

"Weight of Money" Pricing and Speculative Dynamics

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Abstract

We investigate the role of "weight of money" pricing in generating speculative dynamics. "Weight of money pricing" means that the market price of an asset is driven by the amount of liquidity that accumulates in the market for the asset. In our benchmark model, a contagion argument rules out speculation in presence of weight of money pricing. However, in asset markets associated with leveraged trades, where maintaining a trading position creates funding externalities for other traders, speculative dynamics may take hold. Our results suggest that markets that combine low "resiliency" and significant costs of carry (such as the foreign exchange market) have the pre-conditions for large and persistent deviations of price from fundamentals followed by abrupt reversals. We show how the unique equilibrium trading strategies can be obtained through iterated deletion arguments, and how the asset's stochastic price paths follow from the dominance solvable trading strategies combined with fundamental shocks.

Introduction

Financial assets differ in how well their prices are anchored to their respective fundamental values. For instance, while short-maturity government bonds will be tied very closely to the central bank's policy rate and expectations of future central bank actions, long-maturity government bonds will be less well anchored to their fundamental values (as given by the expected path of future central bank policy rates). In the short run, the prices of assets that are less well-anchored to fundamentals may be sensitive to market liquidity effects, and especially to expectations of short-run price movements resulting from the flow of new funds into and out of the market - that is, the "weight of money" into and out of the market.

The foreign exchange market is perhaps the leading example of an asset market where prices are only loosely anchored to the fundamentals. One indication of the loose anchoring is the poor empirical performance of models of exchange rate determination (see, for instance, the survey by Karen Lewis (1995)). Whatever notion of fundamental value one takes, the long and sizeable swings in exchange rates over time suggest that the prices ruling in the short term are not well anchored to these "fundamental values", and will be influenced instead by short term considerations and by the expectations of other traders' actions.

The foreign exchange market also typifies the importance of the profit or loss that accrues to the traders due to the "cost of carry", and funding costs more generally. When a trader funds her trading strategy through pledging assets and borrowing funds, she will incur a profit or loss even if the price of the asset remains unchanged over time. For instance, if the trader borrows Japanese Yen at 1% and purchases US Dollar assets that yield 5%, then the longer the Dollar/Yen exchange rate remains unchanged, the larger will be the trader's profit.

If a trader incurs a profit or loss even if the price of the asset remains unchanged, then the expected length of time that an asset price deviates from its fundamental value is crucial in the trader's calculations. Even if the asset price will return *eventually* to its fundamental value, if the price deviates long enough, then it may be profitable to enter a speculative trade. For each day that the asset price spends away from its fundamental value, the trader books a profit. The expected accumulated profit from maintaining the trading position may then exceed the expected loss that she would incur when the price reverts to its fundamental value at some point in the future.

But then, this is a recipe for the asset price to deviate further from its fundamental value, since other traders will reason to a similar conclusion, and each trader reasons that other traders have reached similar conclusions. If the short-run price of the asset is sensitive to the flow of funds into the market, then the traders will reason that the asset price will respond to the flow of funds, and may move in the opposite direction from its fundamental value. The incentives to engage in the speculative trade are thus enhanced. Froot and Thaler (1990) suggest that such speculative dynamics is a good candidate to explain the forward discount bias.

The objective of our paper is to investigate how well the informal arguments given above stand up to scrutiny. We develop a dynamic asset pricing model in which traders face a small friction. Instead of trading continuously, traders are locked in their positions for (possibly arbitrarily) small amounts of time. Trading decisions are therefore determined by the traders' beliefs about flows of liquidity into or out of the market during this short lock-in period. The market price of the asset is given by the resulting amount of liquidity that accumulates in the market.

In our benchmark result, speculative dynamics is precluded no matter how

loosely anchored the asset price is to its fundamental value. In fact, we can rule out speculative behavior in quite a strong sense. The only outcome that survives the iterated deletion of strictly dominated actions for the traders is for the price to be equal to the fundamental value *at all times*.

This result is an example of the *stabilizing* role of speculation, as emphasized by Friedman (1953). If the price were not equal to its fundamentals, the anticipation of the price reverting to its fundamentals hastens the return of the price to the fundamentals, as the traders sell over-priced assets and buy up under-priced assets. Not only is this a possible scenario, it is the *only scenario* that is possible. The impossibility of speculation without funding costs sets an important benchmark. It suggests that illiquid markets and “weight of money” pricing are not, by themselves, enough to generate speculative dynamics.

The introduction of funding costs changes everything if traders create funding externalities for each other, namely, if they reduce the cost of funding a long position in the asset by injecting cash in the market for the asset. *Even for arbitrarily small funding externalities*, if the tendency of the market to revert to its fundamental value is weak enough, then speculative dynamics may take hold.

Kyle (1985) distinguished three notions of market liquidity - the market’s *tightness* (the bid-ask spread), its *depth* (the sensitivity of the price to the size of trade) and its *resiliency* (the speed with which the market price recovers from a demand/supply shock). Our focus is on the last of these. In our benchmark model, market resiliency *originates* from rational speculation. With the introduction of funding externalities, we obtain that conversely, rational speculation may dramatically *degrade* market resiliency.

Our results have several points of contact with the empirical literature, as well as suggesting some novel empirical implications. In the large price fluctuations generated by our model, extended periods of slowly rising prices are punctuated

by sharp falls in price. Currency traders refer to such price patterns as “going up by the stairs and coming down in the elevator” (see Breedon (2001)). These dynamics is reminiscent of the rational stochastic bubbles in Blanchard and Watson (1982). The probabilities that the bubbles burst in our model are intrinsic, however, and depend on the magnitude of the deviation of the price from the "fundamental" value. Also, the equilibrium is unique, and these endogenous "bubbles and crashes" are generated by purely static externalities in an economy with finite wealth, and a finite horizon.

The paper is organized as follows. We begin in the next section by outlining the basic building blocks of our model, and then addressing the benchmark case without funding costs, and in which the fundamental value is known and constant. In this case, we show that speculation can be precluded in a very strong sense. The only outcome that is consistent with the iterated deletion of dominated actions is for the price to be equal to the fundamental value of the asset. We then follow by introducing funding externalities. Funding externalities have the potential to generate speculative dynamics in which the price moves away from fundamentals.

