ACCOUNTING WORKSHOP

The Effect of Exogenous Information on Voluntary Disclosure and Market Quality

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The Effect of Exogenous Information on Voluntary Disclosure and Market Quality

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Abstract

We analyze a disclosure model in which information may be voluntarily disclosed by a firm and/or by a third party such as a financial analyst. Under plausible assumptions, analyst coverage crowds out disclosure by the firm. Despite this crowding out, we show that an increase in analyst coverage increases the quality of public information. While ranking based on Blackwell informativeness cannot be obtained, we base this claim on two measures of public information. The first, price efficiency, is statistical in nature while the second, expected bid-ask spread, is based on liquidity in a trading stage that follows the information disclosure.

JEL Classification: G14, D82, D83.

Keywords: information disclosure, voluntary disclosure, price efficiency, liquidity, analysts.

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1 Introduction

There is a growing literature on voluntary disclosure that studies how agents or firms strategically decide whether to disclose or withhold their private information. Public companies, for example, are mandated to disclose certain information in their periodic reporting, but some information is disclosed at the discretion of the manager. For example, a firm does not have to disclose that a major customer is negotiating a deal with one of its competitors. Hence, corporate voluntary disclosure is a major source of information in capital markets.\(^1\) Another example is an entrepreneur who seeks funding from investors (VC funds, angels, etc.); the entrepreneur can choose whether to disclose or conceal the results of previous attempts to raise funding and acquire new customers. Examples are not limited to financial markets. An incumbent politician may obtain private information about the success or failure of policies he has supported, and can choose whether to disclose or conceal these results. In all of these examples, informed agents are reluctant to lie because of severe or even criminal punishment, or because once the information is voluntarily disclosed it can be easily verified. Instead, they can choose to withhold negative information, taking advantage of public uncertainty about the possibility that they have private information.

The current literature covers extensively settings where a single agent chooses whether to disclose or withhold private information. This literature has not considered the possibility of an additional source of information that may discover and reveal the agent’s private information. Such sources, however, are very common. For example, financial analysts and rating agencies provide additional information about public firms, investors can gather information through their social network about an entrepreneur, and the media and independent think-tanks can assess public policies.

In this paper, we examine the effect of such additional sources of information. Our main question is how the probability of arrival of information from external source affects

\(^1\)Beyer et al. (2010) find that approximately 66% of accounting-based return variance is generated by voluntary disclosures, 22% is due to analyst forecasts, 8% is due to earnings announcements, and 4% is due to SEC filings.
to the aggregate amount of information that becomes public. In order to answer this question, we need to understand how this external source affects the agent’s disclosure policy, and account for the reaction of the agent.

Our model departs from a standard voluntary disclosure setting with uncertain information endowment (a la Dye, 1985 and Jung and Kwon, 1988). A manager of a public firm, who wishes to maximize his firm’s price, may be endowed with private value-relevant information. The financial market prices the firm based on all publicly available information. If the manager is informed, she can credibly and costlessly disclose her information to the market. The novelty of our model is the additional external source of information, e.g., an analyst, who may discover and publish the information held by the manager. We assume the analyst may discover and publish information when the manager is informed as well as when she is uninformed, and allow for correlation in the manager’s and analyst’s endowment of information.

We first show that, as standard in this literature, the game has a unique equilibrium, in which the manager discloses the realization of her private information if and only if it is higher than an equilibrium threshold. We then study how the firm’s disclosure strategy changes in response to an increase in analyst coverage, i.e., an increase in the probability that the analyst discovers and publishes the private information. We show that, under plausible assumptions, analyst coverage crowds-out corporate voluntary disclosure, i.e., firms respond to an increase in analyst coverage by increasing the disclosure threshold and decreasing the amount of information they disclose. This result, which is new to the theoretical literature, is consistent with the empirical evidence in Anantharaman and Zhang (2011) and Balakrishnan et al. (2014) (see more details on the empirical literature below).

Given the crowding-out result, a second and more challenging question is what is the effect of an increase in analyst coverage on the overall amount of public information - including both the information disclosed by the analyst and by the manager. We find that the information cannot be ranked using the Blackwell informativeness criterion because more exogenous information does not lead to a finer information partition. Instead, we
use two separate measures to capture the overall information available to the market. First, we consider a quadratic loss function, which equals the expected squared difference between the firm’s actual and perceived value. This measure has a natural interpretation in terms of price efficiency or ex-post return volatility. It can also represent the utility function of the receiver, and is consistent with the assumption that such receivers set prices to be equal to the expected value, conditional on all available information.

Our second measure of information quality is more specific to the capital market example and can be linked to empirical findings. We use the expected bid-ask spread as a measure that reflects the extent of information asymmetry in the market. We augment the disclosure model by introducing a trading stage a la Glosten and Milgrom (1985) that follows the disclosure stage by the manager and the analyst. The trade and pricing in this stage are affected by the information revealed in the disclosure stage.

Our analysis establishes that both price efficiency and liquidity increase as a result of an increase in analyst coverage, that is, the overall effect of an increase in third party disclosure on market quality is always positive.\(^2\) This result arises from the qualitative difference between voluntary disclosure and information provided by third parties. Firms always choose to disclose news that is strictly good and hide news that is strictly bad, so the disclosure threshold is the news that will result in a price that equals expected price given no disclosure.\(^3\) A slight decrease in the firm’s disclosure threshold will have only a minor effect on the price of firms that disclose only under the new threshold. In contrast, a slight increase in the probability of disclosure by third parties (such as financial analysts) may result in discovery of news that has a more significant effect on prices. Thus, a change in the probability of disclosure by a third party will, at the margin, affect prices more than a change in the probability of voluntary disclosure.

Our results can be used to assess certain policies to increase market transparency. Such

\(^2\) In contrast to our results, Goldstein and Yang (2017) analyze an REE model a la Grossman and Stiglitz (1980) and demonstrate that the effect of an increase in the precision of public information may not be positive. Their result arises due to weaker incentives of traders to acquire information, an aspect that is not present in our model.

\(^3\) This is not necessarily true in dynamic voluntary disclosure models.
policies often focus on increasing the information provided by one market participant. Financial analysts have an important role in revealing firms’ private information to capital market, but there are other sources of exogenous revelation, e.g., media, social media, competitors, suppliers and government. Our results show that an improvement in one information source may crowd-out information from another source, and that different parties affect the overall public information differently. Our model suggests that to the extent that increasing the likelihood of such information discovery is not too costly, it is beneficial in terms of price efficiency and liquidity.

Unlike the theoretical literature (see the review in the next subsection), the empirical literature has studied the effect of analyst coverage on a firm’s voluntary disclosure and on the liquidity of the firm’s stock. The empirical literature supports the predictions of our model. For example, Kelly and Ljungqvist (2012) show that following an exogenous decrease in analyst coverage, due to mergers and closings in the brokerage industry, the affected firms’ information asymmetry increased and their stocks’ liquidity decreased.

Anantharaman and Zhang (2011) and Balakrishnan et al. (2014) use the same exogenous negative shock to analyst coverage that is used in Kelly and Ljungqvist (2012) to establish the effect of a decrease in analyst coverage on firms’ voluntary disclosure. Balakrishnan et al. (2014) show that one quarter following the decrease in analyst coverage, the affected firms increased their voluntary disclosure (earning guidance) to mitigate the increased information asymmetry and the decreased liquidity. This increased disclosure partially reverses the decrease in liquidity, although the overall effect remains negative, consistent with the prediction in our model.

There is a more extensive empirical literature that studies how disclosure and transparency affect the informational environment in general and the bid-ask spread in particular. While the results are mixed, many papers find that increased disclosure increases infor-

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4Examples of regulations that focus on information provision include: the Sarbanes-Oxley Act attempted to increase the mandated reporting of firms; the Williams Act of 1968 limits the ability of investors to trade anonymously on their private (optimistic) information; the regulation on analyst certification (Reg AC) requires analysts to disclose possible conflicts of interests and prevent biased reports; the Dodd-Frank Act includes several measures aimed at improving the transparency and viability of credit ratings. See also Goldstein and Yang (2017).
mativeness and decreases the bid-ask spread (e.g. Welker 1995; Healy et al. 1999; Leuz and Verrecchia 2000; Heflin et al. 2005; Leuz and Wysocki 2016).

