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A Theory of Board Control and Size

by

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ABSTRACT

We extend the traditional view of corporate boards as monitors to include a role for outside board members as suppliers of expertise or information. Indeed, both outsiders and insiders may have private information relevant to the decision. Because of the agency problem between managers and owners (who are assumed to be represented by the outside directors), neither party will communicate his or her information fully to the other. Outsiders in our model control agency problems by making some decisions themselves. When they do, the refusal of insiders to communicate their information fully becomes costly. Therefore, shareholders can sometimes be better off by having boards controlled by insiders. We characterize whether the board is optimally controlled by insiders or outsiders, the optimal number of outsiders, and resulting profits as functions of the importance of insiders' and outsiders' information, the extent of agency problems, and some other factors. This leads to an endogenous relationship between profits and the number of outside directors that furthers our understanding of some documented empirical regularities.

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A Theory of Board Control and Size

1 Introduction

Corporate governance, and in particular the issue of control of corporate boards by independent directors, has received considerable attention recently, in the wake of corporate scandals afflicting the likes of Enron, Tyco, Adelphi and others.[2] In 2002, Congress passed the Sarbanes-Oxley act mandating that the audit committees of the boards of directors of firms listed on national exchanges have a majority of independent members. And, in 2003, both the New York Stock Exchange and NASDAQ amended their rules to require the boards of listed firms to have a majority of independent members.[3] Meanwhile, the Securities and Exchange Commission has proposed rules that would require mutual funds to have an independent chairman and that at least 75% of directors be independent (they are already required to have a majority of independent directors). Despite the attention given this issue in the popular press, by Congress, and by the major stock exchanges, there has been little theoretical work on this issue.[4] This paper attempts to contribute toward filling that gap.

The developments mentioned in the previous paragraph suggest that the prevailing view among regulators is that it is in the interest of shareholders for corporate boards to be controlled by independent directors. This view seems to be driven by agency considerations, i.e., that only outsiders can effectively curtail agency problems. Although agency problems are clearly important, other considerations may affect the conclusion that boards should be outsider-controlled. In particular, the board's decisions are based on the information available to its members, information provided both by insiders and by outsiders. Since board control affects the strategic interaction between insiders and outsiders, it also affects the board's decisions and hence shareholder value. For example, when outsiders control the board, insiders may not provide full or completely accurate information. Obviously, this can have an adverse effect on the board's decisions, reducing shareholder value.

In this paper, we take account both of the direct effects of agency problems on corporate decision-making and the indirect effects of agency problems on communication to determine optimal board control. In our model, both outsiders and insiders may have private information relevant to a decision. Because of the agency problem between managers and owners (who are assumed to be represented by the outside directors), neither party will choose to communicate his or her information fully to the other. Outsiders in our model control agency problems by making some decisions themselves. When they do, the failure of insiders to communicate their information fully becomes costly. Since outsiders don't always delegate the decision to insiders optimally, shareholders can sometimes be better off by having boards controlled by insiders. This will be the case when insiders' information is important relative to the direct agency costs.

There is another reason why insider-control may be preferred by shareholders. This reason has to do with outsiders' incentives to become informed. In particular, we assume that, while insiders obtain their information "for free" in the process of performing their duties, outside directors must expend effort to apply their expertise to the specific issues faced by this firm. For example, a board member who is also a professor of finance may understand the theory of corporate investment decisions in general much better than firm insiders but will typically need to invest considerable effort to apply that understanding to the issues that arise on the board. When outsiders control the board, they can make decisions even when

[2] See, for example, Burns (2004), Luchetti and Lublin (2004), and Solomon (2004).

[3] See New York Stock Exchange Rule 303A and NASDAQ Rule 4350(c).

[4] One important exception that is related to the current paper is Hermalin and Weisbach (1998) which will be discussed below. Hermalin and Weisbach (2003) provide an excellent survey of economic research on boards of directors. As they point out most of this work is empirical.

uninformed. When insiders are in control, however, insiders will not delegate decisions to outsiders if they are uninformed. Consequently, insider-control of the board may increase outsiders' incentives to exert the effort required to specialize their expertise, and this may make insider-control attractive to shareholders.

The assumption that outside directors must exert effort to apply their expertise also allows us to determine the number of outsiders, along with board control, endogenously. The idea is simple. As the number of outsiders increases, their value to the company in providing expertise increases, provided each outsider continues to expend the same effort. But increasing their number aggravates a free-rider problem for outsiders. That is, when there are more outsiders, each outsider views the importance of his or her contribution as being reduced and, therefore, expends less effort in specializing his or her expertise. Increasing the number of outsiders results in more "heads" but also less informed heads. The optimal number of outsiders appropriately balances the two effects.

Since our model determines both the number of outside directors and profits endogenously, it has implications for the relationship between them. The model can thus help us understand one of the questions posed by Hermalin and Weisbach (2003), namely why does board size appear to affect performance. Indeed, Yermack (1996) and Eisenberg, *et al.* (1998) document a negative correlation between profits and the number of board members. Jensen (1993) and Lipton and Lorsch (1992) interpret this evidence as an indication that large boards are less effective. As Hermalin and Weisbach (2003) point out, this interpretation raises the question why do we observe large boards. In fact, in our model, board size does not affect performance but both are driven by other, exogenous factors such as the importance of the various parties' information, profit potential, and the opportunity cost of outside directors. We show how certain movements in these factors can induce a negative correlation between profits and the number of outside board members. Other movements in these factors can induce a positive correlation or no correlation between profits and the number of outsiders on the board. Thus our model is consistent with the many studies that fail to find a correlation between performance and the fraction of independent directors (see below for citations). The model also generates a number of other predictions detailed below.

Hermalin and Weisbach (1998) provide a model similar in some respects to ours. They model a board's decision about whether to retain or replace the CEO. This decision is based on firm performance and a costly additional signal. More independent boards are more inclined to obtain the additional signal. The degree to which the board is independent depends on the firm's past performance; good prior performance enables the CEO to reduce the board's independence. A number of interesting results are obtained regarding CEO compensation and turnover and the degree of independence of the board, but the model does not address the issue of the relation between the number of outside board members and contemporaneous profits or optimal board control.

Another related paper is Adams (2002) which also considers directors' role as advisors to management and management's incentive to provide information to the board. To obtain better information from managers, the board may want to commit not to use this information in evaluating managerial performance. Adams identifies empirically the extent of the boards' commitment not to use the information the manager supplies against him with the fraction of insiders on the board. This implies a relationship between the extent of managers' career concerns and the fraction of insiders on the board.[5]

This paper is organized as follows. The model is described in section 2. In section 3, we analyze boards controlled by outsiders, and in section 4, we analyze boards controlled by insiders. We then compare outsider-controlled and insider-controlled boards to determine optimal board control in section

[5] Two other, less closely related, theoretical papers on corporate boards are Hirshleifer and Thakor (1994) and Warther (1998).

5. Section 6 presents some comparative statics results and their empirical predictions. Section 7 concludes.

2 The Model

We consider a firm whose profits depend on a strategic decision denoted *s* to be determined by the board of directors. For concreteness, we refer to this decision as the scale of the firm, but it could be interpreted as any strategic decision. Board members may be firm employees, whom we call insiders, or independent directors, whom we call outsiders. Either insiders or outsiders may control the board. Control of the board allows the controlling group to choose scale themselves or to delegate this decision to the other group. In the terminology of Aghion and Tirole (1997), the controlling group has "formal" authority but may delegate "real" authority to the other group.

The firm's optimal scale depends on private information of insiders ("agents"), $\tilde{a}$, and private information of outsiders ("principles"), $\tilde{p}$. In particular, profit, $\tilde{\pi}$, is given by

$$\tilde{\pi} = \pi_0 - \left(s - \left(\tilde{a} + \tilde{p}\right)\right)^2 . \quad (1)$$

Equation (1) implies that ***potential profit***, π_0, is reduced to the extent that the scale is chosen to be other than the "first-best" scale, $\tilde{a} + \tilde{p}$. The quadratic term in (1) is the loss in profits from the choice of scale. We assume that potential profit is sufficiently large that expected profit is always positive. This requires that $\pi_0 > \sigma_p^2 + b^2$, where σ_p^2 is the variance of $\tilde{p}$, and b^2 is a parameter, to be introduced shortly, that measures the extent of the agency problem. Moreover, we assume that the compensation paid to outsiders is negligible compared to expected profits and is ignored when choosing the number of outsiders. Clearly, this assumption is appropriate only for large corporations.

We assume that insiders learn their information, $\tilde{a}$, in the course of their normal duties. Outsiders are presumed to have expertise about the other component of the optimal scale, which may include factors external to the firm. This expertise is costly to specialize to the firm's situation, however. For example, a finance professor who sits on a corporate board may have a deeper understanding of how to evaluate investment projects generally than do insiders but applying that understanding to the company's investment opportunities will require costly study of the particulars of those opportunities. Formally, we model this by assuming that, if any outsider invests non-contractible effort *e* at personal cost *c*(*e*), he or she learns $\tilde{p}$ with probability *e* (we refer to this as "becoming informed"). Thus the cost function should be interpreted as a measure of the uniqueness of the firm's activities, the degree of difficulty of adapting general business lessons to this particular firm. Outsiders are assumed to have identical interests and, therefore will share any information they produce among themselves (but cannot enter into contracts with each other, for example, to compensate each other for investing effort in becoming informed). As a result, if at least one outsider becomes informed, this information can be used to choose scale. The outsiders' information cannot be obtained by insiders (but may be communicated to them by outsiders). Similarly, outsiders cannot obtain the insiders' information directly (but insiders may communicate it to them). The extent to which one party will communicate its information to the other is limited by an agency problem which we now describe.

All board members are risk-neutral. Insiders have an equity stake and obtain a private benefit from larger scale. In particular, we assume the combined effect results in a payoff to insiders given by[6]

[6] The constant π_{I0} in equation (2) can be different from potential profit to allow insiders' preferences to diverge from profits in addition to their preference for larger scale. This constant plays no role in the analysis.

$$\tilde{\pi}_I = \pi_{I0} - \left(s - \left(\tilde{a} + \tilde{p} + b\right)\right)^2. \tag{2}$$

Note that the optimal scale from the point of view of insiders, given $\tilde{a}$ and $\tilde{p}$, is $\tilde{a} + \tilde{p} + b$. The parameter $b > 0$ measures the extent to which insiders prefer larger scale at the expense of profits, i.e., the extent to which they are biased toward larger scale. Because of the quadratic cost functions in (1) and (2), the difference between the profits that result when the insiders choose scale and maximal profit, for any given information, is b^2. We therefore refer to b^2 (and sometimes b) as the ***agency cost***.

We assume outsiders' compensation consists entirely of an equity stake that is independent of outsiders' effort, since effort is assumed to be non-contractible.[7] Consequently, outsiders prefer the profit-maximizing scale. We denote by $\alpha \in (0,1)$ the share of equity given to each outside board member. Outsiders' equity stake is determined by a participation constraint,

$$\alpha\pi - c(e) = \bar{U}, \tag{3}$$

where π is expected profit, and $\bar{U} > 0$ represents the value of an outside board member's opportunity cost of serving on the board.

We make the following assumptions regarding the distributions of $\tilde{a}$ and $\tilde{p}$:

Assumption 1. $\tilde{a}$ and $\tilde{p}$ are independent. $\tilde{a}$ is uniformly distributed on $[0,A]$; $\tilde{p}$ is uniformly distributed on $[0,P]$.

In some cases, it will be more convenient to work with the standard deviations of the random variables $\tilde{a}$ and $\tilde{p}$, as well as those of other uniformly distributed random variables, instead of the parameters A and P. Consequently, for any $x \geq 0$, denote by $\sigma(x)$ the standard deviation of a random variable uniformly distributed on an interval of width x, i.e.,

$$\sigma(x) = x/\sqrt{12}.$$

We will use σ_a to denote $\sigma(A)$ and σ_p to denote $\sigma(P)$.

Because of the quadratic cost function in (1), it turns out that, if an unbiased decision-maker chooses scale, the difference in profits between knowing $\tilde{p}$ (respectively, $\tilde{a}$) and having no information about $\tilde{p}$ (respectively, $\tilde{a}$) is exactly σ_p^2 (respectively, σ_a^2). We will therefore refer to σ_p^2 (σ_a^2) as the ***full value of the outsiders' (insiders') information***. Many of our results depend on a comparison between agency costs and the value of one (or both) party's information. Accordingly, a firm for which $b \geq \sigma_p$ ($b \geq \sigma_a$) is referred to as one for which ***agency costs are more important than outsiders' (insiders') information***. We refer to a firm for which agency costs are more important than either party's information ($b \geq \max\{\sigma_a, \sigma_p\}$) as one for which ***agency costs are critical***. We label a firm as one for which ***agency costs are important*** if agency costs are more important than at least one party's information. Finally, we call a firm for which agency costs are less important than either party's information one in which ***agency costs are not important***. This terminology is illustrated in Figure 1.

Insert Figure 1 Here

[7] We could also include a salary without affecting the results. In fact, since outsiders' incentives to exert effort to become informed are greater, the larger their equity stake, for a given total expected compensation, shareholders would prefer that the compensation be entirely through equity. Since board members are assumed to be risk-neutral, this would be optimal.

3 Outsider-Controlled Boards

We begin by analyzing the scale and delegation decisions of boards that are controlled by the outside directors. We refer to such a board as an ***outsider-controlled board or OCB***. The results will be used later when we consider whether boards controlled by insiders may be preferred to those controlled by outsiders. These results are interesting in their own right, however, in situations in which government regulation or other requirements (e.g., exchange listing requirements) mandate that corporate boards be controlled by independent directors. Indeed the New York Stock Exchange and NASDAQ have recently amended their rules to require the boards of listed firms to have a majority of independent directors.

Recall that control of the board empowers the outside directors to choose scale or to delegate the choice of scale to insiders. If outsiders do not delegate, they will choose scale based on their own information and any information communicated to them by insiders. Denote by $r(a)$ the report of insiders to outsiders if insiders observe $\tilde{a} = a$. If outsiders delegate, insiders will choose scale based on their own information and any information communicated to them by outsiders. Denote by $t(p)$ the report of outsiders to insiders if outsiders observe $\tilde{p} = p$. Of course, outsiders will have nothing to report if they do not become informed. Moreover, because of the agency problem, the reports will not fully communicate the reporting party's information, as will be seen below.

The sequence of events in this case is assumed to be the following. First, outsiders simultaneously choose how much effort to invest in becoming informed. Next, either they become informed or they do not, and then decide whether to delegate the scale decision to insiders. It will be shown below, that if outsiders learn that $\tilde{p} = p$, they will delegate if and only if $p \geq p^*$, for some cutoff p^*. If outsiders fail to become informed, they will delegate the scale decision to insiders, unless agency cost is sufficiently large. How large will be determined below.[8] Finally, depending on whether outsiders delegate or not, either insiders or outsiders choose scale and profits are realized. We start by analyzing the scale choice, given a delegation decision, then return to the delegation decision.