The main part of our paper examines the general case where the fundamental value is stochastic, and follows a Brownian motion process. The shocks to fundamentals modifies the traders’ optimal trading strategies so that a unique, dominance solvable trading equilibrium can be derived in a manner similar to Frankel and Pauzner’s (2000) refinement of multiplicity of equilibria in coordination games.

1. Baseline Model

We begin with the baseline model in which there are no flow costs or benefits to maintaining a trading position. Time is continuous and is indexed by $t \in [0, +\infty)$. There is a continuum of risk-neutral traders. The traders do not discount the

future. There are two assets, a risky asset and a riskless asset, which we will simply call “cash”. The riskless asset is the *numéraire*, and the price of the risky asset at date t is denoted as p_t . Traders are endowed with one unit of cash and cannot borrow either asset. This assumption is only made to ensure bounds on trading positions in spite of risk neutrality.

For the discussion in this section and the next, the *fundamental value* of the risky asset is assumed to be fixed through time, and is denoted by v . The sense in which v is the fundamental value follows from our modelling of the resiliency properties of the risky asset. As noted earlier, the *resiliency* of an asset (as coined by Kyle (1985)) is the speed at which the price of an asset adjusts back after a demand/supply shock to its price. In our setup, the change in price between date t and date $t + \Delta$ is proportional to the new purchases of the asset by the traders, unless there is a shock that takes the asset price to its fundamental value. The probability that the asset price snaps back to its fundamental value between dates t and $t + \Delta$ is given by $\rho\Delta$, where Δ is a small increment in time. Thus, the price of the risky asset at date $t + \Delta$ is related to its price at t as follows.

$$p_{t+\Delta} \simeq \begin{cases} p_t + a(x_{t+\Delta} - x_t) & \text{with prob. } 1 - \rho\Delta \\ v & \text{with prob. } \rho\Delta \end{cases} \quad (1)$$

where x_t is the mass of traders who hold the risky asset, and market depth a is a positive constant. We assume that once the price of the risky asset snaps back to v , then it remains there forever. The idea here is similar to the notion of a “day of reckoning” in Duffie, Gârleanu, and Pedersen (2002) on which there is an exogenous public announcement, or the intervention of a large arbitrageur or of a central bank, that reveals the value of the future consumption claims generated by the risky asset to all market participants.

The arrival rate ρ is our formalization of the notion of resiliency. The larger is ρ , the more resilient is the market for the risky asset, and the better anchored it is

to its fundamental value. The assumption that the risky asset’s price remains at v forever once it has snapped back to v is offered as a simplification. Our focus is on how traders behave *in anticipation* of the anchor. Traders aim at maximizing their expected trading profits before the day of reckoning, or equivalently at maximizing their accumulated wealth at the day of reckoning.

Traders face a small friction in how often they can trade. A trader can only buy or sell at designated trading dates, and such trading dates arrive randomly given by a Poisson process with intensity λ . The processes are independent across traders, so that a fraction λdt of the traders gets a chance to trade between t and $t + dt$. The parameter λ should be construed as a large positive number, and certainly much larger than the parameter ρ . Indeed, the ratio given by

$$\lambda/\rho \tag{2}$$

indicates the number of times a trader may be expected to get an opportunity to trade the asset before the asset price snaps back to fundamentals. In frictionless markets, we would expect the traders to have a free hand in trading the asset, and so for this reason our main focus will be on the limiting case where the ratio λ/ρ becomes arbitrarily large.

The reason for the assumption that traders only trade at their designated trading dates is to overcome the technical problems associated with the sequencing of trades when time is continuous. For instance, we want to rule out non-measurable trading strategies, or strategies such as when a trader wants to remain in the market as long as possible, but strictly before some given date t^* . Also, by imposing the assumption that trading opportunities arrive with Poisson rate λ , we preclude “clumping” of trades, and ensure (from (1)) that the price paths are Lipschitz-continuous. The device of Poisson arrival rates for decision dates has been used, for instance, by Matsuyama (1991) for coordination problems.

The price path given by (1) raises the question of who takes the other side of the trade - that is, who the counterparties are to the sales and purchases of the active traders. The implicit assumption is that the traders we focus on face a set of passive counterparties who, as a group, present a residual demand/supply schedule against which the active traders make their trading decisions. In the appendix, we develop a price formation mechanism that generates such price paths. For the types of markets we have in mind - the fixed income market and the foreign exchange market - it would be quite natural to suppose that active traders are in the market alongside more passive traders who are in the market for hedging purposes, for transactions associated with international trade in goods or hold-to-maturity investments in fixed income instruments.

Any capital gains or losses realized by a trader between two trading dates are accumulated in a cash account: We assume that there is no compounding of the cumulative gains or losses. In what follows, we normalize the problem so that market depth a is equal to 1. Thus, if the risky asset has not snapped back to its fundamental value by date t , its price is given by

$$p_t = \underline{v} + x_t \tag{3}$$

where x_t is the proportion of the traders who hold the risky asset, and \underline{v} is a positive constant that gives the price of the risky asset in case $x_t = 0$. This is the sense in which our model generates “weight of money” pricing. The price of the risky asset is just the weight of money that has accumulated in the market at that time. Since x_t lies between zero and one, p_t lies in the interval:

$$[\underline{v}, 1 + \underline{v}]$$

At trading date t , a trader who holds the risky asset faces a binary decision - to hold the asset or sell the asset for p_t units of cash. For a trader who does not

already hold the risky asset, the binary decision is either to buy the asset at price p_t , or to maintain her cash holding.

At the time of making a decision, the trader can condition on the realized price path of the asset as well as the calendar date t . Thus, the trading strategy (in the game-theoretic sense) of a trader is a mapping:

$$(t, (p_u)_{u < t}) \mapsto \{\text{risky asset, cash}\} \quad (4)$$

that specifies whether a trader will hold cash or the risky asset for all pairs of dates and price histories. An *equilibrium* is a set of trading strategies (one for each trader) such that the binary decision given by (4) maximizes the trader's expected payoff given all other traders' trading strategies. An *equilibrium price path* is the price path implied by an equilibrium.

Dominance Solvable Outcome

Our baseline model allows us to draw a very strong conclusion - starting from any price p_t , the price until the day of reckoning returns to the fundamental value at the fastest possible rate. Any other outcome can be ruled out by the iterated deletion of strictly dominated strategies.

