phi_F\).

(i) If \(\Delta := \Delta_A \equiv \Delta_B\), then \(\phi_F\)-shocks and \(\Delta\)-shocks are equivalent in the following sense: the equilibrium only depends on \(\Delta^* := 1 - (1 - \Delta)(1 - \phi_F)\) and not \(\phi_F\) or \(\Delta\) independently.

(ii) Suppose \(\sigma_A = \sigma_B, \sigma_A = \hat{\sigma}_A, \phi_F = \phi_B, \Delta_A = \Delta_B, \beta = 1 - \beta\). Then, \(\kappa_\infty\) is independent of \(\phi_F\).

Capital requirement shocks (\(\phi_F\)) are similar to diversification shocks (\(\Delta\)) in that both provide ways for financiers to diversify idiosyncratic risks. This is why both parameters appear together in the expression for financiers’ idiosyncratic risk prices, e.g.,

\[
\hat{\pi}_{F \to A} = \frac{\kappa \phi_A (1 - \Delta_A)(1 - \phi_F)\hat{\sigma}_A}{\eta} \quad \text{and} \quad \hat{\pi}_{F \to B} = \frac{(1 - \kappa)\phi_B (1 - \Delta_B)(1 - \phi_F)\hat{\sigma}_B}{\eta}.
\]

Indeed, Proposition 4.2 shows that looser capital requirements act like broad, sectorally-agnostic increases in diversification (part (i)). It follows that looser capital requirements will generate financial leverage. But key distinction is that \(\phi_F\) applies symmetrically to both sectors, while \(\Delta_A, \Delta_B\) can be asymmetric. While a sector-specific diversification shock generates a reallocation, looser capital requirements will tend to raise asset prices and allocations across the board (part (ii)).

This is empirically relevant: referring back to the motivational figure 1, we see that household credit rose as a share of total private non-financial credit. From the multi-asset perspective, diversification shocks are more likely to generate these features than a general capital requirement shock.

This is also relevant to theory. Justiniano et al. (2015a) adopt a reduced-form credit supply shock, a relaxation of “lending constraints,” as a plausible explanation for why house prices rose. But in that paper, the only positive net supply asset is housing, so lending constraints do indeed raise house prices. With multiple assets, house prices may rise with relaxed lending constraints, but they will rise in concert with other asset prices.

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28This outside equity is assumed to be pooled, thus perfectly diversified, and sold to the market. The equilibrium of this modified economy is detailed in the appendix.
4.3 Risk-Tolerance Shock

A popular culprit of boom-bust cycles has been excessive optimism or excessive risk-tolerance. Because of the nature of asset pricing, beliefs and risk-tolerance always enter risk premia jointly. I thus consider shocks to risk-tolerance in this section.

To model this, I endow agents with recursive utility as in (39). See Appendix A.1 for details on solving agents’ portfolio problems under these preferences. For simplicity, I assume all agents have unitary elasticity of intertemporal substitution, but they differ in their risk aversion parameters, $\gamma_A, \gamma_B, \gamma_F$. Risk-tolerance shocks are shocks to these parameters individually.

**Proposition 4.3 (Risk-Tolerance Shock).** Consider equilibrium with risk aversions $\gamma_A, \gamma_B, \gamma_F$.

(i) Suppose $\sigma_A = \sigma_B, \hat{\sigma}_A = \hat{\sigma}_B, \phi_A = \phi_B, \Delta_A = \Delta_B, \beta = 1 - \beta$. Suppose at time $t$, $\gamma_A = \gamma_B = \gamma_F$ and the economy is in steady-state. The following hold:

- (insider risk-tolerance) $\frac{dk_t}{d\gamma_A} > 0$ and $\frac{d\mu^u(\eta_t)}{d\gamma_A} > 0$
- (financier risk-tolerance) $\frac{dk_t}{d\gamma_F} = 0$ and $\frac{d\mu^u(\eta_t)}{d\gamma_F} < 0$
- (both risk-tolerance) $\frac{dk_t}{d\gamma_A} + \frac{dk_t}{d\gamma_F} > 0$ and $\frac{d\mu^u(\eta_t)}{d\gamma_A} + \frac{d\mu^u(\eta_t)}{d\gamma_F} < 0$.

(ii) Suppose the assumptions of part (i) hold, except $\hat{\sigma}_A > \hat{\sigma}_B = 0$. Then, $\frac{d\mu^u(\eta_t)}{d\gamma_A} + \frac{d\mu^u(\eta_t)}{d\gamma_F} = 0$.

Intuitively, an increase in $\gamma_A^{-1}$ lowers discount rates in sector $A$, which generates a sectoral allocation. However, with lower discount rates, insiders are willing to pay higher spreads to financiers, increasing their long-run wealth share. In this sense, a $\gamma_A$-shock is a credit demand shock, just like the LTV shock to $\phi_A$. An increase in $\gamma_F^{-1}$ is a credit supply shock, as it lowers required lending spreads. But because $\gamma_F$ applies symmetrically to both sectors, lending spreads decrease across the board. A sectoral reallocation is less likely, as with the capital requirement shock to $\phi_F$. Only if $\gamma_A^{-1}$ and $\gamma_F^{-1}$ both increase, with $\gamma_B^{-1}$ left unchanged, can the model generate both reallocation and leverage. That said, the leveraging effect is muted by the fact that both lender and borrower idiosyncratic risk

---

29 For example, Kindleberger (1978) says, “The monetary history of the last four hundred years has been replete with financial crises. The pattern was that investor optimism increased as economies expanded, the rate of growth of credit increased and economic growth accelerated, and an increasing number of individuals began to invest for short-term capital gains rather than for the returns associated with the productivity of the assets they were acquiring. The increase in the supply of credit and more buoyant economic outlook often led to economic booms as investment spending increased in response to the more optimistic outlook and the greater availability of credit, and as household spending increased as personal wealth surged.”

30 Note that with identical risk preferences, even if they are non-log preferences, and only fundamental shocks, the economy features a non-stochastic equilibrium that converges onto a balanced-growth-path. This is for the same reasons as in Proposition 2.5.

31 If sector $A$ is interpreted as housing, such a shock corresponds most closely to the survey evidence in Case and Shiller (2003) and the evidence in Foote et al. (2012). Kaplan et al. (2017) and Glaeser and Nathanson (2017) have model economies where agents become optimistic only about housing. Even though Landvoigt (2016) incorporates securitization, a key element of his story is the underpricing of mortgage risk by lenders.
prices are reduced by the risk-tolerance shock. As part (ii) of Proposition 4.3 shows, this offsetting can be complete if the other sector has no idiosyncratic risk. Finally, one must ask what sprouted the sector-specific optimism, whereas diversification shocks are more readily measurable.

4.4 Uncertainty Shock

Uncertainty shocks have been proposed as a possible driver of cycles: when uncertainty is low, banks may take greater leverage, and the economy suffers when uncertainty reverts. A sectoral uncertainty shock would be a reduction in $\hat{\sigma}_A$. We have the following result, which shows that lower sectoral uncertainty generates a reallocation but may not generate financier leveraging.

**Proposition 4.4** (Uncertainty Shock). Suppose $\sigma_A = \sigma_B$, $\phi_A = \phi_B$, $\Delta_A = \Delta_B$, $\beta = 1 - \beta$. Suppose the economy is in steady-state at time $t$. If $\hat{\sigma}_A = \hat{\sigma}_B$, then

$$\frac{dk_t}{d\hat{\sigma}_A} < 0 \quad \text{and} \quad \frac{d\mu_t^n(\eta_t)}{d\hat{\sigma}_A} = 0.$$  

If $\hat{\sigma}_A > \hat{\sigma}_B = 0$, then

$$\frac{dk_\infty}{d\hat{\sigma}_A} < 0 \quad \text{and} \quad \frac{d\eta_\infty}{d\hat{\sigma}_A} = 0.$$  

To understand this result, consider a hypothetical economy with no diversification ($\Delta_A = \Delta_B = 0$) but two values of idiosyncratic volatility that apply to insiders and financiers separately, i.e., $\hat{\sigma}_{A,A}$ and $\hat{\sigma}_{A,F}$. The economy is otherwise exactly identical. One can show that the equilibrium of this economy is isomorphic to the equilibrium of Proposition 2.4, if $\hat{\sigma}_{A,A} = \hat{\sigma}_A$ and $\hat{\sigma}_{F,A} = (1 - \Delta_A)\hat{\sigma}_A$. Therefore, a diversification shock operates by lowering $\hat{\sigma}_{F,A}$ and keeping $\hat{\sigma}_{A,A}$ fixed.

This contrasts with an uncertainty shock, which would have the effect of lowering both $\hat{\sigma}_{F,A}$ and $\hat{\sigma}_{A,A}$ proportionally. The result of this type of shock is to scale down all agents’ idiosyncratic risk premia equally. The long-run effect of low idiosyncratic uncertainty is ambiguous in the sense that $\eta_t$ could be higher or lower, precisely because both insiders and financiers are affected.\(^{32}\)

4.5 Foreign Savings Shock

A final alternative to consider is an increase in demand for safe assets, which tends to reduce interest rates and may fuel the boom, e.g., Bernanke (2005). Because much of this safe-asset demand manifested empirically as foreign agents buying US Treasury securities and other close substitutes, I call this a foreign savings shock. This is also consistent with the documented increase in foreign demand for highly-rated securitized products, which behave like safe assets.

\(^{32}\)This speaks to an important difference between how I am modeling the financial sector and how it has been modeled in the literature. Because both financiers and insiders are taking idiosyncratic risks, they both demand idiosyncratic risk compensation that rises with higher uncertainty. In Appendix C.2, I show that my way of setting up the model leads to substantively different conclusions about uncertainty shocks than Di Tella (2017). Indeed, I show that uncertainty shocks do not lead to excessive intermediary risk concentration, because both insiders and financiers have negative hedging demands against high uncertainty states. From a deeper theoretical perspective, diversification shocks, which are uncertainty shocks aimed directly at the financial sector, are more likely to be a source of risk concentration.
To model this, I introduce a wedge into the bond market clearing condition, which now becomes

\[ N_{A,t} + N_{B,t} + N_{F,t} + N^*_t = K_t \]

I assume \( N^*_t \) follows some exogenous process. A foreign savings shock can be modeled as an exogenous change to \( N^*_t \). Note that this also affects goods market clearing, because net interest payments to foreigners must come out of consumption. In this modified economy, there are three state variables, the relative wealth between financiers, insiders, and foreigners:

\[ \alpha_t := \frac{N_{A,t}}{N_{A,t} + N_{B,t}}, \quad \eta_t := \frac{N_{F,t}}{N_{F,t} + N_{A,t} + N_{B,t}}, \quad \text{and} \quad \eta^*_t := \frac{N^*_t}{K_t}. \]

The equilibrium of this modified economy is detailed in the appendix. We have the following result.

**Proposition 4.5** (Foreign Savings Shock). Suppose \( \sigma_A = \sigma_B, \hat{\sigma}_A = \hat{\sigma}_B, \phi_A = \phi_B, \Delta_A = \Delta_B, \beta = 1 - \beta. \) Suppose there is a one-time increase, \( \eta^*_t - \eta^*_{t-} > 0 \), in foreign savings. Suppose \((\alpha_{t-}, \eta_{t-}, \kappa_{t-}) = (\alpha_\infty, \eta_\infty, \beta)\) prior to the shock. Then, \( \kappa_t = \kappa_{t-} \) and \( \eta_t = \eta_{t-} \).

The key to Proposition 4.5 is that foreign inflows raise all domestic agents’ leverage proportionally. Foreign savings of \( \eta^*_t \) per unit of domestic wealth result in leverage of \((1 - \eta^*_t)^{-1}\) for the domestic representative agent. In particular, financier leveraging does occur after a foreign savings shock.

But the leverage is distributed equally across all domestic agents. As a result, all idiosyncratic risk prices are given by the formulas in (12)-(13), with an additional scaling by \((1 - \eta^*_t)^{-1}\). Formulas (10)-(11) then show that the dynamics of \((\alpha, \eta)\) are merely scaled by \((1 - \eta^*_t)^{-2}\), explaining why \( \eta_t \) is unaffected by foreign savings near steady-state. Applying this logic to formula (14) also explains why \( \kappa_t \) is unaffected by foreign savings near steady-state. Intuitively, foreign savings are not directed toward any particular sector, so reallocation does not occur.

### 5 Quantification: US Housing Cycle

Section 2 showed that better sectoral diversification generates a reallocation and financier leverage. The objective of this section is to quantify these effects in the context of the 1990s-2000s US housing cycle. The first step is to determine a reasonable size for the diversification shock (Section 5.1). The second step is to calibrate the model to fit this particular episode (Section 5.2).

