

Information, trade and common knowledge with endogenous asset values*

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November 18, 2005

Abstract

We study the possibility of trade for purely informational reasons when the final payoff of the object being traded depends in part on an action taken by its eventual owner. A leading example is the trade of a controlling stake in a corporation. We establish conditions under which trade between *ex ante* identical parties with common preferences is possible. Empirically, our model predicts that the action taken is correlated with whether or not trade occurs.

*Preliminary and incomplete. Comments welcome. Working title (subject to change). We thank Franklin Allen, Gary Gorton and Richard Kihlstrom for stimulating conversations. Any errors are our own.

1 Introduction

Following Grossman and Stiglitz (1980) and Milgrom and Stokey (1982), economists have reached a consensus that under many circumstances it is impossible for an individual to profit from superior information. This result is often described as the “no trade” or “no speculation” theorem. The underlying argument is, at heart, straightforward. If a buyer is prepared to buy an asset from a seller for price p , then the buyer must believe that conditional on the seller agreeing to the trade, the asset value must exceed p in expectation. But conversely, knowing this the seller is at least as well off keeping the asset.

This insight has had important consequences for financial economics. In particular, the vast majority of theoretical studies concerning the role of information in financial markets introduce “noise traders” who trade for (typically exogenous) non-informational reasons.¹ If strategic agents are unable to observe the volume of trade generated by noise traders, then information-based trade is possible.

In this paper we develop a distinct and hitherto neglected reason for trade between differentially informed parties: if information allows superior productive decisions to be made, then the information released in trade is socially valuable. This possibility, which is implicitly ruled out in Milgrom and Stokey’s otherwise general framework, is enough to generate the possibility of trade under some circumstances.

AN EXAMPLE

The intuition for our results is best illustrated by an example. A risk neutral agent owns an asset, the value of which depends on two factors: an underlying but currently unobservable state variable $\omega \in \{\alpha, \beta\}$, and what the asset-owner chooses to do with the asset. For specificity, one can think of the asset in question as a controlling

¹See, for example, Kyle (1985), and Glosten and Milgrom (1985).

interest in a firm.

The best action for the asset owner to take depends on ω . Suppose that if $\omega = \alpha$ the best action is A , and the asset is worth 2 if this action is taken. Likewise, if $\omega = \beta$ the best action is B , and the asset is worth 1 if it is taken. For simplicity, we assume that the asset is valueless if any action other than the (state-contingent) best action is taken.

Both the initial asset owner (the *seller*) and a second party (the *buyer*) privately receive signals that are partially informative about the true state ω . We assume that conditional on the state these signals are distributed independently and identically. Specifically, we assume that if the true state is α (respectively, β) then each party observes signal a (respectively, b) with probability $3/4$. We assume also that the unconditional probability of state α is $1/2$.

Consider the following trading game: after observing his signal, the seller decides whether or not to offer to sell the asset, and if so, the price p at which he offers to sell. For notational convenience, we write $p = \infty$ for the cases in which the seller chooses not to offer to sell the asset. The buyer either signals his intention to accept the price, or else rejects the offer. In the later case no trade occurs. In the former case both the buyer and seller have a final opportunity to back out of the trade: if neither vetoes, trade occurs at the terms agreed.

Under the assumptions above, we claim the following is an equilibrium: the seller offers to sell the asset for $p = 9/10$ when he sees signal b , and for $p = \infty$ when he sees signal a . The buyer accepts the offer $p = 9/10$ (and rejects the offer $p = \infty$).

First, consider the situation faced by the seller. If he ends up with the asset, he must make the decision of what to do with the asset using only his own information. Given our assumptions, this means that if he sees signal a and does not sell, his expected payoff is $3/2$, while if he sees signal b and does not sell his expected payoff is

$3/4$.² Consequently, after signal b the seller prefers to sell at a price $p = 9/10$ rather than keep the asset; and after signal a , prefers to keep the asset rather than sell at this price.

Next, consider the buyer. Is he prepared to buy at the price $p = 9/10$? The key point to note is that receiving the offer $p = 9/10$ reveals, in equilibrium, that the seller has observed signal b . The buyer can use this information to make a better decision.

Specifically, if the buyer observes signal a , and receives the offer $p = 9/10$, then he knows the seller observed signal b . As such, the buyer regards $\omega = \alpha$ and $\omega = \beta$ as equally likely. Consequently he will choose action A , giving an expected payoff of $2 \times 1/2 = 1$. On the other hand, if the buyer observes signal b , then given the seller also observed signal b the buyer's probability assessment that $\omega = \beta$ is

$$\frac{\Pr(\beta) \Pr(b|\beta)^2}{\Pr(\alpha) \Pr(b|\alpha)^2 + \Pr(\beta) \Pr(b|\beta)^2} = \frac{\left(\frac{3}{4}\right)^2}{\left(\frac{1}{4}\right)^2 + \left(\frac{3}{4}\right)^2} = \frac{9}{10}.$$

Given this, he chooses action B , giving an expected payoff of $1 \times 9/10 = 9/10$. In both cases, the buyer's expected payoff exceeds the price at which he can buy ($p = 9/10$). As such, the behavior described is indeed an equilibrium.³

In this example both parties are strictly better off under the trade. The reason this is possible is that the information revealed by the agents' equilibrium actions enhances the asset's value for its eventual owner.

Asides from the possibility of trade, two further features of the example are worth noting:

1. Even though the buyer and seller are *ex ante* identical, they take different actions. That is, while the seller always takes action A if he ends up owning

²Note that since $3/4 \times 1 > 1/4 \times 2$, action B is the better action to take if the only information available is that one of the signals is b .

³In Proposition 1 below we show that there exist out-of-equilibrium beliefs under which the seller cannot profitably deviate and offer $\tilde{p} \notin \{4/5, \infty\}$.

the asset, the buyer sometimes takes action B . Put differently, the action taken is correlated with whether trade takes place. This is true even though both agents are equally able to take actions A and B .

2. The unconditional value of the asset in the equilibrium described is

$$\begin{aligned} & \Pr(\alpha) (\Pr(a|\alpha) + \Pr(b|\alpha) \Pr(a|\alpha)) \times 2 + \Pr(\beta) \Pr(b|\beta) \Pr(b|\beta) \times 1 \\ = & \frac{1}{2} \times \left(\frac{3}{4} + \frac{1}{4} \times \frac{3}{4} \right) \times 2 + \frac{1}{2} \times \left(\frac{3}{4} \times \frac{3}{4} \right) \times 1 = \frac{39}{32}. \end{aligned}$$

This is greater than the expected valuation of the asset conditional on trade occurring.