Suppose that the price of the asset is p_t . The most pessimistic scenario for the holder of the asset is that all future traders either switch out of the asset, or refrain from buying into the asset so that the price path is declining over time. Under this most pessimistic scenario, the price path of the asset is given by $\{p_{t+u}\}_{u \geq 0}$, where

$$p_{t+u} = p_t e^{-\lambda u} + \underline{v} (1 - e^{-\lambda u}) \quad (5)$$

In other words, the price converges to its lower bound \underline{v} at the rate λ , as each trader whose trading date arrives switches out of the risky asset.

However, even under this most pessimistic scenario, there is a price at which a trader is better off holding the risky asset. Consider a possible strategy of the trader where she purchases the asset in order to re-sell the asset at the first opportunity in the future (i.e. at the next designated trading date). Let us call this the “flipping strategy”. If the price path from date t onward is given by $\{p_{t+u}\}_{u \geq 0}$ then the expected return from the flipping strategy is:

$$\int_0^{\infty} \frac{\lambda p_{t+u} + \rho v}{p_t} e^{-(\lambda+\rho)u} du \quad (6)$$

The expected return on cash is just 1. Thus, if the future price path is given $\{p_{t+u}\}_{u \geq 0}$, the trader can do better than holding cash whenever (6) is greater than 1. Of course, there may be even better strategies than the flipping strategy, but the important point is that holding cash is dominated.

By substituting (5) into the expression for expected payoff given by (6) we can obtain the price \underline{p}^0 at which a trader is indifferent between holding cash and following the flipping strategy under this most pessimistic scenario. This threshold price \underline{p}^0 is given by

$$\underline{p}^0 \equiv \frac{(1 + 2\theta)v + \theta^2 \underline{v}}{(1 + \theta)^2} \quad (7)$$

where θ is defined as the ratio λ/ρ as commented above on (2). If the price falls below \underline{p}^0 , then holding cash is dominated. Note that \underline{p}^0 tends to \underline{v} as $\theta \rightarrow \infty$.

But then, the most pessimistic price path given by (5) is *too pessimistic* in that it assumes that some future traders may choose dominated actions. By ruling out trading strategies that are dominated, the most pessimistic price path now becomes:

$$\{\max(\underline{p}^0, p_t e^{-\lambda u} + \underline{v}(1 - e^{-\lambda u}))\}_{u \geq 0} \quad (8)$$

Since (8) implies strictly higher prices than (5) beyond some date in the future, we can define a new threshold price given by \underline{p}^1 at which a trader is indifferent

between cash and the risky asset. Clearly, $\underline{p}^0 \leq \underline{p}^1$. If the price is below \underline{p}^1 , the trader will not hold cash. Thus, any trading strategy in which a trader chooses cash at a price below \underline{p}^1 is ruled out after *two* rounds of deletion of dominated strategies.

We can iterate this argument. After $n + 1$ rounds of deletion of dominated strategies, the most pessimistic price path for the risky asset starting from p_t is given by:

$$\left\{ \max \left(\underline{p}^n, p_t e^{-\lambda u} + \underline{v} (1 - e^{-\lambda u}) \right) \right\}_{u \geq 0} \quad (9)$$

This sets a new threshold \underline{p}^{n+1} for the trading strategy, in which choosing cash for any price below \underline{p}^{n+1} is ruled out by $n + 2$ rounds of deletion of dominated strategies. We thus obtain the increasing sequence:

$$\underline{p}^0 \leq \underline{p}^1 \leq \underline{p}^2 \leq \dots \leq \underline{p}^n \leq \dots$$

Since price is bounded above, this sequence converges to some limit, denoted by \underline{p} . No trader will choose the safe asset below \underline{p} in any equilibrium, since such an action is ruled out by iterated dominance. Thus, \underline{p} constitutes a floor for the price of the risky asset in any equilibrium path, so that any equilibrium path $\{p_{t+u}\}_{u \geq 0}$ must be on or above this floor.

Analogously, we can define a *decreasing* sequence of thresholds that corresponds to the most *optimistic* price paths that are consistent with n rounds of deletion of dominated strategies. If the price is sufficiently close to the upper bound $1 + \underline{v}$, then selling is strictly preferred since the price will never rise sufficiently to compensate for the risk that it could possibly fall to its fundamental value v . Let \bar{p}^0 be the price above which selling is dominant. Thus, the price path will never rise above this level. We can then iterate the argument to derive the decreasing sequence:

$$\bar{p}^0 \geq \bar{p}^1 \geq \bar{p}^2 \geq \dots$$

Denote by \bar{p} the limit of this sequence. This limit would constitute a ceiling for any equilibrium price path. Clearly,

$$\underline{p} \leq \bar{p} \tag{10}$$

We will now show that the reverse inequality must hold, too. Consider the floor price \underline{p} . We must have $\underline{p} \geq v$. To see this, suppose (for the sake of argument) that $\underline{p} < v$. Since no trader sells below \underline{p} , the future path $\{p_{t+u}\}_{u \geq 0}$ lies on or above \underline{p} . Thus, conditional on a price \underline{p} , the expected return on the risky asset is *strictly* greater than one since all possible future values of the asset are larger than \underline{p} . But this contradicts the fact that \underline{p} is the limit of the sequence of thresholds. Hence, we must have

$$\underline{p} \geq v \tag{11}$$

From an exactly analogous argument, we conclude that $v \geq \bar{p}$. Thus, we have

$$\underline{p} \geq v \geq \bar{p} \tag{12}$$

From (12) and (10), we conclude that $\underline{p} = \bar{p} = v$. We have thus proved the following theorem.

Theorem 1. *In any subgame, the only trading strategy that survives the iterated deletion of dominated strategies is to hold the risky asset when $p_t \geq v$ and hold the safe asset when $p_t < v$.*

Corollary 2. *In the unique equilibrium price path in the subgame that starts with price p_t , the price converges to the fundamental value at the maximum speed that trading constraints allow for.*

Our benchmark theorem shows the power of the stabilizing role of speculation, as argued by Friedman (1953). No matter how loose the anchor is to the fundamentals, the speculative behavior of traders push the price to coincide with the fundamentals.

Notice that our result does not depend on the size of the parameters λ and ρ , as long as they are positive. Thus, our conclusions hold in the limiting case where $\lambda/\rho \rightarrow \infty$. This is the limiting case in which the trading frictions disappear, and traders can expect to trade an arbitrarily large number of times before the fundamentals assert themselves. Thus, in this limiting case the risk associated with short-term liquidity flows is much more important than fundamental risk. Even in this extreme case, liquidity risk is not priced. As in standard consumption-based asset pricing models, the price reflects only the utility that agents derive from holding the asset.