3.1 Outsiders Do Not Delegate

Suppose outsiders choose scale themselves. Since they have identical preferences and information, outsiders behave as a single agent. Given their compensation, outsiders will maximize expected profits. It is easy to check that, if outsiders are informed, they will choose

$$s\left(p,r\right) = \overline{a}\left(r\right) + p\,, \tag{4}$$

where $\overline{a}\left(r\right) = E\left(\tilde{a}\middle|r\right)$, and the expectation is with respect to outsiders' posterior belief about $\tilde{a}$, given insiders' report, r. Thus, $\overline{a}\left(r\right)$ is a value of $\tilde{a}$ estimated by outsiders based on what is communicated by insiders. The game in which outsiders choose s is analyzed formally in Harris and Raviv (2004). There it is shown that, because of the agency problem, insiders will not fully reveal their information. They will instead inform outsiders only that $\tilde{a}$ lies in some range.

More precisely, in the Pareto-best Bayes equilibrium of the game in which outsiders choose s, insiders will partition the support of $\tilde{a}$, $\left[0, A\right]$, into cells $\left[a_i, a_{i+1}\right]$ and report a value that is uniformly distributed on the cell in which the true realization of $\tilde{a}$ lies. Thus outsiders learn only the cell in which the true value of insiders' information lies, and their posterior belief is that $\tilde{a}$ is uniformly distributed on

[8] This is also shown in Dessein (2002).

that cell. It follows that, if the report r is in $\left[a_i, a_{i+1}\right]$, $\bar{a}(r) = \frac{a_i + a_{i+1}}{2}$. The number of cells is given by $N(b, A)$, where, for any $x > 0$,

$$N(b,x) = \left\langle \frac{1}{2}\left(\sqrt{1+2x/b} - 1\right)\right\rangle, \tag{5}$$

and, for any real number x, $\langle x \rangle$ is the smallest integer greater than or equal to x. Note that the number of cells is a measure of the extent to which insiders communicate their information to outsiders. For example, if there is only one cell, $N(b, A) = 1$, insiders communicate nothing outsiders don't already know. On the other hand as $N(b, A)$ gets very large (as will be the case if agency cost, b, approaches zero), the information communicated approaches perfect information about $\tilde{a}$. The endpoints of the cells satisfy

$$a_i = \frac{iA}{N(b,A)} - 2i\left(N(b,A) - i\right)b, \text{ for } i = 0, \ldots, N(b,A). \tag{6}$$

If outsiders are not informed, they will choose

$$s(p,r) = \bar{a}(r) + \bar{p}, \tag{7}$$

where $\bar{p}$ is the unconditional mean of $\tilde{p}$. The equilibrium report of insiders is the same as before.

3.2 Outsiders Delegate

Now suppose insiders are allowed to choose scale. They will choose

$$s(a,t) = a + \bar{p}(t) + b, \tag{8}$$

where $\bar{p}(t) = E\left(\tilde{p} \mid t\right)$, and the expectation is with respect to insiders' posterior belief about $\tilde{p}$ given outsiders' report, t, and the fact that the decision has been delegated. Since outsiders delegate the scale choice only when $\tilde{p} \geq p^*$, insiders, if given the decision, can infer that $\tilde{p} \geq p^*$. It is shown in Harris and Raviv (2004) that, in the Pareto-best Bayes equilibrium, outsiders will partition the interval $[p^*,P]$ into $N(b, P - p^*)$ cells and report a value that is uniformly distributed on the cell in which the true realization of $\tilde{p}$ lies. Thus insiders' posterior belief about $\tilde{p}$ is that it is uniformly distributed on the cell that contains the true value. It follows that, if the report t is in $\left[p_i, p_{i+1}\right]$, $\bar{p}(t) = \frac{p_i + p_{i+1}}{2}$.

The endpoints of the cells satisfy

$$p_i = \frac{iP + (N - i)p^*}{N} + 2i(N - i)b, \text{ for } i = 0, \ldots, N, \tag{9}$$

where $N = N(b, P - p^*)$.

3.3 Outsiders' Delegation Decision

Here we analyze the outsiders' decision of whether to delegate the scale choice to insiders. We show that, when outsiders are informed, they will delegate if and only if $\tilde{p} \geq p^*$, and when outsiders are not informed, they will delegate if and only if $b \leq \sigma_a$. Intuitively, when outsiders learn that $\tilde{p}$ is large, the optimal scale is likely to be large. There is less to lose from delegating to insiders biased in favor of larger scale. When outsiders are uninformed, they will delegate to informed insiders unless agency costs exceed the full value of insiders' information. We also characterize the cutoff value, p^*.

First suppose outsiders are informed. To determine when informed outsiders will delegate the scale decision to insiders, the following notation will be useful. Let $L(b,X)$ be the expected loss in profits of having only information that can be transmitted in equilibrium about a random variable that is uniformly distributed on an interval of width X. That is,

$$L(b,X)=E\left[\bar{x}\left(r(\tilde{x})\right)-\tilde{x}\right]^2, \tag{10}$$

where $\tilde{x}$ is uniform on an interval of width X, r is the equilibrium report of the party that observes $\tilde{x}$, and $\bar{x}(r)$ is the decision-maker's equilibrium posterior mean of $\tilde{x}$.

It is shown in Crawford and Sobel (1982) that

$$L(b,X)=\begin{cases}\left(\dfrac{\sigma(X)}{N(b,X)}\right)^2+\dfrac{b^2\left(N(b,X)^2-1\right)}{3}, \forall X>0,\\ 0,\ \text{for } X=0.\end{cases}$$

Thus $L(b,X)$ depends on how much information can be transmitted in the report, as measured by $N(b,X)$. In particular, if $N(b,X)=1$, i.e., no information can be transmitted to the decision-maker, then $L(b,X)$ is the entire variance, $\sigma^2(X)$, of the unobserved variable. If some information can be transmitted ($N(b,X)>1$), the expected cost is smaller than the variance.

Next, suppose outsiders are in control of the board and x is such that outsiders will delegate the scale choice to insiders whenever outsiders' private information is at or above $P-x$. Furthermore, suppose insiders know that the choice will be delegated to them only when outsiders' private information is at or above $P-x$. Define $f(b,x)$ as the expected loss in profits due to the scale choice when this choice is delegated, given that the outsiders' private information is exactly x, and delegation involves communication as in the above equilibrium.[9] It is shown in Harris and Raviv (2004) that

$$f(b,x)=\begin{cases}\left[\dfrac{x}{2N(b,x)}+N(b,x)b\right]^2, \forall x>0,\\ b^2,\ \text{for } x=0.\end{cases} \tag{11}$$

Some useful properties of f and L are shown in Lemma 1 (in the appendix).

If the scale choice is not delegated to insiders, the actual loss in profits from the scale choice is given by the square of the deviation of outsiders' choice of s, $\bar{a}\left(r(\tilde{a})\right)+\tilde{p}$, from the true realization of $\tilde{a}+\tilde{p}$, i.e, $\left[\bar{a}\left(r(\tilde{a})\right)-\tilde{a}\right]^2$. Thus the expectation of this loss over $\tilde{a}$, using the above equilibrium value for r, is $L(b,A)$. Essentially, the cutoff, p^*, is determined as the value such that the expected cost of

[9] One can also interpret f symmetrically for the case in which insiders control the board. In that case, one assumes insiders will delegate only when their private information is at or below x. Then f is the expected loss to insiders when they delegate the scale choice to outsiders, given that the insiders' private information is exactly x.

delegating if, in fact, $\tilde{p} = p^*$, $f(b, P - p^*)$, is the same as the cost of not delegating, $L(b, A)$. This is stated formally in the following lemma.

Lemma 2: Delegation Decision of Informed Outsiders. If outsiders are informed that $\tilde{p} = p$, they will delegate the choice of scale to insiders if and only if $p \geq p^*$, where p^* is as follows.

a. If $b \geq \sigma_a$, $p^* = P$, i.e., outsiders do not delegate, regardless of their information.

b. If $f(b, P) \leq L(b, A)$, $p^* = 0$, i.e., outsiders delegate the decision to insiders, regardless of their information.

c. Otherwise, $p^* \in (0, P)$ and is defined by

$$f(b, P - p^*) = L(b, A). \tag{12}$$

In this case, $P - p^*$ is independent of *P*.

Proof. This is shown in Harris and Raviv (2004).

We refer to case (b) of Lemma 2 as the case in which ***insiders' information is critical***. This terminology is justified by the fact (implied by Lemma 1) that if $f(b, P) \leq L(b, A)$, then $\max\{b, 6\sigma_p\} < \sigma_a$, i.e., insiders' information is more than six times as important as outsiders' information, and more important than agency costs. It is also justified by its implication that outsiders always delegate. We refer to the symmetric case for insider-controlled boards, $f(b, A) \leq L(b, P)$, as one in which ***outsiders' information is critical***. These terms are illustrated in Figure 2.

Insert Figure 2 Here

Now suppose outsiders do not become informed. If they delegate the scale decision to insiders, insiders choose $s = \bar{p} + \tilde{a} + b$, where $\bar{p}$ is the unconditional mean of $\tilde{p}$, and the expected loss in profits is

$$E\left((\bar{p} - \tilde{p} + b)^2\right) = \sigma_p^2 + b^2.$$

If outsiders do not delegate, the expected loss in profits is

$$E\left(\left(\bar{a}(r(\tilde{a})) - \tilde{a} + \bar{p} - \tilde{p}\right)^2\right) = L(b, A) + \sigma_p^2.$$

Thus, when outsiders are not informed, they will choose to delegate if and only if $b^2 < L(b, A)$. It follows from Lemma 1 that this condition is equivalent to $b < \sigma_a$, which implies that $N(b, A) = 1$ and hence that $L(b, A) = \sigma_a^2$. Consequently, when uninformed, outsiders delegate to insiders if and only if $b < \sigma_a$. The expected loss in profits due to the scale choice in this case, denoted l_U (the subscript *U* is for "uninformed"), is

$$l_U = \sigma_p^2 + \min\{b^2, \sigma_a^2\}. \tag{13}$$

3.4 Outsiders' Equilibrium Effort

Each outsider will choose his or her effort to maximize his or her share of expected profits net of effort costs. Consider the effort investment decision of an individual outsider, given that all $n - 1$ other outsiders invest e.[10] If the given outsider chooses effort x, the probability that at least one outsider will become informed is given by $1-(1-e)^{n-1}(1-x)$. Denote by l_I the expected loss in profits due to the scale choice given that outsiders are informed (hence the subscript I). An explicit expressions for l_I will be developed presently; for now it suffices to note that l_I does not depend on outsider effort. Recall that l_U is the corresponding loss when outsiders are uninformed, defined in (13). Then expected profits are given by

$$\pi_0 - \left[\left(1-(1-e)^{n-1}(1-x)\right)l_I + (1-e)^{n-1}(1-x)l_U\right] = \pi_0 - l_I - (1-e)^{n-1}(1-x)V_{OCB},$$

where $V_{OCB} = l_U - l_I$ is the ***marginal value of outsiders' information*** with an outsider-controlled board. The given outsider therefore chooses effort x to maximize his or her share α of the above expected profits minus his or her effort cost, i.e., to maximize

$$\alpha\left[\pi_0 - l_I - (1-e)^{n-1}(1-x)V_{OCB}\right] - c(x).$$

The first-order condition for the optimal effort is

$$c'(x) = \alpha(1-e)^{n-1}V_{OCB}, \tag{14}$$

provided there is an interior solution. All outsiders choosing e is a Nash equilibrium, therefore, if and only if

$$c'(e) = (1-e)^{n-1}\alpha V_{OCB},$$

or

$$c'(e)(1-e) = (1-e)^{n}\alpha V_{OCB}, \tag{15}$$

again provided there is an interior solution of (15).

To make sure the first-order condition (14) is necessary and sufficient for a maximum, and to obtain an interior solution for the equilibrium effort for at least some parameter values, we require the following assumption regarding the cost function c:

Assumption 2. The cost c is assumed to be strictly increasing and strictly convex in e. Also, assume $c(0)=0$, and the function $c'(e)(1-e)$ is convex and has a unique, interior minimum over $[0,1]$ at $e^* \in (0,1)$.[11]

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[10] We consider only symmetric Nash equilibria of the outsiders' effort choice game.

[11] If $c'(e)(1-e)$ were monotone decreasing, then the optimal number of outsiders would be either one or zero, depending on whether the marginal value of the outsiders' information does or does not exceed the marginal cost of effort at zero effort. If $c'(e)(1-e)$ were monotone increasing, the optimal number of outsider would either not be defined (for any number of outsiders, more outsiders would be better) or be zero, again depending on whether the marginal value of the outsiders' information does or does not exceed the marginal cost of effort at zero effort. An example of a cost function that satisfies Assumption 2 is

$$c(e) = -\gamma\ln(1-e) - \varepsilon\left[\frac{1}{2}e^2 + (1-2\delta)\left(e+\ln(1-e)\right)\right]$$

for $\varepsilon > 0$, $0 < \delta < 1$ and $\gamma > 2\varepsilon\delta$. In this case $c'(e)(1-e) = \gamma - \varepsilon e(2\delta - e)$, $e^* = \delta$, and $c'(0) = \gamma$.

It is easy to check that, under Assumption 2, for $n > 0$, equation (15) has a unique solution, $e_n(\alpha V_{OCB}) \in (0,1)$, that is increasing in αV_{OCB} and decreasing in n, provided that the marginal cost of effort for an outsider at zero effort is less than the outsider's share of the marginal value of becoming informed, i.e., provided that $\alpha V_{OCB} > c'(0)$ [see Figure 3]. If this condition is satisfied, we say that ***effort cost is not prohibitive***. If effort is prohibitively costly, the only Nash equilibrium is $e = 0$, i.e., outsiders will not invest any effort to become informed (and, therefore, will not become informed for sure). Otherwise, outsiders will invest more effort if the marginal value of becoming informed, V_{OCB}, is larger, outsiders' share of profits, α, is larger, and if the number of outside board members, n, is smaller, other things equal. The last implication is the result of free riding by outside board members. This free riding induces a tradeoff in choosing the optimal number of outside board members. For given effort, more outsiders increases the chances that at least one outsider will become informed. Because of free riding, however, having more outsiders on the board reduces the effort of all outsiders. We will exploit this tradeoff in choosing the optimal number of outsiders.