1.1 Related Theoretical Literature

Our study of voluntary disclosure in the presence of potentially informed traders contributes to two streams of the theoretical literature. The first is the voluntary disclosure literature. To the best of our knowledge, only a few theoretical papers study voluntary disclosure in the presence of a potentially informed trader/receiver. Langberg and Sivaramakrishnan (2008, 2010) offer two models with a firm that can voluntarily disclose information and strategic analysts that can disclose additional information. In these papers, the analyst’s information is orthogonal to the information of the firm; in our model, analysts and the firm potentially learn the same information. This makes the analysis very different. Moreover, in these papers, by construction, greater firm disclosure encourages the analysts to obtain more information. Einhorn (ming) also explores the effect of additional information sources on voluntary disclosure. She focuses on an equilibrium where the firm’s disclosure strategy is independent of the fundamental value and thus cannot be affected by other sources of information. The closest work with an informed receiver is Ispano (2016), whose model, while very different, can be seen as a simplified version of our model with three states and a specific analyst technology. He shows, in his discrete example, that the utility of the receiver – which is equivalent to price efficiency in our setting – is increasing with the probability that the receiver is informed. He does not discuss liquidity. Dutta and Trueman (2002) study a setting in which firm’s manager can credibly disclose verifiable private information, but cannot disclose additional information about how to interpret this information. The manager, not knowing whether the market will interpret the disclosed information as good news or bad news, faces uncertainty about the market reaction. In this setting, Dutta and Trueman (2002) show that the equilibrium disclosure strategy is not necessarily a threshold strategy. Banerjee and Kim (2017) explore a model where a manager may disclose information to the public and
may also use private cheap-talk communication to contact her employees. With some probability, private communication is “leaked” and becomes public. It turns out that the possibility of private communication becoming public has very different implications than the possibility of the manager’s information becoming public, as in our model. Finally, several papers deal with disclosure of two strategic firms/experts (Bhattacharya and Mukherjee, 2013; Bhattacharya et al., 2018; Kartik et al., 2017). In those papers, as in ours, agents consider the possibility of additional information due to disclosure by their peers, but these papers do not focus on the change in overall information due to an increase in the quality of information from one of the agents.  

The second stream of literature studies how changes in one source of information affect the incentives for acquiring information among other parties, and the overall effect on public information. Goldstein and Yang (2017) present a noisy Rational Expectation Equilibrium (REE) model with a public signal that can be interpreted either as corporate mandatory disclosure or as disclosure by a third party, such as an analyst. They show that when keeping traders’ information constant, a more precise public signal improves market liquidity and price efficiency. However, better public information undermines the incentives for traders to acquire information, so the overall effect is ambiguous and depends on how market quality is measured. By contrast, we endogenize the corporate disclosure decision and allow for voluntary rather than mandatory disclosure. Another related paper is Gao and Liang (2013), which studies how a firm’s commitment to disclosure affects investors’ incentives to acquire information. Their focus is on the feedback effect, whereby the firm’s manager learns from prices.

In the next section, we describe the setting of our model. Our objective is to address three questions pertaining to the voluntary disclosure setting with the possibility of an exogenous signal. First, how the introduction of an exogenous signal affects the equilibrium of the disclosure game, in particular the likelihood of voluntary disclosure.

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5 The effect of an informed receiver on the sender’s strategy is also explored in the literature that follows the “Bayesian Persuasion” paradigm, that is, assume the sender commits ex-ante on a disclosure strategy. See, for example, Rayo and Segal (2010); Kolotilin (2018); Azarmsa and Cong (2018).
and the price given no disclosure. This analysis is presented in Section 3. Second, since the presence of an exogenous signal affects the manager’s disclosure strategy, how does a change in the probability of an exogenous signal, e.g., through a change in analyst coverage, affect price efficiency. We answer this in Section 4. Finally, we study how changes in analyst coverage affect the liquidity of the firm’s stock, as captured by the expected bid-ask spread. To do that, in Section 5 we introduce, and analyze, and extended model that includes a stylized trading stage a la Glosten and Milgrom (1985).

2 Setting

Our model builds on the voluntary disclosure literature initiated by Grossman (1981), Milgrom (1981), and Dye (1985). We consider a firm that is involved in a project, e.g., drug R&D, which will either succeed or fail. We denote the terminal value of the firm by $x \in \{0, 1\}$ where $x = 1$ following success and $x = 0$ following failure. The ex-ante probability of success is $\mu_0 \equiv \Pr (x = 1)$ and the probability of failure is $1 - \mu_0 \equiv \Pr (x = 0)$.

**Information Structure** With probability $q \in (0, 1)$, the manager of the firm observes additional information about the possible outcome of the project, in the form of a signal $s$. With probability $1 - q$ the manager does not observe a signal. Information endowment is independent of the realization of $s$, and therefore the ex-ante expected value of $s$ (or $x$) conditional on an information event equals the expected value conditional on no information event and equals $\mu_0$. The signal may represent, for example, the results of a clinical trial or an oil exploration, information about competing projects/firms or information about relevant macroeconomic conditions. We assume that all players in the game are risk neutral, and thus it is without loss of generality to assume that the signal $s$ is simply the updated probability of success. That is, the posterior belief given the realization of $s$ is $\Pr (\tilde{x} = 1|s) = s$. Hence, we assume that $\tilde{s} \in [0, 1]$, with a PDF $f(s)$, a continuous CDF $F(s)$, and $E[\tilde{s}] = \mu_0$. 

Remark 1 (Alternative Information Structures). Most of our results extend to arbitrary continuous distributions of \( \tilde{x} \) and \( \tilde{s} \). This include all the results in Sections 3 and 4. The binary structure is only used to simplify the trading stage in Section 5.

**Disclosure and Pricing** If the manager learns the realization of the private signal \( s \), she can voluntarily disclose it to the market. Disclosure is assumed to be costless and credible (verifiable at no cost). As standard in the voluntary disclosure literature, if the manager does not obtain the private signal, she cannot credibly convey that she is not informed. The manager seeks to maximize the market value, or price, of the firm.\(^6\) For now, assume that risk neutral investors set the market price, \( P \), equal to the expected value conditional on all the available public information, \( I \). That is, \( P = E [s | I] = E [x | I] \).

Later, in Section 5, we introduce a trading stage that follows Glosten and Milgrom (1985) where prices are set by a centralized market maker.

The setting introduced so far is similar to a standard voluntary disclosure setting with uncertainty about information endowment, which has been studied extensively. The main innovation of our setting is the possibility that the signal \( s \) will become public by an external source.

**Analyst (Exogenous Signal)** We use financial analysts as our main motivating example, however, any mechanism that induces stochastic public supply of the firm’s information, such as media, competitors, suppliers, social media, regulators etc., will have a similar effect in our model.