Our result can be understood as the resolution of two competing externalities generated by the predecessors of the date t trader. As the predecessors throw more “weight of money” into the asset, there are two effects. First, the positive externality is that the future resale values $(p_{t+u})_{u \geq 0}$ will be high, other things being equal. But the negative externality is of course that the asset is currently expensive. Because of the risk that the asset reverts to its fundamental value, the negative externality ultimately wins out. Thus, a trader has no incentive to join in pushing the price away from its fundamental value. Instead, the trader will seek to trade against her predecessors, and “catch the falling knife” to bring the price back into line with fundamentals. All this changes when we have positive funding externalities that tip the balance toward conditions that are more fertile to the emergence of speculation, as we see now.

2. Funding Externalities

We introduce funding externalities in the baseline model. For concreteness, we interpret the model in the context of the Yen carry trade mentioned in the Introduction. Traders have a given dollar endowment that they do not trade, but can use as collateral to enter a Yen carry trade, in which they borrow yens from a

Japanese bank and deposit in dollars. Thus, the correspondence with the baseline model is the following:

<u>Baseline Model</u>	<u>Yen Carry Trade</u>
Cash	Yen
Risky Asset	Dollar
p_t	Dollar/Yen exchange rate

The yen carry trade generates an instantaneous interest rate of r on the dollar leg and costs an instantaneous interest rate \hat{r} on the yen leg. Our crucial assumption is that for a trade entered into at date t , the yen interest rate \hat{r} is a decreasing function of the FX rate p_t :

$$\hat{r} = \hat{r}(p_t).$$

This captures that Japanese banks find the yen value of traders' collateral higher when p_t is high, and therefore offer them better credit conditions. As a result, traders create positive funding externalities for each other when structuring a Yen carry trade. Brunnermeier and Pedersen (2005) present an analysis of the endogenous nature of funding liquidity and asset market liquidity based on this feature.¹ We show that this funding externality, *even arbitrarily small*, is sufficient to generate speculation, namely *arbitrarily large* price fluctuations, provided the asset is sufficiently loosely anchored to its fundamental. For the sake of simplicity, we assume a linear form for $\hat{r}(\cdot)$

$$\hat{r}(p_t) = \hat{r} - \varepsilon (p_t - \underline{v})$$

where

$$\hat{r} > r > \hat{r} - \varepsilon.$$

We let

$$\pi(p_t) = (r - \hat{r}(p_t)) p_t$$

¹Kiyotaki and Moore (1997) and Shleifer and Vishny (1992) apply a related argument to industrial assets to analyze the interplay of business and credit cycles.

denote the "carry", namely the per dollar flow generated by entering the carry trade at date t , and assume that π is nondecreasing:

$$\varepsilon > \frac{\widehat{r} - r}{\underline{v}}.$$

Note that the magnitude of the funding externalities, measured by ε , can be arbitrarily small provided \widehat{r} and r are sufficiently close.

We begin by identifying steady state prices. A *steady state price* is defined as an exchange rate which is sustainable indefinitely into the future until the "day of reckoning" arrives. To be sustainable means consistent with the rational trading strategies of the traders.

One way of obtaining a steady state price is to find a price such that if this price is maintained indefinitely (until the day of reckoning arrives) a trader is indifferent between maintaining the carry trade forever and unwinding it now. If the price is constant until the day of reckoning, the expected accumulated flow benefit per dollar is:

$$\int_0^\infty \rho \pi(p) u e^{-\rho u} du = \frac{\pi(p)}{\rho} \quad (13)$$

The dollar gives v yens as the long-run payoff, since the day of reckoning must come eventually. Hence, in order for the trader to be indifferent, the price must be equal to the expected long-run payoff including the accumulated flow benefit. In other words,

$$p = v + \frac{\pi(p)}{\rho} \quad (14)$$

Note that the fundamental value v is a steady state price if

$$v = \underline{v} + \frac{\widehat{r} - r}{\varepsilon}.$$

(14) defines a relationship between the price and the fundamental value. Let us define the function

$$G(p) \equiv p - \frac{\pi(p)}{\rho} \quad (15)$$

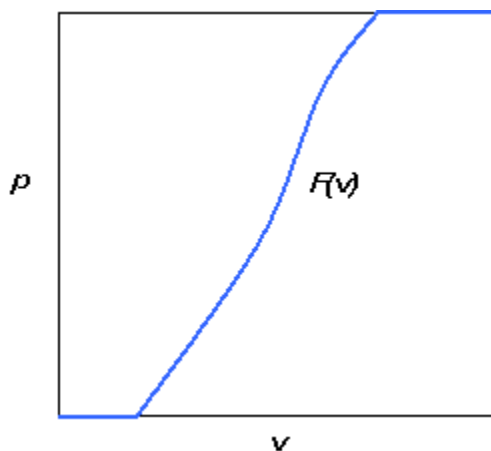


Figure 1: Unique steady state prices

so that $G(p)$ is the fundamental value v that makes traders indifferent if the price remains at p indefinitely until the day of reckoning. Since π is increasing in p , the function $G(p)$ has slope less than one. Another way that we can have a steady state price is if the price is at the lower bound \underline{v} and everyone strictly prefers to unwind the carry trade, or at the upper bound $1 + \underline{v}$, and everyone strictly prefers the carry trade.

It is useful to introduce the notation $F(v)$ for the inverse function of $G(\cdot)$. Thus, $F(v)$ is the price p for which $v = G(p)$. The function $F(v)$ is illustrated in figure 1.

The analysis of our model with funding costs depends on the underlying parameters λ and ρ , in contrast to the analysis of our benchmark case. The parameter ρ in particular is crucial, since this parameter determines the speed of reversal to fundamentals. If ρ is small enough, the effects of the funding externalities dominate the traders' decisions.