Insert Figure 3 Here

We now develop an explicit expression for the marginal value of outsiders' information, V_{OCB}. If outsiders delegate, the loss is given by the square of the difference between the insiders' choice of s, $\tilde{a} + \overline{p}(t(\tilde{p})) + b$, and the true realization of $\tilde{a} + \tilde{p}$, i.e, $\left[\overline{p}(t(\tilde{p})) - \tilde{p} + b\right]^2$. Since outsiders delegate if and only if $\tilde{p} \in [p^*, P]$, the expected loss when outsiders delegate is

$$E\left(\left[\overline{p}(t(\tilde{p})) - \tilde{p} + b\right]^2 \middle| \tilde{p} \in [p^*, P]\right) = L(b, P - p^*) + b^2 .$$

Given that outsiders become informed, the probability that they do not delegate to insiders is $\frac{p^*}{P}$. Recall that if outsiders do not delegate when informed, the expected loss is $L(b, A)$. Therefore, conditional on outsiders becoming informed, but before knowing the value of $\tilde{p}$, the expected loss is

$$l_I = \frac{p^*}{P} L(b, A) + \left(1 - \frac{p^*}{P}\right)\left[L(b, P - p^*) + b^2\right]. \tag{16}$$

From(13) and (16), it follows that the marginal value of outsiders' information with an outsider-controlled board is given by

$$V_{OCB} = l_U - l_I = \sigma_p^2 + \min\left\{b^2, \sigma_a^2\right\} - \left[\frac{p^*}{P} L(b, A) + \left(1 - \frac{p^*}{P}\right)\left[L(b, P - p^*) + b^2\right]\right]. \tag{17}$$

We summarize the results of this subsection and the previous subsection in the following proposition.

Proposition 1: Outsiders' Equilibrium Effort and Delegation Decision When They Are In Control. Assume the board is outsider-controlled.

- If effort cost is not prohibitive ($\alpha V_{OCB} > c'(0)$), then equilibrium effort of outsiders when there are n outsiders, $e_n(\alpha V_{OCB}) \in (0,1)$, and is given by the unique solution of (15); otherwise the equilibrium effort of outsiders is zero.
- If agency costs are more important than insiders' information ($b \geq \sigma_a$), then outsiders never delegate, whether they become informed ($p^* = P$) or not, and insiders do not communicate any information about $\tilde{a}$ ($N(b, A) = 1$). Consequently, the expected loss

in profits if outsiders fail to become informed is the full value of both outsiders' and insiders' information, i.e., $l_U = \sigma_p^2 + \sigma_a^2$. For the same reasons, the expected loss in profits if outsiders become informed is the full value of insiders' information, σ_a^2. Thus, the marginal value of outsiders' information is the full value of outsiders' information ($V_{OCB} = \sigma_p^2$).

- If insiders' information is critical ($f(b,P) \le L(b,A)$), then outsiders always delegate, whether they become informed ($p^* = 0$) or not, so the expected loss in profits if outsiders do not become informed is $l_U = \sigma_p^2 + b^2$. Since outsiders always delegate, the expected loss in profits if outsiders become informed is the cost of imperfectly communicating this information to insiders, $L(b,P)$ (since $p^* = 0$, insiders learn nothing from the fact that outsiders delegate), plus the agency cost, b^2. Thus the marginal value of outsiders' information is $V_{OCB} = \sigma_p^2 - L(b,P) \ge 0$.

- If agency costs are less important than insiders' information ($b < \sigma_a$), and insiders' information is not critical ($f(b,P) > L(b,A)$), then outsiders sometimes delegate when they are informed ($p^* \in (0,P)$ and is given by the solution to (12)) and always delegate when they are uninformed. Consequently, the loss in profits if outsiders do not become informed is $l_U = \sigma_p^2 + b^2$. In this case, $P - p^*$ is independent of *P*.

Note that p^* is not the optimal cutoff. The reason is that the choice of p^* affects insiders' inference about $\tilde{p}$. In fact p^* is larger, relative to *P*, than is optimal, i.e., outsiders, when in control, fail to delegate sufficiently often relative to the importance of their information. This can lead to the counter-intuitive result that the marginal value of outsiders' information may be negative. In particular, when $p^* \in (0,P)$, *not* becoming informed may improve expected profits, because it leads outsiders to delegate to insiders for sure, whereas if outsiders become informed, they will delegate a suboptimal fraction of the time. Obviously, for this to be the case, it must be true that agency costs are relatively small and insiders' information relatively important. Indeed, we see from Proposition 1, that when agency costs are large, so that outsiders fail to delegate even if they do not become informed, the marginal value of outsiders' information is positive. And, when insiders' information is critical, so that outsiders always delegate, the marginal value is non-negative.

3.5 Optimal Number of Outsiders for Outsider-Controlled Boards

We assume shareholders choose the number of outside board members to maximize the expectation of their share of profits, i.e., profits net of payments to outsiders. Recall that the compensation paid to outsiders is assumed to be negligible compared to expected profits and, hence, is ignored when choosing the number of outsiders. Shareholders will therefore choose the number of outsiders, n_{OCB}, to solve the following problem:

$$\max_{n \in \mathbb{Z}} \pi_0 - l_U + \left[1 - \left(1 - e_n\left(\alpha V_{OCB}\right)\right)^n\right] V_{OCB}, \tag{18}$$

where $\mathbb{Z}$ is the set of non-negative integers. One may interpret the objective function in (18) as potential profits, π_0, minus the loss from not observing $\tilde{p}$, l_U, plus the amount of that loss that is saved if $\tilde{p}$ is observed times the probability of observing $\tilde{p}$. This problem is equivalent to

$$\min_{n \in \mathbb{Z}} \left(1 - e_n\left(\alpha V_{OCB}\right)\right)^n. \tag{19}$$

Thus the object is to minimize the probability that outsiders fail to become informed, given that this probability is determined by the number of outsiders and their share of the marginal value of becoming informed.

Assuming that effort cost is not prohibitive, we can use (15) to write the objective function in (19) as

$$\left(1 - e_n\right)^n = \frac{c'\left(e_n\right)\left(1 - e_n\right)}{\alpha V_{OCB}}. \tag{20}$$

Thus, in this case, n_{OCB} can be chosen to minimize $c'\left(e_n\right)\left(1 - e_n\right)$. Using Assumption 2, if n were a continuous variable, the optimal n, denoted n^*, would be such that $e_{n^*} = e^*$ [see Figure 3], i.e.,

$$n^*\left(\alpha V_{OCB}\right) = \frac{\ln \dfrac{c'\left(e^*\right)\left(1 - e^*\right)}{\alpha V_{OCB}}}{\ln\left(1 - e^*\right)}. \tag{21}$$

It follows that the optimal number of outsiders with an outsider-controlled board, n_{OCB}, is either $\left\langle n^*\left(\alpha V_{OCB}\right)\right\rangle$ or $\left\langle n^*\left(\alpha V_{OCB}\right)\right\rangle - 1$. Since n_{OCB} is approximately equal to $n^*\left(\alpha V_{OCB}\right)$, and incorporating the integer constraint makes subsequent calculations intractable, hereafter, we ignore the integer constraint on the number of outsiders.

If effort is prohibitively costly, the equilibrium effort of outsiders is zero, and the optimal number of them is one.[12] Therefore,

$$n_{OCB} = \begin{cases} n^*\left(\alpha V_{OCB}\right), \text{ if } c'\left(0\right) < \alpha V_{OCB}, \\ 1, \text{ otherwise.} \end{cases} \tag{22}$$

It is easy to check that n^* is increasing. Intuitively, in our model, regardless of α and V_{OCB}, as long as effort cost is not prohibitive, the equilibrium effort will be e^*. That is, the number of outsiders is determined to make equilibrium effort of outsiders e^*, which depends only on the effort cost function. When outsiders' share of the marginal value of becoming informed, αV_{OCB}, increases, outsiders will increase their effort. To prevent this, the number of outsiders must increase so that the free-rider problem causes them to continue to choose e^*. This result follows from three key assumptions of the model: (i) the function $c'\left(e\right)\left(1 - e\right)$ has a unique minimum on $\left[0,1\right]$, (ii) to take advantage of the private information available to outsiders requires only that at least one of them learn it, and (iii) in computing profits, the cost of outside board members is negligible.

It also follows from this observation and (18) that the maximum expected profit with an outsider-controlled board, obtained by choosing the optimal number of outsiders, is given by

$$\pi^*\left(OCB\right) = \pi_0 - l_U + \begin{cases} V_{OCB} - M, \text{ if } c'\left(0\right) < \alpha V_{OCB}, \\ 0, \text{ if } c'\left(0\right) \geq \alpha V_{OCB}, \end{cases} \tag{23}$$

[12] Since we ignore the cost of outsiders in choosing the optimal number of them, strictly speaking, any positive number of outsiders would be equally good (at least one outsider is needed for outsider control). If we use the (negligible) cost to break ties, however, the optimal number of outsiders would be one.

where

$$M = \frac{c'(e^*)(1-e^*)}{\alpha}. \tag{24}$$

Note that if $c'(0) < \alpha V_{OCB}$, then $M = (1-e^*)^{n_{OCB}} V_{OCB}$. Consequently, in this case $V_{OCB} - M > 0$.

4 Insider-Controlled Boards

Our goal is to determine board control endogenously by comparing the profitability of outsider- and insider-controlled boards. Consequently, having analyzed outsider-controlled boards in the previous section, we now analyze insider-controlled boards (ICBs).

4.1 Outsiders' Equilibrium Effort and Insiders' Delegation Decision

If insiders control the board, they may choose scale or delegate the choice of scale to outsiders. The sequence of events in this case is assumed to be the following. First, outsiders choose effort and become informed or not. Next, insiders observe $\tilde{a}$ and whether outsiders are informed and decide whether to delegate the choice of scale to outsiders. They will not delegate if outsiders fail to become informed. Finally, either outsiders or insiders choose scale, depending on the outcome of the insiders' delegation decision.

It can be shown, using the same argument as in Theorem 2 of Harris and Raviv (2004), that insiders will prefer to delegate if and only if $a \le a^*$, for some cutoff a^*. The analyses of outsiders' choice of scale and insiders' choice of scale proceeds as in the case in which outsiders are in control of the board as discussed above, with obvious modification.[13] It follows that outsiders' equilibrium effort choice with an insider-controlled board is determined by

$$c'(e) = (1-e)^{n-1} \alpha V_{ICB}, \tag{25}$$

where V_{ICB} is the marginal value of outsiders' becoming informed with an insider-controlled board, i.e., $V_{ICB} = l'_U - l'_I$, and provided (25) has an interior solution, i.e., provided $\alpha V_{ICB} > c'(0)$. The quantities l'_U and l'_I are the counterparts, for insider-controlled boards, of l_U and l_I, i.e., the expected information losses given outsiders do not (do, respectively) become informed. Formally,

$$l'_U = \sigma_p^2 + b^2, \tag{26}$$

and

$$l'_I = \frac{a^*}{A} L(b, a^*) + \left(1 - \frac{a^*}{A}\right)\left(L(b,P) + b^2\right). \tag{27}$$

The basic results for insider-controlled boards are given in the following proposition (the proof is omitted, since it is quite similar to that of Proposition 1).

Proposition 2: Outsiders' Equilibrium Effort and Insiders' Delegation Decision When Insiders' Are in Control. Assume the board is insider-controlled.

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[13] The analysis makes use of the fact that, even though outsiders could delegate the choice of s back to insiders when outsiders become informed, they will not choose to do so. This is because, in any situation in which insiders prefer outsiders to choose scale despite losing the benefits of choosing a larger-than-optimal scale, outsiders will prefer to choose scale themselves even more strongly.

- If effort cost is not prohibitive ($\alpha V_{ICB} > c'(0)$), then equilibrium effort of outsiders when there are *n* outsiders, $e_n(\alpha V_{ICB}) \in (0,1)$, and is given by the unique solution of (25); otherwise the equilibrium effort of outsiders is zero.
- If outsiders' information is less important than agency costs ($\sigma_p \le b$), then insiders never delegate (*a** = 0), and outsiders do not communicate any information ($N(b,P)=1$). In this case, there is nothing to be gained by outsiders' becoming informed ($V_{ICB} = 0$).
- If outsiders' information is critical ($f(b,A) \le L(b,P)$), then insiders always delegate (*a** = *A*). In this case, the marginal value of outsiders' information is

$$V_{ICB} = \sigma_p^2 + b^2 - L(b,A) \ge \sigma_p^2 + b^2 - \sigma_a^2 > 0 .$$

- If outsiders' information is more important than agency costs ($\sigma_p > b$), but not critical ($f(b,A) > L(b,P)$), then insiders sometimes delegate if outsiders are informed, i.e., $a^* \in (0,A)$ and is given by the solution of

$$f(b,a^*) = L(b,P), \tag{28}$$

which implies $6\sigma(a^*) \le \sigma_p$. In this case, *a** is independent of *A*, and the marginal value of outsiders' information is positive ($V_{ICB} > 0$).

Note that, as is the case with *p**, *a** is an equilibrium cutoff, not the optimal cutoff. In fact *a** is smaller than is optimal, i.e., insiders, when in control, fail to delegate sufficiently often relative to the importance of outsiders' information. Nevertheless, when insiders are in control, the marginal value of outsiders' information is always non-negative (and is zero only when insiders never delegate). This is because outsiders have no choice of whether to delegate to insiders when insiders control the board.

4.2 Optimal Number of Outsiders for Insider-Controlled Boards

We assume that the number of outsiders must be chosen independently of *a*. Therefore, with an insider-controlled board, the optimal number of outsiders, n_{ICB}, solves

$$\min_{n \in \mathbb{Z}} \left(1 - e_n(\alpha V_{ICB})\right)^n V_{ICB} . \tag{29}$$

Thus, ignoring the integer constraint as mentioned above, the optimal number of outsiders with an insider-controlled board, is

$$n_{ICB} = \begin{cases} n^*(\alpha V_{ICB}), \text{ if } c'(0) < \alpha V_{ICB}, \\ 0, \text{ otherwise,} \end{cases} \tag{30}$$

and the equilibrium effort of outsiders is *e** if $c'(0) < \alpha V_{ICB}$ and zero otherwise.

The maximum profit obtainable with an insider-controlled board is

$$\pi^*(ICB) = \pi_0 - \left(\sigma_p^2 + b^2\right) + \begin{cases} V_{ICB} - M, \text{ if } c'(0) < \alpha V_{ICB}, \\ 0, \text{ if } c'(0) \ge \alpha V_{ICB}. \end{cases} \tag{31}$$

As is the case for outsider-controlled boards, if $c'(0) < \alpha V_{ICB}$, then $M = (1-e^*)^{n_{ICB}} V_{ICB}$, so $V_{ICB} > M$. If, in addition $c'(0) < \alpha V_{OCB}$, then $M = (1-e^*)^{n_{OCB}} V_{OCB}$, and the expected cost of not observing outsiders' information is the same for both board types.

5 Outsider- vs. Insider-Controlled Boards

In this section we derive the optimal board control from the point of view of shareholders. Our goal is to relate the profit-maximizing board control to the primary exogenous variables σ_a, σ_p, and *b*. It is convenient to divide the population of firms into types based on whether each party's information is critical and whether, if it is not critical, it is more or less important than agency costs. This results in four types of firms as shown in Figure 4. For both type-I and type-IV firms, neither party's information is critical. For type-I firms, agency costs are more important than at least one party's information. For type-IV firms, agency costs are less important than either party's information. Insiders' information is critical for firms of type II, and outsiders' information is critical for firms of type III.