To study the interaction between firm’s voluntary disclosure and the potentially informed market, we add to the above setting a financial analyst, who may also learn the realization of the updated probability of success, \( s \). We abstract from strategic considerations of the analyst, and assume that whenever analysts discover information they publish

\(^6\) As standard in the literature, we take the performance-based compensation of the manager as given. Such compensation may be an optimal contract when the manager has additional activities, which are left unmodeled, that demand effort (as in Holmström, 1979, 1999). Such compensation is also optimal when the market / receiver wishes to price the firm “correctly” (Hart et al., 2017).
it truthfully.\textsuperscript{7}

The likelihood that the analyst will discover this information may depend on whether the manager is informed or not. For example, if the information $s$ is the result of a clinical drug trial, it is unlikely that the analyst will discover this information before the manager does. However, if the signal $s$ is information about market conditions, the analyst may discover this information even when the manager is uninformed. To allow for both types of information, we assume a relatively non-restrictive analyst’s information production technology. In particular, assume that the analyst’s information production technology is reflected by a pair of conditional probabilities $(g_I(r), g_U(r))$, where $g_I(r) \in [0, 1)$ and $g_U(r) \in [0, 1)$ are the probabilities that the analyst discovers $s$ conditional on the manager being informed and uninformed, respectively. We introduce the parameter $r$ to capture the overall quality and/or quantity of analysts that cover the firm. We refer to $r$ as “analyst coverage.” An increase in analyst coverage weakly increases the probability that the analyst becomes informed when the manager is informed and when the manager is uninformed. For simplicity, we assume that $g_I$ and $g_U$ are differentiable, and thus assume $g'_I(r) \geq 0$ and $g'_U(r) \geq 0$, with at least one strict inequality. Note that the ex-ante probability that the analyst issues a report is $q \cdot g_I(r) + (1 - q) \cdot g_U(r)$.

\textbf{Timeline} To summarize our disclosure game, the timeline is as follows.

1. With probability $q$ the manager privately learns the signal $s$.

2. If the manager is informed, she decides whether to publicly disclose $s$ or not.

3. Analysts learn the signal $s$ with probabilities $g_I(r)$ or $g_U(r)$, depending on the outcome of stage 1. An informed analyst immediately discloses $s$ to the market.

4. Following the disclosure or lack of disclosure by both the manager and the analyst, market participants update their beliefs about the expected value of the firm/project.

\textsuperscript{7}It is immediately obvious that all of our results are robust to an analyst’s reporting strategy that is potentially biased, as long as the analyst always issues a report when obtaining information and the analyst’s forecast follows a separating strategy. For an example and additional references see Beyer and Guttman (2011).
5. The price of the firm is determined, and the manager is compensated accordingly.

We first assume risk neutral pricing, and in Section 5 we specify a market mechanism that generates the price.

The setting and all the parameters of the model are common knowledge.

Remark 2 (Alternative Timing). The information that the manager and the analyst may learn and disclose is identical. Thus, the manager’s disclosure is relevant only in the case the analyst has not published a report. This implies that even if the manager observes the report by the analyst before making her disclosure decision (that is, even if stage 3 is before stage 2), the equilibrium is essentially the same: following a disclosure by the analyst the manager is indifferent whether to disclose or not, and following no report by the analyst the manager’s strategy is identical to her strategy in the current model.

3 The Disclosure Game

3.1 Equilibrium Disclosure Strategy

Given the realized signal, an informed manager chooses a disclosure strategy that maximizes the expected firm price. If \( s \) is publicly disclosed either by the manager or by the analyst – an event we denote by “D” – the price of the firm equals its expected value, i.e.,

\[
P^D(s) \equiv E[\tilde{x}|s] = s.
\]

Denote by “ND” the event that neither the manager nor the analyst disclosed \( s \), and by \( P^{ND} \) the price following such an event. \( P^{ND} \) is the market’s belief about the firm’s expected value following no disclosure., i.e., \( P^{ND} \equiv E[\tilde{x}|ND] \).

The manager’s disclosure decision affects the price only when \( s \) is not disclosed by the analyst. Thus, though an informed manager does not know whether the analyst will be informed or not, she conditions her decision only on the event that the analyst will not be informed, taking onto account the probability of this event. When the analyst
is not informed, an informed manager’s optimal strategy is to disclose $s$ if and only if $P^D(s) > P^{ND}$. While $P^D(s)$ is increasing in $s$, $P^{ND}$ is independent of the manager’s type. Therefore, any equilibrium disclosure strategy is characterized by a threshold signal - which we denote by $\sigma$ - such that an informed manager discloses if and only if $s \geq \sigma$.

The price following no disclosure by the manager or the analyst, $P^{ND}$, depends on the market’s belief about the manager’s disclosure strategy. If the market believes the manager uses a disclosure threshold $\sigma$, then the price following no disclosure is given by

$$P^{ND}(\sigma) \equiv E[\hat{s}|ND, \sigma] = \frac{(1 - q) \cdot (1 - g_U(r)) \cdot E[\hat{s}] + qF(\sigma) \cdot (1 - g_I(r)) \cdot E[\hat{s}|s < \sigma]}{(1 - q) \cdot (1 - g_U(r)) + qF(\sigma) \cdot (1 - g_I(r))}.$$  

(1)

The price is a weighted average of the prior mean and the mean conditional on withholding signals below $\sigma$, with weights representing the conditional probabilities that the manager is informed and uninformed. Thus, for any exogenously given disclosure threshold $\sigma \in (0, 1)$ the price given no disclosure is lower than the prior mean, that is, $P^{ND}(\sigma) < E[\hat{s}] = \mu_0$.

Our disclosure model generalizes Dye (1985) and Jung and Kwon (1988) to a setting that contains an additional stochastic public revelation mechanism. Formally, those models are a particular case of our setting in which $g_I(r) = g_U(r) = 0$. It is easy to extend the analysis in Jung and Kwon (1988) and show that a threshold equilibrium exists, and that it is unique.

**Fact 1.** There exists a unique equilibrium to the disclosure game, in which an informed manager discloses if and only if the signal $s$ is greater than a disclosure threshold $\sigma^*$. $\sigma^*$ is the unique solution of the manager’s indifference condition - between disclosing and not disclosing her signal $s = \sigma^*$

$$\sigma^* = P^{ND}(\sigma^*).$$

(2)

An additional useful property of disclosure games that also holds in our model is the Minimum Principle property, first described by Acharya et al. (2011). This property shows that $P^{ND}(\sigma)$ is minimized under the equilibrium threshold.

**Fact 2** ("The Minimum Principle," Acharya et al. 2011, Proposition 1). The equilibrium
threshold $\sigma^*$ is the unique disclosure threshold that minimizes the price given no disclosure, that is, $\sigma^* = \min_\sigma P^{ND}(\sigma)$.

An immediate corollary of the minimum principle is that a change in any parameter, for example $r$, that increases or decreases the function $P^{ND}(\sigma)$ for any exogenous disclosure threshold $\sigma$, will increase or decrease the equilibrium threshold $\sigma^*$. If, for example, a change in $r$ increases the price following no disclosure for any disclosure threshold, then, by the minimum principle, it must increase the equilibrium threshold (that is, decrease disclosure). This is formalized in the following corollary.

**Corollary 1.** The equilibrium disclosure threshold $\sigma^*$ is increasing (decreasing) in $r$, if and only if $P^{ND}(\sigma)$ is increasing (decreasing) in $r$.

### 3.2 The Effect of Analyst Coverage on the Disclosure Strategy

In this section we analyze the main comparative static of the disclosure game – how the level of analyst coverage, $r$, affects the manager’s equilibrium disclosure threshold, $\sigma^*$.

Based on Corollary 1, to study the effect of analyst coverage on corporate disclosure, we need to study how analyst coverage affects the price given no disclosure for an exogenous disclosure threshold $\sigma$, i.e., $\frac{\partial P^{ND}(\sigma)}{\partial r}$. Note from (1) that, for any exogenous disclosure threshold $\sigma$, $\frac{\partial P^{ND}(\sigma)}{\partial g_I(r)} > 0$ and $\frac{\partial P^{ND}(\sigma)}{\partial g_U(r)} < 0$. Higher $g_I(r)$ means that the analyst is more likely to discover and publish $s$ when the manager is informed. Thus, no disclosure when $g_I(r)$ is higher implies that it is less likely that the manager is informed and withholds negative information. Therefore, an increase in $g_I(r)$ increases $P^{ND}$. In contrast, higher $g_U(r)$ means that the analyst is more likely to discover and disclose $s$ when the manager is uninformed. Thus, no disclosure when $g_U(r)$ is higher implies that it is more likely that the manager is informed and is withholding negative information. Therefore, an increase in $g_U(r)$ decreases $P^{ND}$. The overall effect of an increase in $r$ on the price given no disclosure is

$$
\frac{\partial P^{ND}(\sigma)}{\partial r} = \frac{\partial P^{ND}(\sigma)}{\partial g_I(r)} g_I'(r) + \frac{\partial P^{ND}(\sigma)}{\partial g_U(r)} g_U'(r).
$$
Since both $g_I(r)$ and $g_U(r)$ increase in $r$, the overall effect of changes in $r$ on $P_{ND}$ is not clear. Without further assumptions about the functions $g_I(r)$ and $g_U(r)$, one cannot conclude whether an increase in analyst coverage increases or decreases voluntary disclosure. Next, we provide the condition that determines whether the manager’s disclosure threshold is increasing or decreasing in $r$.