RELATED LITERATURE

The key assumption of our model is that the economic agent who decides how to use an asset is able to infer useful information from the trading process. In different contexts, Dow and Gorton (1997) and Subrahmanyam and Titman (1999) have explored theoretical models which make the same assumption. More recently, Chen *et al* (2005) and Durnev *et al* (2004) present supporting evidence that managers are able to make better decisions as a result of information obtained from stock prices.

We have already discussed the relation between our paper and the (extremely large) literature on when information-based trade is possible. To the best of our knowledge, the only prior study of trade when asset owners must decide how to use their assets is a chapter of Diamond's (1980) dissertation. He derives conditions under which a rational expectations equilibrium (REE) with trade exists when there are two types of agents, one of which is uninformed and one of which observes a noisy signal. The main differences between our paper and his are that (i) we study trade between agents who both possess information, (ii) as a consequence, information is never fully revealed, and (iii) instead of restricting attention to the competitive (REE)

outcome, in the spirit of Milgrom and Stokey (1982) we allow for all possible trading mechanisms.

2 The model

Our model is a generalization of the example above. There are two risk neutral agents, who we refer to as a buyer (agent 2) and a seller (agent 1). As in the example discussed above, there is an unobserved state variable $\omega \in \{\alpha, \beta\}$. Also as in the example, the eventual asset owner must decide between taking actions A and B . Let $v(A, \alpha)$ etc. denote the payoff of the asset when the asset owner takes action A and $\omega = \alpha$. Without loss, we assume that the highest asset payoff is achieved when action A is selected and $\omega = \alpha$:

$$v(A, \alpha) = \max \{v(A, \alpha), v(B, \alpha), v(A, \beta), v(B, \beta)\}. \quad (1)$$

Although neither agent is able to directly observe ω , both agents $i = 1, 2$ receive noisy and informative signals $s_i \in \{a, b\}$. Conditional on ω , the signals are distributed independently and identically. Without loss, we assume that $\Pr(a|\alpha) > \Pr(a|\beta)$, so that observing signal a raises the probability that $\omega = \alpha$ relative to the prior. For use below, observe that

$$\Pr(\alpha|ab) = \frac{\Pr(\alpha) \Pr(a|\alpha) \Pr(b|\alpha)}{\Pr(\alpha) \Pr(a|\alpha) \Pr(b|\alpha) + \Pr(\beta) \Pr(a|\beta) \Pr(b|\beta)}.$$

If the probabilities of receiving the “wrong” signal when $\omega = \alpha$ and $\omega = \beta$ are the same, i.e., $\Pr(b|\alpha) = \Pr(a|\beta)$, then $\Pr(\alpha|ab) = \Pr(\alpha)$.

In the analysis that follows, we will often be interested in the expected value of the asset to an agent with certain information. There are two possible cases. First, the asset owner may know only his own private signal s . In this case, we write $V(s)$ for the expected payoff of the asset, given that the asset owner takes the best action

given his information. That is,

$$V(s) = \max \{ \Pr(\alpha|s) v(A, \alpha) + \Pr(\beta|s) v(A, \beta), \Pr(\alpha|s) v(B, \alpha) + \Pr(\beta|s) v(B, \beta) \}.$$

The second possibility is that even though the asset owner cannot directly observe the signal received by the other party, in equilibrium he is able to infer it. In this case the asset owner effectively knows both s_1 and s_2 , and we write $V(s_1 s_2)$ for the associated expected payoff. That is, for any pair of signals s_1 and s_2

$$V(s_1 s_2) = \max \{ \Pr(\alpha|s_1 s_2) v(A, \alpha) + \Pr(\beta|s_1 s_2) v(A, \beta), \Pr(\alpha|s_1 s_2) v(B, \alpha) + \Pr(\beta|s_1 s_2) v(B, \beta) \}.$$

Finally, to rule out trades in which both parties are exactly indifferent between trading and not trading, we assume that a small cost $\gamma > 0$ is associated with transferring the asset from agent 1 to agent 2. Specifically, we assume that whenever the asset changes hands its final payoff is reduced by γ .

Since the realization of ω is not revealed until after decisions are taken, the relevant state space is $\Theta = \{aa, ab, ba, bb\}$. Here, ab , for example, denotes the state in which agent 1 (the seller) receives signal a and agent 2 (the buyer) receives signal b .

EXAMPLES

Examples which satisfy our assumptions include:

1. The asset in question is a large block of shares in a troubled firm. Action A is a decision to restructure the firm, while action B is a decision to liquidate it by selling off all its divisions.
2. The asset in question is a distressed debt claim. The owner of the claim must engage in restructuring negotiations with the issuer. Action A is a refusal to accept any write-down of the debt, while action B is an agreement to postpone some the payments.

Many more examples along these lines exist.

3 Conditions for trade

The main goal of our analysis is to characterize when trade can — and cannot — occur for purely informational reasons. The answer to this question clearly depends to some extent on the institutional environment. However, we clearly want our results to be as independent as possible of *a priori* assumptions about the trading environment.

To meet this goal, we adopt the following strategy. First, we study specific and simple institutional settings, and establish *sufficient* conditions for trade to occur. Second, we switch gears and establish a set of *necessary* conditions that must be satisfied for trade to occur, regardless of the institutional framework. This parallels the approach adopted by Milgrom and Stokey (1982). The necessary conditions we establish coincide with the sufficient conditions for at least some specific institutional settings.

The first specific trading mechanism we consider is one in which: (1) a non-strategic third-party (the broker) sets a price p , (2) the seller can place his asset for sale at price p , in which case (3) the buyer indicates his intention to buy at this price, and finally (4) both the buyer and the seller can back out of deal. If neither does so, the buyer pays the seller p and receives the asset.

For this trading mechanism, we establish:

Proposition 1. *Suppose that either*

$$V(aa) > V(a) > V(ab) > V(b) \text{ and } V(b) < V(bb), \quad (2)$$

or

$$V(bb) \geq V(b) \geq V(ab) > V(a) \text{ and } V(a) < V(aa). \quad (3)$$

Then for γ sufficiently small there exists an equilibrium of the “third-party posts a price” mechanism described in which trade occurs.