We can see the effect of ρ in figures 1 and 2. In our benchmark case without

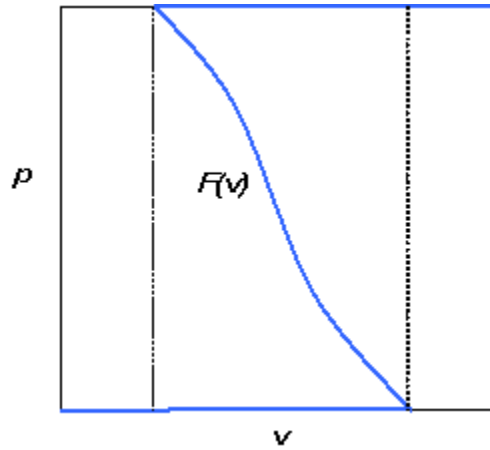


Figure 2: Multiple steady state prices

funding costs, $F(v)$ is just the 45 degree line, and so is upward-sloping. Also, when ρ is sufficiently large (so that fundamentals assert themselves quickly), the $F(v)$ curve is upward-sloping, as shown in figure 1. In this case, the steady state prices consist of the $F(v)$ curve, together with the horizontal strip in the bottom left hand side and the horizontal strip to the top right hand side in figure 1. However, when ρ is sufficiently small, the second term in (15) dominates, and $F(v)$ becomes downward-sloping. In this case, there is more than one steady state for some values of v . In the middle region for v in figure 2 between the dotted lines, each value of v is associated with three steady state prices - namely, the price implied by $F(v)$ together with the lower and upper bounds. This suggests the possibility of multiple equilibria, and the following set of results confirm this suspicion. The role of the parameter ρ thus turns out to be crucial.

Theorem 3. *If $F(v)$ is upward-sloping, for instance because ρ is sufficiently large other things being equal, then the only trading strategy that survives the iterated deletion of dominated strategies is to unwind the carry trade if p_t is below the*

(uniquely defined) steady state price, and to enter the carry trade otherwise.

Corollary 4. *Starting from any price p_t , the unique equilibrium price path converges at the rate λ to the steady state price until the day of reckoning.*

Theorem 5. *Suppose $F(v)$ is downward-sloping, for instance because ρ is sufficiently small all else equal. Then for any v for which $F(v)$ is well-defined, there is a threshold value $\hat{\lambda}$ such that, when λ is larger than this threshold value, starting from any price p_t there are multiple equilibria. In particular, there is both an equilibrium in which all traders enter the carry trade and also an equilibrium in which all traders de-leverage.*

Corollary 6. *Under the conditions of theorem 5, there are at least two equilibrium price paths until the day of reckoning - one that converges to the upper bound price, and one that converges to the lower bound price, both at the rate λ .*

We can paraphrase these results as follows. Theorem 3 states that when the funding externalities are sufficiently small or the anchor to the fundamental sufficiently important, then the results from the benchmark case still hold, and we have a unique, dominance solvable outcome. Thus, if the price of the risky asset starts away from its (uniquely defined) steady state price, it will converge back to the steady state at the fastest rate possible (i.e. at the rate λ).

In contrast, theorem 5 states that when funding externalities are important or the parameter ρ is small enough, then some values of v are associated with more than one steady state price. In this case, we have multiple equilibria, starting from any price p_t , provided that the trading frictions disappear (i.e. λ is large). So, we can have both an upward price path and a downward price path as equilibrium outcomes.

The argument for theorem 3 follows the identical line of reasoning as for our benchmark case except for the observation that the unique steady state price is not v itself, but is given by $F(v)$ or by one of the two bounds. Otherwise the steps are identical, and so the proof is omitted to avoid duplication.

We proceed to prove theorem 5. The expected excess return per dollar from entering a carry trade at date t is:

$$\int_0^{+\infty} [\lambda(p_{t+u} - p_t) + \rho(v - p_t) + (\lambda + \rho)\pi(p_t)u] e^{-(\lambda+\rho)u} du. \quad (16)$$

Based on the expectations that other traders will always enter carry trades, so that the price path is given by (5), it becomes

$$\frac{\theta^2}{(1+\theta)(1+2\theta)}(1+\underline{v}-p_t) + \frac{1}{1+\theta} \left(v - p_t + \frac{\pi(p_t)}{\rho} \right).$$

By assumption, the second term is nonnegative for all $p_t \geq F(v)$. For any p_t below $F(v)$, the first term is positive strictly, the second term negative. But the first term can be made dominant for all p_t below $F(v)$ provided λ is chosen sufficiently large since $\theta \rightarrow +\infty$ then. Thus, switching to the carry trade yields an expected profit for any starting price if λ is sufficiently large. That all traders hold the carry trade forever is therefore a possible self-justified outcome.

A symmetric reasoning applies to show that the no-carry trade equilibrium is sustainable for a sufficiently large λ ($1+\underline{v}$ is just replaced by \underline{v} in the first term). This concludes the proof of theorem 5.

Interpreting the Results

The contrast between the stabilizing role of speculation in the benchmark case and the de-stabilizing role of speculation in theorem 5 is very striking. We can give an alternative interpretation of why we have multiple trading equilibria. When λ becomes large, we get closer to a single-shot game between the traders since

they can trade very frequently. When the $F(v)$ curve slopes downwards, the two extreme steady states (all sell, all hold) resemble the Nash equilibria of a binary action game between the traders. The interior steady state price given by $F(v)$ is akin to the mixed strategy equilibrium in this binary action game.

The fact that the two extreme steady states resemble Nash equilibria in the single-shot game suggests that trading decisions are strategic complements - that is, the more other traders buy, the greater my incentive is to buy (and conversely, the greater the other traders sell, the more I want to sell). Thus, the strategic incentives become inverted, as compared to the benchmark case. We commented after our benchmark theorem in the previous section that the reason why speculation is stabilizing comes from the fact that the *negative* externality from the other players outweigh the *positive* externalities. In theorem 5, the roles are reversed. The positive externality of raising the price higher is larger than the negative externality, since raising the price also raises the flow benefit of holding the asset. If the anchor to the fundamentals is sufficiently loose, very small funding externalities are sufficient to trigger large price fluctuations because the prospect from enjoying small flow benefits for a sufficiently long period overcomes fundamental risk.

When λ is not large, then the evolution of the price path itself will influence expected payoffs, and we cannot come to any firm conclusions without additional argument. In general, we can envisage very complicated dynamic strategies that try to balance the negative and positive externalities between traders, and we cannot say much more without additional structure on the problem.

Rather than going further in investigating complex dynamics, we will now go in a different direction. We will now examine what happens when the fundamental value v itself is stochastic.