### 3.2.1 Condition for the Crowding Out Effect of Analyst Coverage

In order to study the effect of analyst coverage on the equilibrium disclosure strategy, it is useful to consider the following function

$$m(r) \equiv \frac{\Pr(\text{analyst is uninformed} \mid \text{manager is uninformed})}{\Pr(\text{analyst is uninformed} \mid \text{manager is informed})} = \frac{1 - g_U(r)}{1 - g_I(r)}. \quad (3)$$

$m(r) \in [0, \infty)$ is the ratio between the likelihood that the analyst does not discover and discloses $s$ when the manager is uninformed and the likelihood the analyst does not disclose $s$ when the manager is informed. For convenience, we henceforth refer to $m(r)$ as the “informed analyst ratio.”

Denote by $\sigma_D^*$ the disclosure threshold in a model with no analyst, i.e., when $g_U = g_I = 0$. This is the classic Dye (1985) model. We first show that the size of $m(r)$ determines whether the presence of an analyst increases or decreases voluntary disclosure compared to the standard Dye (1985) model.

**Lemma 1.** The firm discloses less information compared to the case where an analyst is not available if and only if the informed analyst ratio is greater than one; that is

$$\sigma^*(r) > \sigma_D^* \iff m(r) > 1.$$

**Proof.** Rewrite 1 as

$$P_{ND}(\sigma, r) = \frac{(1 - q)E[\tilde{x}] + q \cdot m(r)^{-1} \cdot F(\sigma) E[\tilde{x} \mid s < \sigma]}{1 - q + q \cdot m(r)^{-1} \cdot F(\sigma)}. \quad (4)$$
Clearly from (3), when \( g_I = g_U = 0 \) then \( m = 1 \). Thus \( P^{ND}(\sigma, r) > P^{ND}(\sigma, r) \mid_{g_I=g_U=0} \) if and only if \( m(r) > 1 \). The lemma then follows from Corollary 1.

We now turn to the effect of changes in analyst coverage on the level of voluntary disclosure, i.e., on the disclosure threshold. The following proposition shows that this effect depends on the directional change in \( m(r) \) when \( r \) changes.

**Proposition 1.** In equilibrium, analyst coverage crowds out corporate voluntary disclosure if and only if \( m'(r) > 0 \), that is,

\[
\frac{\partial \sigma^*}{\partial r} > 0 \iff m'(r) > 0.
\]

**Proof.** By (4) and the fact that \( E[\tilde{s}] > E[\tilde{s} \mid s < \sigma] \), it is clear that \( P^{ND}(\sigma, r) \) is increasing in \( m(r) \). Thus, \( \frac{\partial P^{ND}(\sigma)}{\partial r} > 0 \) iff \( m'(r) > 0 \). This, together with Corollary 1, gives the desired result.

Higher \( m(r) \) means that the analyst is relatively more likely to be uninformed when the manager is uninformed than when the manager is informed. Thus, if the analyst does not report, this signals that the manager is more likely to be uninformed. Formally, as shown by (4),

\[
\Pr(\text{manager is uninformed} \mid \text{ND}) = \frac{1 - q}{1 - q + q \cdot m(r)^{-1} \cdot F(\sigma)}.
\]

Therefore, higher \( m \) gives the manager a higher payoff in the case that the analyst does not publish a report, and thus a higher incentive to withhold. Note that, as discussed above, the probability that the analyst becomes informed does not enter the manager’s payoff function in any way except through \( P^{ND} \).

### 3.2.2 Information Structure Examples

Since the effect of analyst coverage on voluntary disclosure depends on \( m(r) \), i.e., on the information structure, we offer two relatively simple examples of information structures,
that we find appealing and realistic.

**Example 1** (Private Inquiry andLeaks). Suppose that the manager learns $\tilde{s}$ with probability $q$. The analyst has two potential sources of information, one within the firm and the other external. Examples for external sources could be information about the industry or macroeconomic conditions. Further assume that the probability that the analyst learns $s$ from an external source is $r$ and this probability is independent of whether the manager is informed or not. One interesting case of this example is $r = 0$, which may represent the results of a clinical trial or oil and gas drilling, that are unlikely to be available to the analyst and not to the manager.

The inside source of information captures information that is “leaked” to the analyst from within the firm. Such information can be observed by the analyst only when the manager is informed. Suppose that the probability that the analyst learns $s$ from insiders, conditional on the manager being informed, is $\delta(r) \in (0, 1)$. Naturally we assume that an increase in analyst coverage increases the probability of leaks. For simplicity, we assume that $\delta(r)$ is differentiable, and $\delta'(r) > 0$. In this example, we obtain $g_U(r) = r$ and $g_I(r) = r + (1 - r)\delta(r)$.

**Example 2** (Conditionally Independent Information Endowment). Suppose that with probability $\omega \in (0, 1)$ some information event occurs and with probability $1 - \omega$ no information event occurs. If no information event occurs, the firm’s expected value remains the prior mean ($\mu_0$). However, if an information event occurs, it generates a new probability of success $s$, which is the posterior expected value of the firm.

Conditional on an information event occurring, the probability that the analyst discovers $s$ is $r$, and the probability of the manager discovering $s$ is $\frac{q}{\omega}$ (so the overall probability that the manager discovers $s$ is $q$). Assume that the information endowment events of the manager and the analyst are independent, conditional on an information event. This structure implies

\[
g_U(r) = \frac{\omega(1 - \frac{q}{\omega})r}{1 - q} = \frac{\omega - q}{1 - q} \cdot r \quad \text{and} \quad g_I(r) = \frac{\omega q r}{q} = r.
\]
One can easily verify that \( m'(r) > 0 \) in both examples.\(^8\) Given Proposition 1, this implies that an increase in analyst coverage increases the manager’s disclosure threshold, i.e., \( \frac{\partial \sigma^*}{\partial r} > 0 \). In other words, in both of these examples an increase in analyst coverage crowds out voluntary disclosure.

### 3.3 Assumption about Analyst’s Information Production

Following the two examples above, in what follows we focus our attention on the case where analyst coverage crowds out disclosure. That is, we assume the following regarding the analyst’s information production technology \((g_I(r), g_U(r))\):

**Assumption 1.** The informed analyst ratio \( m(r) \), as calculated in (3), is increasing in \( r \).

Note that this assumption is supported by the empirical literature presented above (Anantharaman and Zhang, 2011; Balakrishnan et al., 2014). Moreover, in the case where \( m(r) \) is decreasing in \( r \), and thus voluntary disclosure is increasing in analyst coverage, the main results of the paper regarding price efficiency and liquidity will trivially hold.

### 4 Price Efficiency

An increase in analyst coverage, \( r \), by definition increases the probability that the signal will be disclosed, and thus directly provides more public information. However, as established above, an increase in analyst coverage also affects voluntary disclosure. In particular, given Assumption 1, an increase in \( r \) decreases corporate voluntary disclosure (Proposition 1). As such, the overall effect of changes in analyst coverage on investors’ information, or price informativeness, is not clear. In this section we assess the overall effect of an increase in analyst coverage. This effect can be decomposed into two parts:

\(^8\)Note that \( m'(r) > 0 \) if and only if \[ \frac{g'_I(r)}{1-g_U(r)} < \frac{g'_U(r)}{1-g_I(r)}. \]
• A change in the probability that the signal \( s \) is made public, either by the manager and/or by the analyst. The probability of this event is given by

\[
q \cdot g_t(r) + q (1 - g_t(r)) (1 - F(\sigma^*)) + (1 - q) g_U(r).
\]

As mentioned before, since the manager’s equilibrium disclosure threshold, \( \sigma^* \), is increasing in analyst coverage \( r \), it is not clear whether this probability increases or decreases following an increase in \( r \).