Proof of Proposition 1

We focus on the case in which (2) holds. (The case in which condition (3) holds is symmetric.)

We claim that for all posted prices $p \in [V(b), \min\{V(ba), V(bb)\} - \gamma]$, there exists an equilibrium in which agent 1 (the seller) places the asset for sale if and only if he observes signal b ; and whenever the seller offers to sell, the buyer agrees to buy.

In the equilibrium described the seller learns nothing. Since $p \in [V(b), V(a))$, he is happy to sell after seeing signal b , but strictly prefers to keep the asset after seeing signal a . Turning to the buyer, he knows that the seller only offers the asset for sale when he has observed signal b . So if the buyer himself observes signal a he values the asset at $V(ab) - \gamma$, while if he observes signal b he values the asset at $V(bb) - \gamma$. By construction both are higher than the price p , and so he will agree to buy whenever the seller places his asset on the market. Finally, neither agent has any incentive to back out. **QED**

Proposition 1 establishes a set of sufficient conditions under which trade occurs. One natural question to ask is whether or not the third party is necessary. To this end, we next consider the following trading mechanism: (1) the seller can place his asset for sale at any price p he chooses; (2) the buyer indicates his intention to buy at this price; and finally (3) both the buyer and the seller can back out of the deal. If neither does so, the buyer pays the seller p and receives the asset.

Proposition 2. *Suppose that either (2) and $V(bb) \leq V(ab)$, or (3) and $V(aa) \leq V(ab)$. Then for γ sufficiently small there exists an equilibrium of the “seller posts a price” mechanism described in which trade occurs.*

Proof of Proposition 2

We focus on the case in which (2) holds and $V(bb) \leq V(ab)$. (The other case is symmetric.) We claim that the following is an equilibrium. In stage 1 the seller offers to sell at price $p = V(bb) - \gamma$ after seeing signal b , but not after seeing signal a . In

stage 2, the buyer accepts offers $\tilde{p} \leq V(bb) - \gamma$ always, accepts offers $\tilde{p} \in (V(bb) - \gamma, V(ab) - \gamma]$ after signal a only, and never accepts offers $\tilde{p} > V(ab) - \gamma$. Whenever the seller places the asset on the market, the buyer believes that the seller observed signal b . In stage 3, the seller vetoes accepted offers $\tilde{p} \in (V(bb) - \gamma, V(ab) - \gamma]$, while the the buyer never vetoes.

Since $p = \min\{V(ba), V(bb)\} - \gamma$, from the proof of Proposition 1 we know that after signal b the seller prefers to place the asset on the market at price p than to withhold the asset, and after signal a he prefers to withhold the asset than to place the asset on the market at price p . Moreover, the buyer's response to these offers is a best response. As such, we need only check that the seller prefers to offer p over all alternative $\tilde{p} \neq p$.

It is easily verified that the buyer's stage 2 acceptance strategy is a best response. Moreover, given that he can always decline an offer at this stage, his stage 3 veto option is redundant. Clearly the seller prefers the offer p to any offer $\tilde{p} < p$ (the buyer accepts in the same states, and the seller gets less), and to any offer $\tilde{p} > V(ab) - \gamma$ (the buyer never accepts). Finally, the buyer accepts offers $\tilde{p} \in (V(bb) - \gamma, V(ab) - \gamma]$ in stage 2 only after seeing signal a . Given this, acceptance by the borrower of offers in this range conveys information to the seller. Depending on whether his own signal was a or b , the seller values the asset at $V(aa)$ and $V(ba)$ respectively. Since both are larger than $V(ab) - \gamma$, he will veto the trade. As such, his payoff is (weakly) lower than that obtained by offering p after signal b and withholding the asset after signal a . **QED**

A third natural trading mechanism to consider is one in which the buyer initiates trade. Consider the following trading mechanism: (1) the buyer chooses a price p to offer; (2) the seller indicates whether or not he is willing to accept; and finally (3) both the buyer and the seller can back out of the deal. For this, we reach exactly the same conclusions as for the third-party mechanism (see Proposition 1): if either

condition (2) or (3) holds, there exists an equilibrium in which trade occurs.⁴

A feature of all the trading equilibria above is that while when the buyer acquires the asset he learns the seller's signal, when the seller keeps the asset *he does not learn the buyer's signal*. We will return to this point below.

Are the conditions of Proposition 1 necessary as well as sufficient? As discussed, to answer this question we consider a broad class of possible institutional frameworks in which trade may occur. Following Milgrom and Stokey (1982), we assume only that regardless of the institutional framework in which a trade occurs, it must satisfy the following property: each agent must, conditional on the combination his private information, the terms of trade, and any other information revealed in the course of trade, prefer the trade to the status quo. Under this assumption, we will prove:

Proposition 3. *Trade is possible only if either (2) or (3) holds. If the former is satisfied, trade can occur only in $\{ba, bb\}$. If the latter is satisfied, trade can occur only in $\{aa, ab\}$.*

That is, the sufficient conditions we established in Proposition 1 for a specific trading environment are in fact necessary conditions for any trading environment.

We prove Proposition 3 in Section 4 below. Before turning to the details, we pause to interpret conditions (2) and (3).

First, observe that if $V(a) > V(b)$ then we can say that signal a is the *good* signal; conversely, if $V(b) > V(a)$ then we say that signal b is the good signal. Consequently:

Corollary 1. *Trade occurs only when the asset owner observes the bad signal.*

It is easiest to relate conditions (2) and (3) to the underlying asset payoffs when signals are accurate. In particular, we assume that $\Pr(a|\beta) = \Pr(b|\alpha) = \varepsilon$, so that ε is a measure of signal accuracy, and moreover $\Pr(\alpha|ba) = \Pr(\alpha)$ for all $\varepsilon > 0$.

⁴A proof is available from the authors.