#### 5.1 Measuring Diversification

In this section, I construct a quantitative measure of mortgage market diversification. At a high level, the steps involved are as follows. First, I construct synthetic mortgage portfolios for mortgage lenders, using originations data in the HMDA dataset. For loans that are sold or securitized, I conservatively assume they are 100% diversified. Loans that are held on the lender’s balance sheet are imperfectly diversified, and computing the exact degree of diversification follows the instructions
below. Note that this is therefore a holistic measure of diversification. It accounts for loan sales to Fannie/Freddie, securitizations, and geographic diversification.

Second, I compute the one-year-ahead volatility of each lender’s mortgage portfolio, using location-specific house price changes as the proxy for each loan’s return. The lender’s portfolio return is simply a weighted average of these loan-level returns, and I compute the volatility of this return. Importantly, this automatically accounts for the empirical correlation between loans held on a lender’s balance sheet. Call this lender-level volatility \( \hat{\sigma}_\Delta \). At the same time, I proxy loan-level risk by measuring the average of all locations’ one-year-ahead house price volatility. Call this loan-level volatility \( \hat{\sigma} \). Finally, I back out a lender-time panel of diversification \( \Delta_{i,t} \) using the model-implied relationship 
\[
(1 - \Delta)\hat{\sigma} = \hat{\sigma}_\Delta.
\]
More details on this procedure are presented in Appendix D.2.

In figure 11, I plot the average diversification level, \( \Delta_t \), which takes a weighted average of lender-specific diversifications, where the weights are given by mortgage portfolio size. In 1990, under 60% of housing risk was diversified by lenders. By 2005, over 90% of such risk was diversified. During the same time period, the idiosyncratic volatility of housing was not significantly reduced, indicating that lenders faced lower housing risks primarily due to diversification.

Why did diversification increase so dramatically? It turns out that both securitization and geographic diversification were significant factors. Figure 12 shows that the number of counties represented by loans in an average lender’s portfolio increased from 10 to 30 during the boom. During the same time, the fraction of mortgage loans sold (either to Fannie/Freddie or to private label securitizations) increased from 45% to 60%. The geographic diversification seems to have been under-appreciated during this episode.

\[33\] In doing this, I am assuming that the risk on lender’s mortgage portfolios can be proxied by the risk inherent in the house prices that the mortgages are attached to (or at least assuming the mortgage risk is proportional to the house price risk). This proportionality assumption is incorrect per se, mainly because mortgages are debt contracts, which you could think of as nonlinear functions of the local house price (e.g., default in bad states). But my assumption is reasonable as long as the covariances between the house prices in different locations is similar to the covariances between mortgage payments in different locations, because these covariances are the key inputs into how I measure diversification.
5.2 Calibrated Model

In this section, I interpret sector A as the household sector, with its capital asset as a proxy for housing. Sector B can be interpreted as all other productive capital. The parameters for this model are listed in table 2 and their targets in the caption.

| Sector | \( G_A \) | \( 1 - \beta \) | \( ||\sigma_A|| \) | \( \hat{\sigma}_A \) | \( \phi_A \) | \( \Delta_A \) | \( \Delta_A \) |
|--------|----------|---------------|----------------|-----------------|----------|--------|--------|
| A      | 0.10     | 0.30          | 0.04           | 0.20            | 0.60     | 0.50   | 0.50   |

| Sector | \( G_B \) | \( \beta \) | \( ||\sigma_B|| \) | \( \hat{\sigma}_B \) | \( \phi_B \) | \( \Delta_B \) | \( \Delta_B \) |
|--------|----------|---------------|----------------|-----------------|----------|--------|--------|
| B      | 0.10     | 0.70          | 0.04           | 0.20            | 0.30     | 0.99   | 0.99   |

<table>
<thead>
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<th>Other</th>
<th>( \rho )</th>
<th>( \rho_F )</th>
<th>( \chi )</th>
<th>( \lambda )</th>
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<td>0.08</td>
<td>0.05</td>
<td>10</td>
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</table>

Table 2: Parameter values used in this section. Calibration targets: capital-to-housing wealth ratio of 4/1.3 \( \approx 3 \) \( [1 - \kappa/\kappa] \); stock price-dividend of 35 \( [(1 - \kappa)/\beta \hat{\rho}] \); housing price-rent of 12 \( [\kappa/(1 - \beta)\hat{\rho}] \); housing consumption share of 22% \( [1 - \beta] \); commercial banks’ share of housing assets in total assets \( \phi_A \kappa/(\phi_A \kappa + \phi_B (1 - \kappa)) \); housing wealth-to-output ratio of 1.4 \( [\kappa/(G_A \kappa)]^{1-\beta} (G_B (1 - \kappa))^{\beta} \) in non-distress times; output growth rate of 2% \( [\approx \iota \text{ in non-distress times}] \); risk-free rate of 2% \( [r] \); housing idiosyncratic volatility of 14% (adjusted upwards to account for marginal borrowers) \( [\hat{\sigma}_A] \); aggregate house price volatility of 3% \( [||\sigma_A||] \); mortgage-to-housing wealth ratio of 0.4 (adjusted upwards to account for higher LTV for higher-risk borrowers and for marginal borrowers) \( [\phi_A] \); market Sharpe ratio of 0.3 \( [||\pi||] \). Housing-specific moments are taken from Piazzesi and Schneider (2016) and Davis and Van Nieuwerburgh (2015).

Into the model, I feed in a series for \( \Delta_{A,t} \) that approximately matches figure 11. I assume \( \Delta_{A,0} = 0.5 \), where \( t = 0 \) corresponds to 1980. Then, \( \Delta_{A,t} \) begins increasing in 1987 until it reaches 0.99 in 2003. The resulting series is depicted in the left panel of figure 13.

To extract the two-dimensional Brownian shocks \( (Z^A_t, Z^B_t) \), I approximately match two model-implied series to the data, from 1980 to 2015: (a) log GDP, ignoring financial distress costs, i.e., constant \( + (1 - \beta) \log \kappa + \beta \log(1 - \kappa) \) as in equation (38); and (b) the household credit share, i.e., \( \frac{\kappa \phi_A}{\kappa \phi_A + (1 - \kappa) \phi_B} \). The extracted shock series are depicted in the right panel of figure 13. In all figures, “Factual” refers to the model with shocks to both \( \Delta_A \) and \( (Z^A, Z^B) \). “Counterfactual” refers to the
model with the same shocks to \((Z^A, Z^B)\) but assumes \(\Delta_A\) constant.

\[
\begin{align*}
\text{Time-Path of } \Delta_A & \\
& \text{Factual} & \text{Counterfactual}
\end{align*}
\]

\[
\begin{align*}
\text{Time-Path of } Z^A, Z^B & \\
& Z^A & Z^B
\end{align*}
\]

Figure 13: Shocks. Parameters are in table 2. Initial values of the state variables are given by \((\alpha_0, \eta_0, x_0) = (1 - \beta, 0.2, 0.5)\).

The data series used for shock extraction, and their model counterparts, are depicted in figure 14. Both models roughly match aggregate output, but the counterfactual model does poorly in matching sectoral credit. Without diversification shocks, the model does not generate any reallocation at all.

\[
\begin{align*}
\text{Time-Path of log } Y, \text{ ignoring costs of financial distress} & \\
& \text{Factual} & \text{Counterfactual}
\end{align*}
\]

\[
\begin{align*}
\text{Time-Path of Household Credit Share} & \\
& \text{Factual} & \text{Counterfactual} & \text{Data}
\end{align*}
\]

Figure 14: Log GDP without financial distress costs (left panel) and household credit share (right panel). Parameters are in table 2. Initial values of the state variables are given by \((\alpha_0, \eta_0, x_0) = (1 - \beta, 0.2, 0.5)\).

Without diversification shocks, financier leverage does not build up, and leverage constraints are no concern. Figure 15 compares the significant model-generated financial distress in the 2007-09 period, compared to a complete lack of distress in the counterfactual model. Referring back to figure 14, notice that the distress period generates an acute housing bust, even more so than in the data.

Before the distress, during the boom years, diversification improvements reduce sector \(A\) spreads. This force operates somewhat independently of sector \(B\) spreads, as shown in figure 16. But as distress arises, \(s_A\) and \(s_B\) move together, nearly one-for-one. Spreads move more closely together in busts, because their behavior is determined by financiers’ health issues, rather than sectoral concerns. Both spreads spike in the model’s peak period of distress, reflecting almost exactly the behavior of the shadow funding cost \(\zeta\).

Distress leads to real costs: when distressed investors participate, the pecuniary cost \(\chi\) is incurred, and either consumption or investment must fall. The left panel of figure 17 shows this
Figure 15: Distressed investment (left panel) and shadow funding cost (right panel). Parameters are in table 2. Initial values of the state variables are given by \((\alpha_0, \eta_0, x_0) = (1 - \beta, 0.2, 0.5)\).

Figure 16: Lending spreads. Parameters are in table 2. Initial values of the state variables are given by \((\alpha_0, \eta_0, x_0) = (1 - \beta, 0.2, 0.5)\).

occurring in the financial crisis. The right panel shows the riskless rate, which falls dramatically in financial crisis. Distress initiates a “flight-to-quality-like” episode that is less acute in the counterfactual model.

Figure 17: Output-to-capital ratio, net of financial distress costs (left panel), and riskless rate (right panel). Parameters are in table 2. Initial values of the state variables are given by \((\alpha_0, \eta_0, x_0) = (1 - \beta, 0.2, 0.5)\).
6 Conclusion

I show that a diversification shock generates a sectoral boom followed by a broader bust. The recent US housing cycle appears to be a good example, with evidence of rising diversification in mortgage markets, followed by high house prices and rising intermediary leverage. The key to these dynamics is that the diversification shock be both asset- and agent-specific. Future research on this subject can go in several directions, and I briefly mention a few.

Going beyond the recent housing boom, credit booms are pervasive and share common features like asset price booms, low risk premia, financial instability, and slow recoveries after the bust. These are phenomena resembling the diversification-induced dynamics studied in this paper. Are diversification-type shocks the root causes of many credit cycles throughout history?

A weakness of my framework is the exogeneity of diversification. In reality, marketing of securitized products, creation of robust banking networks, and even financial deregulations are endogenous. Furthermore, there are likely to be linkages between diversification and other financial variables, such as credit standards and collateral constraints. Understanding these dynamics requires more detailed theoretical analysis of the interplay between economic conditions and diversification.

My quantitative application focuses on the recent housing boom, since interstate branching deregulations disproportionately increased mortgage credit supply, relative to firm credit supply. But as Mian, Sufi and Verner (2017a) show, household credit is generally a better predictor of future recessions than non-household credit. What is special about housing, as it pertains to boom-bust cycles? Future work should go beyond exogenous housing-specific shocks and try to understand why the effects of neutral-seeming credit supply shocks (like interstate branching deregulations) might be stronger in housing markets.

In my model, consistent with Mian et al. (2017a), growing household debt does predict low growth and macroeconomic instability. However, the channel operates entirely through financial intermediary reallocation and leverage, as opposed to household distress and defaults. Mian, Sufi and Verner (2017b) show that the 1980s housing boom, like the 2000s boom, was accompanied both by a significant amount of bank failures and household defaults. Quantitatively, are busts more sensitive to weak household balance sheets or weak intermediary balance sheets?