• Market uncertainty regarding \( s \) in case it does not become public. An increase in \( r \) affects the manager’s disclosure strategy and as a result also affects the distribution of types given no disclosure, and hence the uncertainty given no disclosure.

As one might guess, due to the effect on disclosure strategy one cannot use Blackwell informativeness criteria as a way to measure the effect of an increase in analyst coverage on the amount of public information. This is because more coverage increases the probability that the value of some types will be disclosed (low types that are disclosed only by the analyst), but decreases this probability for other types (types between the previous and the new disclosure threshold who now choose to withhold). In the next section we suggest a measure of price information efficiency, which is the inverse of the expected squared distance of prices from the fundamental value. We then show that an increase in analyst coverage always increases price efficiency according to this measure. In section 5 we obtain a similar result when developing a liquidity measure in an extended model.

4.1 A Measure of Price Efficiency

In our model, when information is made public either by the manager or by the analyst, the price perfectly reflects the underlying value, i.e., the price is \( P^D = E[\bar{x}|s] = s \). When information is not made public the price is on average correct, and it is a noisy measure of the signal (that the manager might be withholding), \( P^{ND} = E[s | ND] \).

To measure how efficiently prices reflect information about future cash flows, we adopt
the commonly used expected squared deviation between the market price and the signal $s$. Our price efficiency measure, which we refer to as PEF, is given by

$$PEF \equiv -E [(s - P)^2].$$

(5)

PEF may represent the “social” benefit from having a price that is close to the fundamental, or the externalities and gains that are obtained from the informativeness of prices. Note that this measure is in line with our assumption of risk neutral pricing: a social planner who wishes to maximize efficiency will choose $P = E [\tilde{s} | I]$, where $I$ is all the available information.

Another interpretation of PEF is that it is the variance of the noise in the price relative to the true underlying value $s$. Thus, higher price efficiency means a decrease in the residual uncertainty of prices (the future movement of prices when the real cash flows $x$ will be realized or revealed).

### 4.2 Analyst Coverage and Price Efficiency

We have discussed above the difficulty in determining even the directional effect of changes in analyst coverage, $r$, on price efficiency. One of our main results is that an increase in analyst coverage always increases price efficiency.

**Proposition 2.** Price efficiency increases in analyst coverage, i.e.,

$$\frac{dPEF(r)}{dr} > 0.$$  

The formal proof of the Proposition is quite involved, and hence is delegated to the appendix. The intuition for the result is as follows. In equilibrium, whenever the manager obtains a signal below the disclosure threshold, $s < \sigma^*$ she does not disclose, and if the analyst does not reveals $s$, the resulting price is $P^{ND} = \sigma^*$.

Consider a change from $r$ to $r + \Delta$ for some small $\Delta > 0$. This will lead to a change in the disclosure threshold from $\sigma^*$ to $\sigma^* + \Delta'$, where $\Delta'$ reflects the effect on the disclosure
strategy following this increase in \( r \). Given Assumption 1, \( \Delta' > 0 \). One can examine the total effect on price efficiency by looking at each of the following effects separately: (i) the effect of changing \( r \) to \( r + \Delta \) without changing \( \sigma^* \) (ii) changing \( \sigma^* \) to \( \sigma^* + \Delta' \) without changing \( r \).

Our claim follows from the fact that the first effect is positive and is of the order of \( O(\Delta) \), while the second effect is negative but in the order of at most \( O(\Delta'^2) \). The fact that \( \Delta' = O(\Delta) \) implies a positive effect so the derivative is positive. The first effect is clear: an increase in the probability that \( s \) is revealed by the analyst increases the probability that the price is equal to the true type, \( s \). This has a first order effect \(- O(\Delta)\).

The second effect is an increase in the manager’s disclosure threshold, that is, a decrease in corporate disclosure. This increase in the disclosure threshold means that signals \( s \in (\sigma^*, \sigma^* + \Delta') \), which originally were disclosed and priced correctly when the manager was informed, are now withheld, and thus receive, with some positive probability, a price \( P_{ND} \). For the moment, suppose that \( P_{ND} \) does not change and remains \( \sigma^* \). There is a decrease in price efficiency because types \( s \in (\sigma^*, \sigma^* + \Delta') \) are not always priced correctly when the manager is informed. However, this is a \( O(\Delta'^2) \) effect, because even when types \( s \in (\sigma^*, \sigma^* + \Delta') \) are not priced correctly, they obtain a price of \( \sigma^* \) that is still very close to their fundamental value. Moreover, the price following no disclosure, \( P_{ND} \), changes, and thus the pricing of all types whose value is not disclosed changes. By definition, the price following no disclosure \( P_{ND} = E[\tilde{s} | \text{ND}] \) maximizes price efficiency following no disclosure. Thus the new \( P_{ND} \), that reflects the additional types who do not disclose, as described above, increases the overall price efficiency compared to keeping the old price. This means that the negative effect is even smaller.

Proposition 2 implies that although analyst coverage has an adverse effect on corporate voluntary disclosure, the overall effect of analyst coverage on public information is positive.
5 Informed Trading and Liquidity

The results in the previous section examine the effect of analyst coverage on a theoretical measure of price efficiency. While price efficiency is a very appealing theoretical construct, empirically measuring or estimating it is not easy. In this section, we study the effect of analyst coverage on liquidity, which is a measure of information asymmetry that is common in the empirical literature. Our measure of liquidity is the bid-ask spread, which is relatively easy to estimate. We analyze how the expected bid-ask spread, which reflects the information asymmetry that remains after the disclosure game, is affected by analyst coverage.\(^9\)

We extend our disclosure model by adding a stylized trading stage. Trading occurs after the manager’s potential voluntary disclosure decision and after the potential release of the analyst’s report. The trading stage is a static version of the Glosten and Milgrom (1985) model (henceforth GM). There are two players: a competitive market maker and a single trader. The trader can either buy or sell one unit (share) of the firm’s stock. With probability \(\gamma \in (0, 1)\) the trader is strategic and informed, and knows the firm’s terminal value, \(x\). An informed trader maximizes his value from trading. With probability \(1 - \gamma\) the trader is a “liquidity trader”, who sells or buys independently of the firm’s value (for example, due to a liquidity shock). The liquidity trader chooses to sell or to buy one unit with equal probabilities.\(^{10}\)

The risk neutral market maker does not have private information about the firm value or whether the trader is informed or not. He operates in a competitive market (which is not modeled), and sets prices that lead to zero expected profit. The bid price, \(b\), equals the expected value of the asset conditional on the trader selling a share. The ask price, \(a\), equals the expected value of the asset conditional on the trader buying a share. The

---

\(^9\)Note that the bid-ask spread in our model reflects the information asymmetry between traders, where the price efficiency reflects the uncertainty of the market regarding the fundamental value. While these two constructs are related, they capture different aspects of the information environment.

\(^{10}\)The assumption that the strategic trader is always informed assures that trade always takes place. Adding an uninformed strategic trader results in a possibility of no trade and a third price \(E[\hat{x} | I, \text{no-trade}]\), but does not change our results.
term \( a - b \) is the bid-ask spread, and we show below it is always positive.