Insert Figure 4 Here

Whether shareholders prefer an outsider- or an insider-controlled board depends on the sign of the ***profit differential***, $\Delta\pi$, defined as

$$\Delta\pi = \pi^*(OCB) - \pi^*(ICB).$$

If $\Delta\pi > 0$, an outsider-controlled board is optimal, and if $\Delta\pi < 0$, insider control is preferred. Of course, if $\Delta\pi = 0$, shareholders are indifferent.

To compare the two board types, we must simplify the model by assuming that the share of profits going to outsiders, α, is exogenous. We also assume that $c'(0) < \alpha b^2$. This eliminates some uninteresting cases in which effort cost is prohibitive.

5.1 Type-I Firms: Neither Party's Information is Critical, But Agency Costs Are Important

Recall that agency costs are important if they are more important than at least one party's information. In this case, agency costs are the overriding consideration. In accordance with conventional wisdom, outsider-control of the board is optimal.

Proposition 3: Optimal Board Control When Neither Party's Information is Critical, But Agency Costs Are Important. Shareholders prefer an outsider-controlled board. This preference is strict unless effort cost is prohibitive if outsiders are in control.[14]

Proof. See the appendix.

First, suppose that agency costs are critical (more important than either party's information). In this case, agency problems are so severe that neither outsiders, whether they become informed or not, nor insiders ever delegate (Propositions 1 and 2). Moreover, neither side will communicate anything to the other (Propositions 1 and 2). Therefore, if insiders control the board, outsiders will not become informed. If the board is insider-controlled, shareholders lose both the full value of outsiders' information and the agency costs. If the board is outsider-controlled, shareholders lose the full value of insiders' information

[14] Even if effort cost is prohibitive for an outsider-controlled board, the preference is still strict if $b > \sigma_a$ and $b \geq \sigma_p$.

and may also lose the full value of outsiders' information. Since agency costs are critical, the former loss exceeds the latter, so outsider-control is preferred.

Next consider the case in which agency costs are more important than insiders' information, but less important than outsiders' information. In this case, outsiders never delegate if they are in control, and insiders do not communicate any information (Proposition 1). Therefore, if outsiders control the board, shareholders lose the full value of insiders' information. If insiders' are in control, losses are a weighted average of the loss if insiders delegate and the loss if they do not. Since agency costs are more important than insiders' information, this weighted average is larger than if insiders were forced to delegate, namely the full value of their information. In addition, regardless of board control, shareholders lose the marginal value of the outsiders' information times the probability that they fail to become informed. Since this quantity is the same for either board type, we can ignore it in the comparison. Consequently, shareholders prefer an outsider-controlled board.

Finally suppose agency costs are more important than outsiders' information, but less important than insiders' information. For this type of firm, insiders never delegate and outsiders do not communicate any information to insiders (Proposition 2). Consequently, if insiders are in control, outsiders have no incentive to exert effort and will, therefore, not become informed. If effort cost is not prohibitive when outsiders are in control, the profit differential between an outsider- and an insider-controlled board is the probability that outsiders will become informed times the marginal value of their information when they are in control. This quantity is, of course, positive. If effort cost is prohibitive when outsiders are in control, they will not become informed regardless of board control. In that case, outsiders will always delegate, and insiders will never delegate, so insiders always have real authority and learn nothing from outsiders. Hence profits are the same for both board types, when effort cost is prohibitive.

5.2 Type-II Firms: Insiders' Information is Critical

Proposition 4: Optimal Board Control When Insiders' Information is Critical. Shareholders prefer an insider-controlled board. This preference is strict unless agency cost is more important than outsiders' information, or effort cost is prohibitive for an insider-controlled board.

Proof. See the appendix.

If outsiders' information is less important than agency costs, outsiders will not become informed whether they are in control or not. In the former case, outsiders will always delegate to insiders (Proposition 1), and, in the latter case, insiders will never delegate (Proposition 2). Since outsiders will never have real authority and do not communicate anything to insiders (Proposition 2), there is no gain to becoming informed in either case. Since insiders always have real authority for this type of firm, regardless of who has formal authority, shareholders don't care who controls the board.

If outsiders information is more important than agency costs, it turns out that outsiders' information is more valuable when they are *not* in control. The reason for this is that, when outsiders are in control, they always delegate (Lemma 2), so their information is not fully used. On the other hand, when insiders are in control, they sometimes delegate (Proposition 2), so, at least some of the time, outsiders' information is fully used. This causes profits to be higher under insider-control than under outsider-control, provided that outsiders have some incentive to exert effort to become informed when insiders are in control. That requires that effort cost is not prohibitive for an insider-controlled board.

5.3 Type-III Firms: Outsiders' Information is Critical

Proposition 5: Optimal Board Control When Outsiders' Information is Critical. Shareholders prefer an outsider-controlled board. This preference is strict if agency costs are less important than insiders' information, and effort cost is not prohibitive for an outsider-controlled board.

Proof. See the appendix.

First suppose agency costs are more important than insiders' information. Then outsiders never delegate, even when uninformed, but insiders always delegate, unless outsiders fail to become informed (Propositions 1 and 2). Moreover, insiders do not communicate any information (Proposition 1). Since they never delegate, the marginal value of outsiders' information if they are in control, V_{OCB}, is the full value of their information, σ_p^2. Since insiders always delegate if outsiders become informed but do not communicate any information to outsiders, the marginal value of outsiders' information if insiders are in control is the full value of outsiders' information, σ_p^2, plus agency costs, b^2, minus the full value of insiders' information, σ_a^2, i.e., $V_{ICB} = \sigma_p^2 + b^2 - \sigma_a^2$. This marginal value is greater than the marginal value of outsiders' information if they are in control, since insiders' information is assumed to be less important than agency cost ($\sigma_a < b$) for this type of firm. That is, the fact that outsiders will have real authority when they are in control but will have real authority only if they become informed when insiders are in control, creates an extra incentive for outsiders to become informed when insiders are in control.

Regardless of who controls the board, when outsiders are informed, profits are reduced by the full value of insiders' information, σ_a^2 (remember, outsiders do not delegate, insiders do not communicate any information, and insiders always delegate if outsiders are informed). When outsiders are uninformed, however, information and agency costs depend on who controls the board. If outsiders control the board, they do not delegate, so profits are reduced by the full value of both party's information, $\sigma_p^2 + \sigma_a^2$. If insiders control the board, they do not delegate, so profits are reduced by the full value of outsider's information plus agency costs, $\sigma_p^2 + b^2$. For this type of firm, the former loss is smaller than the latter ($\sigma_a < b$), so it would seem that, on average, profits are higher for the outsider-controlled board. This reasoning, however, ignores the difference between the two regimes in the probability of outsiders becoming informed. Because the marginal value of outsiders' information is higher when insiders control the board than when outsiders control the board, there will be more outsiders and a greater probability of becoming informed when insiders control the board. It turns out that the greater probability of becoming informed for the insider-controlled board just offsets the greater loss when outsiders are uninformed, making shareholders indifferent between the two regimes.

Now suppose agency costs are less important than insiders information. Here, as in the previous case, insiders always delegate provided outsiders become informed (Proposition 2). Outsiders always delegate if they are uninformed but may not delegate if they are informed (Proposition 1). It turns out, for this type of firm, that the marginal value of outsiders' information is greater when they are in control than when they are not. Since insiders always delegate, information costs when they are in control and outsiders are informed are $L(b, A)$, the cost due to imperfect communication of insiders' information. When outsiders are in control, if they are informed and do not delegate, information costs are also $L(b, A)$. If they are informed and delegate, however, information costs are $L(b, P - p^*) + b^2$, i.e., the costs of imperfect communication of outsiders' information plus agency costs. Because of the way p^* is chosen, this latter cost is smaller than the former. Thus, on average, information costs when outsiders are informed and in control are smaller than when they are informed but not in control. Since information costs when outsiders are uninformed are the same for the two board types, the marginal value of outsiders' information is greater when they are in control. If effort cost is not prohibitive for either board type, then profits are a weighted average of profits when outsiders are informed and when they are not. Since information costs are lower when outsiders are informed and in control than when they are informed but not in control, and information costs are the same when outsiders are not informed, profits are higher when outsiders are in control. This is reinforced if effort cost is prohibitive when insiders are in control but not when outsiders are in control. If effort cost is prohibitive for both board types, outsiders never become informed, and, hence, always delegate. Since insiders never delegate for this type of firm, regardless of who is in control, insiders have real authority and outsiders have no information to communicate. Thus profits are the same for the two board types in this case.

5.4 Type-IV Firms: Neither Party's Information is Critical and Agency Costs Are Not Important

Recall that agency costs are not important if they are less important than either party's information. This is the most complicated case, and, consequently, little can be said in general about the relation between board preference of shareholders and the exogenous parameters in this case. We can, however, relate shareholders' preferences over board control to the marginal value of outsiders' information.

Proposition 6: Optimal Board Control When Neither Party's Information is Critical and Agency Costs Are Not Important. If effort cost is not prohibitive for at least one type of board, then shareholders prefer the board type that results in the largest marginal value of outsiders' information. If effort cost is prohibitive for both board types, shareholders are indifferent.

Proof. See the appendix.

In one special case of particular interest, however, we can say which board type is preferred. This is the case in which outsiders and insiders have more or less equally important information. In that case, one might expect that, since insiders are biased while outsiders are not, that shareholders would prefer an outsider-controlled board. This is indeed the case as is shown in the next result.

Corollary. If effort cost is not prohibitive for an outsider-controlled board, and if insiders and outsiders have information of similar importance, i.e., σ_p and σ_a are sufficiently close, then shareholders prefer an outsider-controlled board.

Proof. see the appendix.

Numerical examples suggest that, for this type of firm, when outsiders' information is more important, or only slightly less important, than insiders' information, shareholders prefer an outsider-controlled board. Otherwise, they prefer an insider-controlled board (see the next subsection).

5.5 Summary

This section establishes which type of board is optimal for firms of various types, determined by the importance of insiders' and outsiders' information, agency cost and the outsiders' effort cost. The theoretical results are shown in Figure 5. Since, we are not able to relate the optimal board type to the exogenous parameters for most type-IV firms, we also present, in Figure 6, the results on board type for a numerical example.[15] The figures show, essentially, that outsider-control is preferred when outsiders' information is more important or not too much less important than insiders' information, except when agency costs are more important than at least one party's information. In the latter case, shareholders are indifferent when one party's information is critical or when effort cost is prohibitive.

Insert Figure 5 and Figure 6 Here

[15] Figure 5 assumes that effort cost is not prohibitive, but in the numerical example of Figure 6, for some parameter values, effort cost is prohibitive. In particular, for some type-I firms for which insiders' information is more important than agency costs, prohibitive effort costs causes shareholders to be indifferent instead of preferring an outsider-controlled board.

Conventional wisdom has it that outside control of boards, or at least of key committees such as the audit committee, is always preferred. This thinking underlies recent legislation, in particular the Sarbanes-Oxley Act, and changes in New York Stock Exchange and NASDAQ listing rules. Our results show that outside board control may, in fact, be value reducing. In particular, if insiders have important information relative to that of outsiders, giving control to outsiders may result in a loss of information that is more costly than the agency cost associated with inside control.

6 Comparative Statics and Empirical Implications

In this section, we examine the effects of changes in the various exogenous parameters of the model on the endogenous variables. We consider only changes in the parameters that do not affect whether effort cost is prohibitive. We also assume either that the parameter changes do not change the optimal board type or that the board type is constrained by regulation or some other factor that is outside our model. We also consider the empirical implications of these results.

6.1 Comparative Statics

We begin our comparative statics analysis by investigating the effect of changes in the importance of outsiders' information on the number of outsiders, profits, outsiders' equity share, and outsiders' effort.

Proposition 7 (OCB): The effect of changes in the importance of outsiders' information for outsider-controlled boards. An increase in the importance of outsiders' information, σ_p, results in

- An increase in the number of outsiders;[16]
- A decrease in profits;
- An increase in the share of outsiders;
- No change in the equilibrium effort of outsiders.

Proof. See the appendix.

It is not surprising that, when outsiders' information becomes more important, it is optimal to have more of them. Since their effort doesn't change (recall that the number of outsiders is determined to keep effort constant), this increases the probability that they become informed and, by itself, would increase profits. There are two opposing effects, however. If either outsiders do not become informed or become informed but delegate to insiders the loss due to their failure to become informed or their refusal to communicate the information fully increases. It turns out that the profit-reducing effects dominate. Since profits decrease, outsiders must be given a larger share of profits to assure their participation.

The corresponding result for insider-controlled boards is given in the next proposition.

Proposition 7 (ICB): The effect of changes in the importance of outsiders' information for insider-controlled boards. If effort cost is prohibitive, an increase in the importance of outsiders'

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[16] If effort cost is prohibitive, the number of outsiders is not affected. Also, if effort cost is not prohibitive, and insiders' information is not critical but is more important than agency costs, we assume that $\sigma_p > b\sqrt{12}$. If this inequality does not hold, V_{OCB} can be decreasing in σ_p, as the following example shows. If b =0.05 and $\sigma_a = 0.425$, then V_{OCB} = 0.000441655789903148 at $\sigma_p = 0.0626275316444043$ and V_{OCB} = 0.000077441977273082 at $\sigma_p = 0.0651275316444044$. In this case, it is unclear whether αV_{OCB} increases or decreases, and hence whether the number of outsiders increases or decreases.

information, σ_p, decreases profits. Since there are no outsiders in this case, their share is irrelevant, and there is no effect on the number of outsiders. If effort cost is not prohibitive, and outsiders' information is critical, the number of outsiders increases, but profits and the share of outsiders do not change. There is no change in the equilibrium effort of outsiders.

Proof. See the appendix.

If effort cost is prohibitive, outsiders never become informed, and insiders never delegate to outsiders. Since insiders make the decision with no information about $\tilde{p}$, profits are reduced by the full value of outsiders' information. Any increase in the importance of outsiders' information raises this value and lowers profits.

If effort cost is not prohibitive, and outsiders' information is critical, insiders always delegate if outsiders are informed. Consequently, the increase in the importance of outsiders' information has no effect on profits if outsiders become informed. If they do not become informed, however, insiders choose the scale with no information about $\tilde{p}$, so profits decline with an increase in the importance of outsiders' information. Thus, on average profits decline. To satisfy the participation constraint of outsiders, (3), their share must rise. Moreover, the increase in the importance of outsiders' information also increases the marginal value of their becoming informed. Consequently, since both their share of the marginal value of becoming informed and the marginal value itself increase, it is optimal to have more outsiders.

As is the case for outsider-controlled boards, the number of outsiders is chosen so that their equilibrium effort is e^*, independently of the importance of outsiders' information (unless, of course, effort cost is prohibitive).

Note that the proposition says nothing about the case in which effort cost is not prohibitive, and outsiders' information is not critical. It turns out that, in this case, the number of outsiders, profits and the share of outsiders are not monotone in the importance of outsiders' information.