What happens to the conditions for trade as information grows precise, i.e., $\varepsilon \rightarrow 0$. Clearly $V(aa)$ and $V(a) \rightarrow v(A, \alpha)$, while $V(bb)$ and $V(b) \rightarrow v(B, \beta)$. Recall that by a normalization $v(A, \alpha) \geq v(B, \beta)$. Consequently, as information grows precise the trade condition (3) can never hold. As such, trade can occur if and only if (2) holds. Note that (2) can hold only if A is preferred after ab — for if B is preferred, then $V(ab) \leq V(b)$. For ε sufficiently small condition (2) is thus equivalent to

$$V(b) \rightarrow v(B, \beta) < \Pr(\alpha)v(A, \alpha) + \Pr(\beta)v(A, \beta) = V(ab).$$

Rearranging, this inequality becomes

$$\Pr(\alpha)(v(A, \alpha) - v(B, \alpha) + v(B, \alpha) - v(B, \beta)) > \Pr(\beta)(v(B, \beta) - v(A, \beta)). \quad (4)$$

As we show below (see Lemma 6) trade can occur only if $v(B, \alpha) \leq v(B, \beta)$, and so from (4):

Corollary 2. *As signals become precise, trade can occur only if*

$$\Pr(\alpha)(v(A, \alpha) - v(B, \alpha)) > \Pr(\beta)(v(B, \beta) - v(A, \beta)).$$

Corollary 2 says that trade can occur only if it is more important to make the correct decision in state α than state β , at least when weighted by the unconditional probabilities of the two states. Recall that α is the state in which the asset value is maximized (provided the correct decision is taken).

4 Proof of Proposition 3

PRELIMINARIES

Recall that the relevant state space is $\Theta = \{aa, ab, ba, bb\}$. Represented as information partitions, agent 1's information prior to trade is $\hat{\mathcal{P}}_1 = \{\{aa, ab\}, \{ba, bb\}\}$ and agent 2's information prior to trade is $\hat{\mathcal{P}}_2 = \{\{aa, ba\}, \{ab, bb\}\}$.

An allocation in our economy is a pair of mappings $\kappa : \Theta \rightarrow \{1, 2\}$ and $\tau : \Theta \rightarrow \mathfrak{R}$ where κ specifies which agent owns the asset, and τ specifies a transfer from agent 2 to agent 1. Let $(\hat{\kappa}, \hat{\tau})$ denote the initial allocation, in which agent 1 owns the asset and no transfer takes place: $(\hat{\kappa}, \hat{\tau}) \equiv (1, 0)$.

A trade is an allocation (κ, τ) distinct from $(\hat{\kappa}, \hat{\tau})$. Trades potentially reveal information. Formally, let $\mathcal{P}^{\kappa, \tau}$ be the information revealed by the trade. The information available to agents 1 and 2 respectively after conditioning on information revealed by the trade is thus $\mathcal{P}_1^{\kappa, \tau} \equiv \hat{\mathcal{P}}_1 \vee \mathcal{P}^{\kappa, \tau}$ and $\mathcal{P}_2^{\kappa, \tau} \equiv \hat{\mathcal{P}}_2 \vee \mathcal{P}^{\kappa, \tau}$, where \vee denotes the coarsest common refinement.

For any partition \mathcal{P} of Θ let $\mathcal{P}(\theta)$ denote the partition element containing θ . With slight abuse of notation, we write $V(\mathcal{P}(\theta))$ for the expected payoff of the asset for an agent whose information is given by \mathcal{P} when the state is θ . Agents' payoffs from the allocation (κ, τ) in a state θ are thus

$$\begin{aligned} u_1(\theta; \kappa, \tau) &\equiv \begin{cases} V(\mathcal{P}_1^{\kappa, \tau}(\theta)) + \tau(\theta) & \text{if } \kappa(\theta) = 1 \\ \tau(\theta) & \text{if } \kappa(\theta) = 2 \end{cases} \\ u_2(\theta; \kappa, \tau) &\equiv \begin{cases} -\tau(\theta) & \text{if } \kappa(\theta) = 1 \\ V(\mathcal{P}_2^{\kappa, \tau}(\theta)) - \tau(\theta) & \text{if } \kappa(\theta) = 2 \end{cases}. \end{aligned}$$

As discussed above, we assume that allocations (κ, τ) distinct from $(\hat{\kappa}, \hat{\tau})$ are attainable only if both agents prefer (κ, τ) to $(\hat{\kappa}, \hat{\tau})$ after conditioning on all available information: for agents $i = 1, 2$ and all states $\theta \in \Theta$,

$$\sum_{\phi \in \mathcal{P}_i^{\kappa, \tau}(\theta)} \Pr(\phi) u_i(\phi; \kappa, \tau) \geq \sum_{\phi \in \mathcal{P}_i^{\hat{\kappa}, \hat{\tau}}(\theta)} \Pr(\phi) u_i(\phi; \hat{\kappa}, \hat{\tau}). \quad (5)$$

(The lefthand side is an agent's expected payoff in state θ under allocation (κ, τ) ; the righthand side is an agent's expected payoff in state θ under the initial allocation $(\hat{\kappa}, \hat{\tau})$.)

We refer to an allocation (κ, τ) as an *individually rational trade* if (5) is satisfied and if agent 2 acquires the asset at least sometimes, i.e., $\kappa(\theta) = 2$ for some $\theta \in \Theta$.

As noted repeatedly above, trade potentially reveals information. There are three (possibly overlapping) cases: trades in which agent 1 learns nothing, trades in which agent 2 learns nothing, and trades in which both agents learn something. We begin our analysis of this section by examining each case in turn, and deriving necessary conditions for individually rational trade.

TRADES IN WHICH THE SELLER (AGENT 1) LEARNS NOTHING

Formally, a trade (κ, τ) in which agent 1 learns nothing is one that is $\hat{\mathcal{P}}_1$ -measurable. Equivalently, after the trade, agent 1's information is $\mathcal{P}_1^{\kappa, \tau} = \{\{aa, ab\}, \{ba, bb\}\} = \hat{\mathcal{P}}_1$.

There is no individually rational trade (κ, τ) that is $\hat{\mathcal{P}}_1$ -measurable. For in this case (κ, τ) is the same in all states. Since agent 1 has the asset under $(\hat{\kappa}, \hat{\tau})$, the only possibility is that under (κ, τ) agent 2 acquires the asset for a price p . But agent 2 is only better off if $p \leq \min\{V(a), V(b)\} - \gamma$, and agent 1 is only better off if $p \geq \max\{V(a), V(b)\}$.

Suppose instead that (κ, τ) is not $\hat{\mathcal{P}}_2$ -measurable. That is, either $(\kappa, \tau)(aa) \neq (\kappa, \tau)(ba)$, or $(\kappa, \tau)(ab) \neq (\kappa, \tau)(bb)$. Since (κ, τ) is $\hat{\mathcal{P}}_1$ -measurable, both inequalities must hold, and so $\mathcal{P}_2^{\kappa, \tau} = \{\{aa\}, \{ba\}, \{ab\}, \{bb\}\}$.