3. Stochastic Fundamentals

It turns out that the multiplicity of equilibria in theorem 5 is not robust to the addition of some variation in the fundamental value v . Adding (possibly arbitrarily small) shocks on v , we obtain, as in the baseline model, a unique dominance-solvable equilibrium. We draw on the work of Brudzy, Frankel and Pauzner (2001) and Frankel and Pauzner (2000), who showed that in binary action coordination games with strategic complementarities, the addition of small stochastic shocks to the fundamentals of the payoffs generates a unique, dominance solvable outcome. The arguments in these papers are similar to the “global game” arguments of Morris and Shin (1998), but the key difference is that the global game argument relies on the uncertainties over other players actions, while the Brudzy et al. papers rely on uncertainty over the *fundamentals*. The paper by Frankel, Morris and Pauzner (2003) gives a unified treatment of the two approaches.

Thus, we assume in this section that the fundamental value of the risky asset is stochastic, and follows the Brownian motion process:

$$dv_t = \mu dt + \sigma dW_t, \tag{17}$$

where W_t is a standard Brownian motion, μ is the trend and σ is the instantaneous volatility. We restrict the analysis to the case where the funding costs are large, so that we are in the realms of theorem 5. In particular, we will consider funding externalities that are large enough that the following feature holds.

Condition 7. $\frac{d\pi}{dp} > \rho(1 + \theta)$.

The exact role of Condition 7 will be explained shortly. Note that it does not put a restriction on the expected number of trades θ provided ρ and λ are sufficiently small. Under condition 7, we have the following result.

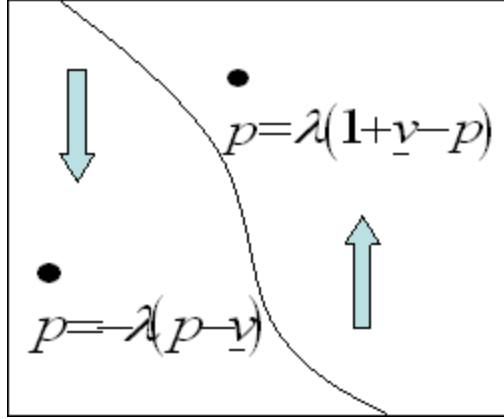


Figure 3: Unique equilibrium with stochastic v_t

Theorem 8. *There is a Lipschitz downward-sloping function $Z(\cdot)$ such that in any subgame starting at date t with any fundamental value v_t and any price p_t , there is a unique, dominance solvable solution to the trading game. In this solution, a trader who trades at date t holds the risky asset if $v_t \geq Z(p_t)$ and holds cash otherwise.*

Theorem 8 states that the multiplicity of equilibria that we saw in the previous section disappears when the fundamentals move around stochastically. Not only is the equilibrium unique, but it is dominance solvable. This theorem can be illustrated in figure 3. The curve $Z(p_t)$ divides the square into two regions. Theorem 8 states that in the unique equilibrium, any trader decides to hold the risky asset to the right of the $Z(\cdot)$ curve, and holds cash to the left of the $Z(\cdot)$ curve. Thus, the price will tend to rise in the right hand region, and tend to fall in the left hand region, as indicated by the two arrows in figure 3.

The price dynamics implied by the unique equilibrium is given by:

$$dp_t = 1_{\{v_t > Z(p_t)\}} \lambda(1 + \underline{v} - p_t) dt - 1_{\{v_t < Z(p_t)\}} \lambda(p_t - \underline{v}) dt. \quad (18)$$

where $1_{\{\cdot\}}$ denotes the indicator function that takes the value 1 when the condition inside the curly brackets are satisfied. These processes are known as *stochastic bifurcations*, and are studied in Bass and Burdzy (1999) and Burdzy et al. (1998). From Theorem 1 in Burdzy et al. (1998), for a given initial price p_0 , and for almost every sample path of v , there exists a unique Lipschitz solution $(p_t)_{t \geq 0}$ to the differential equation (18) defining the price dynamics for Z Lipschitz decreasing.

We provide a sketch of the proof of theorem 8 that follows closely the argument given by Frankel and Pauzner (2000) for their discussion of binary coordination games. The only minor difference between our setup and the game studied in Frankel and Pauzner (2000) is that viewed from date t , the future instantaneous profits at date $t + u$ depend on p_{t+u} , but also on p_t (see expression (16)). It is easy to see that their proofs apply identically, however. This is because condition 7 ensures that $\frac{\pi(p)}{\lambda + \rho} - p$ is nondecreasing, so that the expected excess profit (16) is increasing in p_t , and in p_{t+u} .

Denote by $Z_0(p_t)$ the boundary of the dominance region to the right of which it is dominant to hold the risky asset. Condition 7 ensures that Z_0 is decreasing, and Z_0 is quadratic thus Lipschitz. Refer to figure 4. Ruling out any strategy in which the trader holds cash to the right of Z_0 , we can derive a boundary Z_1 for the second-round dominance region which indicates the region where it is dominant to hold the risky asset in the absence of any first-round dominated trading strategies. We skip the proof that $Z_1(\cdot)$ is Lipschitz with at least the same constant as $Z_0(\cdot)$ (identical to Frankel Pauzner 2000). Condition 7 ensures that Z_1 is decreasing. Other things being equal a higher v_t makes holding the stock strictly preferable. If the other traders use $Z_0(\cdot)$ as a buy/sell frontier, all else equal, a higher p_t makes the stock strictly cheaper once net funding costs are factored in, and increases the probability of future cash injections because $Z_0(\cdot)$ is nonincreasing. By iterating this process, we can obtain the boundary Z_∞ for the region where a trader holding

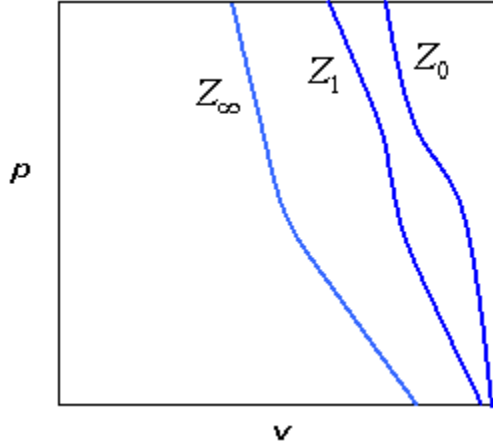


Figure 4: Iterative dominance from right

cash can be eliminated by iterated dominance. Z_∞ is decreasing Lipschitz as a limit of decreasing Lipschitz functions. The boundary Z_∞ defines an equilibrium strategy since, if all traders hold cash to the left and hold the risky asset to the right, the boundary between cash and the risky asset for the trader also lies on Z_∞ .