### 5.1 Disclosure Decision in the Extended Model

In this section we analyze the manager’s disclosure strategy in the extended model. The analysis of the manager’s disclosure strategy in Section 3 assumes the price is the expected terminal value given all publicly available information. Let the public belief about the firm’s terminal value at the beginning of the trading stage, after the disclosure stage, be \( \mu = \Pr (x = 1 \mid I) \). In the basic model, the risk neutral price is simply \( P = \mu \). In the extended model, however, the price is determined by the trading that takes place. The price is either \( a(\mu) \equiv E[\tilde{x} \mid \mu, \text{purchase}] \) or \( b(\mu) \equiv E[\tilde{x} \mid \mu, \text{sale}] \), where “purchase” and “sale” denote events where the trader buys or sells one unit, respectively. We derive \( a(\mu) \) and \( b(\mu) \) analytically in the next section. The expected price following a trade, from the market maker’s point of view, is always \( \mu \). This can be easily seen using the law of iterated expectation:

\[
E[P; \mu] = \Pr (\text{purchase}; \mu) \cdot a(\mu) + \Pr (\text{sale}; \mu) \cdot b(\mu) = E[\tilde{x} \mid \mu] = \mu.
\]

If an informed manager chooses to disclose her signal \( s \), then this leads to a public belief \( \mu = s \). In that case, because the manager and the market maker hold the same beliefs regarding the value of the firm, the manager also expect a price, or payoff, of \( U^{D}(s) \equiv E[P; s] = s \). This is not the case, however, if neither the manager nor the analyst disclose. Let the market maker’s expectation of \( \tilde{x} \) in that case be given by \( \mu = \sigma^* = E[\tilde{x} \mid \text{ND}] \). Our aim is to show now that \( \sigma^* \) is the same threshold that we have found in Section 3. For an exogenous belief \( \sigma^* \), a manager that observes a signal \( s \) can expect, in case neither she nor the analyst disclose, a payoff of

\[
U^{\text{ND}}(s, \sigma^*) \equiv \Pr (\text{purchase}; s) \cdot a(\sigma^*) + \Pr (\text{sale}; s) \cdot b(\sigma^*).
\]

Due to the law of iterated expectations, \( U^{\text{ND}}(s, s) = s \). Moreover, since both the bid and
the ask prices are increasing in the belief of the market maker $\mu$ (we show that formally in the next section), we have $U^{ND}(s, \sigma^*) \leq s = U^D(s)$ if and only if $\sigma^* \leq s$. Thus, we obtain the same disclosure equilibrium strategy as in the basic model: the manager discloses if and only if $s \geq \sigma^*$, where $\sigma^*$ is defined in Equation (2).

5.2 Prices and the Bid-Ask Spread

In this section we provide a short derivation of the bid and ask prices and the bid-ask spread in a standard static GM setting. Readers who are familiar with this derivation can skip directly to Lemma 2.

The public belief in the beginning of the trading stage $\mu$ is between zero and one, and given that $\gamma < 1$, the bid and ask prices are also between zero and one. Thus, an informed trader always buys if $x = 1$ and sells if $x = 0$. The uninformed market maker believes a “purchase” event occurs with probability $Pr(\text{purchase}; \mu) = \gamma \mu + (1 - \gamma) \frac{1}{2}$, and that, conditional on such an event, the probability that the trader is informed is $Pr(\text{informed} \mid \text{purchase}) = \frac{\gamma \mu}{\gamma \mu + (1 - \gamma) \frac{1}{2}}$. Thus, the market maker sets an ask price that equals

$$a(\mu) = E[\tilde{x} \mid \mu, \text{purchase}] = \frac{\gamma \mu}{\gamma \mu + (1 - \gamma) \frac{1}{2}} \cdot 1 + \frac{(1 - \gamma) \frac{1}{2}}{\gamma \mu + (1 - \gamma) \frac{1}{2}} \cdot \mu = \frac{1 + \gamma}{1 + \gamma (2\mu - 1)} \cdot \mu.$$  

A similar calculation result in a bid price of

$$b(\mu) = E[\tilde{x} \mid \mu, \text{sale}] = \frac{1 - \gamma}{1 - \gamma (2\mu - 1)} \cdot \mu.$$  

It is easy to see that $b < \mu < a$ for any $\mu \in (0, 1)$, and that both $a(\mu)$ and $b(\mu)$ are strictly increasing in $\mu$.

\footnote{For simplicity, assume that in the zero probability events that there is no uncertainty in the beginning of the trading stage, that is, $s = \mu = 1$, and $s = \mu = 0$, the informed trader still chooses to buy and sell, respectively, for a fair price.}
The bid-ask spread, which we denote by $\Psi(\mu)$, is the difference between the ask and the bid prices above, and is given by

$$
\Psi(\mu) \equiv a(\mu) - b(\mu) = \frac{4\gamma(1 - \mu)\mu}{1 - \gamma^2 (2\mu - 1)^2}.
$$

(6)

The following Lemma provides some properties of the bid-ask spread.

**Lemma 2.** The bid-ask spread, $\Psi(\mu)$, has the following properties:

1. It is a strictly concave inverse U-shape function of $\mu$.
2. For any $\gamma \in (0, 1)$, the spread is maximized at $\mu = 0.5$.
3. $\Psi(0) = \Psi(1) = 0$.

The proof is trivial and merely involves differentiation of (6) and thus is omitted. The main characteristic of the bid-ask spread that we will be using is the concavity in the beliefs, $\mu.$

5.3 Disclosure and Liquidity

The public information available to the market maker is the result of the disclosure model we studied before. We now study how the parameters of the disclosure game affect illiquidity that results from information asymmetry.

Our measure of illiquidity, $IL(q,r)$, which depends on the parameters of the disclosure game, $q$ and $r$, is the expected bid-ask spread, and is given by

$$
IL(q,r) \equiv E[\Psi(\mu) \mid q,r].
$$

When we refer to liquidity we refer to $L(q,r) = IL(q,r)^{-1}$.

\[\text{Footnote: For simplicity we assume that a liquidity trader buys and sells with equal probabilities. Relaxing this assumption and allowing for this probability to be anywhere between zero and one does not affect our main results. In particular, the bid-ask spread remains a concave inverse U-shape function of } \mu.\]
We can identify three mutually exclusive events that lead to different amounts of public information following the disclosure stage:

1. With probability \( q \cdot g_I(r) + (1 - q) g_U(r) \) the analyst observes and publishes \( s \). In this case all realizations of the signal become public.

2. With probability \( q (1 - g_I(r)) (1 - F(\sigma^*)) \) the analyst is uninformed, but the manager is informed and discloses all realized signals above \( \sigma^* \).

3. With probability \( 1 - (q \cdot g_I(r) + q (1 - g_I(r)) (1 - F(\sigma^*)) + (1 - q) g_U(r)) \) there is no disclosure; the analyst is uninformed, and the manager is either informed, or withholds signals that are below \( \sigma^* \). The expectation of public belief in this case is \( \sigma^* \) (Fact 1).

Given these events, and the resulting distribution of beliefs, we can write the expected bid-ask spread as

\[
IL(q, r) = [1 - (q \cdot g_I(r) + q (1 - g_I(r)) (1 - F(\sigma^*)) + (1 - q) g_U(r)))] \cdot \Psi(\sigma^*) \\
+ [q \cdot g_I(r) + (1 - q) g_U(r)] \cdot E[\Psi(s)] \\
+ q (1 - g_I(r)) (1 - F(\sigma^*)) \cdot E[\Psi(s) | s \geq \sigma^*] .
\]

We are interested in the effect of analyst coverage, \( r \), on liquidity. The difficulty in proving this is similar to the one we described in the previous section, and stems from the fact that an increase in \( r \) has an ambiguous effect on the probability that the signal becomes public, as well as the effect of the underlying uncertainty following no disclosure. \( IL \), however, captures a different economic construct than \( PEF \). In particular, expected liquidity is not a linear function of \( PEF \), and hence Proposition 2 does not imply that the expected liquidity increases in \( r \). For example, if a certain signal \( s \) is disclosed with higher probability following an increase in \( r \), then this clearly has a positive effect on price efficiency because disclosure results in \( P = s \). However, since the spread is non-monotone (Lemma 2), disclosure of \( s \) may actually decrease liquidity if \( \Psi(\sigma^*) < \Psi(s) \). Thus, the
direct effect of an increase in coverage on IL is more nuanced than the effect on PEF. Nevertheless, it is possible to show that analyst coverage always has a total positive effect on liquidity:

**Proposition 3.** The expected bid-ask spread, $IL(q,r)$, is decreasing in $r$ for any $q \in (0,1)$, that is

$$\frac{dIL(q,r)}{dr} < 0.$$ 

The proof of proposition 3 is in the Appendix. The key part of the proof is to show that an increase in $r$ has a direct effect of decreasing illiquidity, despite the fact that for some values $s$, as described above, $\Psi(\sigma^*) < \Psi(s)$. This proof relies on the concavity of the bid-ask spread function (Lemma 2). The proof uses similar intuition as in the proof of Proposition 2 to show that the change in disclosure threshold plays a second order effect where the direct effect is of first order.