Proposition 8 (OCB): The effect of changes in the importance of insiders' information for outsider-controlled boards. If insiders' information is less important than agency costs, an increase in the importance of insiders' information, σ_a, results in an increase in the number of outsiders, a decrease in profits, and increase in the equity share of outsiders.[17] If insiders' information is critical or if it is more important than agency costs and effort cost is prohibitive, an increase in the importance of insiders' information has no effect on the number of insiders, profits, or the equity share of outsiders. If insiders' information is more important than agency costs but not critical (and effort cost is not prohibitive), the number of outsiders and profits move in the same direction as each other, while the equity share of outsiders moves in the opposite direction. In any case, there is no effect on equilibrium effort of outsiders.

Proof. See the appendix.

The intuition for Proposition 8 (OCB) is as follows. If insiders' information is less important than agency costs, then outsiders never delegate, even if they're uninformed. Consequently, the marginal value of becoming informed is the full value of outsiders' information, which is not affected by the importance of insiders' information. Profits, however, decrease as a result of the increase in the importance of insiders' information, since outsiders will have less information about $\tilde{a}$ when they choose scale. This requires an increase in outsiders' equity stake to keep their compensation the same. The increased equity stake increases outsiders' share of the marginal value from becoming informed, so to keep outsiders' effort constant at e^*, one must increase the number of outsiders. If either insiders' information is critical or it is more important than agency costs and effort cost is prohibitive, outsiders

[17] If effort cost is prohibitive, there is no effect on the number of outsiders.

always delegate, so insiders' information is always fully utilized. There is, therefore, no affect on profits, hence no effect on the equity share of outsiders, no effect on the marginal value of outsiders' becoming informed, and no effect on the optimal number of outsiders.

Unfortunately, one cannot pin down the sign of the effect of a change in the importance of insiders' information when insiders' information is more important than agency costs but not critical, since there are conflicting effects on profits (numerical examples show profits can go either up or down). If, say, profits increase when the importance of insiders' information changes, then so too will the marginal value of outsiders' information. Indeed, the latter will increase by a greater relative amount than the former. Meanwhile, outsiders' share of profits must decrease by the same percentage that profits increase. Consequently, outsiders' share of the marginal benefit of becoming informed increases. This results in an increase in the optimal number of outsiders. Thus the number of outsiders and profits move in the same direction as each other.

The corresponding result for insider-controlled boards is given in the next proposition.

Proposition 8 (ICB): The effect of changes in the importance of insiders' information for insider-controlled boards. If effort cost is prohibitive, changes in the importance of insiders' information has no affect on profits, the share of outsiders, or the number of outsiders. If effort cost is not prohibitive, an increase in the importance of insiders' information results in

- A decrease in the number of outsiders;
- A decrease in profits;
- An increase in the share of outsiders;
- No change in the equilibrium effort of outsiders.

Proof. See the appendix.

As in the previous result, if effort cost is prohibitive, outsiders never become informed, and insiders never delegate to outsiders. Since insiders make the decision with full information about $\tilde{a}$, profits are unaffected by the change in the importance of insiders' information. As before, the share of outsiders is irrelevant, and the number of outsiders' is zero, independently of the importance of insiders' information.

Next we consider the effects of changes in effort cost, potential profits, and the opportunity cost of outsiders. To consider comparative statics on effort cost, we introduce a scale factor $g > 0$ for the cost function, i.e., we write $c(e) = gk(e)$, where the function k satisfies Assumption 2.[18] We also consider the effects of changes in potential profit and the opportunity cost of outside directors. These result are valid for both outsider- and insider-controlled boards.

Proposition 9:

- **The effect of changes in effort cost.** If effort cost is prohibitive, changes in effort cost, g, have no effect. Otherwise, an increase in effort cost results in a decrease in the number of outsiders, a decrease in profits, and a less-than-proportionate increase in the equity share of outsiders. There is no effect on equilibrium effort of outsiders.
- **The effect of a change in potential profit.** An increase in potential profits, π_0, results in a decrease in the number of outsiders (or no change if effort cost is prohibitive), an

[18] Changes in effort cost of the type we consider do not change equilibrium effort when effort cost is not prohibitive, e^*.

increase in profits, and a decrease in the equity share of outsiders. There is no effect on equilibrium effort of outsiders.

- **The effect of a change in the opportunity cost of outsiders.** An increase in the opportunity cost of outsiders, $\bar{U}$, results in an increase in the number of outsiders (or no change if effort cost is prohibitive), an increase in profits, and an increase in the equity share of outsiders. There is no effect on equilibrium effort of outsiders.

Proof. See the appendix.

The intuition for the results in Proposition 9 is straightforward. An increase in effort costs forces an increase in outsiders' equity share (unless effort cost is prohibitive). The increase in effort costs also dictates a reduction in the number of outsiders to keep them from reducing their effort. The increase in outsiders' equity share partially reverses this effect, but not fully, since the increase in share is less than proportionate to the increase in cost. Since the number of outsiders falls, but their effort stays constant, they are less likely to become informed, and profits fall. An increase in the opportunity cost of outsiders also requires an increase in outsiders' equity share, leading to an increase in the number of outsiders. This increases the probability of outsiders becoming informed (more outsiders, same effort) and hence increases profits. The opposite occurs if potential profits increase.

6.2 Empirical Implications

The results of the previous subsection predict relationships between profitability and the number of outside directors caused by variation in one or more of the underlying parameters of the model. In particular, for outsider-controlled boards, one expects to observe a negative relationship between profitability and the number of outside directors if one controls for everything except the importance of outsiders' information, the importance of insiders' information and restricts the sample to firms in which agency costs are more important than insiders' information, or potential profits. On the other hand, one expects a positive relationship between profitability and the number of outside directors if one controls for everything except effort costs, the opportunity cost of outside directors, or the importance of insiders' information and restricts the sample to firms in which agency costs are less important than insiders' information. Results for insider-controlled boards are similar, except that the negative relationship between profits and the number of outside directors due to changes in the importance of outsiders' information requires that their information be critical. Also, for insider-controlled boards, one expects a *positive* relationship between profits and the number of outsiders if one controls for everything except the importance of insiders' information, regardless of whether agency costs are more important than insiders' information. None of these results, of course, implies that more or fewer outside directors improves profitability *per se*.

Many researchers have looked for, but failed to find, a relationship between performance, variously measured, and the fraction of outside board members.[19] Yermack (1996) and Eisenberg, *et al.* (1998) provide evidence that performance is negatively related to board size. This empirical regularity has been taken to be evidence for the view espoused by Jensen (1993) and Lipton and Lorsch (1992) that when boards become too big, agency problems render them less effective. But, as noted by Hermalin and Weisbach (2003), interpreting the empirical results in this way raises the question why do we observe large boards. In our model, profits, the fraction of outside board members, and board size are endogenous.[20] In particular, we see that, depending on which exogenous variables are driving the

[19] See MacAvoy, *et al.* (1983), Hermalin and Weisbach (1991), Mehran (1995), Klein (1998), and Bhagat and Black (2000).

[20] The number of insiders on the board is determined by factors outside of our model, so the change in the number of outsiders is also a change in board size by the same amount.

variation in profits, board size, and the number of outsiders, the correlation could be either positive or negative. And, if more than one exogenous variable is changing in the cross section, the effects could cancel out. Thus our model can explain both the lack of an observed relationship between performance and board composition and the negative relationship between performance and board size, depending on what is controlled for, without any implication that large boards are somehow less effective. As always, one must exercise extreme caution in interpreting observed relationships as causal.

Another implication of our approach is that in firms for which insiders' information is very important, boards should have few outsiders and be insider-controlled (Propositions 4, 6, and 8 (ICB)). One might expect startups, especially those in the high-tech industries, to be in that category and thus to have insider-controlled boards with few outsiders.

7 Conclusions

We have presented a model to determine the optimal control of corporate boards of directors and the number of outside directors. Our main result refutes the common view that outsider-controlled boards increase shareholder value. The reason is that the common view does not account for the fact that agency problems affect not only the decisions made by insiders but also the information that they convey to the board. Outsider-controlled boards may be inferior, because some of the decision-relevant information possessed by insiders will be lost.

We also characterize whether the board is optimally controlled by insiders or outsiders, the optimal number of outsiders, and resulting profits as functions of the importance of insiders' and outsiders' information, the extent of agency problems, and some other factors. This leads to an endogenous relationship between profits and the number of outside directors that furthers our understanding of some documented empirical regularities. In particular, our model can account for the observed absence of a relationship between the fraction of outsiders on the board and profits, as well as the observed negative relationship between board size and profits. Since the latter relationship is driven by changes in exogenous parameters in our model, there is no causal relationship. The model thus demonstrates the pitfall of inferring from the negative relationship between board size and profits that large boards are less effective.

Appendix

Lemma 1. L is continuous and increasing in x. For all $x \geq 0$, $f(b,x) = 3L(b,x) + b^2 + xb$, f is continuous and increasing in x, $2bx \leq f(b,x) \leq 2bx + b^2$, and $\max\left\{0, \frac{b}{3}(x-b)\right\} \leq L(b,x) \leq bx/3$. Also, for any $x > 0$, $b^2 \geq L(b,x)$ if and only if $b \geq \sigma(x)$. Finally, $L(b,x)$ and $f(b,x)$ f are strictly increasing in b, for any x.

Proof. That L is continuous and increasing in x is shown in Lemma 1 of Harris and Raviv (2004). That $f(b,x) = 3L(b,x) + b^2 + bx$ follows easily from (10) and (11).

For $n \geq 1$, define $x_n(b)$ as in Harris and Raviv (2004) to be the point at which $N(b,x)$ jumps from $n-1$ to n, i.e., $N(b,x_n) = n-1$ and for $x > x_n(b)$ but sufficiently close, $N(b,x) = n$. As shown in the proof of Lemma 1 in Harris and Raviv (2004),

$$x_n(b) = 2bn(n-1),$$

$L(b,x)$ is continuous in x,[21] and

$$L(b,x_n) = bx_n/3$$

(here and below we drop the argument b from the function x_n, since it remains fixed for this argument). It is easy to check that, for all n and $x \in (x_n, x_{n+1})$, $L(b,x) < bx/3$. This proves that $L(b,x) \leq bx/3$, for all $x \geq 0$.

It is also easy to check that f is continuous in x,

$$f(b,x_n) = 2bx_n + b^2, \forall n$$

and

$$f(b,x) < 2bx + b^2, \forall x \in (x_n, x_{n+1}).$$

Since f is continuous and clearly increasing in x on each interval $[x_n, x_{n+1}]$, f is increasing in x.

Consider the following problem:

$$\max_{x \in (x_n, x_{n+1})} 2bx + b^2 - f(b,x).$$

The solution of this problem is $x = 2bn^2$, and the maximized value of the objective function is

$$4b^2n^2 + b^2 - (2bn)^2 = b^2,$$

independently of n. Consequently, we have that, for any x,

$$2bx + b^2 - f(b,x) \leq b^2 \text{ or } f(b,x) \geq 2bx,$$

[21] Continuity at $x = 0$ follows from the fact that the upper branch on the right hand side of (10) approaches zero as x approaches zero.

as claimed. A similar argument shows that $L(b,x) \geq \frac{b}{3}(x-b)$. Since $L \geq 0$, it follows that $L(b,x) \geq \max\left\{0, \frac{b}{3}(x-b)\right\}$.

Now, the above argument implies that $L(b,x_2) = 4b^2/3 > b^2$. Therefore, if x is such that $b^2 \geq L(b,x)$, then $x < x_2$. On the other hand, if $b \geq \sigma(x)$, then $x \leq (2\sqrt{3})b < 4b = x_2$. Thus, in either case, $N(b,x) = 1$, so $L(b,x) = \sigma^2(x)$.

That $L(b,x)$ is strictly increasing in b is obvious if the change in b does not change $N(b,x)$, so consider two values b_0 and $b_k(x)$, for some $k \geq 2$, where $b_k(x) = \frac{x}{2bn(n-1)}$, and $b_0 \in (b_{k+1}(x), b_k(x))$. Then $N(b_0,x) = k$ and $N(b_k(x),x) = k-1$. Therefore, from (10),

$$
\begin{aligned}
L(b_k(x),x) - L(b_0,x) &= \frac{\sigma^2(x)}{k^2(k-1)^2}(2k-1) + \frac{1}{3}\left[b_k^2(x)k(k-2) - b_0^2(k^2-1)\right], \\
&> \frac{\sigma^2(x)}{k^2(k-1)^2}(2k-1) + \frac{b_k^2(x)}{3}(1-2k), \\
&= \left[\frac{\sigma^2(x)}{k^2(k-1)^2} - \frac{x^2}{12k^2(k-1)^2}\right](2k-1), \\
&= 0.
\end{aligned}
$$

Since L is increasing in b and $f(b,x) = 3L(b,x) + b^2 + xb$, f is also increasing in b.

Q.E.D.

Proposition 3: Optimal Board Control When $f(b,A) > L(b,P)$, $f(b,P) > L(b,A)$, and $b \geq \min\{\sigma_p, \sigma_a\}$. Shareholders prefer an outsider-controlled board. This preference is strict unless effort cost is prohibitive if outsiders are in control.

Proof. First, suppose $b \geq \max\{\sigma_p, \sigma_a\}$. From Propositions 1 and 2, we have $p^* = P$, outsiders will not delegate even if uninformed, $a^* = 0$, $N(b,A) = N(b,P) = 1$, $V_{OCB} = \sigma_p^2$, and $V_{ICB} = 0$. It follows from the above, using (23) and (31), that

$$
\Delta\pi = b^2 - \sigma_a^2 + \begin{cases} \sigma_p^2 - M, \text{ if } c'(0) < \alpha\sigma_p^2, \\ 0, \text{ otherwise.} \end{cases}
$$

But $b^2 - \sigma_a^2 \geq 0$ by assumption, and, as noted above, since $V_{OCB} = \sigma_p^2$, $\sigma_p^2 > M$, if $c'(0) < \alpha\sigma_p^2$. Consequently, in this case, $\Delta\pi \geq 0$, with equality if and only if $b = \sigma_a$ and $c'(0) \geq \alpha\sigma_p^2$.

Next suppose $\sigma_p > b \geq \sigma_a$. From Propositions 1 and 2, $p^* = P$, $V_{OCB} = \sigma_p^2$, $a^* \in (0,A)$, and $N(b,A) = 1$. Since $a^* < A$, $N(b,a^*) = 1$. Consequently,

$$V_{ICB} = \sigma_p^2 + b^2 - \lambda(a^*),$$

where

$$\lambda(a^*) = \frac{a^*}{A}\sigma^2(a^*) + \left(1 - \frac{a^*}{A}\right)\left[\left(\frac{a^*}{2} + b\right)^2 + b^2\right].$$

It is easy to check, using the fact that $N(b, A) = 1$ implies that $A \leq 4b$, that $\lambda(a^*)$ is concave in a^*.

Since $c'(0) < \alpha b^2 < \alpha\sigma_p^2$, it follows from (23) and (31) that

$$\Delta\pi \geq \lambda(a^*) - \sigma_a^2. \tag{32}$$

The right hand side of (32) is positive at $a^* = 0$ and zero at $a^* = A$. Consequently, if $\Delta\pi$ were non-positive anywhere on $(0, A)$, it would have an interior minimum, contradicting the fact that $\lambda(a^*)$ is concave. Therefore, we have shown that $\Delta\pi > 0$.