Suppose now that the dominance region for holding cash is given by the dotted line in figure 5. Consider a translation to the left of Z_∞ so that the whole of the curve lies to the left of the cash-dominance region. Call this translation Z'_0 . To the left of Z'_0 , holding cash is dominant. Then construct Z'_1 as the *rightmost translation* of Z'_0 such that a trader must choose cash to the left of Z'_1 if she believes that other traders will play according to Z'_0 . By iterating this process, we obtain a sequence of translations to the right of Z'_0 . Denote by Z'_∞ the limit of the sequence. Refer to figure 6. The boundary Z'_∞ does not necessarily define an equilibrium strategy, since it was constructed as a translation of Z'_0 . However, we know that if all others were to play according to the boundary Z'_∞ , then there

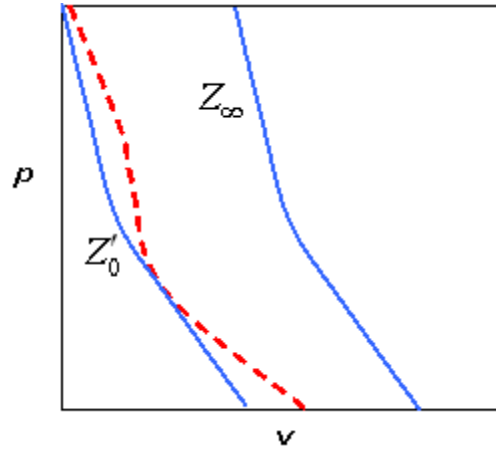


Figure 5: Iterated dominance from left

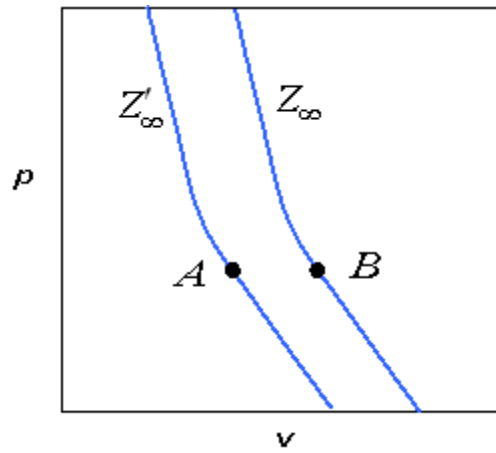


Figure 6: Closing the gap

is at least one point A on Z'_∞ where the trader is indifferent between holding cash and holding the risky asset. If there were no such point as A , this suggests that Z'_∞ is not the *rightmost* translation, as required in the definition.

We claim that Z'_∞ and Z_∞ coincide exactly. The argument is by contradiction. Suppose that we have a gap between Z'_∞ and Z_∞ . Then, choose point B on Z_∞ such that A and B have the same height - i.e. have the same second component. But then, since the shape of the boundaries of Z'_∞ and Z_∞ are identical, the stochastic bifurcation process starting from A must have the same distribution over payoffs as the process starting from B . Thus, the uncertainty governing the expected payoffs are identical at points A and B , except for the fact that B has a higher current fundamental value v_t . This contradicts the hypothesis that a trader is indifferent between the two actions both at A and at B . If she were indifferent at A , she would strictly prefer to hold the risky asset at B , and if she is indifferent at B , she would strictly prefer to hold cash at A . But we constructed A and B so that traders are indifferent. Thus, there is only one way to make everything consistent, namely to conclude that $A = B$. Thus, there is no “gap”, and we must have $Z'_\infty = Z_\infty$. In other words, we have the situation depicted in figure 3 as claimed.

Interpreting the Results

Theorem 8 demonstrates the impact of adding some uncertainty to the fundamental value v_t . The multiplicity of equilibria reported in the previous section resulted from the feature that, if the fundamentals were fixed and known, then one cannot rule out all other players trading in one direction, provided that the fundamentals were consistent with such a strategy. However, the introduction of shocks changes the picture radically. Since v_t follows Brownian motion, while traders must wait for their trading opportunities, the traders are far less nimble

than the shifts in the fundamental value itself. Thus, choosing to hold cash or hold the risky asset entails a substantial degree of commitment over time to fix one's trading strategy.

Suppose that the (v, p) pair is close to a dominance region, but just outside it. If v is fixed, it may be possible to construct an equilibrium for both actions, but when v moves around stochastically, it may wander into the dominance region between now and the next opportunity that the trader gets to trade. This gives the trader some reason to hedge her bets and take one course of action for sure. But then, this shifts out the dominance region, and a new round of reasoning takes place given the new boundary, and so on. Essentially, adding Brownian shocks to the fundamental enables us to extend to the two dimensional space of (v, p) pairs the dominance argument we showed in our benchmark result without funding externalities. Now, we have come full circle in which:

- Without funding externalities, there is a dominance solvable solution.
- With funding externalities and fixed v , there are multiple equilibria
- With the addition of stochastic v , we retrieve, again, a dominance solvable solution.

Extensions and Limiting Results

That $Z(\cdot)$ is nonincreasing implies that price paths exhibit hysteresis. If the dynamic system (p_t, v_t) is in the area where buying is dominant ($v_t > Z(p_t)$), then the buy pressure takes the system away from $Z(\cdot)$, making the continuation of a bullish market even more likely, all else equal. The reader may wonder whether Brownian excursions completely swamp this effect at the proximity of Z , so that runs never develop. The next proposition shows that it is not the case provided μ and σ are sufficiently small.