The result of Proposition 3, which provides additional motivation for the informational benefit of analyst coverage, is consistent with the empirical findings of Kelly and Ljungqvist (2012). Kelly and Ljungqvist (2012) find that following an exogenous negative shock to analyst coverage, there is a decrease in the liquidity of the firms that were affected by the decrease in analyst coverage.

6 Concluding Remarks

The vast theoretical literature on voluntary disclosure has focused on settings with a single information provider. In practice, however, the corporate disclosure environment is complex and often characterized by several agents who may acquire private information. Financial analysts are one example of such agents. In this paper we have studied how the possibility that the firm’s private information may be revealed by a third party (such as an analyst, the media, a regulator, social media, competitors, suppliers, rating agencies) affects the firm’s voluntary disclosure policy and the overall information available to the market. We found that for plausible information structures, an increase in analyst co-
verage crowds out corporate voluntary disclosure. We developed two measures of market quality: the first is the future volatility of prices, which has a natural interpretation of price efficiency in our model as it reflects the extent to which current prices reflect the fundamentals. The second measure is the expected bid-ask spread, which measures illiquidity that arises from information asymmetry. In order to calculate the former measure, we presented a trading stage a la Glosten and Milgrom (1985) that follows the disclosure game. We have shown that an increase in analyst coverage increases market efficiency and liquidity despite the crowding out effect, so that the total effect of public information is positive.

Our results provide potential regulatory implication, by implying that if the regulator can increase the probability of discovery of a firm’s information by various mechanisms, such as analyst coverage, it always has a positive effect on the information environment. Therefore, as long as actions that facilitate more discovery of firm’s private information are not too costly, they are desired.

A Appendix

Proof of Proposition 2

Proof. Denote by $P_{\text{ND}}(\sigma, r)$ the price given no disclosure by the firm or the analyst, as a function of a given disclosure threshold, $\sigma$, and a given analyst coverage $r$. $P_{\text{ND}}(\sigma, r)$ is given by (1). In addition, define $G(r, \sigma)$ as the PEF function (Equation (5)) for a given disclosure threshold $\sigma$ and analyst coverage $r$:

$$G(r, \sigma) = -E\left[(s - P(\sigma, r))^2\right]$$

$$= -\Pr_{\text{ND}}(r, \sigma) \cdot E\left[\left(\tilde{s} - P_{\text{ND}}(\sigma, r)\right)^2\right]_{\text{ND}, \sigma},$$

where

$$\Pr_{\text{ND}}(r, \sigma) = (1 - q)(1 - g_U(r)) + q(1 - g_I(r))F(\sigma)$$
is the probability of no disclosure. Note that in equilibrium the manager’s disclosure threshold is $\sigma = \sigma^*(r)$ and hence, $\text{PEF}(r) = G(r, \sigma^*(r))$.

We need to show that in equilibrium, $\text{PEF}$ is increasing in $r$, that is $\frac{d\text{PEF}}{dr} > 0$. This equals to

$$\frac{d\text{PEF}}{dr} = \frac{dG(r, \sigma^*(r))}{dr} = \frac{\partial G(r, \sigma)}{\partial r} |_{\sigma = \sigma^*(r)} + \frac{\partial G(r, \sigma)}{\partial \sigma} |_{\sigma = \sigma^*(r)} \frac{d\sigma^*(r)}{dr}.$$

A sufficient condition for $\frac{d\text{PEF}}{dr} > 0$ is that (1) $\frac{\partial G}{\partial r} |_{\sigma = \sigma^*(r)} > 0$ and (2) $\frac{\partial G}{\partial \sigma} |_{\sigma = \sigma^*(r)} = 0$. We prove those two properties below.

1. Proof that $\frac{\partial G}{\partial r} |_{\sigma = \sigma^*(r)} > 0$:

$$\frac{\partial G(r, \sigma)}{\partial r} = ((1 - q)g_U'(r) + q \cdot g_I' \cdot F(\sigma)) E \left[ (\tilde{s} - P_{\text{ND}}(\sigma, r))^2 \mid \text{ND}, \sigma \right]$$

$$+ 2 \Pr \text{ND}(r, \sigma) \cdot E \left[ \tilde{s} - P_{\text{ND}}(\sigma, r) \mid \text{ND}, \sigma \right] \frac{\partial P_{\text{ND}}(\sigma, r)}{\partial r}.$$

By definition, $P_{\text{ND}}$ equals the expected value given no disclosure; therefore $E \left[ \tilde{s} - P_{\text{ND}}(\sigma, r) \mid \text{ND}, \sigma \right] = 0$ and the second term equals zero. At $\sigma = \sigma^*(r)$, it becomes

$$\frac{\partial G(r, \sigma)}{\partial r} |_{\sigma = \sigma^*(r)} = [((1 - q)g_U'(r) + q \cdot g_I' \cdot F(\sigma^*(r))) E \left[ (\tilde{s} - P_{\text{ND}}(\sigma^*(r), r))^2 \mid \text{ND}, \sigma^*(r) \right]$$

Since, by definition, $g_U'(r) \geq 0$ and $g_I'(r) \geq 0$, with at least one strict inequality, we obtain $\frac{\partial G(r, \sigma)}{\partial r} |_{\sigma = \sigma^*(r)} > 0$.

2. Proof that $\frac{\partial G}{\partial \sigma} |_{\sigma = \sigma^*(r)} = 0$: 

[^28]:
We can rewrite $G(r, \sigma)$ as

$$G(r, \sigma) = -(1 - q) (1 - g_U(r)) E \left[ (s - P^{ND}(\sigma, r))^2 \right]$$

$$- q (1 - g_I(r)) F(\sigma) E \left[ (s - P^{ND}(\sigma, r))^2 \mid s \leq \sigma \right]$$

$$= -(1 - q) (1 - g_U(r)) \int_0^1 (s - P^{ND}(\sigma, r))^2 f(s) \, ds$$

$$- q (1 - g_I(r)) \int_0^\sigma (s - P^{ND}(\sigma, r))^2 f(s) \, ds.$$

Differentiating with respect to $\sigma$ we obtain

$$\frac{\partial G(r, \sigma)}{\partial \sigma} = -2 (1 - q) (1 - g_U(r)) \int_0^1 (s - P^{ND}(\sigma, r)) f(s) \, ds \cdot \left( -\frac{\partial P^{ND}(\sigma, r)}{\partial \sigma} \right)$$

$$- q (1 - g_I(r)) 2 \int_0^\sigma (s - P^{ND}(\sigma, r)) f(s) \, ds \cdot \left( -\frac{\partial P^{ND}(\sigma, r)}{\partial \sigma} \right)$$

$$- q (1 - g_I(r)) (\sigma - P^{ND}(\sigma, r))^2.$$

To obtain $\frac{\partial G}{\partial \sigma} \big|_{\sigma = \sigma^*(r)}$ observe that: (i) by Fact 1, $\sigma^*(r) = P^{ND}(\sigma^*(r), r)$. Thus, the third term in (8) equals zero; and (ii) by the minimum principle, $\frac{\partial P^{ND}(\sigma, r)}{\partial \sigma} \big|_{\sigma = \sigma^*(r)} = 0$. Therefore, the first two terms in (8) also equal zero. Thus $\frac{\partial G}{\partial \sigma} \big|_{\sigma = \sigma^*(r)} = 0$.