Finally, we can extend the framework of this paper to study an asymmetric equilibrium with location-dependent shocks to diversification, motivated by the real-world staggered implementation of bank branching deregulations. In such a model, we could quantify the spillovers across regions from localized diversification shocks. Studying such an asymmetric equilibrium is challenging, but Khorrami (2018) provides one step in this direction.

\[^{34}\text{Reinhart and Rogoff (2009), Jordà, Schularick and Taylor (2011), and Jordà, Schularick and Taylor (2013) show that credit booms often lead to busts, financial crises, and slow recoveries. López-Salido, Stein and Zakrajšek (2017) and Krishnamurthy and Muir (2016) show that credit spreads tend to be low even though the bust is predictable. Baron and Xiong (2017) show that bank equity provides low returns at the height of the boom, even though bank riskiness is elevated, measured by crash risk.}\]
A  Model Proofs and Derivations

A.1  General Consumption-Portfolio Problem for Recursive Utility Agents

To simplify the analysis, let us cast agents’ portfolio problems in a slightly more general notation. Suppose agents’ have recursive Duffie and Epstein (1992) utility recursions, given by

$$ U_t := \mathbb{E}_t \left[ \int_t^\infty \varphi(c_s, U_s) ds \right], $$

where

$$ \varphi(c, U) := \rho \frac{(1 - \gamma)U}{1 - \varsigma} \left( c^{1 - \gamma} (1 - \gamma)U \right)^{-\frac{\varsigma - 1}{\gamma}} - 1. $$

(39)

In (39), $\rho > 0$ represents the subjective discount rate, $\gamma > 0$ represents the coefficient of relative risk aversion (RRA), and $\varsigma > 0$ represents the elasticity of intertemporal substitution (EIS). Setting $\varsigma = \gamma$, these preferences reduce to von Neumann-Morgenstern preferences. Setting $\varsigma = 1$, the utility aggregator function $\varsigma$ becomes logarithmic over the consumption bundle.\(^{35}\) Consider the general net worth evolution

$$ dn_t = n_t [\mu^n_t dt + \sigma^n_t dZ_t + \hat{\sigma}^n_t dW_t] $$

(40)

where $Z$ and $W$ are two independent standard Brownian motions of dimensions $D$ and $M$ ($Z$ is the vector of aggregate shocks), and

$$ \mu^n_t = r_t - \frac{c_t}{n_t} + \theta_t \pi_t + \lambda_t (a_t - r_t 1), $$

$$ \sigma^n_t = \theta_t + \lambda_t b_t, $$

$$ \hat{\sigma}^n_t = \lambda_t \hat{b}_t. $$

Then, by appropriate definition of the variables $a_t \in \mathbb{R}^S$, $b_t \in \mathbb{R}^S \times \mathbb{R}^D$, and $\hat{b}_t \in \mathbb{R}^S \times \mathbb{R}^M$, all agents’ portfolio problems can be written as

$$ \max_{n_t, c_t, \theta_t, \lambda_t} U_t $$

subject to (40), $n_t \geq 0$, $\lambda_t \in \Lambda$, and $\theta_t \in \Theta$ for some closed, convex sets $\Lambda \subset \mathbb{R}^S$ and $\Theta \subset \mathbb{R}^D$. To simplify the exposition, I will assume $\Lambda = \{ \lambda : \lambda \geq 0, \lambda_0 \leq \lambda \}$ and $\Theta = (\theta_1, \infty) \times \cdots \times (\theta_D, \infty)$ for some vector $\theta := (\theta_1, \ldots, \theta_D) \in \mathbb{R}^D \cup \{-\infty\}^D$. For example, diversification $\Delta$ is accounted for by putting $\hat{b}_{F,t} = (\hat{b}_{F,1,t}, \hat{b}_{F,2,t})$ with $\hat{b}_{F,z,t} = (1 - \Delta_z)\hat{\sigma}_z$ and considering $W_t = (W_{t,1}^{A, \Delta}, W_{t,2}^{B, \Delta})$, with $W_{t,1}^{\Delta} := (1 - \Delta)^{-1} \Delta^{-1} \int_1^{1+\Delta} W_{j,1}^{z} dj$ a Brownian motion by Lemma 2.2.

To solve (41), we first use its scaling properties to simplify the problem. Given the homotheticity of preferences combined with the linearity of wealth evolution, value functions take the form

$$ U_t = \frac{(n_t \xi_t)^{1-\gamma}}{1-\gamma}, $$

where

$$ d\xi_t = \xi_t [\mu^t_* dt + \sigma^t_* dZ_t]. $$

(42)

\(^{35}\)By taking the limit $\varsigma \to 1$ with L’Hôpital’s rule, the aggregator becomes

$$ \varphi(c, U) = \rho (1 - \gamma)U \left[ \log(c) - \frac{1}{1 - \gamma} \log((1 - \gamma)U) \right]. $$
The process $\xi_t$ represents the investment opportunity set of the agent and responds only to the aggregate shock $Z$, due to the free mobility condition, Assumption 2.\(^{36}\)

Then, the HJB equation of such an agent is given by

$$0 = \max_{c,\lambda, \theta, \theta'} \left\{ \phi(c, U) + n \mu^n \partial_{c} U + \frac{1}{2} n^2 \left[ \|\sigma^n\|^2 + \|\hat{\sigma}^n\|^2 \right] \partial_{n} U \ight\}.$$  

Substituting the form of $U$ and its derivatives, then dividing the entire HJB equation by the positive quantity $(n\xi)^{1-\gamma}$, we obtain

$$0 = \max_{c,\lambda, \theta, \theta'} \left\{ \rho^{1/\xi} \left[ \frac{(\xi/\rho)^{1-\gamma} - 1}{1 - \xi} + \mu^n - \frac{\gamma}{2} \left[ \|\sigma^n\|^2 + \|\hat{\sigma}^n\|^2 \right] + \mu^\xi - \frac{\gamma}{2} \|\sigma^\xi\|^2 + (1 - \gamma)\sigma^n(\sigma^\xi)' \right] \right\}.$$  

First-order optimality for this agent implies

$$c_t = \rho^{1/\xi} \left( \frac{(\xi/\rho)^{1-\gamma} - 1}{1 - \xi} \right) n_t$$  

$$0 \geq a_t - (r_t + \zeta_t) \mathbf{1} - \gamma b_t(\sigma^n)' - (\gamma - 1) b_t(\sigma^\xi)' - \gamma \hat{b}_t(\hat{\sigma}^n)'$$  

$$0 \leq \zeta_t$$  

$$0 \geq \pi_t - \gamma (\sigma^n)' - (\gamma - 1)(\sigma^\xi)'.$$  

In the FOC (44) for $\lambda$, notice the introduction of $\zeta_t$, which is an auxiliary variable that satisfies (45). When $\lambda, \mathbf{1} < \lambda$, then $\zeta_t = 0$. Intuitively, the effect of the leverage constraint is to increase the perceived borrowing cost (interest rate) from $r$ to $r + \zeta$. Because of the introduction of the variable $\zeta$, (44) becomes an equality when $\lambda > 0$ (element-wise). Similarly, the FOC (46) for $\theta$ becomes an equality when $\theta > \bar{\theta}$. We write these conditions compactly as the complementary slackness conditions

$$0 = \min \left\{ \lambda', -a + (r + \zeta) \mathbf{1} + \gamma b(\sigma^n)' + (\gamma - 1) b(\sigma^\xi)' + \gamma \hat{b}(\hat{\sigma}^n)' \right\}$$  

$$0 = \min \left\{ \zeta, \lambda - \lambda \mathbf{1} \right\}$$  

$$0 = \min \left\{ \theta' - \theta', -\pi + \gamma (\sigma^n)' + (\gamma - 1)(\sigma^\xi)' \right\}.$$  

Plugging these choices back into the HJB equation, we obtain the following:

$$0 = \rho^{\xi/\rho} \frac{1 - 1}{1 - \xi} - r + \lambda \zeta + \theta \left[ \pi - \gamma (\sigma^n)' - (\gamma - 1)(\sigma^\xi)' \right] + \frac{\gamma}{2} \left[ \|\sigma^n\|^2 + \|\hat{\sigma}^n\|^2 \right] + \mu^\xi - \frac{\gamma}{2} \|\sigma^\xi\|^2,$$  

where $\sigma^n$ and $\hat{\sigma}^n$ are determined using conditions (47)-(49). Note that $\rho^{\xi/\rho} \frac{1 - 1}{1 - \xi} \to \rho(\log(\rho/\xi) - 1)$ as $\xi \to 1$. Because $\xi$ will be a function of aggregate state variables in a Markovian equilibrium, $\mu^\xi$ and $\sigma^\xi$ may be determined in terms of the derivatives of $\xi$ by Itô’s formula. Thus, (50) is a differential equation for $\xi$. Solving this equation is sufficient to demonstrate optimality of the choices outlined above.

Cvitanić and Karatzas (1992) show the convex duality approach yields exactly these optimality conditions for time-separable utility. Therefore, for the model with log utility, the portfolio choices in the text are proven optimal. There is no need to solve (50) in this case.

\(^{36}\) Verifying this equilibrium property is straightforward. Indeed, if $\xi_t$ were affected by idiosyncratic shocks $W$, then different locations would have different levels of $\xi_t$. Free mobility implies agents would immediately migrate to locations with higher levels of $\xi_t$ and attain a higher value function, which is a contradiction.
A.2 Optimal Choices

In this section, I apply the convex duality approach of Cvitanić and Karatzas (1992) to solve agents’ portfolio problems.

A.3 Derivation of Equilibrium

Proof of Lemma 2.3. We prove a more general version of Lemma 2.3, which applies to the general model of Section 3.2. Lemma 2.3 can be deduced by setting \( \bar{\lambda} = +\infty, \chi = +\infty \), and \( \rho_F = \rho \).

First, the capital return distribution from (4)-(5) is location-invariant (“LI” for short in this proof). This implies that all insiders’ consumption and portfolio choices are LI, see Appendix A.2. Consequently, \( \sigma^A_n, \sigma^B_n, \hat{\sigma}^A_n \), and \( \hat{\sigma}^B_n \) must be LI. For a different reason, \( \sigma^F_k \) and \( \sigma^D_k \) must be LI, which is these agents’ ability to hedge aggregate risk with no constraints in a centralized Arrow market (i.e., there is an LI aggregate risk price \( \pi \)).

Second, all agents’ value processes \( \xi_{A,t}, \xi_{B,t}, \xi_{F,t}, \) and \( \xi_{D,t} \) are automatically LI under free mobility, as discussed in footnote 36. As a result, \( (\mu^2, \sigma^2) \) are LI for \( z \in \{A, B, F, D\} \). Using the fact that \( (\sigma^2, \mu^2) \) is LI for \( z \in \{A, B, F, D\} \) in the general HJB equation (50) shows that \( \hat{\sigma}^n_z \) must be LI for \( z \in \{A, B, F, D\} \).

Given \( \hat{\sigma}^F_k = (\lambda^A_n(1 - \Delta_A)\hat{\sigma}^A_A, \lambda^B_n(1 - \Delta_B)\hat{\sigma}^B_B) \) and \( \hat{\sigma}^D_k = (\lambda^D_n(1 - \Delta_A)\hat{\sigma}^A_A, \lambda^D_n(1 - \Delta_B)\hat{\sigma}^B_B) \), the previous result establishes that \( \lambda^F_k \) and \( \lambda^D_k \) are LI for \( z \in \{A, B\} \). Given optimal portfolio choices from Appendix A.2, this implies that spreads \( s_A, s_B \) are LI.

It remains to show that \( k^A/k^B \) is LI. Recall from above that \( k^A/n^A \) and \( k^B/n^B \) are LI. Under the stated assumption that \( n^A/(n^A + n^B) \) is LI, it must be that \( k^A/k^B \) is LI. Under the stated assumption that \( n_F/(n^A + n^B + n^F) \) is LI, it must be that This completes the verification of an LI equilibrium.

Finally, note that it is feasible to have a symmetric equilibrium where all quantities are LI. Indeed, Assumption 2 implies that \( n^A \) can be independent of \( i \) for \( z \in \{A, B, F, D\} \). The fact that capital investment is frictionless implies that \( k^A, k^B \) can be independent of \( i \). The preceding arguments then imply that all other equilibrium objects must be LI. Such mobility of net worth is weakly optimal in a symmetric equilibrium, since it is costless, and similarly for capital since its investment is frictionless.

Proof of Proposition 2.4. This is a special case of Proposition 3.3 with \( \bar{\lambda} = +\infty, \chi = 0 \), and \( \rho_F = \rho \).

To prove existence/uniqueness, we need only consider the equation for \( \kappa_i \) in (14). Indeed, optimal choices exist uniquely by Appendix A.2. Furthermore, all other equilibrium objects are given explicitly.

Proof of Proposition 2.5. Under construction.

Proof of Proposition 2.6. Under construction.

Proof of Proposition 2.7. Under construction.

Proof of Proposition 3.1. This is a special case of Proposition 3.3 with \( \bar{\lambda} = +\infty \).

Proof of Proposition 3.2. Under construction.

Proof of Proposition 3.3. Here, I restrict attention to symmetric equilibria, in which all equilibrium objects are independent of \( i \). To simplify notation, I drop all \( i \) subscripts. Within the class of symmetric equilibria, I solve for the equilibrium objects in two steps. In the first step, I assume \( (\kappa_i, \zeta_i) \) are known and use them to solve for all other objects. In the second step, I solve for \( (\kappa_i, \zeta_i) \) via a system of nonlinear equations.

Step 1. Solving for equilibrium given \( (\kappa, \zeta) \).
Step 2. Solving for \((\kappa, \zeta)\).

A.4 Welfare

A.5 Impulse Responses

Define the IRF of a stationary variable \(Y\) by

\[
\mathcal{I}[Y](t, x) := \mathbb{E}[Y \tau + t - Y \tau - | X \tau - X = x], \quad t \geq 0,
\]  

(51)

where \(X_t\) is the vector of state variables, i.e., \(X_t = (\alpha_t, \eta_t)\) in Section 2. Equation (51) can be decomposed into the sum of an “impact response” \(\mathbb{E}[Y - Y \tau - | X \tau - X = x]\) and a “transition path” \(\mathbb{E}[Y \tau + t - Y | X \tau = x']\).

In the baseline model of Section 2, we have considered one-time unanticipated shocks to \((\Delta A, \Delta B)\). An important simplifying property of this model is that these shocks do not generate any impact response to the state variables in the model, as stated in Lemma 2.8. Thus, \(\mathcal{I}[X](t, x) = \mathbb{E}[X \tau + t - X \tau - X = x]\) for this type of shock. Figure 7 in the main text does this IRF analysis for a one-time shock to \(\Delta A\).