Proposition 9. Assume that the system is in the state (p_t, v_t) such that

$$v_t = Z(p_t).$$

i) For any $\varepsilon > 0$, as $\mu, \sigma \rightarrow 0$, the last time at which the system hits $Z(\cdot)$ before p_{t+u} becomes larger than $1 + \underline{v} - \varepsilon$ or smaller than $\underline{v} + \varepsilon$ tends to 0 in distribution. The probability that the price will go up tends to $1 + \underline{v} - p_t$.

ii) Moreover,

$$\lim_{\mu, \sigma \rightarrow 0} Z(p) = p - \frac{\pi(p)}{\rho} + \frac{2\theta^2}{2\theta + 1} \left(p - \left(\underline{v} + \frac{1}{2} \right) \right). \quad (19)$$

Proof i) The first part of the proposition paraphrases Theorem 2 in Burdzy, Frankel, and Puzner (1998).

ii) From i), as $\mu, \sigma \rightarrow 0$ the expected excess return from entering the yen carry trade (16) tends to

$$\int_0^{+\infty} \left[\lambda \left[\begin{array}{c} (\underline{v} \times (1 - e^{-\lambda u}) + p_t e^{-\lambda u}) \cdot (p_t - \underline{v}) \\ + ((1 + \underline{v}) \times (1 - e^{-\lambda u}) + p_t e^{-\lambda u}) \cdot (1 + \underline{v} - p_t) \\ - p_t \\ + \rho(v_t - p_t) + (\lambda + \rho) \pi(p_t) u \end{array} \right] \right] e^{-(\lambda + \rho)u} du. \quad (20)$$

Simple algebra yields that this is equal to 0 precisely when

$$v_t = Z(p_t),$$

where $Z(\cdot)$ is given in (19). ■

The broad intuition for this result is that when μ and σ are small, the price path around $Z(\cdot)$ is mostly driven by changes in p : Liquidity flows are more important than changes in the fundamentals. The rate at which the price goes up is $\lambda(1 + \underline{v} - p_t)$, while it decreases at the rate $\lambda(\underline{v} - p_t)$. The price path does not

revert to $Z(\cdot)$ once it has headed off towards one direction, and the ratio of the probabilities to go up or down is the ratio of the rates at which the price goes in each direction.

Thus, for μ and σ sufficiently small, the price paths will exhibit 'runs', or long series of identically signed returns, with a nonnegative probability of a large and sudden reversal. Very small variations in traders' opinions may some times trigger very large trading volumes, depending on whether the system is close to Z or not. These dynamics blend several features that are present in models of rational bubbles. The long runs which burst stochastically are reminiscent of the stochastic bubbles in Blanchard and Watson (1982). Like in Froot and Obstfeld (1991), the probabilities that the bubbles burst are 'intrinsic', however: They depend only on the respective values of the price and the fundamental. Unlike in these models, we obtain a unique rationalizable outcome, and our dynamics is the outcome of a *static* funding externality that a fixed number of agents with finite wealth and a finite horizon create for each other.

The reader may find unpalatable that the fundamental has a quadratic variation, while price paths are Lipschitz. This is only due to the very simplifying assumptions that only the active traders' valuations follow a diffusion, and that liquidity flows into the market at a constant rate. Assuming for instance that \underline{v} is hit by Brownian shocks that are not perfectly correlated with the ones on v would be a straightforward way to introduce quadratic variation in the price path. Modelling λ as a diffusion implies mathematical issues that are beyond the scope of this paper.

Points of Contact with Empirical Literature

Our equilibrium has properties that are consistent with some findings in the empirical literature concerned with the (difficult) task of finding evidence of spec-

ulative behavior.

First, McQueen and Thorley (1994) find positive duration dependence in runs of high returns in the U.S. stock market. Namely, they find evidence that the probability that a series of positive returns ends decreases with the length of the run, other things being equal. This is in line with the hysteresis due to the decreasing $Z(\cdot)$ in our setup.

Second, Chen, Hong, and Stein (2001) find that a long series of positive returns together with a high turnover predict crashes in the form of a high skewness in stock returns. In our setup, the returns also exhibit significant skewness when the system hits $Z(\cdot)$ after a long run. The pre-crash returns are much smaller in absolute terms (equal to $\lambda \frac{(1+v-p_t)}{p_t}$ with p_t large) than the post-crash returns equal to $-\lambda \frac{p_t-v}{p_t}$.

Finally, Cutler, Poterba, and Summers (1991) find weak evidence of 'fundamental reversion' in several markets. A proxy for a distance between the price and the fundamental has some predictive power for future returns. In our model if $p - v$ is large, it means that the market is saturated with cash, so that future positive returns will be low, and it also implies that v is low and has therefore a high probability to travel to the left of Z , making a reversal more likely.

Our model predicts symmetric series of positive and negative returns. In some asset markets (e.g., stock markets), casual observation suggests that long series of negative returns followed by a sudden boom are less common than the 'bubble and crash' pattern. Short sales constraint are commonly put forward to explain this asymmetry. In our model, if the (unmodelled) arbitrageur bringing the price back to v at the 'day of reckoning' has a harder time shorting the risky asset than the cash, the arrival intensity of arbitrage ρ would be higher when the stock is underpriced than overpriced, thereby generating a similar asymmetry.

4. Conclusion

We have developed a dynamic asset pricing model in which the asset price is given by the accumulation of liquidity that traders strategically inject into or withdraw from the market. A tension rises between fundamental motives to trade, and the liquidity risk associated with other traders' actions. In our baseline model, the unique equilibrium is equivalent to the Walrasian equilibrium in standard asset pricing models with a competitive representative agent. Liquidity risk is not important, and the asset price reflects the competitive valuation of the future aggregate consumption generated by the asset. This is because the negative externalities associated with liquidity inflows eventually overcome the positive externalities. With the addition of positive funding externalities in this economy, this equilibrium becomes unstable. Taking advantage of recent developments in dynamic coordination games, we pin down unique dynamic price paths that exhibit bubble-like patterns. A natural route for future research is to investigate whether more parameterized versions of the dynamic system driving the price paths can match the data generated by various markets in times when they are suspected of being speculative.

Appendix

The counterparties of the traders are a continuum of agents who have heterogeneous valuations of the risky asset uniformly distributed over $(\underline{v}, \underline{v} + 1)$, where

$$0 < \underline{v} < v < \underline{v} + 1.$$

At each trading date, the trader submits a supply or demand schedule, and the non-filled part is cancelled. Traders and their counterparties can invest up to one unit of cash in the market for the risky asset. They cannot borrow or lend.

We assume that the trading profits or losses realized by a trader between two trading dates are credited to a cash account. Thus, each trader has constantly one unit to invest in the stock market. We normalize the float of the risky asset to be the aggregate competitive demand of the counterparties:

$$F = \int_{\underline{v}}^{\underline{v}+1} \frac{du}{u} = \ln \left(1 + \frac{1}{\underline{v}} \right).$$

The price of the stock at date t , p_t satisfies therefore

$$p_t = \underline{v} + x_t,$$

where x_t is the measure of traders who are invested in the stock at date t .

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