Finally, suppose $\sigma_a > b \geq \sigma_p$. From Proposition 2, $a^* = 0$, $N(b, P) = 1$, and $V_{ICB} = 0$. Consequently,

$$\pi^*(ICB) = \pi_0 - \left(\sigma_p^2 + b^2\right).$$

If $c'(0) < \alpha V_{OCB}$,

$$\pi^*(OCB) = \pi_0 - \left(\sigma_p^2 + b^2\right) + V_{OCB} - M = \pi^*(ICB) + \left[1 - (1 - e^*)^{n_{OCB}}\right]V_{OCB},$$

i.e., $\Delta\pi = \pi^*(OCB) = \left[1 - (1 - e^*)^{n_{OCB}}\right]V_{OCB} > 0$. If $c'(0) \geq \alpha V_{OCB}$, clearly $\Delta\pi = 0$.

Q.E.D.

Proposition 4: Optimal Board Control When $f(b, P) \leq L(b, A)$. Shareholders prefer an insider-controlled board. This preference is strict unless agency cost is more important than outsiders' information, $b \geq \sigma_p$, or effort cost is prohibitive for an insider-controlled board.

Proof. First suppose $b \geq \sigma_p$. From Propositions 1 and 2, we have $p^* = 0$, outsiders delegate if uninformed, $a^* = 0$, $N(b, P) = 1$, and $V_{ICB} = 0$. Also, $V_{OCB} = 0$ follows immediately from Proposition 1 and the fact that $N(b, P) = 1$. Finally, $\Delta\pi = \sigma_p^2 + b^2 - l_U = 0$, since $b^2 < \sigma_a^2$ and $\alpha V_{OCB} = \alpha V_{ICB} = 0 < c'(0)$.

Now suppose $b < \sigma_p$. From, Propositions 1 and 2, we have $p^* = 0$, outsiders delegate if uninformed, $a^* \in (0, A)$, and $V_{OCB} = \sigma_p^2 - L(b, P)$. Therefore, from (27),

$$V_{OCB} - V_{ICB} = \frac{a^*}{A}\left[L(b, a^*) - L(b, P) - b^2\right].$$

From Lemmas 1 and 2,

$$L(b, P) = f(b, a^*) > L(b, a^*) + b^2.$$

Thus, $V_{OCB} - V_{ICB} < -\frac{a^*}{A} 2b^2 < 0$.

Now, from (23) and (31), if $c'(0) < \alpha V_{ICB}$,

$$\pi^*(ICB) = \pi_0 - \sigma_p^2 - b^2 + V_{ICB} - M$$

and

$$\pi^*(OCB) \le \pi_0 - \sigma_p^2 - b^2 + V_{OCB} - M \ .$$

Consequently, $\Delta\pi < 0$.

If $c'(0) \ge \alpha V_{ICB} > \alpha V_{OCB}$, then, it is clear from (23) and (31) that $\Delta\pi = 0$.

Q.E.D.

Proposition 5: Optimal Board Control When $f(b, A) \le L(b, P)$. Shareholders prefer an outsider-controlled board. This preference is strict if agency costs are less important than insiders' information, and effort cost is not prohibitive for an outsider-controlled board.

Proof. First suppose $b \ge \sigma_a$. From Propositions 1 and 2, $p^* = P$, outsiders do not delegate even if uninformed, $a^* = A$, $N(b, A) = 1$, $V_{OCB} = \sigma_p^2$, and $V_{ICB} = \sigma_p^2 + b^2 - \sigma_a^2$. Since $b^2 > \sigma_a^2$ for this type of firm, $V_{ICB} > V_{OCB}$.

Since outsiders' information is critical, $\sigma_p \ge b$, so $c'(0) < \alpha b^2 < \alpha\sigma_p^2$. Therefore,

$$\pi^*(OCB) = \pi_0 - \left(\sigma_p^2 + \sigma_a^2\right) + \sigma_p^2 - M = \pi_0 - \sigma_a^2 - M \ ,$$

and

$$\pi^*(ICB) = \pi_0 - \left(\sigma_p^2 + b^2\right) + \sigma_p^2 + b^2 - \sigma_a^2 - M = \pi_0 - \sigma_a^2 - M \ .$$

Thus $\Delta\pi = 0$.

Now suppose $b < \sigma_a$. From Propositions 1 and 2, p^* is given by (12) and $a^* = A$. Since $b < \sigma_a$,

$$V_{OCB} - V_{ICB} = \left(1 - \frac{p^*}{P}\right)\left[L(b, A) - L(b, P - p^*) - b^2\right] = \left(1 - \frac{p^*}{P}\right)\left[f(b, P - p^*) - L(b, P - p^*) - b^2\right].$$

From Lemma 1 and the fact that $p^* \in (0, P)$, $f(b, P - p^*) > L(b, P - p^*) + b^2$. Hence $V_{OCB} > V_{ICB}$. If $c'(0) < \alpha V_{OCB}$, then

$$\pi^*(OCB) = \pi_0 - \left(\sigma_p^2 + b^2\right) + V_{OCB} - M \ ,$$

and

$$\pi^*(ICB) \le \pi_0 - \left(\sigma_p^2 + b^2\right) + V_{ICB} - M \ .$$

Therefore, $\Delta\pi > 0$. If $c'(0) \ge \alpha V_{OCB}$, then $\pi^*(OCB) = \pi^*(ICB) = \pi_0 - \left(\sigma_p^2 + b^2\right)$.

Q.E.D.

Proposition 6: Optimal Board Control When $f(b,A) > L(b,P)$, $f(b,P) > L(b,A)$, and $\min\{\sigma_a, \sigma_p\} > b$. If effort cost is not prohibitive for at least one type of board, then shareholders prefer the board type that results in the largest marginal value of outsiders' information. If effort cost is prohibitive for both board types, shareholders are indifferent.

Proof. Suppose $c'(0) < \alpha \max\{V_{OCB}, V_{ICB}\}$. If $V_{OCB} > V_{ICB}$, then

$$\pi^*(OCB) = \pi_0 - (\sigma_p^2 + b^2) + V_{OCB} - M,$$

and

$$\pi^*(ICB) \le \pi_0 - (\sigma_p^2 + b^2) + V_{ICB} - M.$$

Thus $\Delta\pi > 0$. If $V_{OCB} < V_{ICB}$, then

$$\pi^*(OCB) \le \pi_0 - (\sigma_p^2 + b^2) + V_{OCB} - M,$$

and

$$\pi^*(ICB) = \pi_0 - (\sigma_p^2 + b^2) + V_{ICB} - M,$$

so $\Delta\pi < 0$.

If $c'(0) \ge \alpha \max\{V_{OCB}, V_{ICB}\}$, then $\pi^*(OCB) = \pi^*(ICB) = \pi_0 - (\sigma_p^2 + b^2)$.

Q.E.D.

Corollary. If effort cost is not prohibitive for an outsider-controlled board, and if insiders and outsiders have information of similar importance, i.e., σ_p and σ_a are sufficiently close, then shareholders prefer an outsider-controlled board.

Proof. First suppose $\sigma_a = \sigma_p$. If $b \ge \sigma_a = \sigma_p$, Proposition 3 implies the result. If $b < \sigma_a$, then a^* is determined by (28) and p^* is determined by (12). Substituting $P = A$ into (12) and (28) shows that $a^* = A - p^*$. Using this result in (27) and (16) shows that,

$$V_{OCB} - V_{ICB} = \left(1 - 2\frac{a^*}{A}\right) b^2.$$

We claim that $a^*/A < 1/2$. Since f is increasing in x, however, it suffices to show that $f(b, A/2) > L(b,A)$. By Lemma 1, $f(b, A/2) \ge 2b\left(\frac{A}{2}\right) = Ab$, and $L(b,A) \le \frac{bA}{3}$, so the claim follows from the fact that $b > 0$. Therefore $V_{OCB} > V_{ICB}$, so the result follows from Proposition 6. For σ_p and σ_a sufficiently close, the result follows from continuity.

Q.E.D.

The following lemma will be used in the proofs of several subsequent propositions.

Lemma 3. For either board type, if effort cost is not prohibitive, then π and l_I or l_I' change in opposite directions as a result of a change in σ_a or σ_p. The share of outsiders, α changes in the opposite direction from π if (3) is required.

Proof. We prove this for the case of an outsider-controlled board; the proof for the other board type is symmetric.

Since effort cost is not prohibitive, $\pi = \pi_0 - l_I - M$. If α is exogenous, i.e., we do not require (3), the result is obvious. If α is endogenous, suppose, for example, l_I falls, and π also falls. Then, to satisfy (3), $\alpha\pi$ cannot change, since e^* does not change. Therefore, α must increase. Also since e^* does not change, M falls. Since both l_I and M decline, π increases, contradicting our supposition that π falls. Thus π must increase when l_I falls (a symmetric argument applies if l_I rises). If (3) is required, then, α must clearly change in the opposite direction from π.

Q.E.D.

Proposition 7 (OCB): The effect of changes in the importance of outsiders' information for outsider-controlled boards. An increase in the importance of outsiders' information, σ_p, results in

- An increase in the number of outsiders (if insiders' information is not critical but is more important than agency costs, we assume that $\sigma_p > b\sqrt{12}$);
- A decrease in profits;
- An increase in the share of outsiders;
- No change in the equilibrium effort of outsiders.

Proof. First suppose effort cost is not prohibitive, so equilibrium effort is e^*, independent of σ_p.

Now suppose $b \geq \sigma_a$, so that $p^* = P$. In this case, an increase in σ_p has no effect on $l_I = L(b, A)$ but increases $l_U = \sigma_p^2 + \sigma_a^2$ and hence increases V_{OCB}. Since the increase in σ_p has no effect on l_I it does not directly affect profits, so that α is not affected. The increase in V_{OCB}, however, requires an increase in n_{OCB}.

Next suppose insiders' information is critical, so that $p^* = 0$. In this case, an increase in σ_p increases $l_I = L(b, P) + b^2$ and, therefore, decreases profits and increases in α by Lemma 3. Now $V_{OCB} = \sigma_p^2 - L(b, P)$. It is clear that this quantity is (weakly) increasing in σ_p as long as the change in σ_p does not change $N(b, P)$ and strictly increasing if $N(b, P) > 1$. If a small increase in σ_p increases $N(b, P)$, then, from the proof of Lemma 1, $P = 2bk(k-1)$ for some $k > 1$, $N(b, P) = k - 1$, and $L(b, P) = \dfrac{2b^2k(k-1)}{3}$. Consider a small increase in P to P'. Then $N(b, P') = k$, and

$$L(b, P') - L(b, P) = \frac{\sigma^2(P')}{k^2} - \frac{b^2(k-1)^2}{3} = \frac{1}{k^2}\left[\sigma^2(P') - \sigma^2(P)\right] < \sigma^2(P') - \sigma^2(P).$$

Thus, in this case V_{OCB} is increasing in σ_p. Since both α and V_{OCB} increase, n_{OCB} also increases.

Finally, suppose $p^* \in (0, P)$. Let $p_0 = P - p^* > 0$. Then, using Lemmas 1 and 2 and (17), we can write

$$l_I = L(b, p_0)\left(3 - 2\frac{p_0}{P}\right) + p_0 b\left(1 - \frac{p_0}{P}\right) + b^2,$$

and

$$V_{OCB} = \sigma_p^2 - \left[L(b, p_0)\left(3 - 2\frac{p_0}{P}\right) + p_0 b\left(1 - \frac{p_0}{P}\right)\right].$$

Since p_0 is independent of P, clearly l_I increases with σ_p, so profits decrease and α increases as before. Moreover,

$$\frac{\partial V_{OCB}}{\partial \sigma_p} = 2\sigma_p - \frac{\sigma(p_0)}{\sigma_p^2}\left[2L(b, p_0) + p_0 b\right].$$

Thus, to show that V_{OCB} increases with σ_p, it suffices to show that

$$\sigma_p^2 > \frac{\sigma(p_0)}{\sigma_p}\left[L(b, p_0) + \frac{1}{2} p_0 b\right], \tag{33}$$

for $\sigma_p \geq \sigma(p_0)$. Since the left side of (33) is increasing in σ_p, and the right side is decreasing in σ_p, it suffices to show that (33) is satisfied at $\sigma_p = \sigma(p_0)$, i.e., that

$$\sigma_p^2 > L(b, P) + \frac{1}{2} Pb. \tag{34}$$

Suppose $N(b, P) = k$. It is easy to check that (34) is satisfied if and only if

$$\sigma_p^2 > \frac{b^2 k^2}{3} + \frac{bPk^2}{2(k^2 - 1)}.$$

From the proof of Lemma 1, we have $P > 2bk(k-1)$. Consequently,

$$\frac{b^2 k^2}{3} + \frac{bPk^2}{2(k^2 - 1)} < \sigma_p^2 \frac{4k + 1}{(k-1)^2 (k+1)}.$$

Therefore, it suffices to show that

$$\frac{4k + 1}{(k-1)^2 (k+1)} < 1,$$

or $k(k-1) > 5$, which is true for all $k \geq 3$. Since we assume that $\sigma_p > b\sqrt{12}$, it follows that $k \geq 3$. Since both α and V_{OCB} increase, n_{OCB} also increases.

If effort cost is prohibitive, then $\pi = \pi_0 - \sigma_p^2 - \min\left\{\sigma_a^2, b^2\right\}$. Therefore, an increase in σ_p reduces profits and, hence, increases α. Since we assume that effort cost remains prohibitive, the increase in σ_p does not affect the number of outsiders.

Q.E.D.

Proposition 7 (ICB): The effect of changes in the importance of outsiders' information for insider-controlled boards. If effort cost is prohibitive, an increase in the importance of outsiders' information, σ_p, results in a decrease in profits. Since there are no outsiders in this case, their share is irrelevant, and there is no effect on the number of outsiders. If effort cost is not prohibitive, and outsiders' information is critical, the number of outsiders increases, but profits and the share of outsiders do not change. There is no change in the equilibrium effort of outsiders.

Proof. If effort cost is prohibitive, outsiders never become informed, and insiders never delegate. Therefore, $n_{ICB} = 0$. The share of outsiders is irrelevant and the result on profits follows from the fact that $\pi = \pi_0 - \sigma_p^2 - b^2$.

If effort cost is not prohibitive, and outsiders' information is critical, $a^* = A$, $\pi = \pi_0 - L(b, A) - M$, and $V_{ICB} = \sigma_p^2 + b^2 - L(b, A)$. The results follow trivially.

Equilibrium effort is e^* (or zero if effort cost is prohibited), which is not affected by changes in σ_p.

Q.E.D.