Alternatively, we may consider fully anticipated diversification shocks, in the sense that news about the future diversification path breaks at time \(\tau\), and after that time, agents know the entire future time-path of diversification. This changes nothing in the analysis.

**Lemma A.1.** Suppose \((\Delta A, \Delta B)\) follows a deterministic path. At time \(\tau\), agents are informed about a new future path \(\{(\Delta A, \Delta B): t \geq \tau\}\). Then, the economy is in the equilibrium of Proposition 2.4 with \((\Delta A, \Delta B)\) representing \((\Delta A, \Delta B)\) at every point in time \(t\). In particular, the impact response of \((\alpha_t, \eta_t)\) is equal to zero.

We may even consider partially anticipated diversification shocks, in the sense that agents know they follow a particular stochastic process. As long as all agents can hedge shocks to the level of diversification, then nothing changes in the analysis in this case either.

**Lemma A.2.** Suppose \((\Delta A, \Delta B)\) follows an arbitrary Markov jump-diffusion. Suppose all agents are unconstrained in markets for Arrow claims on the shocks \(d\Delta z \tau - \mathbb{E}_t[d\Delta z \tau]\). Then, the economy is in the equilibrium of Proposition 2.4 with \((\Delta A, \Delta B)\) representing \((\Delta A, \Delta B)\) at every point in time \(t\). In particular, the impact response of \((\alpha_t, \eta_t)\) is equal to zero.

As a consequence of Lemmas A.1-A.2, doing impulse response analysis to a series of diversification shocks need not be thought of as repeatedly fooling economic actors in the model by hitting them with a sequence of zero-probability events. Instead, all we need to assume is either that agents are perfectly informed about the future diversification path, or that they can hedge future uncertainty to diversification paths. Mathematically, these lemmas show that these various concepts of IRFs are equivalent in this model.
Figure 18 below repeats the analysis of figure 7 with a gradual increase in $\Delta_A$. Specifically, $\Delta_A$ increases from 0.5 to 1 over the course of 10 years, more in line with the data (see Section 5.1). This generates a slower reallocation and financial leveraging, although the convergence appears similar after 20 years.

![Figure 18](image)

Figure 18: IRFs to a gradual increase from $\Delta_A = 0.5$ to $\Delta_A = 1$ linearly from time $t = 0$ to time $t = 10$. In this example, $||\sigma_A|| = ||\sigma_B|| = 0.02$, $G_A = G_B = 0.1$, and $\beta = 0.5$. The blue curves set $\Delta_B = 0.5$, $\hat{\sigma}_A = \hat{\sigma}_B = 0$, and $\phi_A = \phi_B = 0.50$, and the red curves set $\Delta_B = 1$, $\hat{\sigma}_A = 0.40$, and $\phi_A = 0.90$.

Figure 19 repeats the analysis a third time, but with both $\Delta_A$ and $\Delta_B$ increasing gradually over time. The increases are chosen so that $\bar{\Delta}_t := \kappa_t \Delta_{A,t} + (1 - \kappa_t) \Delta_{B,t}$ increases gradually from 0.5 to 1 over the course of 10 years, i.e., $\bar{\Delta}_t \approx \max(0.5, \min(1, 0.5 + 0.05t))$. Since $\kappa_t$ is endogenous, this requires a back-and-forth between picking $(\Delta_{A,t}, \Delta_{B,t})$ and solving for $\kappa_t$. Since this requires adjusting two shocks to match a single time series, the shock extraction is non-unique. I impose that $\Delta_{A,t}$ only increases once $\Delta_{B,t} = 1$. The interest in this comparison stems from the fact that I measure diversification in Section 5.1 at the aggregate level rather than disaggregated by borrower type. Furthermore, casual evidence suggests that loan diversification of high-risk borrowers increased after that of low-risk borrowers.

To match a given increasing path of $\bar{\Delta}_t$, the later rise in $\Delta_{A,t}$ must occur more rapidly than the earlier rise in $\Delta_{B,t}$, because the low-quality sector $A$ obtains a smaller capital share than the high-quality sector $B$. This generates a faster reallocation from $B$ to $A$ (blue curves) than if there was only an increase in sector $A$ diversification (red curves).

![Figure 19](image)

Figure 19: IRFs to a gradual increase from $\bar{\Delta} = 0.5$ to $\bar{\Delta} = 1$ linearly from time $t = 0$ to time $t = 10$. In this example, $||\sigma_A|| = ||\sigma_B|| = 0.02$, $G_A = G_B = 0.1$, $\beta = 0.5$, $\hat{\sigma}_A = 0.40$, $\hat{\sigma}_B = 0.20$, $\phi_A = 0.90$, and $\phi_B = 0.5$. The blue curves represent IRFs to $\Delta_A, \Delta_B$ shocks (dashed lines, left panel), assuming $\Delta_{A,0} = \Delta_{B,0} = \Delta_0$ at time 0, and the red curves represent IRFs to only $\Delta_A$ shocks, assuming $\Delta_B = 1$ is fixed, as in figure 18.

Finally, figure 20 repeats the analysis of figure 19, but with asymmetric starting diversification levels in the two sectors. This is meant to accommodate the empirical regularity that diversification of higher-risk
borrowers’ risks tends to be lower. Evidently, the reallocation and leverage patterns are amplified in this situation, as $\Delta_{A,t}$ must rise more over time to match the same observed pattern in $\bar{\Delta}_t$.

![Graph of shock paths and IRFs](image)

Figure 20: IRFs to a gradual increase from $\bar{\Delta} = 0.5$ to $\bar{\Delta} = 1$ linearly from time $t = 0$ to time $t = 10$. In this example, $||\sigma_A|| = ||\sigma_B|| = 0.02$, $G_A = G_B = 0.1$, $\beta = 0.5$, $\hat{\rho}_A = 0.40$, $\hat{\rho}_B = 0.20$, $\phi_A = 0.90$, and $\phi_B = 0.5$. The blue curves represent IRFs to $\Delta_A, \Delta_B$ shocks (dashed lines, left panel), assuming $\Delta_{A,0} = 0.25 < \bar{\Delta}_0$, and the red curves represent IRFs to only $\Delta_A$ shocks, assuming $\Delta_B = 1$ is fixed, as in figure 18.

### A.6 Other Shocks

This section presents proofs and details for the results of Section 4.

**Proof of Proposition 4.1.** Under construction.

**Proof of Proposition 4.2.** Under construction.

**Proof of Proposition 4.3.** Under construction.

**Proof of Proposition 4.4.** Under construction.

**Proof of Proposition 4.5.** Under construction.
B Results on the Brownian Cylinder $W$

B.1 Aggregate risk along investment arcs

Proof of Lemma 2.2. To examine the degree of aggregate risk in this economy, consider investing one unit of consumption, divided equally amongst each market in $[\frac{1-\Delta}{2}, \frac{1+\Delta}{2}]$ (the fact that it is centered at $1/2$ is without loss of generality, by symmetry). This results in:

$$\text{Var}_i \left( \int_{\frac{1-\Delta}{2}}^{\frac{1+\Delta}{2}} \Delta^{-1} dW_{i,t} di \right) = \Delta^{-2} \text{Cov}_i \left( \int_{\frac{1-\Delta}{2}}^{\frac{1+\Delta}{2}} dW_{i,t} di, \int_{\frac{1-\Delta}{2}}^{\frac{1+\Delta}{2}} dW_{j,t} dj \right)$$

$$= \Delta^{-2} \int_{\frac{1-\Delta}{2}}^{\frac{1+\Delta}{2}} \int_{\frac{1-\Delta}{2}}^{\frac{1+\Delta}{2}} \text{Cov}_i (dW_{i,t}, dW_{j,t}) dj \}

= \left( 1 - 6 \int_{\frac{1-\Delta}{2}}^{\frac{1+\Delta}{2}} \int_{\frac{1-\Delta}{2}}^{\frac{1+\Delta}{2}} \Delta^{-2} |i - j| (1 - |i - j|) \, dj \right) dt$$

$$= \left( 1 - 6 \int_{0}^{1} \int_{0}^{1} \Delta |x - y| (1 - \Delta |x - y|) \, dx \, dy \right) dt$$

$$= \left( 1 - 6 \int_{-1}^{1} (1 - |u|)(1 - \Delta |u|)\Delta |u| \, du \right) dt$$

$$= (1 - \Delta)^2 dt.$$

In the third line, I have substituted the covariance and distance metric: $\text{Cov}_i (dW_{i,t}, dW_{j,t}) = 1 - 6 \min(|i - j|, 1 - |i - j|)(1 - \min(|i - j|(1 - |i - j|))) = 1 - 6 |i - j| (1 - |i - j|)$. In the fourth line, I have performed the change-of-variables $i = \frac{1-\Delta}{2} + \Delta x$ and $j = \frac{1-\Delta}{2} + \Delta y$. In the fifth line, I have substituted $u = x - y$ and used the fact that if $X$ and $Y$ are independent uniform random variables, then $X - Y$ has the triangular distribution. Given this formula, we may take $\Delta \to 1$ to see that $\text{Var}_i (\int_{0}^{1} dW_{i,t} dt) = 0$. As this expectation is zero, this shows that $\int_{0}^{1} dW_{i,t} dt = 0$ almost-surely. \hfill \Box

B.2 Existence of $W$

One may ask whether or not such a stochastic process $W := \{W_{i,t} : i \in [0,1], t \geq 0\}$ exists on any probability space. In other words, are the properties assumed above mutually consistent, or are there any internal contradictions? Below, I prove that such shocks exist by an implicit method, based on the theory of Gaussian processes. This relates to the class of Gaussian random fields that are used to model forward rates in Kennedy (1994), which to my knowledge is the first use of such processes in financial economics. The key property aiding the analysis of that paper, as in this paper, is the independent increments property of the random field in the “time” direction. Santa-Clara and Sornette (2001) study a similar stochastic process, which they call “string” shocks. They obtain these shocks using the theory of stochastic partial differential equations (SPDEs), although I prove existence in a different way. In fact, the $W$ process is a special case of the class of processes they consider, and my existence proof is general enough to apply analogously to their entire class of processes.

First, I build a particular Gaussian process. Second, I show that this stochastic process has the desired properties. Given the construction, which posits the covariance in the $t$-direction and $i$-direction as multiplicatively separable, and the property that the process acts as a continuum of Wiener processes in the $i$-direction, $W$ is thus an example of a cylindrical Wiener process (see a reference on SDEs in infinite dimensions, e.g., Da Prato and Zabczyk (2014)).
Proof of Lemma 2.1. The existence of a mean-zero Gaussian process having covariance function

\[
V((i, s), (j, t)) = \left[1 - 6\text{dist}(i, j)(1 - \text{dist}(i, j))\right] \times \min(s, t)
\]

is guaranteed if and only if \(V\) is symmetric and positive semi-definite (see any reference on Gaussian processes, e.g., proposition I.24.2 in Rogers and Williams (2000)). Clearly, \(V\) is symmetric. To check positive semi-definiteness, construct the Gram matrix: let \(i_1, \ldots, i_N \in [0, 1]\) and \(t_1, \ldots, t_N \in \mathbb{R}_+\), and define the matrix \(G\) by

\[
G := [V((i_m, t_m), (i_n, t_n))]_{m, n \in \{1, \ldots, N\}}.
\]

We need to show that \(G\) is positive semi-definite. To do this, define the “univariate” covariance functions \(v_1(i, j) := V((i, 1), (j, 1))\) and \(v_2(s, t) := V((0, s), (0, t))\), and the associated Gram matrices

\[
G_1 := [v_1(i_m, i_n)]_{m, n \in \{1, \ldots, N\}} \quad \text{and} \quad G_2 := [v_2(t_m, t_n)]_{m, n \in \{1, \ldots, N\}}.
\]

Notice that

\[
G = G_1 \circ G_2,
\]

where \(\circ\) denotes the Schur product (element-wise multiplication). By the Schur product theorem, it suffices to show that \(G_1\) and \(G_2\) are both positive semi-definite, because then so is \(G\).