If agent 2 acquires the asset for a price $p(\{aa, ab\})$ in $\{aa, ab\}$, then we need

$$\min\{V(aa), V(ab)\} - \gamma \geq p(\{aa, ab\}) \geq V(a)$$

while if agent 1 acquires the asset for a price $p(\{ba, bb\})$ in $\{ba, bb\}$, then we need

$$\min\{V(ba), V(bb)\} - \gamma \geq p(\{ba, bb\}) \geq V(b).$$

Consequently, we obtain:

Lemma 1. *A $\hat{\mathcal{P}}_1$ -measurable individually rational trade exists only if either*

$$\min\{V(aa), V(ab)\} - \gamma \geq V(a) \tag{6}$$

or

$$\min \{V (ba), V (bb)\} - \gamma \geq V (b). \quad (7)$$

Trade occurs in $\{aa, ab\}$ only if (6) holds. Trade occurs in $\{ba, bb\}$ only if (7) holds.

TRADES IN WHICH THE BUYER (AGENT 2) LEARNS NOTHING

By a parallel argument,

Lemma 2. *A $\hat{\mathcal{P}}_1$ -measurable individually rational trade exists only if either*

$$V (a) - \gamma \geq \max \{V (aa), V (ba)\} \quad (8)$$

or

$$V (b) - \gamma \geq \max \{V (ab), V (bb)\}. \quad (9)$$

Trade occurs in $\{aa, ba\}$ only if (8) holds. Trade occurs in $\{ab, bb\}$ only if (9) holds.

TRADES IN WHICH BOTH AGENTS LEARN SOMETHING

If both agents learn everything in a trade (κ, τ) then $\mathcal{P}_1^{\kappa, \tau} = \mathcal{P}_2^{\kappa, \tau} = \{\{aa\}, \{ba\}, \{ab\}, \{bb\}\}$.

No individually rational trade exists (due to cost γ).

Next, suppose that agent 2 learns everything, while agent 1 learns something. The possibilities are $\mathcal{P}_1^{\kappa, \tau} = \{\{aa\}, \{ab\}, \{ba, bb\}\}$ and $\mathcal{P}_1^{\kappa, \tau} = \{\{aa, ab\}, \{ba\}, \{bb\}\}$. In the former case an individually rational trade exists only if (6) holds. In the latter case an individually rational trade exists only if (7) holds.

Likewise, if agent 1 learns everything, while agent 2 learns something, the possibilities are $\mathcal{P}_2^{\kappa, \tau} = \{\{aa\}, \{ba\}, \{ab, bb\}\}$ and $\mathcal{P}_2^{\kappa, \tau} = \{\{aa, ba\}, \{ab\}, \{bb\}\}$. In the former case an individually rational trade exists only if (8) holds. In the latter case an individually rational trade exists only if (9) holds.

Finally, consider the cases in which neither agent learns everything, but both agents learn something. Thus at least one of $(\kappa, \tau) (aa) = (\kappa, \tau) (ba)$ and $(\kappa, \tau) (ab) =$

$(\kappa, \tau)(bb)$ must hold, and at least one of $(\kappa, \tau)(aa) = (\kappa, \tau)(ab)$ and $(\kappa, \tau)(ba) = (\kappa, \tau)(bb)$ must hold. This implies that (κ, τ) must be constant over three of the four states.

This leaves four subcases. If (κ, τ) is distinct at aa , then both agents learn aa . Given the cost γ , trade must occur in the other three states. To improve agent 2's expected payoff this requires $V(ba) - \gamma \geq p$ and $V(b) - \gamma \geq p$. To improve agent 1's expected payoff this requires $p \geq V(ab)$ and $p \geq V(b)$. Since these conditions cannot both be met, no individually rational trade that is measurable with respect to $\{\{aa\}, \{ba, ab, bb\}\}$ exists. Similarly, no individually rational trade that is measurable with respect to $\{\{bb\}, \{ba, ab, aa\}\}$ exists.

Next, suppose that (κ, τ) is distinct at ba . To improve agent 2's expected payoff this requires $V(aa) - \gamma \geq p$ and $V(b) - \gamma \geq p$. To improve agent 1's expected payoff this requires $p \geq V(bb)$ and $p \geq V(a)$. Thus we need

$$\min \{V(aa), V(b)\} - \gamma \geq \max \{V(bb), V(a)\}. \quad (10)$$

Finally, suppose that (κ, τ) is distinct at ab . To improve agent 2's expected payoff this requires $V(bb) - \gamma \geq p$ and $V(a) - \gamma \geq p$. To improve agent 1's expected payoff this requires $p \geq V(aa)$ and $p \geq V(b)$. Thus we need

$$\min \{V(bb), V(a)\} - \gamma \geq \max \{V(aa), V(b)\}. \quad (11)$$

with p not equal to at least one of the endpoints.

Lemma 3. *An individually rational trade that is neither $\hat{\mathcal{P}}_2$ -measurable nor $\hat{\mathcal{P}}_1$ -measurable exists only if at least one of conditions (6) - (11) holds. Trade occurs in $\{aa, ab\}$ only if (6) holds. Trade occurs in $\{ba, bb\}$ only if (7) holds. Trade occurs in $\{aa, ba\}$ only if (8) holds. Trade occurs in $\{ab, bb\}$ only if (9) holds. Trade occurs in $\{aa, bb, ab\}$ only if (10) holds. Trade occurs in $\{aa, bb, ba\}$ only if (11) holds.*

NECESSARY CONDITIONS FOR TRADE

From above, an individually rational trade exists only if at least one of conditions (6) - (11) holds. These conditions are stated in terms of $V(aa)$ etc. How do these conditions relate to the underlying asset valuation $v(A, \alpha)$ etc? As a preliminary, we establish:

Lemma 4. *Let $s, s' \in \{aa, a, ab, b, bb\}$ be such that s' indicates α more strongly than s , i.e., $\frac{\Pr(\alpha|s')}{\Pr(\beta|s')} > \frac{\Pr(\alpha|s)}{\Pr(\beta|s)}$. Then if A is preferred after s , it is preferred after s' also. Likewise, if B is preferred after s' it is preferred after s .*

Our first main result is to formally establish that trade is only possible if different actions are optimal after different signals. This result is in fact an immediate implication of Milgrom and Stokey's (1982) analysis: if the same action were preferred after every signal, our model would fall within their general framework. Below, we offer a direct proof, which has the advantage of establishing that action A is preferred after signal combination aa .