\[\square\]

**Proof of Proposition 3**

**Proof.** For a given and constant value of $q$, define a function $H(r, \sigma)$ that equals the expected spread conditional on analyst coverage $r$ and a given disclosure threshold $\sigma$ (which may not be the equilibrium threshold), as follows:

$$H(\sigma, r) \equiv \Pr(ND) (r, \sigma) \Psi \left( P^{ND}(\sigma, r) \right) + (1 - q) g_U(r) + q \cdot g_I(r) \cdot E \left[ \Psi(s) \right]$$

$$+ q \cdot (1 - g_I(r)) \int_\sigma^1 \Psi(s) \cdot f(s) \, ds,$$
where
\[\Pr_{\text{ND}} (r, \sigma) \equiv (1 - q) (1 - g_U (r)) + q (1 - g_I (r)) F(\sigma) \]  
(10)
is the probability of no disclosure, and \(P_{\text{ND}} (\sigma, r)\), given in (1), is the price following no-disclosure by the manager or the analyst. When evaluated at the equilibrium disclosure threshold, \(H (\sigma, r)\) is our measure of illiquidity, that is, \(IL (q, r) = H (\sigma^*(r), r)\). Thus, the total derivative of \(IL (q, r)\) with respect to \(r\) is:
\[
\frac{dIL (q, r)}{dr} = \frac{\partial H (\sigma, r)}{\partial r} \bigg|_{\sigma = \sigma^*(r)} + \frac{\partial H (\sigma, r)}{\partial \sigma} \bigg|_{\sigma = \sigma^*(r)} \frac{d\sigma^* (r)}{dr}.
\]  
(11)
To obtain \(\frac{dIL (q, r)}{dr} < 0\) it is sufficient to show that \(\frac{\partial H (\sigma, r)}{\partial r} \bigg|_{\sigma = \sigma^*(r)} < 0\) and \(\frac{\partial H (\sigma, r)}{\partial \sigma} \bigg|_{\sigma = \sigma^*(r)} = 0\). We establish these sufficient conditions in the two lemmas below.

Lemma 3. \(\frac{\partial H (\sigma, r)}{\partial r} \bigg|_{\sigma = \sigma^*(r)} < 0\).

Proof. We show that \(\frac{\partial H (\sigma, r)}{\partial r} < 0\) for any given \(\sigma\), and hence it also holds for \(\sigma = \sigma^*(r)\). Given the continuity of \(H (r, \sigma)\) in \(r\), it is sufficient to show that \(H (r_h, \sigma) < H (r_l, \sigma)\) for any \(r_h > r_l\) and any \(\sigma\).

1. Using (14), we compute \(H (r_l, \sigma) - H (r_h, \sigma)\):
\[H (r_l, \sigma) - H (r_h, \sigma) = \Pr_{\text{ND}} (r_l, \sigma) \Psi \left( P_{\text{ND}} (\sigma, r_l) \right) - \Pr_{\text{ND}} (r_h, \sigma) \Psi \left( P_{\text{ND}} (\sigma, r_h) \right) \]
\[+ [(1 - q) (g_U (r_l) - g_U (r_h)) + q \cdot (g_I (r_l) - g_I (r_h))] \cdot E [\Psi (s)] \]
\[+ q \cdot (g_I (r_h) - g_I (r_l)) \int_{r}^{1} \Psi (s) \cdot f (s) d s \]
\[= \Pr_{\text{ND}} (r_l, \sigma) \Psi \left( P_{\text{ND}} (\sigma, r_l) \right) - \Pr_{\text{ND}} (r_h, \sigma) \Psi \left( P_{\text{ND}} (\sigma, r_h) \right) \]
\[+ (1 - q) (g_U (r_h) - g_U (r_l)) \cdot E [\Psi (s)] \]
\[+ q \cdot (g_I (r_h) - g_I (r_l)) F (\sigma) \cdot E [\Psi (s) | s < \sigma] \]
We can therefore establish that $H(r_l, \sigma) - H(r_h, \sigma) > 0$ if and only if

$$\Pr_{\text{ND}}(r_l, \sigma) \Psi(P_{\text{ND}}(\sigma, r_l)) > \Pr_{\text{ND}}(r_h, \sigma) \Psi(P_{\text{ND}}(\sigma, r_h))$$

$$+ (1 - q) (g_U(r_h) - g_U(r_l)) \cdot E[\Psi(s)]$$

$$+ q \cdot (g_I(r_h) - g_I(r_l)) F(\sigma) \cdot E[\Psi(s) \mid s < \sigma]. \quad (12)$$

2. Now observe from (1) that

$$\Pr_{\text{ND}}(r, \sigma) \cdot P_{\text{ND}}(\sigma, r) = (1 - q) (1 - g_U(r)) E[s] + q F(\sigma) (1 - g_I(r)) \cdot E[s \mid s < \sigma].$$

This equation, applied to $r_l$ and $r_h$, together with some algebra, leads to

$$\Pr_{\text{ND}}(r_l, \sigma) \cdot P_{\text{ND}}(\sigma, r_l) = \Pr_{\text{ND}}(r_h, \sigma) \cdot P_{\text{ND}}(\sigma, r_h)$$

$$+ (1 - q) (g_U(r_h) - g_U(r_l)) E[s]$$

$$+ q (g_I(r_h) - g_I(r_l)) F(\sigma) \cdot E[s \mid s < \sigma]. \quad (13)$$

Observe the similarity between the LHS and RHS of (12) and (13); in the next step we use (13) to prove that (12).

3. We can use (10) to rewrite (13) explicitly as

$$P_{\text{ND}}(\sigma, r_l) = A \cdot P_{\text{ND}}(\sigma, r_h) + B \cdot E[s] + (1 - A - B) \cdot E[s \mid s < \sigma]$$

where $A = \frac{\Pr_{\text{ND}}(r_h, \sigma)}{\Pr_{\text{ND}}(r_l, \sigma)}$ and $B = \frac{(1-q)(g_U(r_h) - g_U(r_l))}{\Pr_{\text{ND}}(r_l, \sigma)}$. This representation presents $P_{\text{ND}}(\sigma, r_l)$ as an average of $P_{\text{ND}}(\sigma, r_h)$ and various signals. In order to obtain (12) remember that $\Psi(\cdot)$ is a strictly concave function (Lemma 2). Thus, by definition,

$$\Psi(P_{\text{ND}}(\sigma, r_l)) < A \cdot \Psi(P_{\text{ND}}(\sigma, r_h)) + B \cdot E[\Psi(s)] + (1 - A - B) \cdot E[\Psi(s) \mid s < \sigma].$$

This inequality is simply (12), and thus implies that $H(r_l, \sigma) > H(r_h, \sigma)$. 31
Lemma 4. $\frac{\partial H}{\partial \sigma} |_{\sigma=\sigma^*(r)} = 0$.

Proof. Differentiating (9) with respect to $\sigma$ we obtain

$$\frac{\partial H}{\partial \sigma} = q \left(1 - g_I(r)\right) f(\sigma) \left[\Psi\left(P^{ND}(\sigma, r)\right) - \Psi(\sigma)\right] + Pr\;ND\,(r, \sigma) \Psi'(\cdot) \frac{\partial P^{ND}}{\partial \sigma}. \quad (14)$$

To obtain $\frac{\partial H}{\partial \sigma} |_{\sigma=\sigma^*(r)}$ observe that: (i) by Fact 1, $\sigma^*(r) = P^{ND}(\sigma^*(r), r)$. Thus, the first term in (14) equals zero; and (ii) by the minimum principle, $\frac{\partial P^{ND}(\sigma, r)}{\partial \sigma} |_{\sigma=\sigma^*(r)} = 0$. Therefore, the second term in (14) also equals zero. Thus $\frac{\partial H}{\partial \sigma} |_{\sigma=\sigma^*(r)} = 0$. 

This completes the proof of Proposition 3. 

$\square$
References