Consider a standard Brownian bridge process \(\{W^\circ_i : i \in [0, 1]\}\) and define the process

\[
B_i := \sqrt{12} \left[ W^\circ_i - \int_0^1 W^\circ_j \, dj \right].
\]

Note that \(\mathbb{E}B_i = 0\) for all \(i\) and

\[
\mathbb{E}B_i B_j = 12 \mathbb{E}[(W^\circ_i - \int_0^1 W^\circ_k \, dk)(W^\circ_j - \int_0^1 W^\circ_k \, dk)]
\]

\[
= 12 \left[ \mathbb{E}W^\circ_i W^\circ_j + \mathbb{E} \int_0^1 \int_0^1 W^\circ_k W^\circ_l \, dk \, dl - \mathbb{E} \int_0^1 W^\circ_i W^\circ_k \, dk - \mathbb{E} \int_0^1 W^\circ_j W^\circ_k \, dk \right]
\]

\[
= 12 \left[ \min(i, j) - ij + \int_0^1 \int_0^1 [\min(k, l) - kl] \, dk \, dl - \int_0^1 [\min(i, k) - ik] \, dk - \int_0^1 [\min(j, k) - jk] \, dk \right]
\]

\[
= 12 \left[ \min(i, j) - ij + \int_0^1 \frac{l(1 - l)}{2} \, dl - \frac{i(1 - i)}{2} - \frac{j(1 - j)}{2} \right]
\]

\[
= 1 - 6|i - j|(1 - |i - j|).
\]

In the third and fourth equality, I have used the Brownian bridge covariance function to compute \(\mathbb{E}W^\circ_i W^\circ_j = \min(i, j) - ij\), as well as the integral

\[
\int_0^1 [\min(i, j) - ij] \, dj = \int_0^i j(1 - i) \, dj + \int_i^1 i(1 - j) \, dj
\]

\[
= \frac{1}{2} i^2 (1 - i) + \frac{1}{2} i (1 - i)^2
\]

\[
= \frac{i(1 - i)}{2}.
\]

Therefore, \(v_1\) is the covariance function for \(B\). As a valid covariance function, we immediately conclude that \(G_1\) is positive semi-definite. Finally, \(v_2\) is the covariance function of standard Brownian motion, so the matrix
$G_2$ is positive semi-definite.

Thus, define $W$ to be a Gaussian process with covariance function $V$. We want to show that $W$ has the desired properties: (1) at each location, $W$ acts as a Brownian motion; (2) $dW$ has the correct cross-sectional correlations; (3) $W$ has a path-continuous version.

First, fixing $i$, the time-series process $W^{(i)} := \{W_{i,t} : t \geq 0\}$ is a standard Brownian motion. Indeed, $E|W_{i,t}|^2 = 0$ implies $W_{i,0} = 0$ almost-surely. Since $W^{(i)}$ is a centered Gaussian process with $V((i,s),(i,t)) = \min(s,t)$, it has the same probability law as a standard Brownian motion. Having the same probability law, it is well-known that $W^{(i)}$ can be chosen to be path-continuous. Independent increments can be established as follows. Using the covariance function and the Normal distribution, we have $E[(W_{i,t_2} - W_{i,t_1})(W_{i,t_1} - W_{i,t_0})] = 0$ for $t_2 \geq t_1 \geq t_0 \geq 0$. Orthogonality plus joint Normality implies independence of $W_{i,t_2} - W_{i,t_1}$ from $W_{i,t_1} - W_{i,t_0}$.

Second, the increments to $W^{(i)}$ and $W^{(j)}$ have the desired pairwise correlations. Indeed, using the covariance function $V$, we have

$$\frac{1}{s}E[(W_{i,t+s} - W_{i,t})(W_{j,t+s} - W_{j,t})] = 1 - 6\text{dist}(i,j)(1 - \text{dist}(i,j)).$$

As $s > 0$ is arbitrary, and using the Markov property of Brownian motion, we have that

$$\text{corr}(dW_{i,t}, dW_{j,t} | \mathcal{F}_t) = 1 - 6\text{dist}(i,j)(1 - \text{dist}(i,j)).$$

Combining these first two properties, we have shown that $W = \{W_{i,t} : i \in [0,1], t \geq 0\}$ has the properties stated in Assumption 1 of the text.

Third, we can use the Kolmogorov-Chentsov continuity criterion (see any reference on Gaussian processes, e.g., theorem I.25.2 in Rogers and Williams (2000)) to show that $W$ has a version with continuous sample paths.\textsuperscript{37} To do this, we may fix an arbitrary $T > 0$ and show that there exist $C > 0$, $\varepsilon_1 > 0$, and $\varepsilon_2 > 0$ (which may all depend on $T$) such that

$$E|W_{i,s} - W_{j,t}|^{\varepsilon_1} \leq C \times \text{dist}((i,s),(j,t))^{2(1+\varepsilon_2)}, \quad \forall s,t \leq T, \quad (53)$$

where $\text{dist}(\cdot,\cdot)$ is Euclidean distance in $\mathbb{C}^2 \times \mathbb{R}$, where $\mathbb{C}^2$ is the circle of circumference one.\textsuperscript{38} In particular,

$$\text{dist}((i,s),(j,t)) := |s - t|^2 + |i - j|^2(1 - |i - j|)^2.$$

Assume $s > t$ (the opposite case follows symmetrically). Set $\varepsilon_2 > 0$ arbitrarily, and set $\varepsilon_1 = 4(1 + \varepsilon_2)$. Then, because $W$ is Gaussian, there exists a constant $M$ such that

$$E|W_{i,s} - W_{j,t}|^{\varepsilon_1} = E|W_{i,s} - W_{j,t}|^{4(1+\varepsilon_2)} = M E[|W_{i,s} - W_{j,t}|^{2(1+\varepsilon_2)}].$$

\textsuperscript{37}One might think we could prove continuity by appealing to the fact that $\{W_{i,t} : i \in [0,1]\}$ is a translated, scaled Brownian bridge for each $t$, and $\{W_{i,t} : t \geq 0\}$ is a Brownian motion for each $i$. Thus, we could construct continuous versions of each of these at the rational indexes, and use a density argument to construct a continuous $W$ in the limit. The problem with this approach is that we don’t know that the limiting process has the desired distributional properties.

\textsuperscript{38}This is a slight generalization of the conventional Kolmogorov-Chentsov theorem, in which the index set is not $\mathbb{R}^2$. But the exact same condition applies.
Compute, using the triangle inequality, covariance function, and assumption that \( t < s < T \):

\[
\mathbb{E}|W_{i,s} - W_{j,t}|^2 = V((i, s), (i, s)) + V((j, t), (j, t)) - 2V((i, s), (j, t)) \\
\leq |V((i, s), (i, s)) - V((i, s), (j, t))| + |V((j, t), (j, t)) - V((i, s), (j, t))| \\
= 2|\min(s, t)(1 - 6|i - j|(1 - |i - j|)) - t| \\
= 12t|i - j|(1 - |i - j|) \\
\leq 12T[s - t]^2 + |i - j|^2(1 - |i - j|)^2)^{1/2} \\
= 12T\text{dist}((i, s), (j, t))
\]

Consequently,

\[
\mathbb{E}|W_{i,s} - W_{j,t}|^2 \leq M(12T)^{2(1+\varepsilon_2)}\text{dist}((i, s), (j, t))^{2(1+\varepsilon_2)},
\]

which is condition (53) with \( C = M(12T)^{2(1+\varepsilon_2)} \).

\[\square\]

### B.3 Simulating \( W \)

Given that Lemma 2.1 only implicitly defines \( W \), one might wonder how such a process could be simulated. In this section, I provide a method for simulating \( W \). I do not show that this construction satisfies all the technical measurability requirements, but I do show that the simulated process has all of the relevant properties if those technical requirements are satisfied.

Let \( (\Omega, \mathcal{F}, \mathbb{P}) \) be a complete probability space satisfying the usual conditions, on which \( W^* := \{W^*_t : t \geq 0\} \) is a standard Brownian motion adapted to the filtration \( \{\mathcal{F}_t : t \geq 0\} \). In addition, let \( B_i := \{B_{i,t} : i \in [0, 1]\} \) be a standard Brownian bridge for every \( t > 0 \). Then, let \( \{u_i\} \) be iid Uniform(0,1] random variables, independent of \( W^* \) and \( B \), and define

\[
B_{i,t} := \begin{cases} 
B_{u_i+t,t}, & \text{if } i \in [0, 1-u_i] \\
B_{u_i-1,t}, & \text{if } i \in (1-u_i, 1]. 
\end{cases}
\]

Thus, \( B_t \) is essentially a Brownian bridge for every \( t \), but with a random initial index on the circle, \( u_t \). In fact, \( B_{i,t} - B_{0,t} \) is a standard Brownian bridge.\(^{39}\) Assume that the sequence of Brownian bridges \( B := \{B_t : t > 0\} \) is progressively measurable. Define the process

\[
W_{i,t} := \sqrt{12} \int_0^t \left( B_{i,s} - \int_0^s B_{j,s} \,dj \right) dW^*_s 
\]

so that the increment is \( dW_{i,t} = \sqrt{12}(B_{i,t} - \int_0^t B_{j,t} \,dj) \,dW^*_t \). I will show that \( W_{i,t} \) satisfies Assumption 1. In that case, simulating the process requires an approximation to a single aggregate Brownian motion \( \{W^*_t : t = 0, dt, 2dt, \ldots \} \), a sequence of iid uniform random variables \( \{u_t : t = 0, dt, 2dt, \ldots \} \), and a Brownian bridge drawn independently at each time step, \( \{B_{i,t} : i = 0, dt, 2dt, \ldots, 1, \quad t = 0, dt, 2dt, \ldots \} \).\(^{40}\)

\(^{39}\)One can check this by simply by verifying that this process has the correct covariance function, i.e., for \( i < j \), \( \text{Cov}(B_{i,t} - B_{0,t}, B_{j,t} - B_{0,t}) = i(1 - j) \).

\(^{40}\)This last step can even be relaxed for speed. Due to the iid nature of \( u_t \), one may draw a single Brownian bridge only and achieve the desired properties.
The quadratic covariation is (52). By Lévy’s characterization of Brownian motion, so that, using similar moments for the Brownian bridge as above, we have for the points being Brownian motions, we have an unconditional version of this covariation formula:

\[ \mathbb{E}_t [W_{i,t}^2, U_t] = 12 \mathbb{E}_t \left[ \int_t^{t+T} \left( B_{i,s} - \int_0^1 B_{j,s}^* df \right)^2 ds \right] = W_{i,t}^2 + 12 \mathbb{E}_t \left[ \int_t^{t+T} \left( B_{i,s} - \int_0^1 B_{j,s}^* df \right)^2 ds \right] = W_{i,t}^2 + 12 \int_t^{t+T} \left[ \mathbb{E}(B_{i,s} - B_{0,s})^2 + \mathbb{E} \left( \int_0^1 (B_{j,s} - B_{0,s}) df \right)^2 - 2 \mathbb{E}(B_{i,s} - B_{0,s}) \int_0^1 (B_{j,s} - B_{0,s}) df \right] ds = W_{i,t}^2 + \frac{1}{2} \int_t^{t+T} j(1-j) ds \}

\[ \mathbb{E}_t [W_{i,t}^2] = W_{i,t}^2 + t + (T - t). \]

Note that \( \mathbb{E}_t \) denotes conditional expectation using the information set at time \( t \), not including the information on \( U_t \). In the second equality, I have used the independence of \( \{B_{i,t+s} : i \in [0, 1], s > 0 \} \) and \( \{W_{i,t+s}^* : s > 0 \} \) from \( F_t \) as well as Itô’s isometry. In the third equality, I have used Fubini’s theorem. In the fourth equality, I have used the fact that \( \{B_{i,t} - B_{0,t} : i \in [0, 1] \} \) is a standard Brownian bridge, combined with the calculation in (52). By Lévy’s characterization of Brownian motion, \( W_{i,t} \) is a standard Brownian motion for each \( i \).

To prove property (ii) of Assumption 1, recall the quadratic covariation formula: for any two continuous local martingales \( M \) and \( N \),

\[ \mathbb{E}_t [M_{t+T} N_{t+T} - M_t N_t] = \mathbb{E}_t ([M, N]_{t+T}) - [M, N]_t, \]

Using \( M_t = W_{i,t} \) and \( N_t = W_{j,t} \), and noting that these local martingales have the Markov property (by virtue of being Brownian motions), we have an unconditional version of this covariation formula:

\[ \mathbb{E}_t [(W_{i,t+T} - W_{i,t})(W_{j,t+T} - W_{j,t})] = \mathbb{E}_t ([W_{i,t+T} - W_{i,t}, W_{j,t+T} - W_{j,t}]) = \mathbb{E}_t ([W_{i,t} W_{j,t+T}] - [W_{i,t} W_{j,t}]). \]

The quadratic covariation is

\( [W_t, W_j]_t = 12 \int_0^t \left( B_{i,s} - \int_0^1 B_{k,s}^* dk \right) \left( B_{j,s} - \int_0^1 B_{k',s}^* dk' \right) ds, \)

so that, using similar moments for the Brownian bridge as above, we have for \( i < j \),

\[ \mathbb{E}_t [(W_{i,t+T} - W_{i,t})(W_{j,t+T} - W_{j,t})] = 12 \int_t^{t+T} \left( B_{i,s} - \int_0^1 B_{k,s}^* dk \right) \left( B_{j,s} - \int_0^1 B_{k,s}^* dk' \right) ds = 12 \int_t^{t+T} \left[ E B_{i,s} B_{j,s} + \mathbb{E} \left( \int_0^1 B_{k,s}^* dk \right)^2 - E B_{i,s} \int_0^1 B_{k,s}^* dk - E B_{j,s} \int_0^1 B_{k,s}^* dk \right] ds = 12 \left[ i(1-j) + \frac{1}{12} - \frac{i(1-i)}{2} - \frac{j(1-j)}{2} \right] T = [1 - 6(j - i)(1 - (j - i))] T. \]
This shows that
\[
\frac{1}{dt} \mathbb{E}_t[dW_{i,t}dW_{j,t}] := \lim_{T \to 0} \frac{1}{T} \mathbb{E}_t[(W_{i,t+T} - W_{i,t})(W_{j,t+T} - W_{j,t})] = 1 - 6|i - j|(1 - |i - j|).
\]
and verifies property (ii).\(^{41}\) Thus, \(W\) satisfies Assumption 1.