Lemma 5. *An individually rational trade exists only if action A is strictly preferred after seeing signal aa and action B is strictly preferred after seeing signal bb .*

Proof of Lemma 5

Suppose first that action B is weakly preferred after seeing signal aa . Then by Lemma 4, B is preferred after a , ab , b and bb . From this, it follows that if $v(B, \alpha) > v(B, \beta)$ then

$$V(aa) > V(a) > V(ab) > V(b) > V(bb),$$

if $v(B, \alpha) = v(B, \beta)$ then

$$V(aa) = V(a) = V(ab) = V(b) = V(bb),$$

while if $v(B, \alpha) < v(B, \beta)$ then

$$V(aa) < V(a) < V(ab) < V(b) < V(bb).$$

It is then easily checked that in none of these cases are any of the necessary conditions (6) - (11) for an individually rational trade are satisfied.

Likewise, suppose that action A is weakly preferred after seeing signal combination bb . Then by Lemma 4, action A is weakly preferred after seeing b , ab , ab and aa . Since $v(A, \alpha) \geq v(A, \beta)$,

$$V(aa) \geq V(a) \geq V(ab) \geq V(b) \geq V(bb).$$

Moreover, if any of the inequalities are strict, all are. Again, it is then easily checked that none of the necessary conditions (6) - (11) for an individually rational trade are satisfied. **QED**

Lemma 5 simply says that different actions are optimal after different signals. The following establishes the manner in which actions matter.

Proposition 4. *An individually rational trade exists only if $v(A, \alpha) > v(B, \alpha)$ and $v(A, \beta) < v(B, \beta)$.*

Proof of Proposition 4

Since A is strictly preferred after aa and B is strictly preferred after bb ,

$$\Pr(\alpha|aa)v(A, \alpha) + \Pr(\beta|aa)v(A, \beta) > \Pr(\alpha|aa)v(B, \alpha) + \Pr(\beta|aa)v(B, \beta) \quad (12)$$

$$\Pr(\alpha|bb)v(A, \alpha) + \Pr(\beta|bb)v(A, \beta) < \Pr(\alpha|bb)v(B, \alpha) + \Pr(\beta|bb)v(B, \beta) \quad (13)$$

The former implies that $v(A, \alpha) > v(B, \alpha)$ and/or $v(A, \beta) > v(B, \beta)$. The latter implies $v(A, \alpha) < v(B, \alpha)$ and/or $v(A, \beta) < v(B, \beta)$. Consequently either

$$v(A, \alpha) > v(B, \alpha) \text{ and } v(A, \beta) < v(B, \beta)$$

or

$$v(A, \beta) > v(B, \beta) \text{ and } v(A, \alpha) < v(B, \alpha). \quad (14)$$

Rewriting, (12) and (13) respectively are equivalent to

$$\begin{aligned} v(B, \alpha) - v(A, \alpha) &< \frac{\Pr(\beta|aa)}{\Pr(\alpha|aa)} (v(A, \beta) - v(B, \beta)) \\ v(B, \alpha) - v(A, \alpha) &> \frac{\Pr(\beta|bb)}{\Pr(\alpha|bb)} (v(A, \beta) - v(B, \beta)). \end{aligned}$$

Since $\frac{\Pr(\beta|bb)}{\Pr(\alpha|bb)} > \frac{\Pr(\beta|aa)}{\Pr(\alpha|aa)}$, it follows that (14) cannot hold. **QED**

Proposition 4 says that an individually rational trade exists only if A is the better action in state α and B is the better action in state β . The next result establishes that the asset values in one state cannot dominate the the asset values in the other:

Lemma 6. *An individually rational trade exists only if $v(A, \alpha) > v(A, \beta)$ and $v(B, \alpha) \leq v(B, \beta)$.*

Proof: By Assumption 1, $v(A, \alpha) \geq v(A, \beta)$. If $v(A, \alpha) = v(A, \beta)$ then A is preferred after every signal realization, since again by Assumption 1 $v(A, \alpha) \geq \max\{v(B, \alpha), v(B, \beta)\}$. But then by Lemma 5 no individually rational trade exists.

Next, suppose that $v(A, \alpha) > v(A, \beta)$ but $v(B, \alpha) > v(B, \beta)$. We claim that this implies $V(aa) > V(a) > V(ab) > V(b) > V(bb)$. To see this, take $s, s' \in \{aa, a, ab, b, bb\}$ such that s' indicates α more strongly than s , i.e., $\frac{\Pr(\alpha|s')}{\Pr(\beta|s')} > \frac{\Pr(\alpha|s)}{\Pr(\beta|s)}$. We show that $V(s') > V(s)$. Since both $v(A, \alpha) > v(A, \beta)$ and $v(B, \alpha) > v(B, \beta)$, if the same action is preferred after both s and s' the claim follows immediately. By Lemma 4, the only remaining possibility is that A is preferred after s' while B is strictly preferred after s . But then

$$\begin{aligned} V(s') &= \Pr(\alpha|s') v(A, \alpha) + \Pr(\beta|s') v(A, \beta) \\ &\geq \Pr(\alpha|s') v(B, \alpha) + \Pr(\beta|s') v(B, \beta) \\ &> \Pr(\alpha|s) v(B, \alpha) + \Pr(\beta|s) v(B, \beta) = V(s) \end{aligned}$$

and the claim follows. But when $V(aa) > V(a) > V(ab) > V(b) > V(bb)$ it is easily seen that none of conditions (6) - (11) can be satisfied. **QED**

From Lemma 5, we know that an individually rational trade exists only if action A is preferred after aa and action B is preferred after bb . From Lemma 4, the four remaining possibilities are: action A is preferred after b, ab, a, aa ; action A is preferred after ab, a, aa ; action A is preferred after a and aa ; and action A is preferred after aa only. We next analyze each of the cases in turn to obtain necessary conditions for trade.