Proposition 8 (OCB): The effect of changes in the importance of insiders' information for outsider-controlled boards. If insiders' information is less important than agency costs, an increase in the importance of insiders' information, σ_a, results in an increase in the number of outsiders, a decrease in profits, and increase in the equity share of outsiders (if effort cost is prohibitive, there is no effect on the number of outsiders). If insiders' information is critical or if it is more important than agency costs and effort cost is prohibitive, an increase in the importance of insiders' information has no effect on the number of insiders, profits, or the equity share of outsiders. If insiders' information is more important than agency costs but not critical (and effort cost is not prohibitive), the number of outsiders and profits move in the same direction as each other, while the equity share of outsiders moves in the opposite direction. In any case, there is no effect on equilibrium effort of outsiders.

Proof. As in the previous result, if effort cost is prohibitive, then $\pi = \pi_0 - \sigma_p^2 - \min\{\sigma_a^2, b^2\}$. If insiders' information is less important than agency costs, $\pi = \pi_0 - \sigma_p^2 - \sigma_a^2$, so an increase in σ_a reduces profits and, hence, increases α. If insiders' information is more important than agency costs, $\pi = \pi_0 - \sigma_p^2 - b^2$, so an increase in σ_a has no affect on profits or α. In either case, effort remains zero.

When effort cost is not prohibitive, equilibrium effort is e^*, independent of σ_a, since the number of outsiders is chosen to make effort equal to e^*. Consequently, any change in σ_a must leave $\alpha\pi$ unchanged in order to satisfy (3). Profits, in this case, can be written as $\pi = \pi_0 - l_I - M$.

If $\sigma_a \leq b$, $p^* = P$, $l_I = \sigma_a^2$, and $V_{OCB} = \sigma_p^2$. Thus an increase in σ_a reduces profits but does not change V_{OCB}. The results for this case follow immediately.

If insiders' information is critical, $p^* = 0$, so the change in σ_a has no effect on l_I, V_{OCB}, π, α, or n_{OCB}.

Finally, suppose $\sigma_a > b$, but insiders' information is not critical. Since $\alpha\pi$ cannot change, the change in αV_{OCB} is given by $\frac{\alpha}{\pi}(\pi_0 - l_U)$ times the change in π. Since we assume $\pi_0 > \sigma_p^2 + b^2 \geq l_U$, αV_{OCB} and π must change in the same direction. Consequently, n_{OCB} and π must change in the same direction.

Q.E.D.

Proposition 8 (ICB): The effect of changes in the importance of insiders' information for insider-controlled boards. If effort cost is prohibitive, changes in the importance of insiders' information has no affect on profits, the share of outsiders, or the number of outsiders. If effort cost is not prohibitive, an increase in the importance of insiders' information results in

- A decrease in the number of outsiders;
- A decrease in profits;
- An increase in the share of outsiders;
- No change in the equilibrium effort of outsiders.

Proof. The argument for the case in which effort cost is prohibitive is similar to that in the previous proposition, but uses the fact that, in this case, profits do not depend on σ_a.

Suppose effort cost is not prohibitive. Then, we must have $b < \sigma_p$, since otherwise $V_{ICB} = 0$. If outsiders' information is critical, $a^* = A$, and $l_I' = L(b, A)$. Therefore, an increase in σ_a increases l_I', so, from Lemma 3, reduces π and increases α. It is easy to show, using the fact that $\alpha\pi$ doesn't change, that the change in αV_{ICB} is given by $\sigma_p^2 + b^2 - \pi_0$ times the change in α. Since we assume $\pi_0 > \sigma_p^2 + b^2$, and the change in α is positive, αV_{ICB} decreases. Therefore, n_{ICB} decreases.

If outsiders' information is not critical, but $b < \sigma_p$, then an increase in σ_a increases A, but does not affect a^* (Proposition 2). Since $L(b, P) + b^2 > L(b, a^*)$, the increase in σ_a increases l_I' [equation (27)], thus reducing π and increasing α (Lemma 3). As in the previous case, αV_{ICB} decreases so, n_{ICB} decreases.

As in the previous proposition, equilibrium effort of outsiders is unaffected.

Q.E.D.

Proposition 9:

- **The effect of changes in effort cost for outsider-controlled boards.** If effort cost is prohibitive, changes in effort cost, g, have no effect. Otherwise, an increase in effort cost results in a decrease in the number of outsiders, a decrease in profits, and a less-than-proportionate increase in the equity share of outsiders. There is no effect on equilibrium effort of outsiders.
- **The effect of a change in potential profits for outsider-controlled boards.** An increase in potential profits, π_0, results in a decrease in the number of outsiders (or no change if effort cost is prohibitive), an increase in profits, and a decrease in the equity share of outsiders. There is no effect on equilibrium effort of outsiders.
- **The effect of a change in the opportunity cost of outsiders for outsider-controlled boards.** An increase in the opportunity cost of outsiders, $\bar{U}$, results in an increase in the number of outsiders (or no change if effort cost is prohibitive), an increase in profits, and an increase in the equity share of outsiders. There is no effect on equilibrium effort of outsiders.

Proof. If effort cost is prohibitive, equilibrium effort is zero, so the change in g has no effect on outsiders' effort cost (remember $c(0) = 0$), hence no effect on profits or outsiders' share.

Now suppose effort cost is not prohibitive. Since g is a scale factor, the minimum point of $c'(e)(1-e)$ is not affected, and, hence, the equilibrium effort of outsiders does not change. From (3), α must increase, but, since $\bar{U} > 0$, α increases less than in proportion to g. It follows that M increases, so π decreases. Since α increases less than in proportion to g, and V_{OCB} does not change, n_{OCB} decreases.

The second and third bullets follow from (3) and the fact that V_{OCB} does not change.

Q.E.D.

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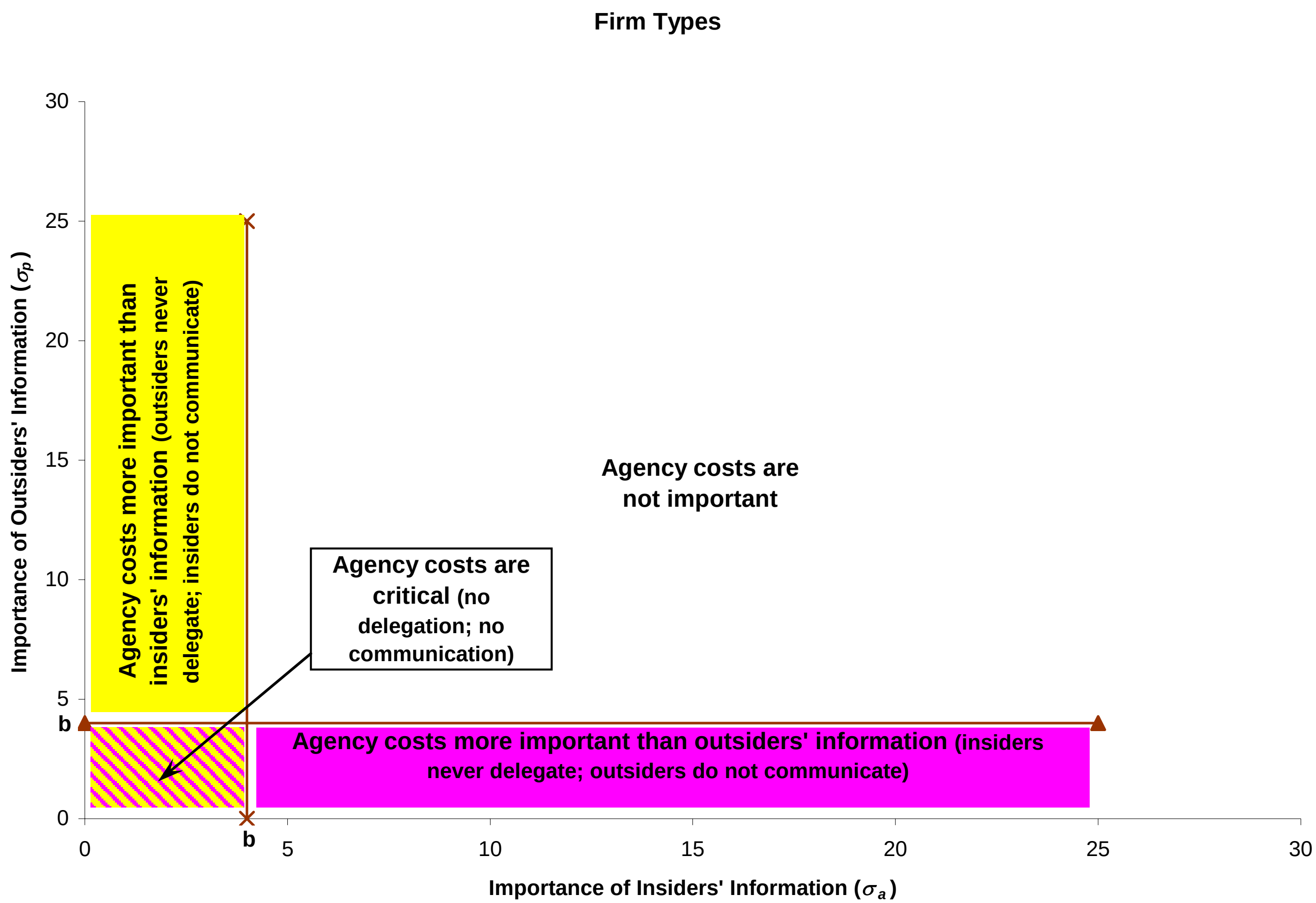

Figure 1

This graph shows the definitions of the terms "Agency costs are more important than insiders' (outsiders') information," $b \geq \sigma_a \left(\sigma_p\right)$, "Agency costs are critical," $b \geq \max\left\{\sigma_a, \sigma_p\right\}$, and "Agency costs are not important," $b < \min\left\{\sigma_a, \sigma_p\right\}$.

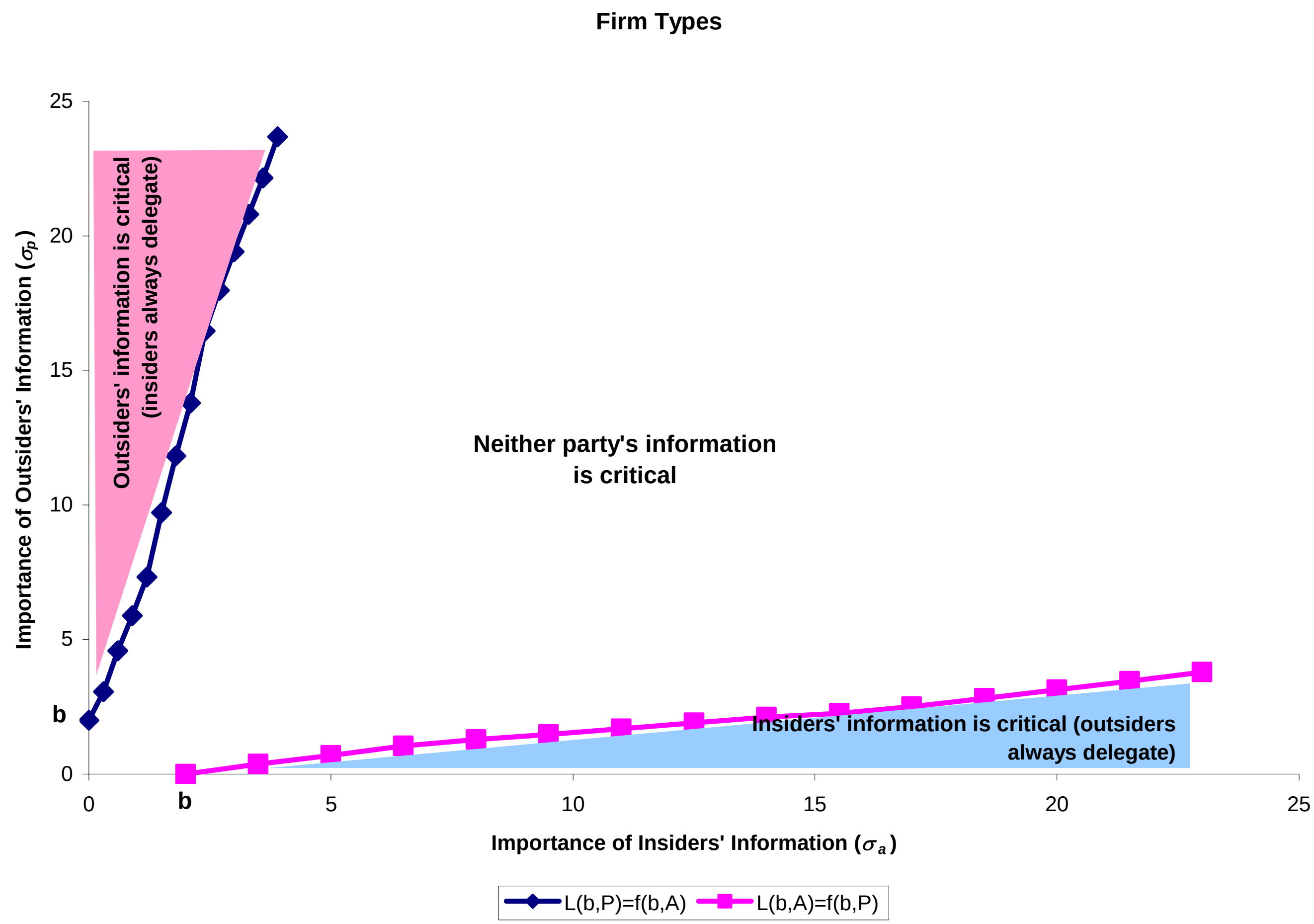

Figure 2

This graph illustrates the definitions of the terms "Insiders' (Outsiders') information is critical," $f(b,P) \le L(b,A)$ ($f(b,A) \le L(b,P)$), and "Neither party's information is critical," i.e., neither of the previous two inequalities is satisfied.