---

\(^{41}\)Property (ii) of Assumption 1 was made to ensure \(\int_0^1 (W_{i,t+T} - W_{i,t})di = 0\) almost-surely for any \(T > 0\). Note that, with the definition of \(W_{i,t}\) above, this integration can be verified directly.
C Extensions and Auxiliary Results

C.1 Moral Hazard and Skin-in-the-Game

In the model of Section 2, an insider sells an exogenous fraction $\phi$ of the capital stock to outsiders (financiers). The insider keeps a fraction of $1 - \phi$ of the capital risk on his own balance sheet. In this appendix, I derive this risk-sharing arrangement from a standard moral hazard problem.

Let the capital stock of a generic insider evolve as follows:

$$ dk_{i,t} = k_{i,t}[(c_{i,t} - \delta_{i,t})dt + \sigma dZ_t + \delta dW_{i,t}] . $$

The new object is $\delta_{i,t}$, which captures hidden diversion. In particular, insiders may divert $\delta k$ units of capital to obtain $(1 - \phi)\delta q k$, where $q$ is the unit price of that capital and $\phi$ determines the inefficiency from diversion. This may also be thought of as diverting effort away from capital upkeep.

Insiders hold assets, borrow/lend in risk-free debt markets, and make contractual payments to outsiders. Contractual payments are $-q_i k_id\Omega_{i,t}$ per unit of time, where $q_i$ is the market price of capital. Since diversion is unobservable, $\Omega_i$ must be adapted to $(Z, W^\delta_i)$, where $dW^\delta_{i,t} = dW_{i,t} - \delta_{i,t}dt$. Given that agents may frictionlessly trade claims on the aggregate shock $Z$, it suffices to assume $\Omega_i$ is adapted to $W^\delta_i$. Contract payments thus take the form $d\Omega_{i,t} = \zeta_{i,t}[(\hat{\sigma} \hat{\pi}_{F,t} - \delta_{i,t})dt + \hat{\sigma} dW_{i,t}]$.

Incorporating these contract payments, insiders experience the following net worth evolution:

$$ dn_{i,t} = \left( n_{i,t}r_t - c_{i,t} \right) dt + (1 - \phi)\delta_{i,t}q_i k_{i,t} dt + q_i k_{i,t}(dR_{i,t} - r_t dt) $$

$$ - q_i k_{i,t}\zeta_{i,t} \left( \hat{\hat{\sigma}} \hat{\pi}_{F,t} - \delta_{i,t} \right) dt + \hat{\sigma} dW_{i,t} \right) + \theta_{i,t}n_{i,t}(\hat{\pi}_t dt + dZ_t) . $$

This budget constraint has a simple interpretation. Insiders retain a stake $1 - \zeta$ in their asset risks and issue $\zeta \hat{\sigma}$ of their capital risk. This can be thought of as an equity stake, which has market price of risk $\hat{\hat{\pi}}_F$.

**Definition 2.** Optimal contracts consist of possible risk exposures and promised payments (i.e., $\zeta_{i,t}, \hat{\pi}_{F,t}$) that implement no diversion (i.e., $\delta_{i,t} \equiv 0$ for all $i, t$) and maximize total surplus in the following sense. Taking as given future contracts $\{\zeta_{i,t+s}, \hat{\pi}_{F,t+s}\}_{s>0}$, time-$t$ contracts $(\zeta_{i,t}, \hat{\pi}_{F,t})$ maximize total instantaneous surplus among contracting parties.

An important feature of these contracts is that they are short-term, which is captured by the last statement in Definition 2. Contracts are chosen to maximize instantaneous surplus, rather than total long-term surplus, which aids tractability. These short-term contracts would be optimal long-term contracts as well, if agents cannot commit to future contracts.

To heuristically derive optimal contracts, note that diversion of $\delta k$ units of capital yields $(1 - \phi)\delta q k$ in net worth to the insider. On the other hand, the insiders’ return-on-assets is reduced by $\delta$, which translates into a $(1 - \zeta)\delta q k$ lower payoff from inside equity. Consequently, the insider will not divert any capital as long as $\zeta_{i,t} \leq \phi$, which is insiders’ incentive-compatibility constraint. In other words, $1 - \phi$ is the minimum skin-in-the-game requirement. Moreover, the insider has no diversification technology, whereas financiers do, so it is optimal to set $\zeta = \phi$ (recall: experts and financiers have identical risk aversions). Finally, given competition in the financier sector, $\hat{\hat{\pi}}_F$ is determined by their marginal utility process, as in the main text.

---

Adaptability to $W^\delta_i$ implies the weights of $d\Omega_{i,t}$ on $\hat{\sigma} dW_{i,t}$ must be identical. This weight is $\zeta_{i,t}\hat{\sigma}$. The additional term $\zeta_{i,t}\hat{\hat{\pi}}_{F,t} dt$ allows for time-varying flow payments.
C.2 Financial Shocks and Risk Concentration

In the baseline model of Section 2, so-called risk concentration channel is absent, i.e., the wealth distribution evolves deterministically, which makes the entire economy evolve deterministically. Exactly as in Di Tella (2017), this conclusion arises due to the combination of agents’ symmetric risk preferences and their ability to frictionlessly access markets for trading aggregate risk. However, as Di Tella (2017) goes on to show, idiosyncratic uncertainty shocks may break this neutrality result and lead financiers to take excess risk. In my model, excess financier risk is more likely to generated by diversification shocks than by uncertainty shocks (or LTV shocks).

To demonstrate this, I allow \((\Delta, \hat{\sigma}, \phi)\) to be truly stochastic aggregate shocks and show that shocks to \(\Delta\) incentivize financiers to disproportionately incur aggregate risk, whereas shocks to \(\hat{\sigma}\) and \(\phi\) do not. Since financiers wealth tends to be more cyclical than households’ empirically, this suggests that financial shocks like \(\Delta\) are needed in model economies.

Consider the following setting. Take the economy from Section 2 and set \(\beta = 1\) so that there is a single capital stock and single consumption good. Since there is only one type of insider, call them “households” and label their variables with the subscript “\(H\)”. Endow financiers and households with symmetric Epstein-Zin preferences, as in (39). Suppose all agents have unitary EIS, i.e., \(\varsigma = 1\).

Let \((\Delta, \hat{\sigma}, \phi)\) follow the following exogenous stationary Markov processes:

\[
\begin{align*}
    d\Delta_t &= \mu^{\Delta}(\Delta_t)dt + \sigma^{\Delta}(\Delta_t)dZ_t \\
    d\hat{\sigma}_t &= \mu^{\hat{\sigma}}(\hat{\sigma}_t)dt + \sigma^{\hat{\sigma}}(\hat{\sigma}_t)dZ_t \\
    d\phi_t &= \mu^{\phi}(\phi_t)dt + \sigma^{\phi}(\phi_t)dZ_t
\end{align*}
\]

Note that shocks to \((\Delta_t, \hat{\sigma}_t, \phi_t)\) are locally perfectly correlated with aggregate TFP shocks, which is only for simplicity and does not affect the results of this section. In fact, if TFP shocks are shut down completely, everything here goes through. Assume \(\sigma^{\Delta} \geq 0, \sigma^{\hat{\sigma}} \leq 0, \sigma^{\phi} \geq 0\) in accordance with conventional wisdom and empirical evidence.\(^{43}\)

Because shocks to \((\Delta, \hat{\sigma}, \phi)\) are driven by the same aggregate shock as capital, the optimality conditions outlined in Appendix A.1 continue to hold here. The additional complication is that \(\xi_F\) and \(\xi_H\), the processes measuring financier and household marginal utility of wealth, satisfy PDEs in \((\eta, \Delta, \hat{\sigma}, \phi)\) rather than ODEs in \(\eta\). In equilibrium, we have the following idiosyncratic risk prices earned by financiers and households,

\[
\pi_{F,t} = \gamma \frac{(1 - \Delta_t)\phi_t \hat{\sigma}_t}{\eta_t} \quad \text{and} \quad \pi_{H,t} = \gamma \frac{(1 - \phi_t)\hat{\sigma}_t}{1 - \eta_t},
\]

which are generalized from Section 2 because risk aversion \(\gamma \neq 1\) and because \((\Delta_t, \hat{\sigma}_t, \phi_t)\) are non-constant. The idiosyncratic risk premia (risk price times risk quantity) earned on idiosyncratic risks are given by

\[
\alpha_{F,t} = \gamma \left(\frac{(1 - \Delta_t)\phi_t \hat{\sigma}_t}{\eta_t}\right)^2 \quad \text{and} \quad \alpha_{H,t} = \gamma \left(\frac{(1 - \phi_t)\hat{\sigma}_t}{1 - \eta_t}\right)^2.
\]

\(^{43}\)There is some controversy over \(\sigma^{\phi} \geq 0\). In this model, external finance is done via equity contracts, and higher \(\phi\) is a sign of a more efficient financial sector. From this perspective, the statement that \(\phi\) is procyclical is intuitive. In the data, stock market equity-issuance is procyclical. Similarly, the loan-to-value ratios of the marginal borrower tend to be procyclical (e.g., Pavlikis et al. (2017)). On the other hand, aggregate loan-to-value ratios in the housing market tend to be countercyclical (e.g., Davis and Van Nieuwerburgh (2015)). That said, this discrepancy can be partially accounted for by the fact that borrowers typically issue debt contracts, whose values are mechanically less procyclical than levered equity. Still, note that if \(\sigma^{\phi} \leq 0\), the results of this section on hedging demands for \(\phi_t\) all flip signs, and shocks to \(\phi_t\) can also be a source of risk concentration.
At this point, we can understand the intuition for how $\Delta_t$ generates financial risk concentration. Notice that $\alpha_{H,t}$ is unaffected by $\Delta_t$, whereas $\alpha_{F,t}$ is decreasing in $\Delta_t$. If $\sigma_{F,t}^\varepsilon > 0$, negative shocks $dZ_t < 0$ will decrease $\Delta_t$ and lead to higher financier expected returns going forward, relative to households. Such an improvement of the relative investment opportunity set of financiers is a hedge and suggests that financiers’ utility diffusion is smaller than households’, i.e., $\sigma_{F,t}^\varepsilon < \sigma_{H,t}^\varepsilon$.

Shocks to $\Delta_t$ have aggregate implications through this mechanism. Indeed, clearing the aggregate risk market, we obtain

$$\pi_t = \gamma \sigma + (\gamma - 1) \left[ (1 - \eta_t) \sigma_{H,t}^\varepsilon + \eta_t \sigma_{F,t}^\varepsilon \right].$$

Applying Itô’s formula to $\eta_t$, we find that its local volatility is

$$\sigma_{\eta,t}^\eta = \eta_t (1 - \eta_t) \frac{1 - \gamma}{\gamma} \left[ \sigma_{F,t}^\varepsilon - \sigma_{H,t}^\varepsilon \right].$$

If $\sigma_{F,t}^\varepsilon < \sigma_{H,t}^\varepsilon$, as conjectured above, then $\sigma_{\eta,t}^\eta > 0$ if and only if $\gamma > 1$. Hence, the hedging demands induced by financial shocks $\Delta_t$ create risk concentration in the sense that financiers buy Arrow claims on $dZ_t$ from households. This stands in contrast to the situation in which $\Delta_t$ is non-stochastic as in Section 2. In that case, no shocks affect the relative investment opportunity sets of experts and financiers, so $\sigma_{\eta,t}^\eta \equiv 0$. This result echoes the results of Di Tella (2017): $\Delta_t$ impacts the uncertainty faced by financiers, just as a direct uncertainty shock does.

An exact opposite logic is true for shocks to $\phi_t$. When $\sigma^\phi > 0$, negative shocks $dZ_t < 0$ will decrease $\phi_t$ and lead to lower financier expected returns going forward, relative to households. This exposure is the opposite of a hedge, and suggests $\sigma_{F,t}^\phi > \sigma_{H,t}^\phi$. If $\gamma > 1$ so that hedging demands are strong, then it is households who will hold concentrated aggregate risk positions and $\sigma^\eta < 0$!