Lemma 7. *Suppose that A is preferred after b, ab, a, aa . Then an individually rational trade exists only if $V(aa) > V(a) > V(ab) > V(b)$ and $V(bb) > V(b)$. If trade occurs, it does so in $\{ba, bb\}$.*

Proof: From Lemma 6, $V(aa) > V(a) > V(ab) > V(b)$. None of conditions (6) and (8) - (11) is satisfied. Condition (7) is satisfied if and only if $V(bb) > V(b)$. **QED**

Lemma 8. *Suppose that A is preferred after ab, a, aa . Then an individually rational trade exists only if $V(aa) > V(a) > V(ab) > V(b)$ and $V(bb) > V(b)$. If trade occurs, it does so in $\{ba, bb\}$.*

Proof: From Lemma 6, $V(aa) > V(a) > V(ab)$ and $V(bb) \geq V(b)$. Since $V(aa) > V(a)$, it is immediate that conditions (8) and (11) cannot hold. Since $V(a) > V(ab)$ condition (6) cannot hold. Since $V(bb) \geq V(b)$ condition (9) cannot hold. Likewise, condition (10) implies $V(b) - \gamma \geq V(bb)$ and so (11) cannot hold. This leaves only condition (7), which requires $V(ab) > V(b)$ and $V(bb) > V(b)$. **QED**

Lemma 9. *Suppose that A is preferred after a, aa . Then an individually rational trade exists only if $V(aa) > V(a)$ and $V(bb) \geq V(b) \geq V(ab) > V(a)$. If trade occurs, it does so in $\{aa, ab\}$.*

Proof: From Lemma 6, $V(aa) > V(a)$ and $V(bb) \geq V(b) \geq V(ab)$. Since $V(aa) > V(a)$, it is immediate that conditions (8) and (11) cannot hold. Since $V(b) \geq V(ab)$ condition (7) cannot hold. Since $V(bb) \geq V(b)$ neither condition (9) nor (10) cannot hold. This leaves only condition (6), which holds only if $V(ab) > V(a)$. **QED**

Lemma 10. *Suppose that A is preferred after aa only. Then an individually rational trade exists only if $V(bb) \geq V(b) \geq V(ab) > V(a)$ and $V(aa) > V(a)$. If trade occurs, it does so in $\{aa, ab\}$.*

Proof: From Lemma 6, $V(bb) \geq V(b) \geq V(ab) \geq V(a)$. Since $V(b) \geq V(ab)$ condition (7) cannot be satisfied. Since $V(ab) \geq V(a)$ condition (8) cannot be satisfied. Since $V(bb) \geq V(b)$ neither condition (9) nor (10) can be satisfied. Since $V(b) \geq V(a)$ condition (11) cannot be satisfied. This leaves only condition (6), which holds only if $V(aa) > V(a)$ and $V(ab) > V(a)$. **QED**

Combined, Lemmas 7 - 10 are enough to establish Proposition 3.

5 Equilibrium information revelation

As we noted following Proposition 1, a characteristic of the particular trade equilibrium we described is that while when the buyer acquires the asset he learns the seller's signal, when the seller keeps the asset he does not learn the buyer's signal. Is this a feature of all trade equilibria, or is it a feature of this particular equilibrium of the take-it-or-leave-it game?

To answer this question, we need to specify a general class of trading mechanisms in slightly more detail than before. We consider any mechanism of the following type:

1. Agents submit reports, $m_i \in M_i$ for $i = 1, 2$, after observing their signals.

2. A mechanism designer announces an allocation $g(m_1, m_2)$, and possibly some additional information.
3. Agents have the right to unilaterally quit the mechanism, in which case agent 1 keeps the asset and no transfer takes place. If neither agent quits, the outcome imposed by the mechanism designer obtains.

For example, in the take-it-or-leave-it trading game used in Proposition 2, the seller submits an “offer” $m_1 \in M_1 = \mathfrak{R}$ and the allocation $g(m_1, m_2)$ is that the buyer sends m_1 to the seller in return for the asset. The seller is free to decline this allocation.

The main restriction imposed is that, as discussed previously, agents have the right to quit the mechanism *after* observing the allocation proposed by the mechanism designer. Additionally, we rule out the existence of non-refundable “entry fees” paid prior to the allocation announcement.

Within this class of trading mechanisms it is possible to establish:

Proposition 5. *No trading mechanism exists in which trade occurs, and the seller (agent 1) learns the buyer’s (agent 2) information in states in which the seller retains the asset.*

Proposition 5 implies that even when trade is possible, information is not revealed all the time. However, no welfare loss is associated with this lack of revelation.

To see this, suppose that the trade condition (2) holds. (As usual, the case (3) is covered by symmetric arguments.) In this case, the lack of information revelation occurs in states $\{aa, ba\}$, where the seller keeps the asset.

What decision does the seller make in these states, and what decision is socially optimal? Since $V(ab) > V(b)$ it must be the case that action A is preferred by an asset owner who knows one signal is a and the other is b . (If instead action B were preferred, then $V(b) \geq V(ab)$.) Consequently, action A is also preferred if the asset

owner knows one signal is a , and if the asset owner knows both signals are a . As such, the seller will take action A in both states aa and ba , and this action is the efficient one even after conditioning on information not possessed by the asset owner.

Consequently:

Corollary 3. *Suppose that (2) or (3) holds. Then in any equilibrium in which trade occurs, for all $\theta \in \Theta$ the asset owner makes the same decision as he would if he had all information available to him.*

6 The correlation of actions with trade

The strongest empirical implication of our model is that the action taken is correlated with whether or not trade takes place. This implication is obtained even though the two agents have exactly the same ability to take the two actions under consideration, A and B .

To derive this implication, suppose first that condition (2) holds. If trade occurs, it does so in $\{ba, bb\}$. Moreover, this condition is only consistent with trade if action A is preferred after ba, a, aa (see Lemmas 7 and 8). Recall that agent 1 starts with the asset; so when trade occurs agent 2 ends up holding the asset. Since trade is measurable with respect to $\{\{ba, bb\}, \{ab, aa\}\}$, then agent 2 learns agent 1's signal. Consequently, when trade has taken place the asset owner (agent 2) knows either ba or bb . He takes action A in the former case, and action B in the latter.

What happens when trade does not take place? In this case, agent 1 is the asset owner. From Proposition 5, agent 1 does not learn agent 2's signal. Thus he observes only his own signal, which is a . He takes action A .