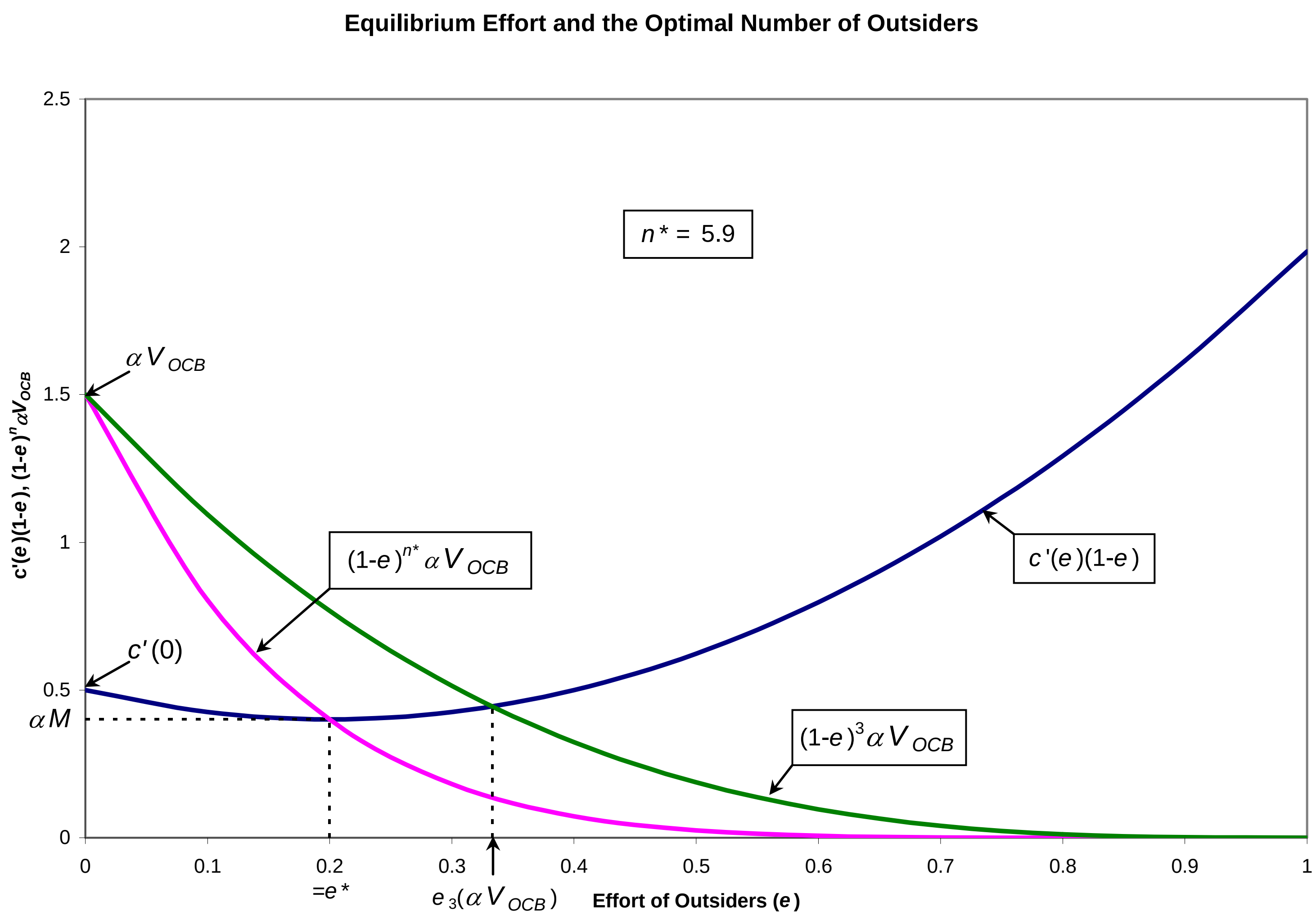

Figure 3

The figure shows the determination of equilibrium effort for an outsider-controlled board with three outsiders, $e_3\left(\alpha V_{OCB}\right)$. It also shows the determination of the optimal number of outsiders, n^*. For this figure, $c'\left(e\right)\left(1-e\right)=0.5-2.475e\left(0.4-e\right)$, and $\alpha V_{OCB}=1.5$.

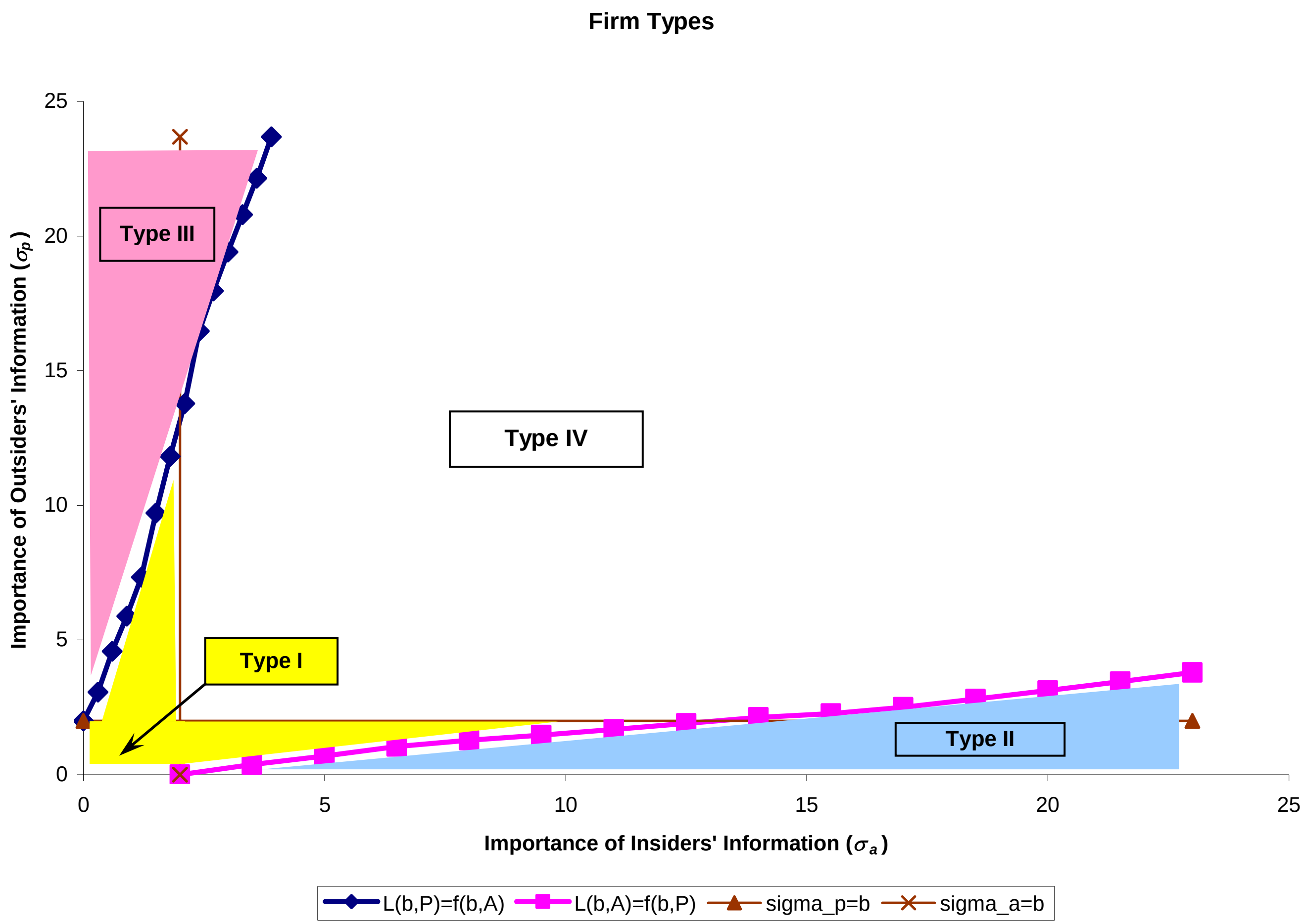

Figure 4

This figure shows the four firm types as defined in section 5. For this diagram, $b = 2$.

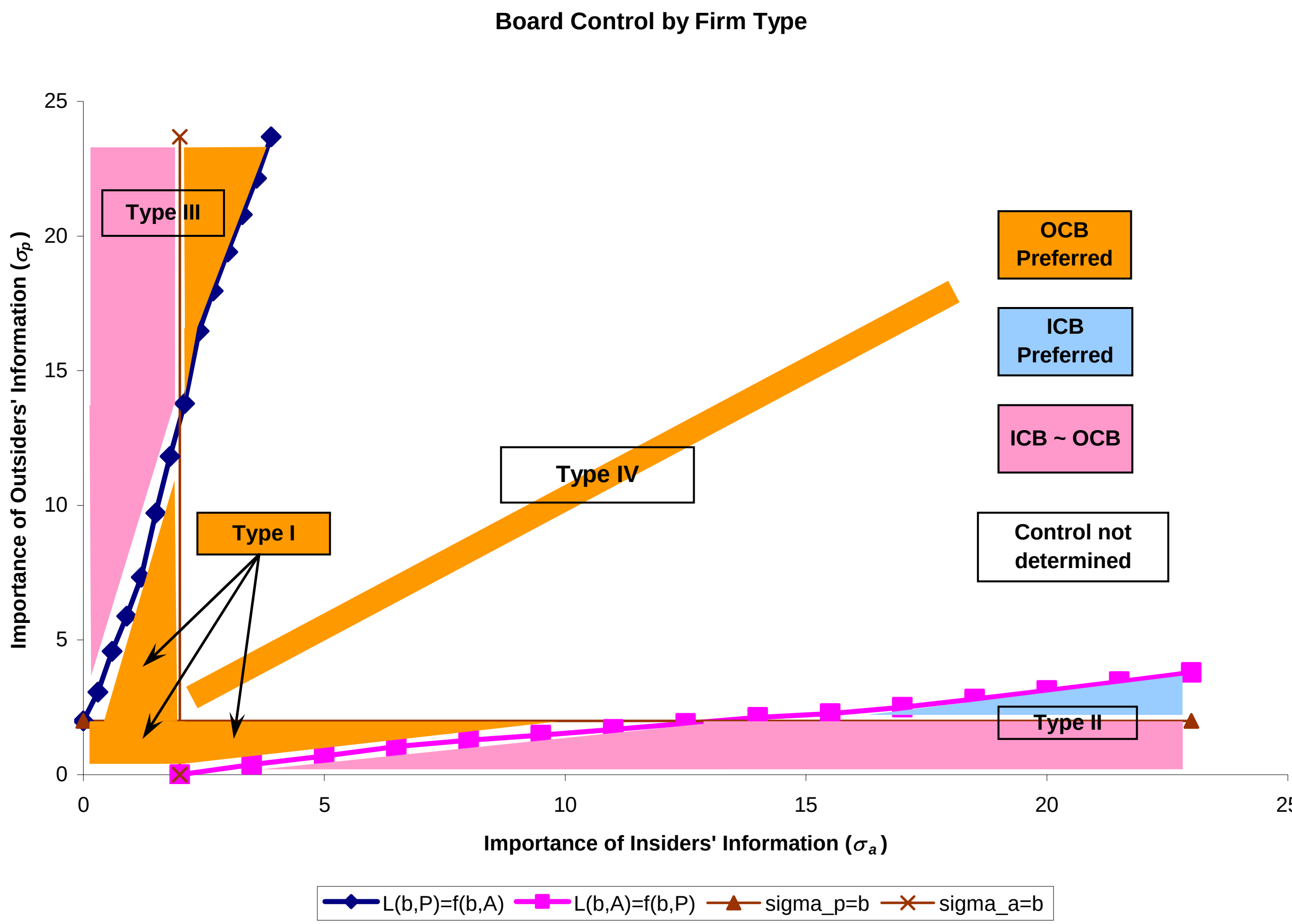

Figure 5

This figure is the same as Figure 4, except the firm types have been colored to show board preference. The board types shown are based on the assumption that effort cost is not prohibitive. For type-II firms for which shareholders are indifferent to board control, insiders' always have real authority, regardless of who is in control. For type-III firms for which shareholders are indifferent, if insiders control the board, outsiders will have real authority if and only if they are informed. If outsiders control the board, they will always have real authority.

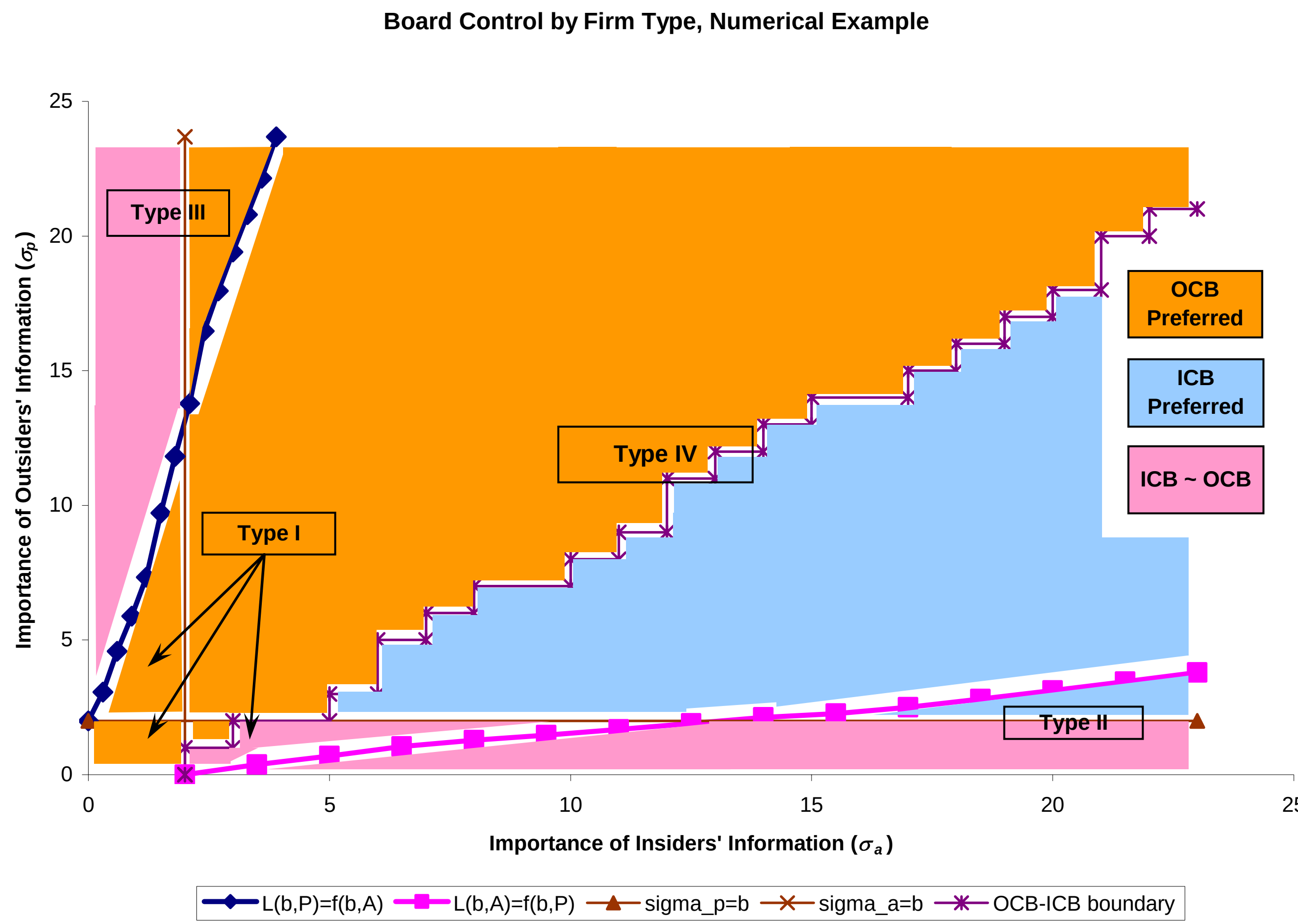

Figure 6

This figure shows, for various combinations of the two parties' information importance, whether shareholders prefer an outsider-controlled board, an insider-controlled board or are indifferent. For this example, agency costs, $b = 2$, and the ratio of the marginal cost of effort at zero effort to the equity share of outsiders, $c'(0)/\alpha = 0.0005$.