The story is slightly different under uncertainty shocks (shocks to $\tilde{\sigma}$). Uncertainty shocks would affect both $\alpha_{F}$ and $\alpha_{H}$ in the same direction, and the strength of this effect depends on the relative size of $(1 - \Delta_t)\phi_t/\eta_t$ and $(1 - \phi_t)/(1 - \eta_t)$. Ultimately, this produces an ambiguous effect on financial risk concentration. In fact, we have the following neutrality result.

**Proposition C.1 (Neutrality of Uncertainty Shocks).** Consider an economy with only uncertainty shocks (i.e., $\mu^\Delta, \sigma^\Delta, \mu^\phi, \sigma^\phi \equiv 0$, while $\sigma^\phi < 0$). If at any point of time $\eta_t = \eta_\infty := \frac{\sigma^\phi(1 - \Delta)}{1 - \phi^\phi}$, then there exists an equilibrium in which $\eta_t = \eta_\infty$ thereafter. If $\Delta < 1$ and $\eta < \phi < 1$, in addition, then $\xi_F = \xi_H$ in this equilibrium.

The key to Proposition C.1 is that the drift nets out financier and household risk compensation:

$$\mu^\eta = \gamma^{-1} \eta_t (1 - \eta_t) [\tilde{\pi}_F^2 - \tilde{\pi}_H^2].$$

Since both $\tilde{\pi}_F$ and $\tilde{\pi}_H$ scale with $\tilde{\sigma}$, uncertainty shocks do not affect the “steady-state” of this system (defined by $\mu^\eta = 0$). At this steady-state $\eta_t = \eta_\infty$, idiosyncratic risk prices are equalized, $\tilde{\pi}_F = \tilde{\pi}_H$. Hence, shocks to $\tilde{\sigma}$ affect both agents equally. Formally, one can verify that both agents’ HJB equations and all equilibrium conditions are satisfied if $\xi_F = \xi_H$ at $\eta = \eta_\infty$. This leads to $\sigma_{F,t}^\xi(\eta_\infty) = \sigma_{H,t}^\xi(\eta_\infty)$ and thus a non-stochastic equilibrium with $\sigma^\eta(\eta_\infty) = 0$. This then verifies the conjecture that $\xi_F = \xi_H$, as $\eta_t$ stays constant forever.\textsuperscript{44}

The difference between Di Tella (2017) and this model is that $\Delta < 1$. When $\Delta = 1$, outsiders are unaffected by idiosyncratic risk; only insiders’ investment opportunity sets are affected by uncertainty shocks.

\textsuperscript{44}Furthermore, even starting with $\eta_t \neq \eta_\infty$, I conjecture that there are parameters such that $\eta_t \to \eta_\infty$ almost-surely. Indeed, $\mu^\eta > 0$ for $\eta < \eta_\infty$ and $\mu^\eta < 0$ for $\eta > \eta_\infty$. Thus, $\eta_\infty$ is an attracting point of the dynamical system. Whether the state variable hits the attracting point (and thus stays forever) depends on the relative speed at which $\mu^\eta$ and $\sigma^\eta$ vanish when $\eta \to \eta_\infty$. 
Mathematically, $\xi_H$ contains more exposure to $\hat{\sigma}$ than $\xi_F$. In that case, uncertainty shocks provide insiders a hedge against bad states, and they will take additional aggregate risk ex-ante. But this result is fragile, as it is not generically true for any $\Delta < 1$. A similar discussion applies for the assumption $0 < \phi < 1$.

To provide an analytical result illustrating the induced hedging demands of $(\Delta, \phi)$ shocks, we simplify the setting even further. Suppose at a known time $\tau$, one of the following experiments will occur. We will consider these experiments one-by-one.

\[ (\text{"\Delta experiment"}) : \quad \Delta_{\tau} = \begin{cases} 
\Delta_+ & \text{with probability } 1/2 \\
\Delta_- & \text{with probability } 1/2 
\end{cases} \]

\[ (\text{"\phi experiment"}) : \quad \phi_{\tau} = \begin{cases} 
\phi_+ & \text{with probability } 1/2 \\
\phi_- & \text{with probability } 1/2 
\end{cases} \]

where $\Delta_+ > \Delta_-$ and $\phi_+ > \phi_-$. In each experiment, allow both financiers and households to frictionlessly trade Arrow claims on these shocks. Let the Arrow state prices be given by $q_+$ and $q_-$. After the shock, each variable will remain constant, i.e., $(\Delta_t, \phi_t) = (\Delta_{\tau}, \phi_{\tau})$ for all $t \geq \tau$.

Thus, financiers and households each solve the following problem prior to the shock:

\[
\max_{n_+, n_-} \frac{1}{2} \left( \xi_{n_+}^{1-\gamma} + \xi_{n_-}^{1-\gamma} \right) \geq \frac{1}{2} \left( \xi_{n_+}^{1-\gamma} + \xi_{n_-}^{1-\gamma} \right)
\]

subject to $q_+ n_+ + q_- n_- = n_{\tau}$,

where $n_+, n_-$ are net worths and $\xi_+, \xi_-$ the marginal utility process in the $(+)$ and $(-)$ states at time $\tau$. After the shock, the economy is assumed to be in a Markov equilibrium in the state variable $\eta$. Let $\xi_F(\eta; \Delta_t, \phi_t) := \xi_{F,t}$ and $\xi_H(\eta; \Delta_t, \phi_t) := \xi_{H,t}$ be the equilibrium marginal utility processes in that equilibrium for all $t \geq \tau$.

We have the following results, the proofs of which are omitted because they follow closely the proofs of Di Tella (2017).

**Lemma C.2 (Relative Investment Opportunities).** The function $\Xi(\eta; \Delta, \phi) := \xi_F(\eta; \Delta, \phi)/\xi_H(\eta; \Delta, \phi)$ is strictly decreasing in $\eta$, strictly decreasing in $\Delta$, and strictly increasing in $\phi$.

**Proposition C.3 (Risk Concentration).** Suppose $\gamma > 1$. In the “$\Delta$ experiment”, we have $\eta_+ > \eta_{-\tau} > \eta_-$. In the “$\phi$ experiment”, we have $\eta_+ < \eta_{-\tau} < \eta_-$. In summary, by extending the logic of Proposition C.3, we expect that the financier wealth share $\eta_\tau$ will be positively correlated with $\Delta_\tau$ shocks and negatively correlated with $\phi_\tau$ shocks. Procyclicals of $\Delta_\tau$ induce procyclicality of $\eta_\tau$ – as in canonical models like Basak and Cuoco (1998), Brunnermeier and Sannikov (2014), and He and Krishnamurthy (2013) – but without assuming the typical exogenous trading restrictions. On the other hand, procyclicality of $\phi_\tau$ induces countercyclicality of $\eta_\tau$, opposite to these models. Finally, the cyclicality of $\hat{\sigma}_\tau$ is less likely to matter, due to the neutrality result of Proposition C.1, which is a key difference from Di Tella (2017).

### C.3 Endogenous Financial Innovation

To capture the idea that financial innovation can be an endogenous response to other shocks, I allow for the endogenous choice of diversification $\Delta$. Diversification is chosen optimally by financiers to balance risk and exogenously specified costs. I assume that it is costly for a financier to participate in financial markets at all,
and it is costlier to finance firms located far away from location \( i \), which is embedded in a cost function. In particular, if intermediary \( i \) funds an arc of locations having length \( \Delta_{i,t} \), she incurs the flow non-pecuniary cost \( \frac{1}{2} \zeta(\Delta_{i,t})dt \). I make the following assumptions about the function \( \zeta \), borrowing from Gârleanu et al. (2015).

**Assumption 3 (Diversification Costs).** Assume the function \( \zeta(\Delta) \) has the following properties:

(i) \( \zeta(\Delta) \geq 0 \), \( \zeta'(\Delta) \geq 0 \), and \( \zeta''(\Delta) > 0 \) for all \( \Delta \).

(ii) \( (1 - \Delta)^3 \zeta'(\Delta) \) is increasing in \( \Delta \) for all \( \Delta \).

(iii) \( \zeta(0) = \zeta'(0) = 0 \).

First, whereas Lemma 2.2 shows the benefits of increasing \( \Delta \) in the form of diversification, Assumption 3 describes the costs of diversification. Part (i) of the assumption says that the cost function is positive, and increasing and convex in distance. Part (ii) additionally ensures that the individual financier’s problem is well-defined in that the first-order conditions of the problem are sufficient for optimality. Part (iii) ensures that some diversification is guaranteed.

Now, consider the model of Section 2 augmented with diversification costs. Financier’s problem is augmented by these costs, and they now solve

\[
\max \ U_{t}^{F} - \frac{1}{2} \mathbb{E}_{t} \int_{t}^{\infty} e^{-\rho s} [\zeta_{A}(\Delta_{i,s}^{A}) + \zeta_{B}(\Delta_{i,s}^{B})] ds
\]

subject to (8). Due to orthogonality of sectoral idiosyncratic shocks and additive separability of the diversification cost functions, let us focus on the financier investment in one of the sectors. Drop all sector subscripts/superscripts for the time being.

In a symmetric equilibrium, there is a location-invariant sectoral lending spread, which is unaffected by individual diversification choices. This means diversification is chosen to minimize the sum of portfolio variance and diversification costs \( \zeta \). The portfolio variance is given by \( (\lambda \hat{\sigma} (1 - \Delta))^2 \), where \( \lambda \) is the volume of sector \( z \) lending per unit of net worth. This first-order condition says

\[
\zeta'(\Delta) = (1 - \Delta)(\lambda \hat{\sigma})^2.
\]

Under Part (ii) of Assumption 3 implies that this condition is sufficient for optimality. Next, optimal risk-taking implies \( \lambda = \frac{\hat{\pi}_{F}}{(1-\Delta)\hat{\sigma}} \), so

\[
(1 - \Delta)\zeta'(\Delta) = \hat{\pi}_{F}^2.
\]

Finally, in equilibrium, \( \hat{\pi}_{F} = \eta^{-1} \kappa \phi (1 - \Delta) \hat{\sigma} \), so

\[
\frac{\zeta'(\Delta)}{1 - \Delta} = \left( \frac{\kappa \phi \hat{\sigma}}{\eta} \right)^2.
\]

Assumption 3 implies this equation has a unique interior solution for \( \Delta \). Let this solution be denoted \( \Delta(\alpha, \eta; \zeta) \) since \( (\alpha, \eta) \) are the state variables in equilibrium.

We can do comparative statics on (55) for \( \Delta(\alpha, \eta; \zeta) \). For example, we find that optimal \( \Delta \) falls in response to a proportional reduction in the function \( \zeta(\cdot) \) for a particular sector. A direct financial innovation shock (akin to \( \Delta \uparrow \) in the baseline model) can be achieved by a downward shock to the entire function \( \zeta(\cdot) \). Lower diversification costs induce higher choices of diversification.
Moreover, the endogenous \( \Delta \) now responds to other financial shocks as well. By re-incorporating the second sector and taking ratios, we have

\[
\frac{\zeta_A'(\Delta_A) (1 - \Delta_B)}{\zeta_B(\Delta_B) (1 - \Delta_A)} = \left( \frac{\kappa \phi_A \hat{\sigma}_A}{1 - \kappa) \phi_B \hat{\sigma}_B} \right)^2.
\]  

(56)

The left-hand-side of (56) is increasing in \( \Delta_A/\Delta_B \). Thus, we may think about how relative diversification incentives are affected by other shocks.

For example, consider \( \phi_A \uparrow \) and \( \hat{\sigma}_A \downarrow \), both of which generate reallocation towards sector \( A \) (i.e., \( \kappa \uparrow \)), as discussed in Section 4. An increase in \( \phi_A \), which increases sectoral borrowing from intermediaries, unambiguously increases relative diversification \( \Delta_A/\Delta_B \). Conversely, a decrease in \( \hat{\sigma}_A \), which increases all market participants’ incentives to invest in the affected sector, may actually decrease financiers’ relative diversification \( \Delta_A/\Delta_B \). Endogenous diversification thus amplifies reallocation when \( \phi_A \uparrow \) and dampens reallocation when \( \hat{\sigma}_A \downarrow \). Intuitively, \( \phi_A \uparrow \) is a credit demand shock, which induces a credit supply response, while \( \hat{\sigma}_A \downarrow \) is a credit supply shock, which induces an endogenous retrenchment of credit supply. Lastly, any neutral credit supply shock, i.e., a shock that leaves \( \kappa \) relatively unaffected, cannot affect relative diversification motives.

### C.4 Debt-Like Contracts

A key assumption in Section 2, motivated by standard agency issues and tractability considerations, has been that insiders share risk with financiers through equity-like contracts. In reality, many firms and households borrow in debt contracts. Debt contracts have many distinguishing features, but chief among them is the default option, which generates put-option payoff structure, see Merton (1974). In this section, I introduce a parsimonious device to model this feature and show that it amplifies diversification-induced cycles.