Combined with a symmetric argument for the case in which (3) holds, we obtain:

Proposition 6. *Suppose that trade is possible. Then either:*

(I) Trade occurs when agent 1 observes signal b . When trade does not occur, the asset owner takes action A . When trade does occur, the asset owner takes either action A or B .

(II) Trade occurs when agent 1 observes signal a . When trade does not occur, the asset owner takes action B . When trade does occur, the asset owner takes either action A or B .

Proposition 6 says that the action taken by the asset owner is correlated with whether or not trade occurs. In particular, in any equilibrium in which trade is possible, there exists an action which is taken *only if* trade occurs. Possible examples include:

1. Vulture investors in debt restructuring:

It is widely perceived that vulture investors behave in ways in restructuring negotiations that the original creditors do not. See, e.g., Morris (2002).

2. Corporate raiders:

There is evidence that large scale layoffs and divestitures follow takeovers. See, e.g., Bhagat *et al* (1990).

7 Concluding remarks

We conclude by briefly discussing the implications of our model for observable price responses. Focusing on the case in which signals are precise (i.e., $\Pr(a|\beta) = \Pr(b|\alpha) \rightarrow 0$), trade can only occur if condition (2) holds. When trade occurs the expected value of the asset is either $V(ab)$ or $V(bb)$; while when trade does not occur, its expected value is $V(a)$. Under condition (2), $V(a) > V(ab)$. Moreover, since by normalization $v(A, \alpha) \geq v(B, \beta)$, when signals are precise either $V(a) > V(bb)$, or else $|V(a) - V(bb)| \rightarrow 0$.

Combining these observations, one can see that trade is associated with a reduction in the expected valuation of the asset. This prediction is consistent with at least some empirical claims. For example, in the context of mergers between firms this prediction implies that conditional on a merger occurring the combined valuation of the merging firms should fall. Moeller *et al* (2005) document just such value destruction in recent mergers.⁵

Likewise, many observers have expressed the view that Chapter 11 allows too many firms to reorganize.⁶ One way to view negotiations in Chapter 11 is that a group of creditors “owns” the right to liquidate the firm, and management is allowed to make an offer to purchase it from them. As such, our model predicts that conditional on exiting Chapter 11 (that is, on “trade”) the value of the firm is low. Note that while this prediction matches what many observers have claimed, it does not stem from a bias towards too much reorganization.

Nonetheless, one needs to be very careful in taking our stylized model to the data. In particular, consider the following minor modification. In place of two underlying states, actions and signal realizations, suppose instead that there are three of each: $\omega \in \{\alpha, \beta, \gamma\}$, actions $\{A, B, C\}$, and signals $\{a, b, c\}$. We assume that states α and β are as before, while state γ has the following characteristics. First, action C is best in state γ . Second, the asset valuation is low: $v(C, \gamma) < \min\{v(A, \alpha), v(B, \beta)\}$. Third, agents know when the state is γ : $\Pr(c|\gamma) = 1$ and $\Pr(c|\alpha) = \Pr(c|\beta) = 0$.

Under these assumptions, over large segments of the state space both agents observe γ , there is no scope for trade, and the asset value is low. Trade is only feasible when $\omega \neq \gamma$ and agents observe signals a or b . Consequently, although trade is associated with lower asset values when conditioned on agents receiving signals a or

⁵Studies of previous U.S. merger waves generally found that the combined valuation of merged firms rose. See, e.g., Bradley *et al* (1988).

⁶See, e.g., Baird (1986), along with other references cited by Hotchkiss (1995). Hotchkiss herself presents quantitative evidence that firms exiting Chapter 11 perform poorly.

b, *unconditionally* trade is associated with an increase in asset valuations.

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A Appendix

PROOF OF LEMMA 4

If action A is preferred after s ,

$$\Pr(\alpha|s)v(A, \alpha) + \Pr(\beta|s)v(A, \beta) \geq \Pr(\alpha|s)v(B, \alpha) + \Pr(\beta|s)v(B, \beta),$$

or equivalently,

$$\frac{\Pr(\alpha|s)}{\Pr(\beta|s)}(v(A, \alpha) - v(B, \alpha)) \geq v(B, \beta) - v(A, \beta).$$

By assumption 1, $v(A, \alpha) - v(B, \alpha) \geq 0$. Consequently, if A is preferred after s it is preferred after s' also.

Similarly, if B is preferred after s' ,

$$\Pr(\alpha|s')v(B, \alpha) + \Pr(\beta|s')v(B, \beta) \geq \Pr(\alpha|s')v(A, \alpha) + \Pr(\beta|s')v(A, \beta),$$

or equivalently,

$$\frac{\Pr(\beta|s')}{\Pr(\alpha|s')} (v(B, \beta) - v(A, \beta)) \geq v(A, \alpha) - v(B, \alpha).$$

Again, since $v(A, \alpha) - v(B, \alpha) \geq 0$ then $v(B, \beta) - v(A, \beta) \geq 0$, and so B is preferred after s also. **QED**

PROOF OF PROPOSITION 5

From Proposition 3, trade can occur only if either (2) or (3) holds. We focus on the case in which (2) holds; the proof of the other case proceeds symmetrically.

Suppose to the contrary that such a trading mechanism exists in which agent 1 learns agent 2's information in states where trade does not take place.

By the revelation principle, we can focus on truth-telling mechanisms: $M_1 = M_2 = \{a, b\}$. From previous results, we know that:

- The outcomes $g(ba)$ and $g(bb)$ are: agent 2 acquires the asset, agent 1 does not learn agent 2's signal, agent 1 receives a transfer p .
- The outcomes $g(aa)$ and $g(ab)$ are: agent 1 keeps the asset. Moreover, since either agent can quit after the allocation is announced, no monetary transfer can take place.

Since agent 2 learns agent 1's signal in equilibrium, the transfer p must satisfy $p < \min\{V(bb), V(ba)\}$. (If this is not the case, agent 2 will quit the mechanism.)

Now, suppose that agent 1 learns agent 2's signal in states $\{aa, ab\}$. Consider agent 1's incentive constraint after observing signal b . If he reports b , under the mechanism agent 2 acquires the asset and agent 1 receives p . On the other hand, if

he reports a he keeps the asset. By supposition he learns agent 2's signal. As such, the asset is worth either $V(ba)$ or $V(bb)$. Since both exceed p , he prefers deviating and reporting a to truthfully reporting b .

This gives a contradiction, and completes the proof. **QED**