In particular, I suppose sector \( B \) has the same contracts as before, while sector \( A \) now has a time-varying loan-to-value (LTV) ratio \( \phi_{A,t} \). Insiders promise to pay lenders \( \phi_{A,t} dR_{A,t}^A \), where

\[
d\phi_{A,t} = -\gamma_0 \phi_{A,t}(1 - \phi_{A,t}) \sigma_A \cdot dZ_t - \gamma_1 (\phi_{A,t} - \phi_A^*) dt, \quad \gamma_0 > 0, \gamma_1 > 0, \phi_A^* \in (0, 1).
\]

(57)

The motivation for equation (57) is as follows. Debt is informationally-insensitive: when a positive shock occurs, asset returns are greater than debt returns, resulting in a decline in LTV ratios. This mechanism is captured by the term \( -\gamma_0 \phi_{A,t}(1 - \phi_{A,t}) \sigma_A \cdot dZ_t \). The scaling of this term with \( \phi_{A,t}(1 - \phi_{A,t}) \) reflects the natural property that debt has a concave price function, induced by its put-option like payoffs.\(^{45}\) This functional form

\(^{45}\)For example, suppose an underlying asset follows a geometric Brownian motion \( dP_t = P_t [\mu dt + \sigma dZ_t] \) and debt value has the typical properties. Debt is increasing, concave, bounded, sub-linear function of the price: \( D_t = D(P_t) \) with \( D' \geq 0, D'' \leq 1, D''(0) = 1, D''(+\infty) = 0 \). Then, the diffusion of \( \phi_t := D_t/P_t \) is given by

\[
\sigma_t^\phi := -\sigma \phi_t \left( 1 - P_t \frac{D'(P_t)}{D(P_t)} \right) dt.
\]

The stated assumptions imply that \( 0 \leq P_t \frac{D'(P_t)}{D(P_t)} \leq 1 \). Furthermore, as \( P_t \to 0 \),

\[
P_t \frac{D'(P_t)}{D(P_t)} \to 1 \text{ and } \phi_t \to 1.
\]

As \( P_t \to +\infty \),

\[
P_t \frac{D'(P_t)}{D(P_t)} \to 0 \text{ and } \phi_t \to 0.
\]

Consequently, replacing quantity \( P_t \frac{D'(P_t)}{D(P_t)} \) with \( \phi_t \) in \( \sigma_t^\phi \) preserves the qualitative properties of the diffusion.
also keeps $\phi_{A,t} \in (0, 1)$ with probability one. To preserve symmetric equilibrium, notice that $\phi_A$ only loads on aggregate shocks and not idiosyncratic shocks.

The drift term $-\gamma_1(\phi_{A,t} - \phi_A^*)dt$ reflects the fact that debt issuance and repurchase can occur when LTV deviates sufficiently from an “optimal LTV” $\phi_A^*$. As a reduced-form, I model this as a continuous process that strengthens as the equilibrium LTV is further from optimum. With the constant LTV assumption of the previous model, such issuance and repurchase are implicitly costless, and occur so fast as to mitigate the Brownian shocks. In reality, issuance and repurchase are costly and tend to be slow.

There are many other interesting aspects of debt which are omitted from this analysis – deadweight losses in default, costs of debt issuance and repurchases, endogenous default, illiquidity, fixed coupon payments, etc. I leave the exploration of these features for future work.

The equilibrium is solved as follows. No new shocks have been introduced by equation (57). All agents have log utility and consequently no hedging demands to $\phi_A$ shocks. Thus, all equilibrium objects remain the same as in Proposition 3.3, but with $\phi_A$ replaced by a time-varying version.

Economically, time-variation in $\phi_A$ generates interesting dynamics purely from TFP shocks. Consider a positive shock $dZ_A > 0$. As sector $A$ insiders there are now wealthier than sector $B$ insiders ($d\alpha_t > 0$), this generates a reallocation toward sector $A$ ($\kappa_t \uparrow$). To the extent that financiers are levered, the equilibrium will feature $\sigma_t^\eta > 0$, so that the financial sector profits ($d\eta_t > 0$). But with lower $\phi_{A,t}$, future financial sector profitability falls ($\mu_t^\eta \downarrow$), so $\eta_t$ will be expected to drift downward below its starting point, a so-called “overshooting.”

At the same time, there is a slow increase of LTV back towards $\phi_A^*$. The continuous increase in $\phi_{A,t}$ amplifies the reallocation channel, as credit demand increases. Although $\phi_{A,t}$ is expected to return to its steady-state level, which dampens the overshooting of $\eta_t$ but does not eliminate it, since profits are accumulated slowly. Consequently, sector-specific TFP-driven boom can generate a credit demand increase and future financial instability. When $\eta_t$ is low, due to overshooting, the economy is excessively sensitive to a negative TFP shock.
D Empirical Analysis

D.1 Qualitative Support: Why the Model Applies to the US Housing Cycle

Figure 1 shows that financial deregulation and securitization were accompanied by an increase of household credit as a share of total credit and an increase in broker-dealer leverage. A diversification increase is consistent with these patterns, which I have termed the “reallocation effect” and the “leverage effect” in discussing my model.

In addition to these patterns, here I provide some more qualitative support for the mechanism of the model. First, the model requires that the increase in securitization actually improves diversification of mortgage loans. This is not necessarily true a priori: one possibility is that securitization of mortgage loans increases simply because the volume of mortgage lending increases. Figure 21 rejects this by showing that RMBS increase dramatically as a share of total household credit in the US. Moreover, non-agency MBS rise as a share of all MBS. Private label securitizations may be particularly important for diversification, because prior to the securitization boom, the types of loans in these pools were those most likely to be held on banks’ balance sheets until maturity.

Second, it is crucial for my results that diversification in the housing market increases more than diversification in the corporate credit market. This turns out to be true, if we measure diversification by securities, which are likely to be broadly held. Figure 22 shows that mortgage securities outstanding were equal to corporate securities outstanding in 1990, but nearly double by 2007.

Third, my model assumes that the financial sector will adapt to an environment with better mortgage diversification by taking more housing-related risks onto their balance sheets. Figure 23 shows that commercial
banks do indeed hold more housing-related assets on their balance sheets through the housing boom.

![Figure 23: Commercial bank risk-taking in housing markets. “RE Loans / Assets” refers to real estate loans held on bank balance sheets, relative by assets. “MBS / Assets” are mortgage-backed securities held, relative to assets. Source: Call Reports.](image)

Similarly, figure 24 shows that price-to-cash-flow ratios in capital and housing markets do not move in lockstep, suggestive of some sectoral asymmetry in this boom period.

![Figure 24: The price-dividend ratio on the S&P 500 and a measure of house prices relative to the rental rate on housing services. The house price-rent ratio is obtained from http://datatoolkits.lincolninst.edu/subcenters/land-values. The plotted ratio is scaled by 3.](image)

Finally, a key reason financial sector capitalization deteriorates in my model is through declining financier profitability. As diversification improves in the model, financiers are willing to accept lower risk premia on mortgages. Figure 25 shows that commercial banks' profitability declined marginally between the boom years 2000-2007.

![Figure 25: Commercial bank profitability. “Operating Inc / Assets” is operating income, relative to assets. “Interest Inc / Assets” is income from interest payments, relative to assets. Source: Call Reports.](image)
D.2 Quantifying Mortgage Diversification

In this appendix, I describe more specifically the methodology to compute the diversification index of Section 5.1. Start by defining an aggregate mortgage return during month $k$ of year $t$:

$$ R_{t + \frac{k-1}{12} \rightarrow t + \frac{k}{12}} := \sum_{\ell} \omega_{\ell,t} R_{\ell,t + \frac{k-1}{12} \rightarrow t + \frac{k}{12}} $$

where $\omega_{\ell,t} := \frac{s_{\ell,t} + m_{\ell,t}}{\sum_{\ell'} s_{\ell',t} + m_{\ell',t}}$ are origination weights:

- $m_{\ell,t} :=$ portfolio mortgages originated to location $\ell$ in year $t$
- $s_{\ell,t} :=$ securitized mortgages originated to location $\ell$ in year $t$.

The location-specific mortgage return $R_{\ell,t + \frac{k-1}{12} \rightarrow t + \frac{k}{12}}$ is proxied by the housing return in location $\ell$ and month $k$ of year $t$, taken from CoreLogic. This is the return building block for all other returns. The aggregate return $R_{t + \frac{k-1}{12} \rightarrow t + \frac{k}{12}}$ allows me to extract the idiosyncratic components of all other returns.

In an analogous fashion, define the mortgage return for intermediary $i$:

$$ R_{t + \frac{k-1}{12} \rightarrow t + \frac{k}{12}}^{(i)} := \sum_{\ell} \omega_{m,\ell,t}^{(i)} R_{\ell,t + \frac{k-1}{12} \rightarrow t + \frac{k}{12}} + \omega_{s,\ell,t}^{(i)} R_{t + \frac{k-1}{12} \rightarrow t + \frac{k}{12}} $$

where

$$ \omega_{m,\ell,t}^{(i)} := \frac{m_{\ell,t}^{(i)}}{\sum_{\ell'} s_{\ell',t}^{(i)} + m_{\ell',t}^{(i)}} \quad \text{and} \quad \omega_{s,\ell,t}^{(i)} := \frac{s_{\ell,t}^{(i)}}{\sum_{\ell'} s_{\ell',t}^{(i)} + m_{\ell',t}^{(i)}} $$

- $m_{\ell,t}^{(i)} :=$ portfolio mortgages originated by lender $i$ to location $\ell$ in year $t$
- $s_{\ell,t}^{(i)} :=$ sold mortgages originated by lender $i$ to location $\ell$ in year $t$.

Note that any mortgages originated by intermediary $i$ which are then sold off within the same year are captured by $s_{\ell,t}^{(i)}$. I make the assumption that these sales return the aggregate return $R_{t + \frac{k-1}{12} \rightarrow t + \frac{k}{12}}$, which is subject to no idiosyncratic risk. Loans not sold are captured by $m_{\ell,t}^{(i)}$. These loans are assumed to get the location-specific return $R_{\ell,t + \frac{k-1}{12} \rightarrow t + \frac{k}{12}}$.

Next, I define “idiosyncratic returns” by subtracting the aggregate return:

$$ R_{t + \frac{k-1}{12} \rightarrow t + \frac{k}{12}}^{(i)} := R_{t + \frac{k-1}{12} \rightarrow t + \frac{k}{12}} - R_{t + \frac{k-1}{12} \rightarrow t + \frac{k}{12}}^{\ast} $$

The (monthly) idiosyncratic variances in year $t$ are then given by

$$ V_{t,t} := \frac{1}{12} \sum_{k=1}^{12} \left( R_{t,t + \frac{k-1}{12} \rightarrow t + \frac{k}{12}}^{(i)} \right)^2 - \left( \frac{1}{12} \sum_{k=1}^{12} R_{t,t + \frac{k-1}{12} \rightarrow t + \frac{k}{12}} \right)^2 $$

$$ V_{i,t} := \frac{1}{12} \sum_{k=1}^{12} \left( R_{t,t + \frac{k-1}{12} \rightarrow t + \frac{k}{12}}^{(i)} \right)^2 - \left( \frac{1}{12} \sum_{k=1}^{12} R_{t,t + \frac{k-1}{12} \rightarrow t + \frac{k}{12}}^{(i)} \right)^2 $$

In this computation, I am using the fact that the returns are computed monthly, while the originations and securitizations data are only available at a yearly frequency.

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I average over locations and intermediaries to get the volatilities that I want:

\[ \hat{\sigma}_t := \sum_{\ell} \omega_{\ell,t} V_{\ell,t} \]

\[ \hat{\sigma}_{\Delta,t} := \sum_i \omega_{i,t} V_{i,t} \]

where

\[ \omega_{i,t} := \sum_{\ell} \frac{m_{\ell,t}^{(i)} + s_{\ell,t}^{(i)}}{\sum_{\ell,j} m_{\ell,j}^{(i)} + s_{\ell,j}^{(i)}} \]

Note that we necessarily have \( \hat{\sigma}_{\Delta,t} \leq \hat{\sigma}_t \), because correlations between the loans in lender’s portfolios are less than 1, while loan-level volatilities are proxied by location-specific volatilities.

Finally, in the symmetric equilibrium, the following equation relates financiers’ housing risk \( \hat{\sigma}_{\Delta} \) to the fundamental housing risk \( \hat{\sigma} \) and the level of diversification \( \Delta \):

\[ (1 - \Delta) \hat{\sigma} = \hat{\sigma}_{\Delta} = \text{idio volatility of unlevered mortgage portfolio}. \]

Thus, I define

\[ \Delta_t := 1 - \frac{\hat{\sigma}_{\Delta,t}}{\hat{\sigma}_t} \]

by inverting this relation. The units of \( \Delta_t \) are the fraction of fundamental housing risk that are eliminated from lender’s portfolios, either through loan sales and securitizations, or through geographic diversification.
References


Risk of Risk-Sharing

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